



Choosing the Optimum Underground Mine Layout with Regard to Metal Price Uncertainty Using Expected Utility Theory

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ABSTRACT

Metal price is one of the most important parameters in the calculation of cut- off grade. The cut- off grade has the main role in determination of mine layout. Mine layout actuates mineable reserve, mine life and economic profitability. Not considering the uncertainty in metal prices can lead to a non-optimal layout. In this paper optimum underground mine layout is determined by expected utility theory with regard to metal price uncertainty. With the proposed approach metal price uncertainty is modeled by Monte Carlo simulation technique and decision maker will be gained probability of underground mine layouts. The utility function of underground mine layouts is defined and by the probability of them, expected utility is determined. Underground mine layout with the maximum expected utility is the optimum layout. Application of this approach in a hypothetical gold mine, in addition to considering metal price uncertainty, leads to 14% more mineable reserve and 18% higher net present value than normal design.

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NOMENCLATURE

UML	Underground Mining Limit	Po	Processing cost per metric tonne of ore
COG	Cut- Off Grade	Oo	Overhead cost per metric tonne of ore
MRO	Mineable Reserve Optimizer	V	Value of one unit of valuable product
EUT	Expected Utility Theory	r	Proportion of valuable product recovered from the mined material (%)
xc	cut- off grade (%)	R	Refining costs, defined as costs that are related to the unit of valuable material produced
Mo	Mining cost per metric tonne of ore		

1. INTRODUCTION

Mining projects require sequential risk assessment. This is because different types of uncertainties affect the value of a mine project. The most important sources of uncertainty can be assorted into three groups: exploration, engineering and economic uncertainties. The economic uncertainties are the main factors which may influence the project evaluation. Economic uncertainty has the greatest impact on the value of a mining project in the form of three sources of

uncertainty (price and income, operating and capital costs, and discount rates). Future metal prices are the premier factors of economic uncertainties. In mining project designs metal prices are usually modeled as the mean price for the last three years, mainly for precious and base metals [1]. Single commodity price restrains the use of extremely optimistic prices and it may be misleading when evaluating mining projects.

Determination of Underground Mine Layout (UML) is initial step of underground mine design. UML is part of the geological reserve that has economic value and

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gives mineable reserve, mine lifetime, requisite capital costs and other design parameters. In other words, UML gives the boundary of underground mineable reserve and is equivalent to ultimate mining limit, ultimate pit limit or final pit limit in open mining. There are few algorithms for determination of UML. Dynamic Programming [2], Floating Stope [3], Maximum Value Neighborhood [4], Probable Stope [5] and other algorithms [6–10] are among the mentioned algorithms. Review of these algorithms may be found in literature [11, 12]. The UML optimization algorithms are commonly divided into rigorous and heuristic. The rigorous algorithms have mathematical proving and give the optimal solution according to the status they are applied. The heuristic algorithms, based on the rules of searching, do not guarantee the true optimum but provide estimated solutions and the precision of the solution depends on potency of the search technique. Each of the UML's algorithms has their advantages and disadvantages. The algorithm which has mathematical proof (rigorous algorithm) can find optimum solution. If the quiddity of the algorithm is 3 dimensional and it is applicable for all of the underground mining method, the limitation of its application decreased. It is worth noting that none of the existing algorithms model the metal price uncertainty and there is only one algorithm, the floating stope algorithm of Datamine, which is computerized in a commercial size and available in the market. Mineable Reserve Optimizer (MRO) in Datamine is based on floating stope algorithm and implemented as a set of scripts supplied with Studio 3. Floating stope algorithm of Datamine by MRO analyzes a geological model by Cut- Off Grade (COG) and delineates optimal mineable reserves using practical mining constraints. Uncertainty effects were studied in surface mining planning more than underground mining. Metal price uncertainty in surface mining optimization has been studied in several researches [13–16]. Among the researches about uncertainty in underground mine planning, only one research is related to stope layout. In the study, the risk of grade uncertainty was calculated by a probabilistic method [6]. Production scheduling and extraction rate of underground mine under the influence of price fluctuation are among the topics that have been further explored [17–22]. None of the above researches considered effect of price uncertainty in UML. The metal price fluctuations can change the COG and different COGs lead to different UML. Accordingly, metal price uncertainty lead to mine layout uncertainty. Ignoring this uncertainty can

lead to determination of the layout that is far from the optimum. With regard to importance of price uncertainty and mine layout, the aim of this paper is to determine the UML with floating stope algorithm considering metal price uncertainty. The Monte Carlo technique is one of the most well-known simulation methods used in this paper. This technique requires the creation of a model and Output of the model (COG) can be predicted by random sampling from price distribution. In this research, distribution of COG obtains based on metal price distribution. Since simulation of COGs, UMLs will be attained using floating stope algorithm of Datamine software. Therefore, Monte Carlo simulation technique provides probability of each UML. For final selection between all of the UMLs, expected utility theory (EUT) has been used. EUT which used in such problems where a decision-maker chooses from a limited set of consequences to equilibrium involved uncertainty. EUT offers a means for rational decision making. EUT and its predecessor expected value has been used as a procedure for determining not only what selection a decision maker “should” make but also to give a framework for better realizing and explaining what is a rational choice. For the first time, Von Neumann and Morgenstern [23] introduced the EUT by exploiting the lottery concept. According to EUT, decision makers should always choose the option that suggests the greatest expected value. Expected value is determined by weighting the value of an option by its probability of success. This analysis is straight and allows for a numerical analogy that can be easily differed from making a plainly preferable option. Expected utility is defined by weighting the utilities of each option by the respective probabilities. Decision makers should then choose the alternative that has the greatest weighted sum. Expected utility maximization has become the most usual decision rule in decision-making research [20]. Expected utility of the alternatives (UMLs) obtained from utility value of the alternative multiplied by their respective probability. In this paper the linear utility function is specified for the UMLs. Minimum and maximum UML have least and most utility values, respectively. Based on EUT best alternative has the highest expected utility.

2. METHODOLOGY

When designing a stope, the restrictions inflicted by mining method and geotechnical situations should be considered. The lower-grade material placed along the

border of the stope, should be mined only if the expected value of the product exceeds all incremental costs, including mining, haulage, processing and other costs. The minimum COG that defines border material which should be mined is the mine COG, and is calculated by a formula like that for material at the bottom of an open pit mine which is given by following expression [21]:

$$xc = \frac{(Mo+Po+Oo)}{[r.(V-R)]} \quad (1)$$

where

xc: cut- off grade (%)

Mo: Mining cost per metric tonne of ore

Po: Processing cost per metric tonne of ore

Oo: Overhead cost per metric tonne of ore

r: Proportion of valuable product recovered from the mined material (%)

V: Value of one unit of valuable product (cost per metric tonne of valuable product)

R: Refining costs, defined as costs that are related to the unit of valuable material produced (cost per metric tonne of valuable product).

In this study, the COG is modeled, and its variables are expenses and incomes. Metal price is one of the variables with special distribution that is simulated by Monte Carlo method. The simulation is carried out by @RISK software. By placing distribution of metal prices (DIST(p)) instead of average value of price (p) in COG formula; distribution of Simulated Cut off Grades are achieved. In other words, by randomly sampling from metal prices distribution using Monte Carlo method, distribution of COGs is achieved.

$$COG = \frac{(Mo+Po+Oo)}{[r.(DIST(V)-R)]} \quad (2)$$

where DIST(V) is distribution of metal prices.

After obtaining simulated COGs, the UMLs are attained with MRO in Datamine software. MRO delineates and evaluates three dimensional envelopes of ore body considering factors such as shape and orientation of stopes, the minimum size of stopes, and the minimum head grade of the mined minerals. In MRO optimization criteria include the maximization of ore tones, grade, and contained metal and accumulated value. An output envelope model is created containing blocks that meet the defined economic and mining criteria. One of three types of envelopes can be calculated based on different algorithms: ranked, minimum or maximum. All three types of envelopes conform to the geometric criteria of size and orientation of the minimum mining unit. Both the ranked and minimum envelopes are a subset of the

maximum envelope. Ranked envelopes only depend on head grade whereas minimum and maximum envelopes are a function of both head grade and cutoff grade [22]. After obtaining simulated UML based on COGs, EUT is used for determining the optimum UML. Von Neumann and Morgenstern proved that if the preferences of a person satisfy the following principles, then the decision maker should choose between lotteries using the EUT [23].

1. Completeness supposes that the decision maker has well defined preferences and can always decide between any two options.

2. Transitivity supposes that, as the decision maker decides according to the completeness principle, also decides systematically.

3. Independence of unrelated options depends on well-defined preferences as well. It supposes that two gambles mixed with an unrelated third one will retain the same priority as when the two are presented separately of the third one. The independence principle is the most eristic principle.

4. Continuity supposes that when there are three options (A, B and C) and the decision maker prefers A to B and B to C, then there should be a feasible compound of A and C in which the decision maker is then unconcerned between this combination and the lottery B.

When the entities x whose value x_i takes one of a set of discrete values and have an effect on the utility of the person, the expected utility equation is stated as follows:

$$E[u(x)] = p_1 \cdot u(x_1) + p_2 \cdot u(x_2) + p_3 \cdot u(x_3) + \dots \quad (3)$$

where x_i is the possible option, $u(x_i)$ is the utility of that option, and p_i is its probability. There could be either a limited or an unlimited set of possible values x_i , in which case the right side of this equation has a limited number or an unlimited number of terms [24]. A flowchart indicating the different steps of the proposed methodology is shown in Figure 1.

3. IMPLEMENTATION OF THE MODEL

In this paper, a hypothetical underground gold mine is used as a case study. This mine is exploited with cut and fills method and the processing method is heap leaching. The economic parameters of the case study were estimated based on similar mines (Table 1). The model used for this case study is derived from model for the MRO tutorial project of Datamine Software. The *in situ* reserves were calculated based on the entire

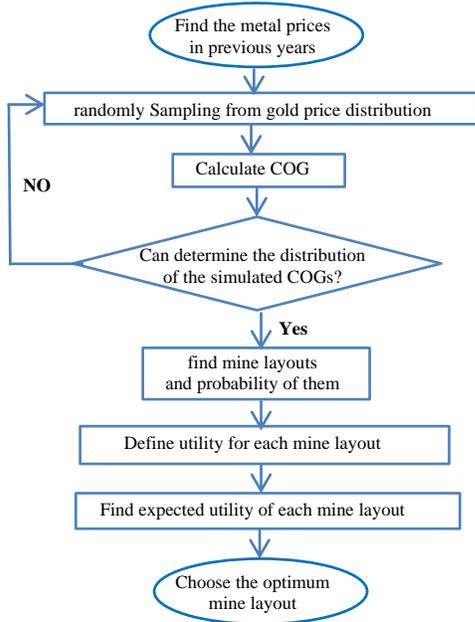


Figure 1. Steps of the proposed methodology

geological model. Using a density of 2.5, the total tonnage of ore reserve is 3.08 MT at 1.23 g/t Au as average grade. Reserves have been calculated for Au cutoffs between 0 and 3 g/t at intervals of 0.25 g/t. The resulting grade and tonnes above each of the 13 cutoffs are shown in Figure 2. The figures in this table are calculated from the grades of the individual cells in the model.

3. 1. Gold Prices in Previous Years Monthly gold price changes over past 3 years shows the gold price was downgraded from the middle of 2014 to early 2016, but this trend is set to rise in 2016, and it slowed down again at the end of 2016 and early 2017 (Figure 3). According to histogram and statistical parameters of the gold price over past 3 years, gold price range is 1068 to 1341 dollar per ounce (Table 2 and Figure 4).

3. 2. Simulation of Cut-Off Grades Simulated COGs are calculated based on the distribution of gold

TABLE 1. Economic parameters used for the case study

Notation	Explanation	Unit	Value
Mo	Extraction costs	\$/tonne of ore	55
Po+Oo	Processing cost and Overhead cost	\$/tonne of ore	28
r	Recovery	%	80
R	Selling costs	\$/tonne of metal	2,000,000

price. The Monte Carlo technique is used to randomly sample from the gold price data, which is repeated 100 times. Each sampled price provides a simulated COG. Repeating the previous steps gives an appropriate number of estimations of the COG (Tables 3 and 4).

3. 3. Find Probability of Mineable Reserves COG is one of the most important parameters that is used in MRO. With simulated COGs, Datamine software tool was used to obtain the corresponding.

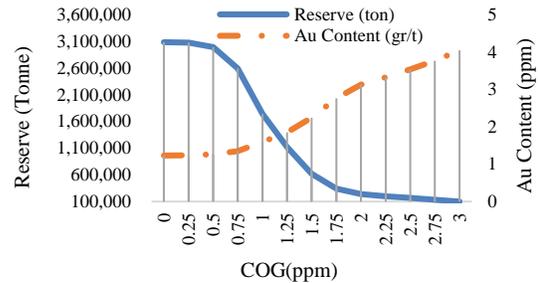


Figure 2. Grade- tonnage curve

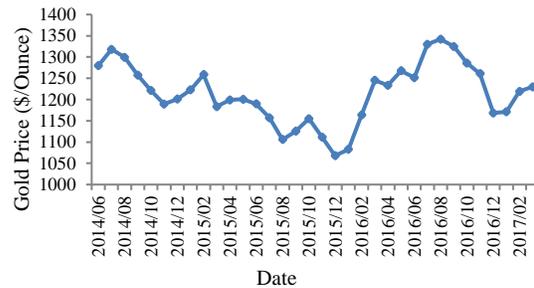


Figure 3. monthly gold price changes over past 3 years [25]

TABLE 2. Statistical parameters of past gold prices

	Fit	Input
Function	Logistic (1216.473, 40.793)	-
α	1216.47	-
β	40.793	-
Minimum	-Infinite	1068.14
Maximum	+Infinite	1341.91
Mean	1216.47	1215.27
Mode	1216.47	1262.36
Median	1216.47	1220.36
Std. Deviation	73.991	70.982
Variance	5474.606	4890.29
Skewness	0	-0.1603
Kurtosis	4.2	2.3932



Figure 4. Distribution of past gold prices

TABLE 3. Simulated COGs (ppm)

2.70	2.70	3.21	2.91
2.70	3.16	2.57	2.78
2.83	2.95	3.16	2.64
2.65	3.04	3.21	2.69
2.83	3.09	2.55	3.16
2.86	2.78	2.76	2.65
2.95	2.57	2.83	2.83
2.88	2.55	2.52	2.64
2.93	2.70	2.86	2.55
2.91	2.70	3.16	2.78
2.78	2.56	2.83	2.70
2.76	2.78	2.61	2.86
2.73	2.70	3.09	2.92
2.83	2.76	2.88	2.71
2.71	3.16	2.86	2.61
3.04	3.08	2.68	2.65
2.56	2.69	2.86	2.93
2.86	2.83	2.56	2.76
2.65	2.68	2.83	2.79
2.71	2.69	2.57	2.86
2.76	2.93	2.92	2.76
2.76	2.88	3.09	2.79
2.69	2.88	3.09	2.84
2.73	2.52	2.52	2.61
2.92	2.70	3.09	2.92

TABLE 4. Statistical parameters of simulated COGs

Minimum	2.52
Maximum	3.21
Mean	2.8046
Mode	2.70
Median	2.78
Std. Deviation	0.17528
Variance	0.030417
Skewness	0.5171
Kurtosis	2.6403

In this research, with 100 random sampling of monthly gold prices, 100 COGs were calculated with the accuracy of two decimal places. Due to the tonnage grade curve, the COG above 3 was removed from the list of simulated COGs. At this stage out of 100 simulated COG, 86 COGs were obtained. Each COG was used to determine the UML using the Datamine software. Each COG lead to a UML and a mineable reserve. In other words, 86 UMLs were obtained from 86 COGs (Table 5) Frequency of UML's mineable reserve is shown in Figure 5. In the following, resulting mineable reserves of minimum, average and maximum simulated COGs are tabulated in Table 6 and Figure 6.

TABLE 5. Data of simulated COGs and UMLs

Simulated COG	No of simulation	Reserve of UML (Mtonne)	Reserve of UML (Tonne)
2.52	1	0.17	166,800
2.52	2	0.17	166,800
2.52	3	0.17	166,800
2.52	4	0.17	166,800
2.55	5	0.16	162,000
2.55	6	0.16	162,000
2.55	7	0.16	162,000
2.56	8	0.16	160,400
2.56	9	0.16	160,400
2.56	10	0.16	160,400
2.57	11	0.16	158,800
2.57	12	0.16	158,800
2.57	13	0.16	158,800
2.61	14	0.15	152,400
2.61	15	0.15	152,400
2.61	16	0.15	152,400
2.64	17	0.15	147,600
2.64	18	0.15	147,600
2.65	19	0.15	146,000
2.65	20	0.15	146,000
2.65	21	0.15	146,000
2.65	22	0.15	146,000
2.68	23	0.14	141,200
2.68	24	0.14	141,200
2.69	25	0.14	139,600
2.69	26	0.14	139,600
2.69	27	0.14	139,600
2.69	28	0.14	139,600
2.70	29	0.14	138,000
2.70	30	0.14	138,000
2.70	31	0.14	138,000
2.70	32	0.14	138,000
2.70	33	0.14	138,000

2.70	34	0.14	138,000
2.70	35	0.14	138,000
2.70	36	0.14	138,000
2.71	37	0.14	136,400
2.71	38	0.14	136,400
2.71	39	0.14	136,400
2.73	40	0.13	133,200
2.73	41	0.13	133,200
2.76	42	0.13	128,400
2.76	43	0.13	128,400
2.76	44	0.13	128,400
2.76	45	0.13	128,400
2.76	46	0.13	128,400
2.76	47	0.13	128,400
2.76	48	0.13	128,400
2.78	49	0.13	125,200
2.78	50	0.13	125,200
2.78	51	0.13	125,200
2.78	52	0.13	125,200
2.78	53	0.13	125,200
2.79	54	0.12	123,600
2.79	55	0.12	123,600
2.83	56	0.12	117,200
2.83	57	0.12	117,200
2.83	58	0.12	117,200
2.83	59	0.12	117,200
2.83	60	0.12	117,200
2.83	61	0.12	117,200
2.83	62	0.12	117,200
2.83	63	0.12	117,200
2.84	64	0.12	115,600
2.86	65	0.11	112,400
2.86	66	0.11	112,400
2.86	67	0.11	112,400
2.86	68	0.11	112,400
2.86	69	0.11	112,400
2.86	70	0.11	112,400
2.86	71	0.11	112,400
2.88	72	0.11	109,200
2.88	73	0.11	109,200
2.88	74	0.11	109,200
2.88	75	0.11	109,200
2.91	76	0.10	104,400
2.91	77	0.10	104,400
2.92	78	0.10	102,800
2.92	79	0.10	102,800
2.92	80	0.10	102,800
2.92	81	0.10	102,800
2.93	82	0.10	101,200
2.93	83	0.10	101,200
2.93	84	0.10	101,200
2.95	85	0.10	98,000
2.95	86	0.10	98,000

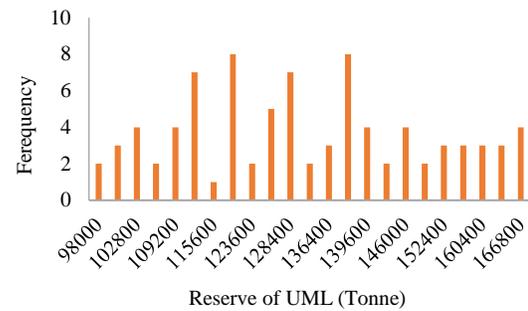


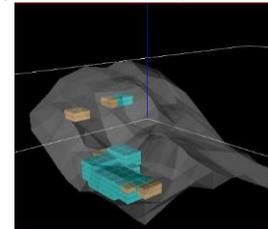
Figure 5. Absolute frequency of reserve of UMLs

Table 6. Specification of 3 cases of obtained mineable reserves

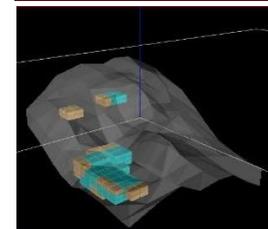
Case	Volume (m ³)	Mineable reserve (Tone)	Average AU Grade (ppm)	COG (ppm)
A	54,000	135,000	3.3	2.5
B	46,000	123,000	3.5	2.8
C	34,500	86,250	3.8	3.1

(Maximum Envelope= Blue blocks, Minimum Envelope= Yellow blocks, Waste=Red blocks)

Case A:
COG=Average simulated COG



Case B:
COG=Minimum simulated COG



Case C:
COG=Maximum simulated COG

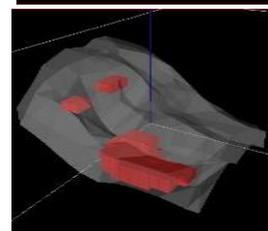


Figure 6. Mineable envelope models of each case

3. 4. Find Expected Utility of Each Mine Layout and Choose the Optimum Mineable Reserve

According to Equation (1), the expected utility of an entity will be counted according to the probability of

occurrence and the utility of each of its consequences or effects. Thus, if we look only at the UML from the economic point of view, we can obtain the expected utility of each UML economically as the probability of occurrence of that UML in its utility. In other words, the right side of the equation will have only one sentence. As the amount of reserve increases and UML getting bigger and economic utility of it increases, this relationship can be considered linear for ease of work. This is assumed to be zero for the lowest reserve and zero for the largest reserve, which is 1 (Table 7). Optimum UML is that provides mineable reserve with maximum expected utility. UML with 138,000 tonnes mineable reserves have maximum expected utility (Table 7 and Figure 7).

4. DISCUSSION

Aim of this paper is to find the UML and subsequently the underground mineable reserve with regard to uncertainty of the metal price. If metal price fluctuation does not consider, optimum price for design is \$1215 per ounce and its corresponding COG is 2.8 ppm. Finally, optimum UML has 123,200 tonnes mineable reserves (scenario 1 in Table 8). Normal design is the design currently being done for mining projects and the average price of metal in the past three years is usually used for mine design. Scenario 1 in Table 8 defined as normal design. Assuming the future behavior of the gold price is the same as its past behavior, most probable UMLs have 123,000 and 138,000 tonnes mineable reserve resulting 2.8 and 2.7 ppm COG, respectively. Accordingly, optimum metal prices as for probability of occurrence are \$1215 and \$1261 per ounce (scenario 2 in Table 8) In this paper, for selection between two prices with the same

probability, EUT was used. As respects further reserve is more desirable, UML with 138000 tonnes mineable

TABLE 7. Data for calculation of EU

Reserve of UML (Tone)	Frequency of UML	Probability of UML	utility of UML	EU of UML
98000	2	2%	0.04	0.001
101200	3	3%	0.09	0.003
102800	4	5%	0.13	0.006
104400	2	2%	0.17	0.004
109200	4	5%	0.22	0.010
112400	7	8%	0.26	0.021
115600	1	1%	0.30	0.004
117200	8	9%	0.35	0.032
123600	2	2%	0.39	0.009
125200	5	6%	0.43	0.025
128400	7	8%	0.48	0.039
133200	2	2%	0.52	0.012
136400	3	3%	0.57	0.020
138000	8	9%	0.61	0.057
139600	4	5%	0.65	0.030
141200	2	2%	0.70	0.016
146000	4	5%	0.74	0.034
147600	2	2%	0.78	0.018
152400	3	3%	0.83	0.029
158800	3	3%	0.87	0.030
160400	3	3%	0.91	0.032
162000	3	3%	0.96	0.033
166800	4	5%	1.00	0.047

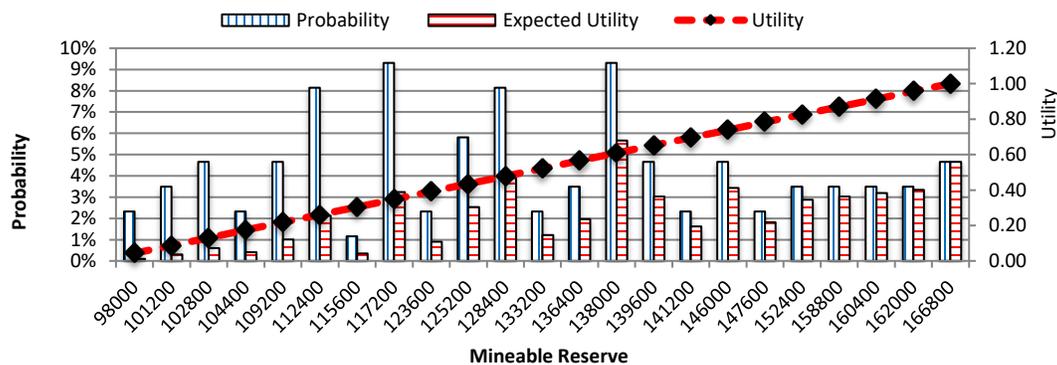


Figure 7. Utility, probability and expected utility of each mineable reserve

reserve has more utility value, so has more expected utility value (scenario 3 in Table 8) With this methodology, both probabilities and utility of UMLs are considered. Always the most probable UMLs are not the optimum and as observed, there may be several UML with equal probability. Even if there is only one UML with the most probability of occurrence, that UML is not optimum because of UMLs with further mineable reserve and subsequently higher utility which may lead to higher expected utility. On the other hand, using this method leads to more mineable reserve due to consideration of utility values. The results in Table 9 show the application of proposed methodology leads to 14% more mineable reserve than normal design. Net Present Value (NPV) is the common criteria for optimum mine design selection in the mining industry.

To verify the proposed methodology, results of the research has compared with NPV analysis. In this

TABLE 8. specifications of 3 scenarios

Scenario No.	Description	Gold price (\$/ounce)	COG (ppm)	Mineable Reserve (Tonne)
1	Average gold price over 3 past years (Normal Design)	1215	2.8	123200
2	Most probable UMLs	1215 1261	2.8 2.7	123200 138000
3	UML with maximum expected utility	1261	2.7	138000

TABLE 9. Economic parameters of two designs

Parameter	Unit	Normal design	Design based on EUT
Gold price	\$/Ounce	1215	1261
COG	ppm	2.8	2.7
Mineable reserve of UML	Tonne	123,200	138,000
Average grade of the reserve	ppm	3.8	3.7
Fixed costs	\$	433,814	437,840
Production costs	\$/year	357,190	373,295
Income	\$/year	1,463,023.5	1,656,062.1
Mine Life	year	5	5
Internal Rate of Return (IRR)	%	1.89	2.17
Net Present Value (VPV)	\$	1,725,408	2,048,993

manuscript, normal design and the design based on proposed approach (EUT) (scenarios 1 and 3 of Table 8) were compared with data presented in Table 9. Net present value of the design based on EUT also confirms the superiority of this methodology.

5. CONCLUSIONS

Mine design regardless of metal price uncertainty, will not be optimal design. It may be misleading when evaluating mining projects. Optimization of stope layout and mineable reserve is one of the important steps in underground mine design. In this paper, mineable reserve of an underground mine was determined regarding metal price uncertainty. Metal price uncertainty was simulated by Monte Carlo technique and cut-off grades was modeled by @Risk software. Mineable reserves corresponding to simulated cut-off grades provided by mineable reserve optimizer script of Datamine software and probability of them were calculated. For selection between most probable reserves expected utility was used. Accordingly, by this method besides the fact that metal price uncertainty is considered, larger mine layout and high NPV is achieved.

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Choosing the Optimum Underground Mine Layout with Regard to Metal Price Uncertainty Using Expected Utility Theory

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قیمت فلز یکی از مهم‌ترین پارامترها در محاسبه عیار حد است. عیار حد نقش مهمی در تعیین محدوده نهایی استخراج ایفا می‌کند. محدوده نهایی معدن؛ مقدار ذخیره قابل استخراج، عمر معدن و بازدهی اقتصادی آن را به دست می‌دهد. لحاظ نکردن عدم قطعیت قیمت فلز می‌تواند منجر به دستیابی به محدوده نهایی غیر بهینه شود. در این مقاله محدوده نهایی یک معدن زیرزمینی با توجه به عدم قطعیت قیمت و با استفاده از تئوری مطلوبیت انتظاری به دست آمده است. با روش پیشنهادی این مطالعه عدم قطعیت قیمت فلز با تکنیک مونت کارلو شبیه‌سازی شده و تصمیم‌گیرنده احتمال دستیابی به هر محدوده استخراج را به دست می‌آورد. تابع مطلوبیت محدوده استخراج، تعریف شده و با استفاده از احتمالات محاسبه شده در مرحله قبل، مطلوبیت انتظاری برای محدوده‌ها محاسبه می‌شود. محدوده‌ای که بیشترین مطلوبیت انتظاری را دارد، محدوده استخراج بهینه است. کاربرد این روش در یک معدن طلای فرضی، علاوه بر لحاظ کردن عدم قطعیت قیمت فلز، منجر به افزایش ۱۴ درصدی ذخیره قابل استخراج و ۱۸ درصد ارزش کنونی خالص بالاتر نسبت به طراحی نرمال دارد.

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