



## Multi-period and Multi-objective Stock Selection Optimization Model Based on Fuzzy Interval Approach

A. Kameli<sup>a</sup>, N. Javadian<sup>\*b</sup>, A. Daghbandan<sup>c</sup>

<sup>a</sup> Department of Financial Engineering, Kooshyar Higher Education Institute, Rasht, Iran

<sup>b</sup> Department of Industrial Engineering, Mazandaran University of Science and Technology, Babol, Iran

<sup>c</sup> Department of Industrial Engineering, Gilan University, Rasht, Iran

### PAPER INFO

#### Paper history:

Received 20 May 2019

Received in revised form 30 June 2019

Accepted 05 July 2019

#### Keywords:

Historical data

Multi-objective

LR-Fuzzy

LP-metrics

Portfoli

### ABSTRACT

The optimization of investment portfolios is the most important topic in financial decision making, and many relevant models can be found in the literature. According to importance of portfolio optimization in this paper, deals with novel solution approaches to solve new developed portfolio optimization model. Contrary to previous work, the uncertainty of future returns of a given portfolio is modeled using LR-FUZZY numbers while the function of its return are evaluated using possibility theory. We used a novel Lp-metric method to solve the model. The efficacy of the proposed model is tested on criterion problems of portfolio optimization on LINGO provides a framework to optimize objectives when creating the loan portfolio, in a search for a dynamic markets decision. In addition to, the performance of the proposed efficiently encoded multi-objective portfolio optimization solver is assessed in comparison with two well-known MOEAs, namely NSGAII and ICA. To the best of our knowledge, there is no research that considered NSGAII, ICA fuzzy simultaneously. Due to improve the performance of algorithm, the performance of this approach more study is probed by using a dataset of assets from the Iran's stock market for three years historical data and PRE method. The results are analyzed through novel performance parameters RPD method. Thus, the potential of our comparison led to improve different portfolios in different generations.

doi: 10.5829/ije.2019.32.09c.11

## 1. INTRODUCTION AND PREVIOUS WORK

Currently, portfolio selection is one of the most complex issues that needs to decide on both the strategic level and the operational level [1]. Modern portfolio selection theory is derived from the mean-variance probabilistic model by Markowitz [2]. Markowitz's classic model shows the involvement of the investor in the amount of investment. This research considered optimization of the three-objective model with constraints simultaneously. Today, in this zoon, there are some models to determine portfolios that over time the defects are specified, and one model is replaced by another one. On the other hand, volatility on the stock exchange is unpredictable and has a random nature. Access to an appropriate portfolio without planning and evaluating investment options is a difficult problem. The most important problem is how much of each asset is allocated in the portfolio of each

investor and the reason for that uncertainty is the return from each asset.

Saborido et al. [3] and Bermudez et al. [4] described the skewness to combine the measurement of fuzzy data asymmetry in a portfolio, and a study of its role in selecting a potential portfolio have been addressed [5]. The model was developed as a robust model and to ensure proper implementation of the model [6]. There are different [7] strategies for solving the portfolio optimization model in fuzzy space. To solve the fuzzy problem, the LR-FUZZY approach was used. In this context, Vercher et al. [8] used probability distributions using fuzzy LR- NUMBERS to determine the amount of portfolio return in an uncertainty. The membership function of the random function was obtained using historical data. This random function was obtained using the features derived from investment inflows [9, 10]. Framework. They used some analysis techniques to solve

\*Corresponding Author Email: [Njavadian@ustmb.ac.ir](mailto:Njavadian@ustmb.ac.ir) (N. Javadian)

this problem. On the other hand Harvey et al. [11] used hybrid algorithms (ICA-FA) to solve multi-period problem. Based on uncertain theory, we present a novel multi-period multi-objective mean-variance-skewness model by considering multiple realistic investment constraints, such as transaction cost, bounds on holdings, and cardinality etc [12]. To this end, Wang et al. [9] have solved multidisciplinary MDRS problems using meta-framework methods that are specifically designed to produce different portfolios with a different substitution than the MDRS model since evolutionary algorithms such as genetics and other algorithms were developed with the propose of optimizing optimal portfolios and optimizing multi-objective problems [9]. Therefore, the MDRS probabilistic [13, 14] model cannot optimize all specially designed for dealing with the difficulties of the cardinality constrained portfolio optimization problem (CCPOP). Also, the proposed algorithm incorporates a new mutation and recombination operator tailor-made to work well with the new encoding scheme [15]. According to the previous work and researches gap,

- Through the total stock income to the stock portfolio
- Resolving a single-period portfolio problem
- Use of evolutionary algorithms like GA, PSO, FA, NSGAI, ant colony.
- Using goal programming and weighting to solve multi-objective problems.
- According to gap in the literature
- Most of the works done in this field comes from the collection of each single stock in the desired portfolio, and in this research, on the contrary, this process has been taken from the hypothetical portfolio to the desired stocks. It means that, we have considered a hypothetical portfolio and reached the portfolio to the required stocks, but on similar issues, the stock will reach the desired portfolio.
- Solving the problem by using the L-Pmetric method in order to change multi-objective problem to single target. So, the novelties of our paper are:
- Changing the single period problem to multi-period one solving the problem with MOEA(ICA). The reason for the use of the ICA algorithm is that: this algorithm provides an excellent answer to the Markowitz (mean-variance) model problem, which is rarely used or never used, NSGAI and lingo in IRAN's market. The reminder of the paper is organized as follows: In section 2, we briefly review of the model. the description of the algorithm and pseudo code of algorithms in section 3. Section 4 presents the proposed ICA and NSGAI algorithms for MDRSMP model And the numerical results for a data set from Iranian market. Then we are provided some managerial results in section 4, and finally, we conclude and give some future directions in section 5.

## 2. PROBLEM DEFINITION

Contrary to previous works, the MDRSMP model is developed on fuzzy MDRS models, is used a novel Lp-metric method to solve the model and is assumed that portfolio optimization model is multi-period. The efficacy of the proposed model is tested on criterion problems of portfolio optimization with LINGO provides a framework to optimize objectives when creating the loan portfolio, default in a search for a dynamic markets decision when the uncertainty of the return on a given portfolio is directly quantified through its possibilistic moments for power LR-fuzzy numbers and the risk of the investment is measured by means of the downside risk.

### 2. 1. Parameters and Variables

$x_{it}$  = The decision variable, the fraction of the total capital, is invested in the purchase of a stock  $i$  in the period  $t$ ,  
 $a_{uit}$  = The upper limit of the price of the  $i$ -th stock in  $t$ ,  
 $a_{lit}$  = Limit the price of the  $i$ -th stock in  $t$ ,  
 $c_{it}, d_{it}$  = Trapezoidal numbers for trapezoidal fuzzy logic that lie between upper and lower bounds,  
 $L_{it}$  = The lowest investment in stock  $i$  in period  $t$ ,  
 $u_{it}$  = The highest investment in stock  $i$  in period  $t$ ,  
 $\sum_{i=1}^n x_{it} = 1$ , the aggregation of investing in stocks must be equal to one.

### 2 .2. Assumptions

- The model is considered periodically,
- The principle that the price of any stock is limit and cannot be exceeded,
- The market assumed efficient,
- The performance of the market is the highest, and stocks traded easily,
- Data are supposed to be normal,
- The stocks selected from four different industries because stocks do not have any correlation with each other.

### 2. 3. Model

This study aims to tackle the Mean-downside risk-skewness-multi period problem for selecting the best portfolio by considering evolutionary algorithms. The proposed model is a multidimensional where the objective functions concord to the crisp values. These objective functions are nonlinear because they depend on the sample percentiles of the returns on the stock  $X$ . According to earlier papers, for selecting efficient portfolios, we propose to maximize the odd moments while minimizing the downside risk value. The portfolio selection problem can be formulated as follows: Where the decision variables are  $x_i$ , the fraction of the portfolio value invested in asset  $i$ , and  $i = 1, 2, \dots, N$  denote the different risky assets:

$$\text{Max } E(\tilde{p}_x) = \frac{\sum_{i=1}^n a_{uit}x_{it} + \sum_{i=1}^n a_{lit}x_{it}}{2} \quad (1)$$

$$\text{Min } w(\tilde{p}_x) = \frac{\sum_{i=1}^n d_{it}x_{it} + \sum_{i=1}^n c_{it}x_{it}}{2} + \frac{\sum_{i=1}^n a_{uit}x_{it} - \sum_{i=1}^n a_{lit}x_{it}}{1} \quad (2)$$

$$\text{Max } \mu(\tilde{p}_x) = \frac{1}{32} (\sum_{i=1}^n d_{it}x_{it} - \sum_{i=1}^n c_{it}x_{it})^3 + \left( \frac{\sum_{i=1}^n a_{uit}x_{it} - a_{lit}x_{it}}{16} \right) ((\sum_{i=1}^n d_{it}x_{it})^2 - (\sum_{i=1}^n c_{it}x_{it})^2) + \frac{1}{8} (\sum_{i=1}^n d_{it}x_{it})(\sum_{i=1}^n c_{it}x_{it})(\sum_{i=1}^n d_{it}x_{it} - \sum_{i=1}^n c_{it}x_{it}) \quad (3)$$

St:

$$\sum_{i=1}^n x_{it} = 1 (\text{Budget constraint}) \quad (4)$$

$$k_l \leq C(X) \leq k_u (\text{Cardinality constraint}) \quad (5)$$

$$L_{it} \leq x_{it} \leq u_{it} (\text{Bound constraint}) \quad (6)$$

$$l_{it} \geq 0, x_{it} \geq 0 \quad (7)$$

difference between performed a thorough comparative [16] assessment of different bi-objective models as well as multi-objective one, in terms of the performance and robustness of the whole set of Pareto optimal portfolios. Barbati et al. [17] proposed a different approach that enables the Decision Maker (DM) to control the distribution of good evaluations on different criteria over the projects composing a portfolio. With this aim, for each criterion we fix a certain number of reference levels corresponding to the qualitative satisfaction degrees. The last example of 2018 works with this title (including the main constraints) were introduced into this study facilitates a more reasonable investment decisions with four objective decision criteria including Burg's entropy [18]. Mokhtarian Asl et al. [15] proposed a novel multiobjective evolutionary Algorithm (MOEA) for the solution of the cardinality constrained portfolio optimization problem (CCPOP). The proposed algorithm introduces an efficient encoding scheme.

Three important components of decision making are risk, returns, and skewness. Most investors are getting a certain amount of return. Markowitz showed his mean-variance model, with the selection of a portfolio of financial assets, is possible to reduce a certain level of risk returns. This possibility is due to the lack of correlation between financial assets. Individuals will invest by their expected utility and will ignore the consumption of today in the future. The utility function of each investor-investor is determined by the preferences of the same person, which is not necessarily the same as other investors [10]. The optimization of the portfolio is to select the best combination of financial assets in the direction that objectives and constraints of multi-objective problems with the explanation that the three objectives of the mean, variance, and the skewness

generated by the data cannot be optimized simultaneously with this explanation [19].

This paper deals with mean, downside risk, and skewness and multi-period called (MDRSMP) model. We use a novel Lp-metric method to solve the model. To optimize objectives when creating the loan portfolio, default in a search for a dynamic markets decision. It optimizes the expected return, the downside risk, and the skewness of a given portfolio taking into account budget bound and cardinality constraints. The nobility of this study dealt simultaneously with the optimization of returns, risk, skewness, and cardinality constraint. In addition to, the uncertainty of future returns of a given portfolio is modeled using LR-FUZZY numbers while the function of its return are evaluated using possibility theory.

The main propose of this paper is solved the MDRSMP portfolio selection model as a whole constrained by three objective functions and relevant limitations. According to literature review to solve this problem, we started with a feasible problem, and then we used NSGAI along with the evolutionary algorithms MOEA-ICA, the reason for the use of the ICA algorithm is that: This algorithm provides an excellent answer to the Markowitz (mean-variance) model problem, which is rarely used or never used- and NSGAI what has not been done before in order to analyze the efficient portfolios which optimize the three criteria simultaneously. Finally, we studied for a data set from Iran's market to find out the trade-off between Lingo software and MATLAB. That maximizes the return on investment of the investor [17].

### 3. COMPUTATIONAL TESTS

After that the original model is solved by MATLAB software and Lingo (the model is presented in the previous section). Given the assumptions of the model and the fact that the shares selected in this research have no dependence:

According to the output of proposed model in Table 1, which has been selected from a Spanish market paper and implemented in the Iranian market and examined in a multi-variate manner in this study, shows that:

1. In constant periods, with the addition of the number of stocks, the values of the return and skewness functions have not changed or have not changed at all, but the risk level of the stock portfolio decreases.

2. According to the definition of Meta-Heuristic, it does not give an accurate value and gives us an answer close to the optimal answer and the results of NSCAII are better than ICA. The advantage of the methods of solving Meta-Heuristic is the time to solve them. As shown in the answer, the time to solve it is better than the time used by lingo to resolve it.

**TABLE 1.** Difference between out puts of LINGO and ICA and NSGAI

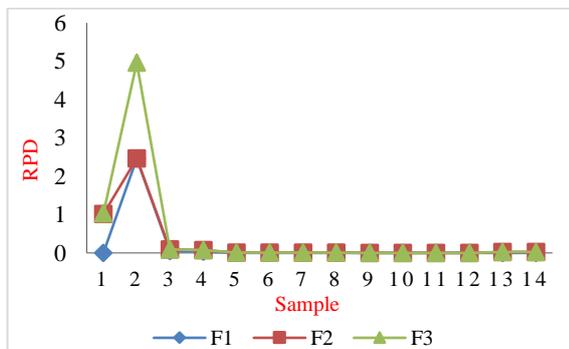
Sample	LINGO					The best answer of ICA				The best answer of NSGAI				
	Number of stock	Period(t)	OFV1*	OFV2	OFV3	Time(s)	OFV1	OFV2	OFV3	SV**	OFV1	OFV2	OFV <sub>3</sub>	SV
1	4	2	3700	878	12691	30	3665	945	11790	113.5	3680	920	11920	12
2	6	2	3700	859	12691	30	3665	959	11790	15	3680	934	11940	15
3	8	3	5845	1949	39712	32	5698	1560	39600	15	5700	1458	39600	14.50
4	10	3	4667	1689	39712	45	5780	1590	39580	17	5800	1460	39600	15
5	12	4	5473.5	1235	99532	52	6000	1600	99526	17	6500	1498	99580	15.30
6	14	5	0	0	0	0	6750	1670	201609	17	6800	1540	201620	15.30
7	16	5	0	0	0	0	6750	1670	201609	20	6800	1578	201628	18
8	18	5	0	0	0	0	7010	1670	201609	21	7400	1597	201628	19.30
9	19	6	0	0	0	0	7300	1720	348645	23	7680	1603	348660	22
10	20	6	0	0	0	0	7960	1723	348645	25	7980	1603	348660	24
11	30	6	0	0	0	0	8000	1800	350000	27	8100	1743	353000	26
12	50	7	0	0	0	0	8500	1970	358000	33	8960	1830	370000	29
13	60	7	0	0	0	0	8768	2300	378000	37	9345	1950	410000	32
14	80	8	0	0	0	0	8793	2700	400000	40	10280	2020	43350	36

\* OFV(Objective Functional Value)      \*\*SV(Solving time)

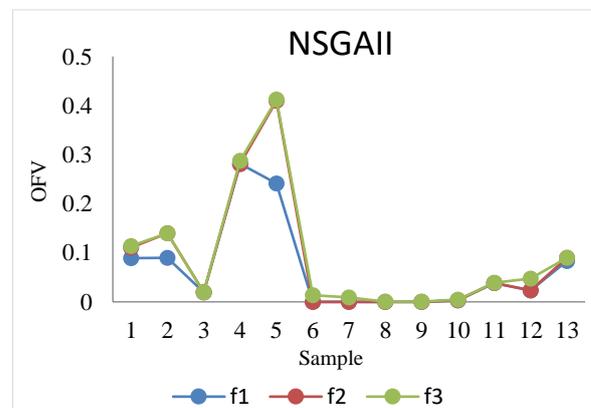
3. Determination of the parameters related to the ICA algorithm. Now, to carry out a difference between three algorithms, the results are of high accuracy and quality. Therefore, in this research, three functions setting RPD Figure 1 was used for this purpose. Further explanations are given about RPD, and then related affiliations will be raised in connection with this research. Finally, the problems were solved by using Lingo's small-scale software and Macro software (meta-heuristic) in large dimensions [10].

In constant periods, with the increase in the number of stocks, the values of the return and skewness functions have not changed or have not changed at all, but the risk level of the portfolio decreases. According to the

definition of meta-heuristic, it does not give a precise amount and gives us the nearest answer to the optimal answer and the results of NSGAI Figure 2 were better (both of time and values) than ICA Figure 3. The advantage of the meta-heuristic solution and the amount of investment in all stocks in all proposed portfolios, as shown in the answer, is the time to solve problems which is better than the time Lingo uses to solve them Figure 4. Results are taken from four randomly selected stocks which are among the active industries in Iran. If they were selected from different companies or the size of the company were changed, type of investment in different parts would also change.



**Figure 1.** The difference between RPD's answers between NSGAI, ICA, and LINGO



**Figure 2.** The output of NSGAI's Algorithm

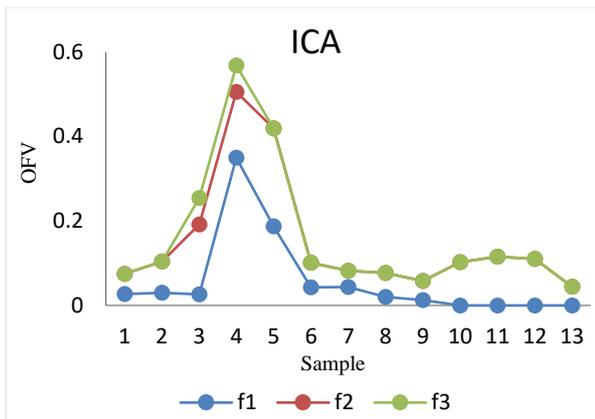


Figure 3. The output of three functions for ICA's Algorithm

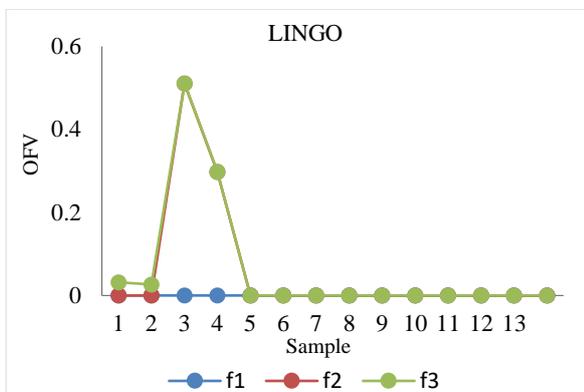


Figure 4. The out put of LINGO's Algorithm

#### 4. MANAGERIAL RESULTS

In order to increase the return of our portfolio the investor could have different approaches: the first one is that, if he is a risk taker, he could choose different stocks from small industries in a short term. Because some of these firms in primarily years of construction are really profitable. Some people are risk averse. It means that they escape from the risk. So, we suggest that they choose heavy stocks. It means that these stocks are from the well-known companies. They are expensive and most of people follow these companies' stocks. Because, in long terms (more than one year) they are profitable. Another solution for risk averse in stock market is better that the investors try to increase the cardinality of their portfolio. If some stocks face different risks in markets risks, other stock's return is covered their risks. So, we suggest that, they make a balanced between their stocks.

#### 5. CONCLUSION

Modern portfolio selection was driven by Markowitz considering that investors would choose their portfolios

based on two criteria of risk and return, and for these purposes, he presented his mathematical model based on selecting the optimal portfolio. One of the biggest problems with his model is that it just only considered two criteria, mean and standard deviation of returns, so investors consider different criteria when selecting portfolios. In this study, portfolio selection is approached and achieving the amount of investment per stock in the fuzzy space was used and finally, the problems were solved by using Lingo's small-scale software and Macro software (meta-heuristic) in large dimensions [10]. In constant periods, with the increase in the number of stocks, the values of the return and skewness functions have not changed or have not changed at all, but the risk level of the portfolio decreases. According to the definition of meta-heuristic, it does not give a precise amount and gives us the nearest answer to the optimal answer and the results of NSGAI were better (both of time and values) than ICA. The advantage of the meta-heuristic solution and the amount of investment in all stocks in all proposed portfolios, as shown in the answer, is the time to solve problems which is better than the time Lingo uses to solve them. Results are taken from four randomly selected stocks which are among the active industries in Iran. If they were selected from different companies or the size of the company were changed, type of investment in different parts would also change.

#### 7. REFERENCES

1. Cheraghalipour, A., Paydar, M.M. and Hajiaghaei-Keshteli, M., "An integrated approach for collection center selection in reverse logistics", *International Journal of Engineering - Transactions A: Basics*, Vol. 30, No. 7, (2017), 1005–1016.
2. Markowitz, H., "Portfolio selection", *The Journal of Finance*, Vol. 7, No. 1, (1952), 77–91.
3. Saborido, R., Ruiz, A.B., Bermúdez, J.D., Vercher, E. and Luque, M., "Evolutionary multi-objective optimization algorithms for fuzzy portfolio selection", *Applied Soft Computing*, Vol. 39, (2016), 48–63.
4. Bermúdez, J.D., Segura, J.V., and Vercher, E., "A multi-objective genetic algorithm for cardinality constrained fuzzy portfolio selection", *Fuzzy Sets and Systems*, Vol. 188, No. 1, (2012), 16–26.
5. Vercher, E. and Bermudez, J.D., "A Possibilistic Mean-Downside Risk-Skewness Model for Efficient Portfolio Selection", *IEEE Transactions on Fuzzy Systems*, Vol. 21, No. 3, (2013), 585–595.
6. Kaviyani-Charati, M., Heidarzadeh Souraki, F. and Hajiaghaei-Keshteli, M., "A Robust Optimization Methodology for Multi-objective Location-transportation Problem in Disaster Response Phase under Uncertainty", *International Journal of Engineering - Transactions B: Applications*, Vol. 31, No. 11, (2018), 1953–1961.
7. Li, X., Qin, Z., and Kar, S., "Mean-variance-skewness model for portfolio selection with fuzzy returns", *European Journal of Operational Research*, Vol. 202, No. 1, (2010), 239–247.
8. Vercher, E. and Bermúdez, J.D., "Portfolio optimization using a credibility mean-absolute semi-deviation model", *Expert Systems with Applications*, Vol. 42, No. 20, (2015), 7121–7131.

9. Wang, S. and Xia, Y., *Portfolio Selection and Asset Pricing*, Springer Berlin Heidelberg, Vol. 514, (2002).
10. Bermudez, J.D., Segura, J.V., and Vercher, E., "A fuzzy ranking strategy for portfolio selection applied to the Spanish stock market", 2007 IEEE International Fuzzy Systems Conference, (2007), 1-4.
11. Harvey, C.R., Liechty, J.C., Liechty, M.W. and Müller, P., "Portfolio selection with higher moments", *Quantitative Finance*, Vol. 10, No. 5, (2010), 469-485.
12. Cao, J.L., "Algorithm research based on multi period fuzzy portfolio optimization model", *Cluster Computing*, (2018), 1-8.
13. Liu, S., Wang, S. Y., and Qiu, W., "Mean-variance-skewness model for portfolio selection with transaction costs", *International Journal of Systems Science*, Vol. 34, No. 4, (2003), 255-262.
14. Gupta, P., Inuiguchi, M., Mehlatat, M.K. and Mittal, G., "Multiobjective credibilistic portfolio selection model with fuzzy chance-constraints", *Information Sciences*, Vol. 229, (2013), 1-17.
15. Mokhtarian Asl, M. and Sattarvand, J., "Integration of commodity price uncertainty in long-term open pit mine production planning by using an imperialist competitive algorithm", *Journal of the Southern African Institute of Mining and Metallurgy*, Vol. 118, No. 2, (2018), 165-172.
16. Cao, L., Huang, J.Z., Bailey, J., Koh, Y.S. and Luo, J., "New Frontiers in Applied Data Mining", PAKDD: Pacific-Asia Conference on Knowledge Discovery and Data Mining, PAKDD 2011 International Workshops, Shenzhen, China, (2012).
17. Barbati, M., Greco, S., Kadziński, M. and Słowiński, R., "Optimization of multiple satisfaction levels in portfolio decision analysis", *Omega*, Vol. 78, (2018), 192-204.
18. Liagkouras, K. and Metaxiotis, K., "A new efficiently encoded multiobjective algorithm for the solution of the cardinality constrained portfolio optimization problem", *Annals of Operations Research*, Vol. 267, No. 1-2, (2018), 281-319.
19. Li, H. and Zhang, Q., "Multiobjective Optimization Problems With Complicated Pareto Sets, MOEA/D and NSGA-II", *IEEE Transactions on Evolutionary Computation*, Vol. 13, No. 2, (2009), 284-302.

## Multi-period and Multi-objective Stock Selection Optimization Model Based on Fuzzy Interval Approach

A. Kameli<sup>a</sup>, N. Javadian<sup>b</sup>, A. Daghbandan<sup>c</sup>

<sup>a</sup> Department of Financial Engineering, Kooshyar Higher Education Institute, Rasht, Iran

<sup>b</sup> Department of Industrial Engineering, Mazandaran University of Science and Technology, Babol, Iran

<sup>c</sup> Department of Industrial Engineering, Gilan University, Rasht, Iran

### P A P E R I N F O

### چکیده

#### Paper history:

Received 20 May 2019

Received in revised form 30 June 2019

Accepted 05 July 2019

#### Keywords:

Historical data

Multi-objective

LR-Fuzzy

LP-metrics

Portfoli

بهینه‌سازی سبد سهام یکی از مهم‌ترین مسائل در تصمیم‌گیری‌های اقتصادی می‌باشد در حالی که بسیاری از مدل‌های مرتبط با این موضوع یافت می‌شود. بر مبنای اهمیت بهینه‌سازی پرتفوی این مقاله با بهره‌گیری از رویکردهای جدید به حل مدل بهینه‌سازی پرتفوی می‌پردازد. برخلاف پژوهش‌های پیشین، این مقاله، عدم اطمینان از درآمد حاصل از پرتفوی را با استفاده از اعداد LR-Fuzzy به ارزیابی درآمد مورد نظر با استفاده از تئوری احتمالی را مدل‌سازی کرده است. در این مدل از رویکرد جدیدی از LP-metric بهره گرفته شده است. اثربخشی این مدل به این شرح است که: این مدل بر مبنای مسایل بهینه‌سازی چندهدفه پرتفوی و با استفاده از نرم‌افزارهای جامع تحقیق در عملیات در فضای پویا حل گردیده است. علاوه بر این، این مقاله به مقایسه حل NSGAII و ICA پرداخته است. بر طبق مطالعات ما، تاکنون پژوهشی که همزمان به مقایسه حل این دو در فضای فازی پردازد انجام نشده است. برای بهبود اثربخشی این تحقیق از داده‌های بازار بورس ایران در سه سال گذشته و تنظیم پارامترها با استفاده از روش تاگوچی بهره گرفته شده است. بنابراین، نقطه‌ی قوت این پژوهش ایجاد پرتفوی‌های مختلف در نسل‌های متفاوت می‌باشد.

doi: 10.5829/ije.2019.32.09c.11