



A Lagrangian Decomposition Algorithm for Robust Green Transportation Location Problem

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ABSTRACT

In this paper, a green transportation location problem is considered with uncertain demand parameter. Increasing robustness influences the number of trucks for sending goods and products, caused consequently, increase the air pollution. In this paper, two green approaches are introduced which demand is the main uncertain parameter in both. These approaches are addressed to provide a trade-off between using available trucks and buying new hybrid trucks for evaluating total costs beside air pollution. Due to growing complexity, a Lagrangian decomposition algorithm is applied to find a tight lower bound for each approach. In this propounded algorithm, the main model is decomposed into master and subproblems to speed up convergence with a tight gap. Finally, the suggested algorithm is compared with commercial solver regarding total cost and computational time. Due to computational results for the proposed approach, the Lagrangian decomposition algorithm is provided a close lower bound in less time against commercial solver.

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Nomenclature

Sets			
I	The set of Origins	cbc^p	Purchasing cost of P th hybrid truck
J	The set of destinations	D_j^l	Demands of l th product in j th destination
P	The set of trucks	Td_{ij}	Maximum allowable pollution emission in link between i and j
L	The set of products	$1-\alpha$	Confidence interval
Parameters		Variables	
c_{ij}^p	Set up cost for link between i th origin to j th destination for truck P	y_{ij}^p	1 If a link between i and j is constructed for truck P and 0, otherwise
q_{ij}^p	Transportation cost for l th product with P th truck from i th origin to j th destination	x_{ij}^p	Flow between i and j by the truck P for l th product
h_i^l	i th origin opening cost for l th product	z_i^l	1 if origin i is used for shipping commodity l , 0 otherwise
w_j^l	Penalty cost for unmet demands for l th product in j th destination	u_j^l	Amount of unsatisfied demand in j th destination for l th product
b^p	Capacity of P th truck for l th product	nc^p	Number of P th needed hybrid trucks
k_i^l	Capacity of i th origin for l th product	$ofv1$	Objective function 1 (Total Cost)
em^p	Pollution emission from P th truck	$ofv2$	Objective function 2 (Total Cost)

1. INTRODUCTION

Increasing the number of required products and

developing transportation systems are the main result of population growth. As a result, it makes the air polluted, and mechanisms for controlling pollution become

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important. When companies produce a particular product for the first time, have not any accessible data about the products' demand. Therefore, they should estimate their volume. There are several ways to dealing with this uncertainty. In recent years, one of the ways that have progressed remarkably is robust optimization. Robust optimization generally divides into two types of interval-based and scenario-based models. In this paper, interval-based robust optimization is considered. In this term, there are some pioneers such as Soyster [1], Ben-Tal and Nemirovski [2], and Bertsimas and Sim [3] approaches. Bertsimas's approach used regarding its flexibility on considering uncertain parameters related to other approaches [4-6]. Budget parameter affected on the price of robustness in this approach. The number of trucks and consequently pollution emission is enhanced by increasing the budget parameters. Some companies eliminate these problems by choosing the costly solution and buy new hybrid trucks. However, controlling pollution created by available trucks is an economical solution against the first approach. These two approaches are examined in this paper. For considering the second approach, suppose that pollution caused by each truck is followed from a distribution function with known mean and variance, and can be controlled with a threshold. Under this assumption, a chance constraint is necessary to be used. The chance constraint is one of the hard and probabilistic restrictions, which can be added to the main problem.

In large-scale mixed integer programming (MIP) problems, commercial solvers' efficiency is reduced. Therefore, decomposition algorithms may be used to solve these problems. Decomposition-based solution methods are employed to find exact solutions for MIP problems. In the contrast of other decomposition-based algorithms, Lagrangian decomposition algorithm is considered to find a tight lower bound for large-scale problems. Lagrangian decomposition is a kind of Lagrangian relaxation algorithm which decomposes the problem into some subproblems after relaxing hard constraints. For application of Lagrangian decomposition, some methods have been introduced previously like the subgradient method, cutting plane and so on which, in this paper, the second one method is applied. In this conception, the master problem is considered for reducing iteration of solving problems after decomposing the model into two subproblems. In the master problem, the main decision variables are fixed, and Lagrange multipliers are found as decision variables.

The rest of this paper is organized as follows: In the next section, a literature review is presented for green transportation location problem with uncertain demands. In section 3, the proposed mathematical model is presented. In the fourth section, the Lagrangian decomposition and steps of this algorithm are discussed. Sensitivity analysis and computational experiments are

examined in section 5. Finally, the paper concludes in the last section.

2. LITERATURE REVIEW

In this section, the published papers were reviewed and contribution of each paper is discussed according to its evaluation. A two stage robust was mentioned by Gabrel et al. [7]. A transportation location problem has been modeled with two stage stochastic programming concept, which distribution channels are a priori decisions to optimized network flow. Second stage variables are flow and origins decisions [8]. A novel mathematical model for transportation location problem has presented in disaster application, which location and origin-destination allocation decisions are priori known [9]. A two stage stochastic programming is used to deal with parameter uncertainty. First stage variables are flow of priori allocation and new allocation decisions. Flow of new distribution channels and shortage or leftovers of distribution channels are second stage variables [10]. A bi-objective mixed integer location/routing model have presented that aims to minimize transportation cost and risks for large-scale hazardous waste management systems (HWMSs), whereas all parameters are known [11]. Also, Lagrangean decomposition has been used in various problems such as Quadratic binary Program [12], location-allocation problem, offshore oilfield development planning [13] as a solution algorithm. Due to mentioned papers, we applied Lagrangean decomposition for this problem which was not used until now.

Main contributions of this study can be summarized as follows ;

- Controlling amount of pollution in the network with a chance constraint concept.
- Using robust optimization for dealing with uncertainty in the green transportation location problem.
- Considering a Lagrangian decomposition algorithm for the robust green transportation-location problem.

3. PROBLEM DEFINITION

This section is divided into two parts, in the first part, a transportation location problem is defined with demand uncertainty. In the second one, green approaches are mentioned.

A transportation-location problem is composed of transportation and location-allocation problems, and its aim is transporting each product due to the amount of demand in each destination with the minimum total cost. The capacity of origins and trucks restrict sending products, and it is assumed that the required vehicles

already exist in the shipping company. Considered problem costs are included:

- A. Shipping costs from the origins to destinations
- B. The cost of linking between origins and destinations
- C. The cost of established origins
- D. The cost of the shortage of products at destinations

For example, a company is planned to produce various products and deliver to customers with regard to the total cost. The company should use different types of trucks for satisfying customers' demands. The capacity of the origins and trucks are playing an important role in the number of delivered products. Due to the mentioned example, suppose that this company produces new commodities; while, market demands are unknown, and company revenue is increased when all market demands are satisfied. If the company wants to satisfy all customer demands, the number of trucks and consequently, pollution emissions are increased. Two approaches are suggested for addressing pollution emissions: Firstly, due to required trucks, the company is decided to purchase hybrid trucks for sending products to destinations. In the second approach, the pollution emission is controlled by adding some limitations regarding as the age of trucks. Assumed that it follows a normal distribution, so related constraints are added as the chance constraint.

4. MATHEMATICAL MODEL

4.1. Mathematical Problem

$$Min\ of\ v1 = \sum_{i=1}^I \sum_{j=1}^J \sum_{p=1}^P c_{ij}^p y_{ij}^p + \sum_{i=1}^I \sum_{l=1}^L h_i^l z_i^l + \sum_{i=1}^I \sum_{j=1}^J \sum_{p=1}^P \sum_{l=1}^L q_{ij}^p x_{ij}^{lp} + \sum_{j=1}^J \sum_{l=1}^L w_j^l u_j^l + \sum_{p=1}^P nc^p nc^p \tag{1}$$

$$s.t. \sum_{i=1}^I \sum_{p=1}^P y_{ij}^p \geq 1, \quad \forall j \tag{2}$$

$$\sum_{l=1}^L x_{ij}^{lp} \leq \sum_{l=1}^L b^{lp} y_{ij}^p, \quad \forall i, j, p \tag{3}$$

$$\sum_{j=1}^J \sum_{p=1}^P x_{ij}^{lp} \leq k_i^l z_i^l, \quad \forall i, l \tag{4}$$

$$u_j^l \geq D_j^l - \sum_{i=1}^I \sum_{p=1}^P x_{ij}^{lp}, \quad \forall j, l \tag{5}$$

$$nc^p = \sum_{i=1}^I \sum_{j=1}^J y_{ij}^p, \quad \forall p \tag{6}$$

$$Pr(\sum_{p=1}^P em^p y_{ij}^p \leq Td_{ij}) \geq 1 - \alpha, \quad \forall i, j \tag{7}$$

$$y_{ij}, z_i^l \in \{0,1\} \tag{8}$$

$$x_{ij}^{lp}, u_j^l, nc^p \geq 0$$

For clarifying the mathematical problem, note that first and second approaches are considered in unified model. Because many constraints are looked at in both approaches.

Equations (1) calculate the total cost of the transportation system that the two first parts are about the cost of establishing origins and destinations. The third part calculates the transportation cost between origin and destination, and the fourth part calculates penalty cost of unmet demands in destinations. Last part calculates total cost for buying new vehicles. This term is added when the first approach are considered. Equation (2) guarantees that each destination should be visited. The capacity limitation for trucks and origins are mentioned in Equations (3) and (4). Amount of unsatisfied demands are determined in Equation (5). Equation (6) calculates the number of each truck is used. In the conception, pollution emission threshold is considered in the Equation (13) which looked at the second approach.

4.3. Robust Optimization in Green Transportation Location Problem

Bertsimas and Sim approach [3] can control effects of demand uncertainty on the network design. This approach was presented by Bertsimas and Sim [3] which was improved by Keyvanshokoo et al. [14]. In the improved paper, a novel approach is used to model the robust green closed loop supply chain problem [14].

Assume that demand interval is $[\underline{D}_j^l + \Delta D_j^{l-}, \overline{D}_j^l + \Delta D_j^{l+}]$ that \overline{D}_j^l it is the nominal value of demand and $\Delta D_j^{l+}, \Delta D_j^{l-}$ are positive and negative deviation from the nominal value, respectively. Assuming that computing unmet demand penalty cost is computed in the following constraint:

$$\sum_{j=1}^J \sum_{l=1}^L w_j^l (D_j^l - \sum_{i=1}^I \sum_{p=1}^P x_{ij}^{lp}) \leq v \tag{9}$$

v is a free variable which used in the objective function instead of $\sum_{j=1}^J \sum_{l=1}^L w_j^l u_j^l$.

Final robust counterparts are written as follows:

$$\sum_{j=1}^J \sum_{l=1}^L ((\overline{D}_j^l + \alpha 1_j^l + \Gamma^l \mu_j^l - \sum_{i=1}^I \sum_{p=1}^P x_{ij}^{lp}) w_j^l) \leq v \tag{10}$$

$$\alpha 1_j^l + \mu_j^l \geq \Delta D_j^{l+}, \quad \forall J, L \tag{11}$$

$$\alpha 2_j^l + \mu_j^l \geq \Delta D_j^{l-}, \quad \forall J, L \tag{12}$$

$$\alpha 1_j^l, \alpha 2_j^l, \mu_j^l, v \geq 0 \tag{13}$$

4. 4. Chance Constraint Programming in Green Transportation Location Problem

Pollution emission constraint is propounded by a chance constraint format, the linearization of these constraints is considered as below:

Suppose that pollution emission of each truck (em^p) has a normal distribution with $(E(em^p), VAR(em^p))$. The linearized constraints are given in Equation (14):

$$\sum_{p=1}^P E(em^p) y_{ij}^p + Z_{1-\alpha} \sqrt{\sum_{p=1}^P VAR(em^p) (y_{ij}^p)^2} \leq Td_{ij}, \quad \forall i, j \tag{14}$$

5. LAGRANGIAN DECOMPOSITION

Used Lagrangian decomposition is based on literature [15], which considered cutting planes to provide a tight lower bound. In this section, both component of Lagrangian decomposition and pseudo-code of the algorithm is presented, respectively. For starting this algorithm, relaxed constraint must be determined. Relaxed constraint is:

$$\sum_{i=1}^L x_{ij}^{lp} \leq \sum_{i=1}^L b^{lp} y_{ij}^p, \quad \forall i, j, p \tag{15}$$

After relaxing mentioned constraint, the main problem decomposed into two sub-problems that presented in the rest of the paper.

Extra parameters that used in this algorithm are mentioned below:

- θ : Upper bound of first sub-problem
- η : Upper bound of second sub-problem
- λ_{ij}^p : Lagrange multipliers for relaxed constraints

5. 1. Lagrangean Sub Problems With relaxing constraint (15), Lagrangean subproblems for both approaches can be demonstrated from their main mathematical models. After relaxing mentioned constraint with Lagrange multipliers, the relaxed problem is divided into two independent problems.

5. 2. Lagrangean Master Problems

5. 2. 1. First Approach Master Problem

$$Max LM = \theta + \eta \tag{16}$$

$$\theta \leq \sum_{i=1}^J \sum_{j=1}^L \sum_{p=1}^P (c_{ij}^p - \lambda_{ij}^p \sum_{l=1}^L b^{lp}) x_{ij}^{lp} + \sum_{p=1}^P cbc^p \dot{n}c^p \tag{17}$$

$$\eta \leq \sum_{j=1}^J \sum_{i=1}^L \sum_{l=1}^L \sum_{p=1}^P (q_{ij}^{lp} + \lambda_{ij}^p) x_{ij}^{lp} + \sum_{i=1}^L \sum_{l=1}^L h_i^l x_i^l + \mathcal{F} \tag{18}$$

$$\lambda_{ij}^p \geq 0 \tag{19}$$

5. 2. 2. Second Approach Master Problem

$$Max LM = \theta + \eta \tag{20}$$

$$\theta \leq \sum_{i=1}^J \sum_{j=1}^L \sum_{p=1}^P (c_{ij}^p - \lambda_{ij}^p \sum_{l=1}^L b^{lp}) x_{ij}^{lp} \tag{21}$$

$$\eta \leq \sum_{j=1}^J \sum_{i=1}^L \sum_{l=1}^L \sum_{p=1}^P (q_{ij}^{lp} + \lambda_{ij}^p) x_{ij}^{lp} + \sum_{i=1}^L \sum_{l=1}^L h_i^l x_i^l + \mathcal{F} \tag{22}$$

$$\lambda_{ij}^p \geq 0 \tag{23}$$

Suggested algorithm Pseudo-code is presented as follow:

5. 3. Lagrangian Decomposition Algorithm Pseudo-code

1) Initialized:

$$Z_{up} = \infty, \quad Z_{lb} = -\infty, \quad iter = 1$$

2) Solve Lagrangian Sub problems:

- Store all variables
- Store Objective functions values
- If sum of the objective values are greater than Z_{lb} , update Z_{lb}

3) Solve Lagrangian Master Problem:

- Store λ_{ij}^p
- If Master problem objective function are lower than Z_{up} , update Z_{up}

4) Convergence test:

- If $Z_{up} - Z_{lb} < \varepsilon$ stop the algorithm
- Else go to step 2

6. NUMERICAL STUDIES

Two approaches propound for dealing with published pollution, which in the first approach, the decision maker must buy new trucks to serves the customers. But, in the second approach, the decision maker tries to design

supply network somehow that the available truck published pollution is not exceeding form the particular threshold. In this section, the algorithm-based results illustrate. In the other words, the Lagrangean decomposition applies for each proper conception and computational and GAP percent of this algorithm are compared.

Algorithm-based results are illustrated in Tables 1-4, which in these tables, computational time and gap of lagrangian decomposition algorithm are calculated. As is clear from the results, mentioned algorithm are provide closed lower bound for proposed model with lower running time, that obtained results are more specifically due to growing size of the problem. GAP measure can be computed as follow:

$$\%GAP = \left(\frac{Z_{M\ Problem} - Z_{P\ Algorithm}}{Z_{M\ Problem}} \right) * 100 \tag{24}$$

Referred by results in below tables, when the sample size is small not only the computational time between each method is closed together, but also the GAP of the Lagrangian decomposition is less than large-scale problem. However, in the small size, the distinction between the two methods according to the time is so hard, but in the large one, the differences are seen obviously. Moreover, provided tables are demonstrated that in all instances Lagrangian decomposition's GAP is less than 1% which the suitability of this algorithm for solving such problems is demonstrated.

TABLE 1. First size comparison computational time for the first approach

NO.	Main Problem		Proposed Algorithm		
	Obj	T(s)	Obj	T(s)	%Gap
1	7258546.335	0.5	7252045.364	0.173	0.089562985
2	7547085.984	0.592	7540585.014	0.109	0.086138819
3	7835625.634	0.484	7829124.663	0.156	0.082966837
4	8124165.283	0.608	8117664.313	0.141	0.080020168
5	8412704.933	0.296	8406203.962	0.14	0.07727563
Average		0.496		0.1438	0.083193

TABLE 2. First size comparison computational time for the second approach

$Z_{1-\alpha} = -3$					$Z_{1-\alpha} = 3$				
Main Problem		Proposed Algorithm			Main Problem		Proposed Algorithm		
Obj	T(s)	Obj	T(s)	%Gap	Obj	T(s)	Obj	T(s)	%Gap
7275656.23	25.65	7271088.77	0.79	0.0628	7275727.26	23.30	7258546.33	0.53	0.2361
7564266.91	23.30	7559634.28	0.62	0.0612	7564266.91	23.88	7547085.98	0.54	0.2271
7852806.56	23.10	7848173.93	0.62	0.059	7852806.56	23.12	7835625.63	0.57	0.2188
8141346.21	24.78	8136713.58	0.62	0.0569	8141346.21	24.24	8124165.28	0.56	0.211
8429885.86	25.42	8425253.23	0.60	0.055	8429885.86	24.55	8412704.93	0.54	0.2038
Average	24.45		0.65	0.0589		23.813		0.548	0.2193

TABLE 3. Second size comparison computational time for the first approach

NO.	Main Problem		Proposed Algorithm		
	Obj	T(s)	Obj	T(s)	%Gap
1	17108168.97	0.312	17081279.72	0.234	0.15717198
2	17880707.73	0.67	17853818.48	0.172	0.15038134
3	18653246.49	0.671	18626357.25	0.187	0.14415318
4	19425785.26	0.452	19398896.01	0.048	0.138420391
5	20198324.02	0.687	20171434.78	0.203	0.133126134
Average		0.5584		0.1688	0.144651

TABLE 4. Second size comparison computational time for the second approach

$Z_{1-\alpha} = -3$					$Z_{1-\alpha} = 3$				
Main Problem		Proposed Algorithm			Main Problem		Proposed Algorithm		
Obj	T(s)	Obj	T(s)	%Gap	Obj	T(s)	Obj	T(s)	%Gap
12691067	135.78	12690733	0.624	0.002635	12691067	4.27	12690733	0.779	0.002635
13222631	138.73	13222296	0.968	0.002529	13222631	4.13	13222296	0.733	0.002529
13754195	103.88	13753860	0.967	0.002431	13754195	4.21	13753860	0.733	0.002431
14285758	51.12	14285424	0.936	0.002341	14285758	4.3	14285424	0.749	0.002341
14817322	201.05	14816988	0.982	0.002257	14817322	4.43	14816988	0.78	0.002257
average	126.112		0.895	0.00243		4.26		0.754	0.00244

7. CONCLUSION

In this paper, a robust green transportation location problem is suggested with uncertain demands. Bertsimas methodology is used for dealing with demand uncertainty. Two approaches are mentioned for controlling pollution emissions. In the first approach, total purchasing cost of new hybrid trucks is examined. In the second one, chance constraints are added to control pollution emission by available trucks. According to numerical examples, a trade-off is performed and it is demonstrated that which one has a lower total cost. Lagrangian decomposition is presented for providing a tight lower bound in a rational time. Computational results confirm that the presented algorithm is efficient besides of low optimally gap. For future research, a conditional value-at-risk instead of robust optimization can be used due to the problem concept. In the problem, a definition can propound multi-period or dynamic system that can adjust published pollution in different time periods. Using exact algorithms such as bender's decomposition, Dantzig-Wolfe decomposition, and so on can reduce computational burden besides solving problems in the exact forms. Moreover, there are some applications such as telecommunication, electricity distribution systems, and production planning which can employ the proposed model to improve the performance of their optimization problems. Employing this formulation is suggested as a future study.

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در این مقاله یک مسئله حمل و نقل مکان‌یابی سبز با پارامتر تقاضای غیرقطعی در نظر گرفته شده است. افزایش استواری بر تعداد کامیون‌ها برای ارسال کالاها و محصولات تاثیر می‌گذارد و در نتیجه باعث افزایش آلودگی هوا می‌شود. در این مقاله، دو رویکرد سبز معرفی شده است که تقاضا پارامتر اصلی غیرقطعی در هر دو رویکرد می‌باشد. این رویکردها طراحی شده‌اند تا مقیاسی میان استفاده از وسایل نقلیه موجود و خرید وسایل نقلیه جدید برای ارزیابی هزینه‌های کل در کنار آلودگی هوا ایجاد شود. با توجه به افزایش پیچیدگی، یک الگوریتم تجزیه لاگرانژ برای پیدا کردن یک حد پایین مناسب برای هر رویکرد استفاده شده است. در الگوریتم پیشنهادی، مدل اصلی به یک مسئله اصلی محدود و دو زیر مسئله تجزیه می‌شود تا سرعت همگرایی الگوریتم برای رسیدن به یک گپ مناسب افزایش یابد. در نهایت، الگوریتم پیشنهاد شده با یک حل‌کننده تجاری با توجه هزینه کل و زمان محاسباتی مقایسه می‌شود. با توجه به نتایج محاسباتی برای رویکرد پیشنهادی، الگوریتم تجزیه لاگرانژ در زمان کمتری حد پایین مناسبی برای مسائل مورد بررسی پیدا می‌کند.

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