



## Surface Energy and Elastic Medium Effects on Torsional Vibrational Behavior of Embedded Nanorods

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### ABSTRACT

In this paper, surface energy and elastic medium effects on torsional vibrational behavior of nanorods are studied. The surface elasticity theory is used to consider the surface energy effects and the elastic medium is modeled as torsional springs attached to the nanorod. At the next step, Hamilton's principle is utilized to derive governing equations and boundary conditions. Then, with the aid of an analytical method, natural frequencies are obtained and effects of various parameters on torsional frequencies are studied in details. It is concluded from the present study that the surface energy can make nanorods unstable depending on the nanorod dimension and frequency number. Results of the present study can be useful in design of nanoelectromechanical systems like drive shafts.

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## 1. INTRODUCTION

Analyzing of vibrational behavior of nano-devices is one of the important steps prior to manufacturing. One of the most useful parts of nano-devices is nanorod (nanobeam). Nanorods have different applications in nanoelectromechanical systems as nanogenerators, nanosensors, transistors, diodes, actuators, and resonators [1-8].

Making a comparison between the results of classical theories and experimental methods shows that mechanical behaviors of nano-structures cannot be analyzed by the classical theories. The reasons of this are small scale and surface energy effects in nanoscale structures. The small scale effect on mechanical behavior of nano-structures is considered in many literatures with the aid of nonlocal theory of Eringen [9], strain gradient theory [10], couple stress theory [11], and modified couple stress theory [12]; and the surface energy effects are investigated with the aid of the surface elasticity theory [13], respectively.

In several applications of nanorods, such as drive shafts [14], torsion bar springs [15, 16], linear nano

servomotors and bearings [17], or torsional actuators [18], tensile and torsional loads are expected to occur. This implies that optimizing the design of new devices requires torsional behavior analysis of nanorods besides analyses of other mechanical behaviors. Based on the non-local theory, Murmu et al. [19] analyzed the torsional vibration of carbon nanotube-buckyball systems; Loya et al. [20] and Arda and Aydogdu [21] investigated the effects of crack on free torsional vibration of nanorods and effects of elastic medium on torsional statics and dynamics of nanotubes, respectively; Khademolhosseini et al. [22] considered the torsional vibration of carbon nanotubes (CNTs); and torsional buckling of a double-walled carbon nanotubes embedded on winkler and pasternak foundations are investigated by Mohammadimehr et al. [23]. With the aid of strain gradient theory, the size-dependent pull-in instability of torsional nano-actuators is modeled [24], and the torsional vibration of carbon nanotubes is studied [10]. Gheshlaghi and Hasheminejad [25] investigated torsional vibration of CNTs using a modified couple stress theory. In another work, based on the surface elasticity theory, Nazemnezhad and Fahimi [26] modeled free torsional vibration of cracked nanobeams incorporating the surface energy effects.

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Literature survey shows that although there are several studies focusing on understanding the torsional responses of nanorods, only one of them has used the surface elasticity theory. The lack prompted the author to explore the torsional behavior of nanorods embedded in elastic medium based on the surface elasticity theory.

The goal of present work is to propose a comprehensive analytical model to study the elastic medium effect on torsional vibrational behavior of nanorods incorporating the surface energy effect. Considering the elastic medium is due to this fact that considerable attention has recently been turned on the mechanical behavior of single- and multi-walled carbon nanotubes (beam-like nanostructures) embedded in polymer or metal matrix. To this end, the elastic medium is modeled as torsional springs attached to the nanorod and the governing equations of nanorod incorporating the surface energy effects are derived by using the Hamilton's principle and solved by using the exact solution for various boundary conditions. Natural frequencies of nanorods with clamped-clamped and clamped-free boundary conditions for various stiffnesses of elastic medium, mode numbers, values of the surface energy, and dimensions of nanorod are calculated.

**2. PROBLEM FORMULATION**

In this section, Hamilton's principle is utilized to derive the torsional governing equation of motion of a nanorod embedded in an elastic medium in presence of the surface energy. To this end, we consider a nanorod with circular cross-section of constant area  $A$ , radius  $R$ , and length  $L$  ( $0 \leq x \leq L$ ) as shown in Figure 1.

The displacement components ( $u_x, u_r, u_\theta$ ) of the nanorod (the bulk and the surface layers) parallel to the three coordinate axes ( $x, r, \theta$ ) can be given by

$$u_x(x,t) = 0 \tag{1}$$

$$u_r(x,t) = 0 \tag{2}$$

$$u_\theta = r\theta(x,t) \tag{3}$$

where  $\theta(x,t)$  is the angle of twist of the cross-section along the coordinate  $x$ .

The strains and stresses in the nanorod bulk can be calculated based on the displacement components (Equations (1)-(3)) as

❖ bulk strains:

$$\epsilon_{x\theta} = \frac{\partial u_x}{r \partial \theta} + \frac{\partial u_\theta}{\partial x} = r \frac{\partial \theta}{\partial x} \tag{4}$$

$$\epsilon_{xx} = \epsilon_{\theta\theta} = \epsilon_{rr} = \epsilon_{xr} = \epsilon_{r\theta} = 0 \tag{5}$$

❖ bulk stresses:

$$\sigma_{x\theta} = G \epsilon_{x\theta} = G r \frac{\partial \theta}{\partial x} \tag{6}$$

$$\sigma_{xx} = \sigma_{\theta\theta} = \sigma_{rr} = \sigma_{xr} = \sigma_{r\theta} = 0 \tag{7}$$

in which  $G$  is the bulk shear modulus.

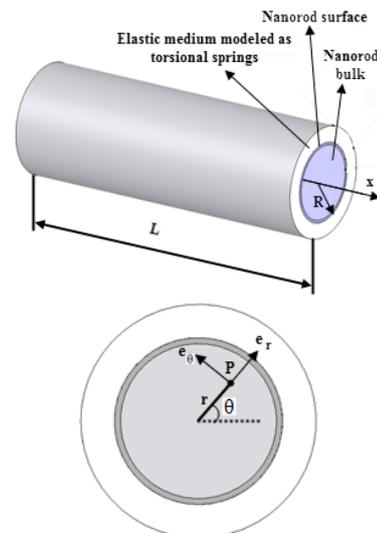
The surface elasticity theory is implemented to obtain the surface stresses. In the micro/nanoscale, the fraction of energy stored in the surface becomes comparable with the same in the bulk, due to the relatively high ratio of surface area to the volume of nanoscale structures; therefore the surface and induced surface forces must be taken into consideration. The constitutive relations of the surface layers,  $S^+$  (upper surface) and  $S^-$  (lower surface), given by [13] are

$$\tau_{\alpha\beta}^\pm = \tau_0^\pm \delta_{\alpha\beta} + (\mu_0^\pm - \tau_0^\pm)(u_{\alpha,\beta}^\pm + u_{\beta,\alpha}^\pm) + (\lambda_0^\pm - \tau_0^\pm)u_{\gamma,\gamma}^\pm + \tau_0^\pm u_{\alpha,\beta}^\pm \tag{8}$$

$$\tau_{\alpha z}^\pm = \tau_0^\pm u_{z,\alpha}^\pm \tag{9}$$

in which  $\tau_0$  is the residual surface tension under unconstrained conditions,  $\lambda_0$  and  $\mu_0$  are the surface Lamé constants,  $\delta_{\alpha\beta}$  is the Kronecker delta and  $u_\alpha$  are the displacement components of the surfaces, and  $\alpha, \beta = x, \theta$ . The signs + and - denote the upper and lower surfaces, respectively. Considering the same material properties for the surface layers of nanorod results in the following surface stress-strain relation

$$\tau_{x\theta} = (\mu_0 - \tau_0) \left( r \frac{\partial \theta}{\partial x} \right) \tag{10}$$



**Figure 1.** Schematic of an embedded nanorod with surface energy

It is worth mentioning here that we know from continuum mechanics the Lamé constant  $\mu$  is identical to the shear modulus  $G$  [27]. Therefore, hereinafter the Lamé constant  $\mu_0$  is shown as  $G_0$ .

At this step, it is possible to derive governing equation of motions of nanorod with the aid of Hamilton's principle [28] (Equation (11)) because the bulk and surface stress-strain relations are available:

$$\delta \int_{t_1}^{t_2} (U - T - W) dt = 0 \quad (11)$$

In Equation (11)  $U$  is the strain energy of nanorod,  $T$  is the kinetic energy of nanorod, and  $W$  is the work done by the elastic medium.  $U$ ,  $T$  and  $W$  are given by

$$U = U_b + U_s = \int (\sigma_{x\theta} \varepsilon_{x\theta}) dx dr d\theta + \int (\tau_{x\theta} \varepsilon_{x\theta}) dx ds \quad (12)$$

$$T = T_b + T_s = \int \rho \left( \frac{\partial u_\theta}{\partial t} \right)^2 dx dr d\theta + \int \rho_0 \left( \frac{\partial u_\theta}{\partial t} \right)^2 dx ds \quad (13)$$

$$W = \int (T_e \theta) dx \quad (14)$$

where,  $U_b$  and  $T_b$  are the bulk strain and kinetic energies, respectively, and  $U_s$  and  $T_s$  are the surface strain and kinetic energies, respectively, and  $T_e$  is the torque due to the elastic medium modeled as torsional springs attached to the nanorod.

The final step of calculating the governing equation of motion and boundary condition is substituting Equations (12)-(14) in Equation (11) and using Equations (4)-(7) and (10). This gives

$$(GI_p)_{eq} \frac{\partial^2 \theta}{\partial x^2} - (\rho I_p)_{eq} \frac{\partial^2 \theta}{\partial t^2} - k_t \theta = 0 \quad (15)$$

$$(GI_p)_{eq} \frac{\partial \theta}{\partial x} \delta \theta \Big|_0^L = 0 \Rightarrow \frac{\partial \theta}{\partial x} \delta \theta \Big|_0^L = 0 \quad (16)$$

where,  $k_t$  is the stiffness of elastic medium per unit length and

$$\begin{aligned} (GI_p)_{eq} &= GI_p + (G_0 - \tau_0) I_{ps}; & (\rho I_p)_{eq} &= I_0 + I_{0s}; \\ I_{0s} &= \rho_0 I_{ps} = 2\pi R^3 \rho_0; & T_e &= k_t \theta; \\ I_0 &= \rho I_p = \rho \iint r^2 dA = \rho \pi \frac{R^4}{2}; \end{aligned} \quad (17)$$

It is seen from Equations (15)-(17) that the torsional governing equations of motion of nanorod are affected by the surface energy parameters, but it is the other way round for the boundary condition. In addition, the equation of torsional motion of the conventional beam [28] can be obtained from Equation (15) by setting  $\rho_0 = \tau_0 = G_0 = 0$ . It is worth noting from Equation (15) that the equivalent mechanical properties of nanorods are different from the mechanical properties of macro

scale rods. This is attributed to the fact that the fraction of energy stored in the surfaces becomes comparable to that in the bulk due to the relatively high surface area to volume ratio. These equivalent mechanical properties cause that the nanoscale structures exhibit unique mechanical behaviors for applications in NEMSs. It is also necessary to mention that the equivalent mechanical properties of nanostructures depend on their crystallographic direction.

In order to obtain natural frequencies, the separation-of-variables method (Equation (18)) can be utilized to solve the governing equation of motion, Equation (15).

$$\theta(x, t) = \Phi(x) e^{i\omega t} \quad (18)$$

where,  $\omega$  is the natural frequency of the torsional vibration.

Substituting Equation (18) in Equations (15) and (16) and using the following dimensionless variables

$$X = \frac{x}{L}; \quad \Omega = \sqrt{\frac{((\rho I_p)_{eq} \omega^2 - k_t) L^2}{(GI_p)_{eq}}} \quad (19)$$

give the equation for the spatial function,  $\Phi(x)$ , and the boundary condition as mentioned below:

$$\frac{d^2 \Phi(X)}{dX^2} + \Omega^2 \Phi(X) = 0 \quad (20)$$

$$\frac{d\Phi(X)}{dX} \delta \Phi \Big|_0^1 = 0 \quad (21)$$

Solving Equation (20) gives

$$\Phi(X) = C_1 \sin(\Omega X) + C_2 \cos(\Omega X) \quad (22)$$

Applying boundary conditions, Equation (21), to Equation (22) yields the mode shapes and natural torsional frequencies of nanorod with clamped-clamped and clamped-free boundary conditions as below:

❖ The clamped-clamped nanorod

$$\begin{aligned} \Phi_{CC}(X) &= C_n \sin(n\pi X) \\ \omega_{CC} &= \sqrt{\frac{(n\pi)^2 (GI_p)_{eq} + L^2 k_t}{L^2 (\rho I_p)_{eq}}} \end{aligned} \quad (23)$$

❖ The clamped-free nanorod

$$\begin{aligned} \Phi_{CF}(X) &= C_n \cos\left(\frac{(2n-1)\pi}{2} X\right) \\ \omega_{CF} &= \sqrt{\frac{((2n-1)\pi)^2 (GI_p)_{eq} + (2L)^2 k_t}{(2L)^2 (\rho I_p)_{eq}}} \end{aligned} \quad (24)$$

As seen from Equations (23) and (24), both the surface energy and the elastic medium affect torsional

frequencies of nanorods while this is not the case for their effects on torsional mode shapes of nanorods.

### 3. RESULTS AND DISCUSSION

**3. 1. Comparison Study** The only studies commensurate with present investigation are torsional vibration of macrorods without surface energy and elastic medium effects by Rao [28], and torsional vibration of nanorods with only the surface energy effect by Nazemnezhad and Fahimi [26].

By ignoring both the elastic medium and surface energy effects in the present work, the obtained first five natural torsional frequencies,  $\omega$ , are compared to their counterparts reported by Rao [28]. Table 1 brings the results for a rod with clamped-clamped and clamped-free boundary conditions and  $L=1\text{ m}$ ,  $G=30\text{ GPa}$ ,  $\rho=2700\text{ kg.m}^{-3}$ . Excellent concordance between the results is observed which substantiates the reliability of present formulation.

As the second comparison, the present results considering only the surface energy effect are compared with those reported by Nazemnezhad and Fahimi [26]. In Table 2, the fundamental frequency ratios of nanorod with clamped-clamped and clamped-free boundary conditions are listed for various nanorod radii and  $G=27\text{ GPa}$ ,  $\rho=2700\text{ kg.m}^{-3}$ ,  $G_0=-5.4251\text{ N/m}$ ,  $\tau_0=0\text{ N/m}$ , and  $\rho_0=5.46\text{E-}7\text{ kg.m}^{-2}$ . Table 2 demonstrates that the present results with those of Ref. [26] are the same. This implies that the presented equations and corresponding results are accurate and reliable.

**3. 2. Developed Results** The surface energy and elastic medium effects on free torsional vibration of nanorods is investigated. A circular cross-section nanorod with clamped-clamped (CC) and clamped-free (CF) boundary conditions, made of aluminum with crystallographic direction of [100] and the bulk and surface properties [28] expressed hereunder is employed for the analysis:

**TABLE 1.** Comparison of the first five natural torsional frequencies of clamped-clamped and clamped-free nanorod when  $L=1\text{ m}$ ,  $G=30\text{ GPa}$ ,  $\rho=2700\text{ kg.m}^{-3}$

Frequency number	clamped-clamped		clamped-free	
	Present study	Rao [29]	Present study	Rao [29]
First	10472.00	10472.00	5235.99	5235.99
Second	20944.00	20944.00	15708.00	15708.00
Third	31415.90	31415.90	26179.90	26179.90
Forth	41887.90	41887.90	36651.90	36651.90
Fifth	52359.90	52359.90	47123.90	47123.90

**TABLE 2.** Comparison of the fundamental frequency ratio of nanorod in presence of the surface energy ( $L=10\text{ nm}$ ,  $k_t=0$ )

Boundary condition type	R (nm)	Frequency ratio	
		Present study	Ref. [26]
clamped-clamped	1	0.560	0.560
	2	0.747	0.747
	3	0.821	0.821
	4	0.861	0.861
	5	0.887	0.887
clamped-free	1	0.560	0.560
	2	0.747	0.747
	3	0.821	0.821
	4	0.861	0.861
	5	0.887	0.887

❖ Aluminum:

$G=27\text{ GPa}$ ,  $\rho=2700\text{ kg/m}^{-3}$ ,  $G_0(=\mu_0)=-5.4251\text{ N/m}$ ,  $\rho_0=5.46\times 10^{-7}\text{ kg/m}^{-2}$ ,  $\tau_0=0.5689\text{ N/m}$

The results are obtained for the variations of frequency ratio with the following definition:

$$FR = \frac{SEF}{CF} \quad (25)$$

where CF and SEF denote the classical natural frequency of nanorod and the natural frequency of nanorod with surface energy and elastic medium effect, respectively. The FR variations are measured with the variations of length and radius of nanorod, mode number, and stiffness of elastic medium. Also it is assumed that the values of surface material properties do not vary in the presence of elastic medium. From this time on the acronyms EFr, SFr, ESFr and CFr are employed for brevity. EFr denotes the natural torsional frequency with only the elastic medium effects, SFr means the natural torsional frequency with only the surface energy effects, ESFr indicates the natural torsional frequency with integrated elastic medium and surface energy effects and finally, CFr represents the classical natural torsional frequency.

The variation of nanorod's length is investigated on torsional frequency ratio in the presence of elastic medium and surface energy effects, first. Figure 2 depicts the variations of fundamental FRs versus nanorod length for CC and CF boundary conditions and  $R=1\text{ nm}$ . The below observations are of paramount importance for this case:

❖ The effects of surface energy and elastic medium on torsional frequencies of nanorods are opposite to each other. The surface energy decreases the frequency while the elastic medium increases it. Since the surface has a negative shear modulus and

its value is amplified by the surface residual stress, the torsional rigidity of nanorod decreases. On the other hand, the surface density increases the kinetic energy of the system. Based on Equations (23) and (24), as the kinetic energy of the system increases, its natural frequency decreases. Consequently, the surface energy has a decreasing effect on the torsional frequency ratio. However, this implication is not general for the mechanical behavior of nanostructures. Based on reported literature [30-33], the surface energy has an increasing effect on transverse frequencies of nanorods and nanoplates. The resistance of the elastic medium against the rotation of nanorods, on the other hand sets forth the increasing effects of the elastic medium. In other words, the elastic medium restricts the rotation of nanorod, and the natural torsional frequencies increase.

- ❖ The effect of the surface energy on the torsional frequencies is independent of the length of nanorod. While the elastic medium effect depends on nanorod's length. As the elastic medium stiffness increases, the increasing effect intensifies. For longer nanorods, the effect of elastic medium decreases in the presence of surface energy. Interestingly, the earlier studies [34, 35] showed that the effect of the surface energy on the transverse vibration of nanorods depends on the length of the nanorod. Based on their observations, the longer the nanorod, the higher the effect of the surface energy is on the transverse frequencies.
- ❖ The softer the boundary condition, the higher the elastic medium effect is on torsional frequencies. On the other hand, it has been observed that the surface energy effect is independent of the boundary condition type. However, for transverse frequencies, the surface energy effect is reduced while the boundary condition grows stiffer [32, 36].

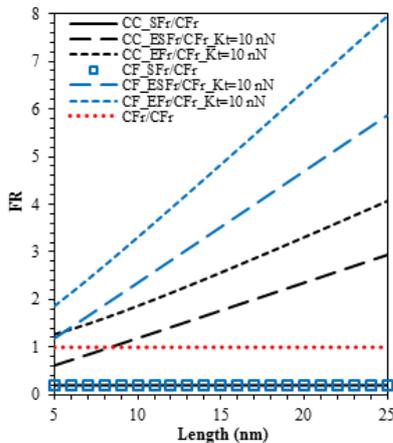


Figure 2. Variations of FR versus the nanorod length

Secondly, the variation of mode number is considered for the problem. The graphical results have been presented in Figure 3 for nanorod with CC and CF boundary conditions,  $R=1$  nm and  $L=10$  nm. According to the figure, the torsional frequency is independent of surface energy for the changes of mode number (CC\_SF/Cr and CF\_SF/Cr curves). This behavior is irrespective of the boundary condition type. On the other hand, the increasing effect of the elastic medium on torsional frequencies depends on both the boundary condition type and frequency number, for low frequency numbers. Furthermore, by increasing the mode number CC\_EFr/Cr and CF\_EFr/Cr curves tend to overlap with CFr/Cr curve which gives rise to an expected decrease in the elastic medium effects. In addition, the torsional frequencies are affected by both the surface energy and the elastic medium, at low mode numbers. By increasing the mode number, the CC\_EFr/Cr and CF\_EFr/Cr curves approach the CC\_SF/Cr curve (or the CF\_SF/Cr curve) expressing the decrease in the elastic medium effects. In other words, at higher mode numbers the surface energy effect becomes dominant. A literature survey shows a fluctuating surface energy effect on transverse frequencies of nanobeams [34]. The effect first decreases and then increases with increasing the mode number.

Thirdly, the effects of elastic medium and surface energy on torsional frequency ratios are depicted in Figure 4 for various radii of nanorod with  $L=10$  nm. The surface energy effect is independent of the type of boundary condition like the previous cases, while this is the other way round for the elastic medium effect. Based on CC\_SF/Cr and CF\_SF/Cr curves of Figure 4, the surface energy effect on torsional frequencies depends on the radius of nanorod. Also, by increasing the nanorod's radius, the decreasing effect of surface energy decreases. In addition, Figure 4 displays that the increasing effect of the elastic medium on torsional frequencies depends on nanorod radius where the effect is reduced by increasing the nanorod radius.

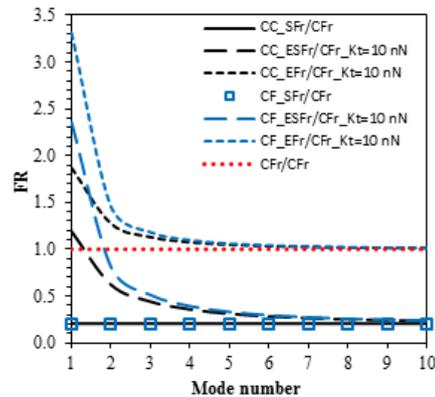
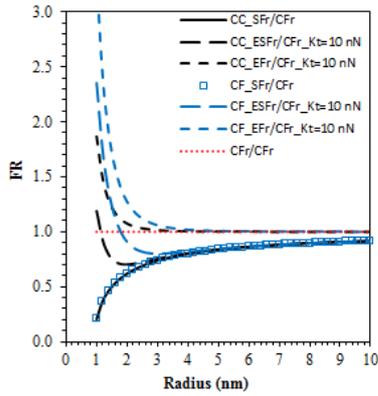


Figure 3. Variations of FR versus mode number

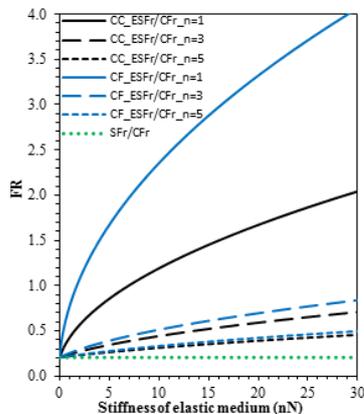


**Figure 4.** Variations of FR versus the nanorod radius

For thin nanorods the torsional frequencies is affected by both the elastic medium and the surface energy (see CC\_ESFr/CFr and CF\_ESFr/CFr curves). By increasing the nanorod radius, the slope of CC\_ESFr/CFr and CF\_ESFr/CFr curves decrease and the curves approach the CC\_SF/CFr and CF\_SF/CFr curves, then. This implies that only the surface energy affects the torsional frequencies of thick nanorods.

The upcoming investigation concerns with the variation of elastic medium and the subsequent effects on the torsional frequency ratios according to Figure 5. The curves of the figure have been plotted for  $L=10 \text{ nm}$  and  $R=1 \text{ nm}$ .

As expected, Figure 5 shows that as the elastic medium grows stiffer, the increasing effect intensifies, where the increasing effect is higher for nanorods with softer boundary conditions. Additionally, as the frequency number increases, the effect of the elastic medium decreases. To elaborate, at higher frequency numbers (frequency numbers more than three) the decreasing effect of the surface energy on the torsional frequencies becomes dominant for all values of the elastic medium stiffness.



**Figure 5.** Variations of FR versus stiffness of elastic medium.

Finally, since it is observed that the surface energy has a decreasing effect on torsional frequencies, it is expected that the surface energy makes nanorods unstable. If the frequency equations (Equations (23) and (24)) are non-positive, this observation will be established. The instability region of nanorods incorporating the surface energy is investigated here for two cases: considering the elastic medium, and ignoring the elastic medium.

❖ clamped-clamped and clamped-free nanorods ( $k_t=0$ )

$$\left\{ \begin{array}{l} \text{CC type : } \frac{(n\pi)^2 (GI_p)_{eq}}{L^2 (\rho I_p)_{eq}} \leq 0 \Rightarrow \\ (GI_p)_{eq} \leq 0 \Rightarrow GI_p + |G_0 - \tau_0| I_{ps} \leq 0 \Rightarrow R \leq \frac{4|G_0 - \tau_0|}{G} \\ \text{CF type : } \frac{((2n-1)\pi)^2 (GI_p)_{eq}}{(2L)^2 (\rho I_p)_{eq}} \leq 0 \Rightarrow \\ (GI_p)_{eq} \leq 0 \Rightarrow GI_p + |G_0 - \tau_0| I_{ps} \leq 0 \Rightarrow R \leq \frac{4|G_0 - \tau_0|}{G} \end{array} \right. \quad (26)$$

❖ clamped-clamped nanorod ( $k_t \neq 0$ )

$$\frac{(n\pi)^2 (GI_p)_{eq} + L^2 k_t}{L^2 (\rho I_p)_{eq}} \leq 0 \Rightarrow (n\pi)^2 (GI_p)_{eq} + L^2 k_t \leq 0 \quad (27)$$

$$\Rightarrow L \leq (n\pi) \sqrt{\frac{|G_0 - \tau_0| I_{ps} - GI_p}{k_t}}$$

❖ clamped-free nanorod ( $k_t \neq 0$ )

$$\frac{((2n-1)\pi)^2 (GI_p)_{eq} + (2L)^2 k_t}{(2L)^2 (\rho I_p)_{eq}} \leq 0 \Rightarrow \quad (28)$$

$$((2n-1)\pi)^2 (GI_p)_{eq} + (2L)^2 k_t \leq 0 \Rightarrow$$

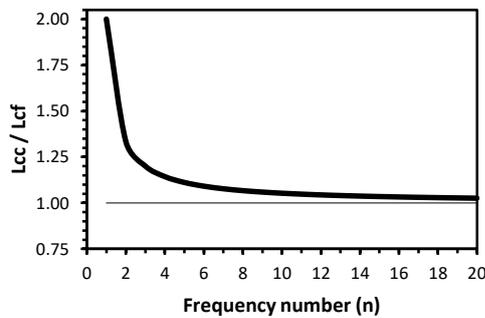
$$L \leq \left( \frac{(2n-1)\pi}{2} \right) \sqrt{\frac{|G_0 - \tau_0| I_{ps} - GI_p}{k_t}}$$

The surface and bulk mechanical properties are the determinants in torsional vibration of nanorods. However, based on the above equations, the surface and bulk density do not have any role in the instability of nanorods. The surface and bulk shear modulus and surface stress are the only effective determinants in the instability of nanorods. According to Equation (26), when the elastic medium effect is not considered, the nanorod's radius is the only geometrical parameter causing the nanorod to become unstable. In the presence of elastic medium effect (Equations (27) and (28)), the instability depends on the nanorod's radius and length, nonetheless. If we compare Equation (26) with Equations (27) and (28) it is concluded that the same instability situation is established for CF and CC boundary conditions when only the surface energy effect is considered while this is not the case when both the surface energy and the elastic medium effects are considered. Based on Equations (27) and (28), for a

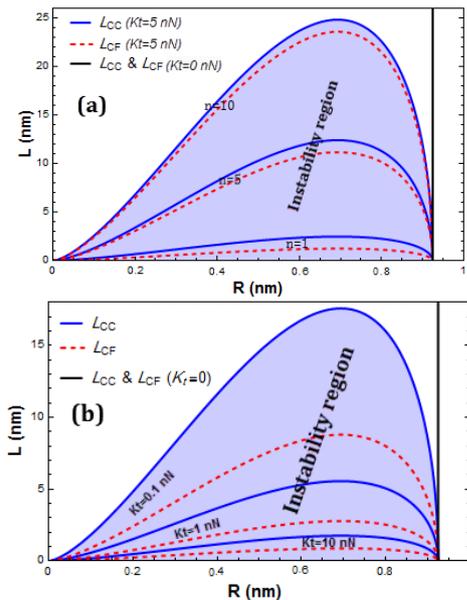
given nanorod radius and material properties the following ratio is established:

$$\frac{L_{CC}}{L_{CF}} = \frac{2n}{2n-1} \quad (29)$$

where,  $L_{CC}$  and  $L_{CF}$  are the critical nanorod length with CC and CF boundary conditions, respectively. The ratio indicates that by increasing the frequency number ( $n$ ), the difference between the critical lengths of nanorod with CC and CF end conditions becomes insignificant (see Figure 6). Finally, Figure 7 depicts Equations (26)-(28) graphically for better understanding of the relation between critical geometrical parameters (radius and length of nanorod) and the stiffness of the elastic medium as well as the frequency number. According to the figure, increasing the frequency number increases the instability region while it is the other way round for increasing the stiffness of elastic medium.



**Figure 6.** Variations of  $L_{CC}/L_{CF}$  versus the frequency number.



**Figure 7.** Instability region, a) for various frequency numbers, b) for various stiffnesses of elastic medium.

## 4. CONCLUSIONS

This study investigates for the first time the surface energy and the elastic medium effects on torsional vibrational behavior of embedded nanorods. The problem is formulated using the surface elasticity theory and the elastic medium is modeled as torsional springs attached to the nanorod. Then, the governing equation of motion is derived by using Hamilton's principle and torsional natural frequencies are analytically obtained for nanorods with clamped-clamped and clamped-free boundary conditions. The numerical results reveal that the surface energy has a decreasing effect on torsional frequencies while it is the other way round for the effect of elastic medium. Results show that the effect of the surface energy is independent from the length, the boundary condition type, the mode number and also the stiffness of the elastic medium. But its effect depends on the radius of the nanorod so that with increasing the radius of the nanorod the surface energy effect decreases. On the other hand, it is seen that the effect of the elastic medium depends on the nanorod length and radius, boundary condition type, mode number and also its stiffness. The results of this study is important because it is concluded that the effect of the surface energy on free torsional vibration of nanorods is different from that has been reported on the effect of the surface energy on free transverse vibration of nanorods (nanobeams). For instance, the surface energy can have destabilizing effect on free torsional vibration of nanorods while this is not the case for its effect on the transverse vibration of nanorods. So it is important to separately and specifically investigate the torsional behavior of nanorods in presence of the surface energy effects.

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## Surface Energy and Elastic Medium Effects on Torsional Vibrational Behavior of Embedded Nanorods

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در این مقاله، اثرات انرژی سطح و محیط الاستیک بر رفتار ارتعاشات آزاد پیچشی نانومیله‌ها مطالعه شده است. به منظور در نظر گرفتن تاثیر انرژی سطح، از تئوری الاستیسیته سطحی استفاده شده است و محیط الاستیک بصورت فنرهای پیچشی متصل به نانومیله مدل شده است. در گام بعدی، با استفاده از اصل همیلتون معادله حرکت و شرایط مرزی استخراج شده است. سپس فرکانس‌های طبیعی با استفاده از روش تحلیلی استخراج شده و تاثیر پارامترهای مختلف بر روی فرکانس‌های پیچشی مطالعه شده است. نتایج این پژوهش نشان می‌دهد که انرژی سطح می‌تواند سبب ناپایداری نانومیله شود که این ناپایداری بستگی به ابعاد و شماره فرکانس نانومیله دارد. نتایج این پژوهش می‌تواند در طراحی سیستم‌های نانوالکترومکانیکی مانند شافت‌های محرک مفید باشد.

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