



## Modelling and Test Verification of Suspension Optimal Damping Ratio for Electric Vehicles Considering Occupant-cushion and In-wheel Motor Effects

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### ABSTRACT

The damping ratio of chassis suspension is a key parameter for damping matching of in-wheel motor vehicles (IWMVs). Because the motor is attached to the driving wheel, the initial design method of the damping ratio for traditional cars is not entirely suitable for IWMVs. This paper proposes an innovative initial design method of the damping ratio for IWMVs. Firstly, a traveling vibration model of occupant-vehicle-road (OVR) for IWMVs is established. The model involves the occupant, cushion, suspension, in-wheel motor, road, and running speed. Secondly, on the basis of the model, using a special form of infinite integral, a mathematical expression of the occupant root-mean-square (RMS) acceleration is derived. Thirdly, based on the RMS optimization criterion for ride comfort, an 8 order polynomial equation about the suspension optimal damping ratio is deduced. Subsequently, through factors analysis, the change principles of the optimal damping ratio versus vehicle parameters are unveiled. Finally, the reliability of the optimal damping ratio is validated by test. The relative deviation of the calculated optimal damping ratio and the tested damping ratio is 5.4%. The results show that the proposed optimal damping ratio can effectively guide the suspension damping matching for IWMVs.

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### 1. INTRODUCTION

For in-wheel motor vehicles (IWMVs), the motor is integrated into the driving wheel and the traditional transmission system is omitted, such as the transmission shaft and the differential mechanism [1, 2]. This kind of high integration makes them have obvious technical advantages in the aspects of the components space layout, energy saving, and environmental protection, etc [3, 4]. However, due to the increase of unsprung mass, the ride comfort of IWMVs is often degraded [5].

The damping matching of chassis suspension system is critical to the improvement of ride comfort for IWMVs [6-8]. For example, Shao et al. researched about the dynamic damping of an active suspension of an IWMV [9]. The damping ratio is a key parameter of suspension system [10]. The design of the damping ratio has an important influence on ride comfort [11]. At present, for IWMVs, the damping matching of suspension system mainly refers to the damping

matching theory for traditional cars [12]. The traditional method is as follows: firstly, according to engineering experience and subjective judgment, using the feasible design range of the damping ratio (0.2~0.4) for traditional cars, an initial value of the damping ratio is selected; then, based on the initial value, a damper considered good is determined by trial and error or optimization algorithm. A good initial value as the starting point of the exact optimal matching can reduce the workload and the design difficulty, so that it can greatly shorten the whole design cycle of suspension system.

The shortcomings of the above traditional method are as follows: (1) the initial value selection of the damping ratio relies on the design experiences to a great degree, which has a great blindness and subjectivity, therefore, it is not easy to guarantee the whole design cycle of suspension system; (2) because the motor is attached to the driving wheel, the feasible design range of the damping ratio for traditional cars is not entirely suitable for IWMVs; (3) the feasible design range does not involve the effects of occupant-cushion system,

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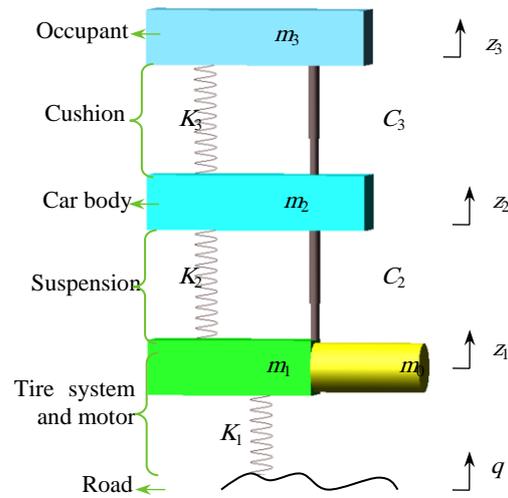
although this system has an important influence on the suspension damping matching [12, 13]. Therefore, it is difficult to provide a good initial design value of the damping ratio for IWMVs by the traditional method, which makes the suspension design cycle longer and the cost higher. In addition, due to the vehicle model complexity and the convergence ability constraints of the optimization algorithm [14, 15], the accurate design of suspension damping is also dependent on the initial design value and the feasible design range of the damping ratio. To effectively guide the suspension damping matching for IWMVs, a reliable initial design method of the damping ratio for suspension system should be created.

The main aim of this paper is to provide a practical and reliable initial design method of the suspension damping ratio for IWMVs. In Section 2, a travelling vibration model of occupant-vehicle-road (OVR) for IWMVs is established. In Section 3, using a special form of infinite integral, a mathematical expression of the occupant RMS acceleration is derived. Moreover, based on the RMS optimization criterion, an 8 order polynomial equation about the optimal damping ratio of suspension system is deduced. In Section 4, through factors analysis, the change principles of the optimal damping ratio versus vehicle parameters are unveiled. In Section 5, the reliability of the optimal damping ratio is validated by test. In Section 6, some useful conclusions are given.

**2. TRAVELLING VIBRATION MODEL OF OVR FOR IWMVS**

The vibration model for IWMVs is the base of suspension damping matching and comfort analysis. It is difficult to carry out dynamic analysis based on complex models in the initial design stage, because of a large number of unknown vehicle parameters [16]. Although the simplified model is different from the actual situation, its analysis results are close to the actual situation to a great extent. The simplified model has important reference value for the solution of some actual problems [17, 18]. Therefore, simplified vehicle models can be used to provide a simple and feasible method for the initial design of the suspension damping ratio for IWMVs. Considering the occupant-cushion system and motor system, a simplified traveling vibration model of OVR for IWMVs is established, as shown in Figure 1.

In Figure 1, the cushion is characterized as a shock absorber with damping  $C_3$  and a spring with stiffness  $K_3$ ; the chassis suspension is simplified as a shock absorber with damping  $C_2$  and a spring with stiffness  $K_2$ ; the tire is characterized as a spring with stiffness  $K_1$ ;  $m_i, i=0,1,2,3$ , represents the in-wheel motor mass, the



**Figure 1.** The travelling vibration model of OVR for IWMVs

tire system mass, the car body mass, and the occupant mass, respectively;  $z_i, i=1,2,3$ , represents the vertical displacement of the corresponding mass and  $q$  represents the road displacement input.

The vibration equation of the model can be expressed as:

$$M\ddot{z} + C\dot{z} + Kz = K_1q \tag{1}$$

where,

$$M = \begin{bmatrix} m_3 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_1 \end{bmatrix}, \quad C = \begin{bmatrix} C_3 & -C_3 & 0 \\ -C_3 & C_3 + C_2 & -C_2 \\ 0 & -C_2 & C_2 \end{bmatrix},$$

$$K = \begin{bmatrix} K_3 & -K_3 & 0 \\ -K_3 & K_3 + K_2 & -K_2 \\ 0 & -K_2 & K_2 + K_1 \end{bmatrix}, \quad z = \begin{bmatrix} z_3 \\ z_2 \\ z_1 \end{bmatrix}, \quad q = \begin{bmatrix} 0 \\ 0 \\ q \end{bmatrix}.$$

The power spectral density (PSD) of the road surface displacement input  $q$  can be expressed as [19]:

$$G_q(\omega) = 4\pi^2 G_q(n_0) n_0^2 \frac{v}{\omega^2} \tag{2}$$

where,  $n_0=0.1m^{-1}$ ,  $v$  is the running speed of IWMVs,  $G_q(n_0)$  is the road irregularity coefficient and  $\omega$  is the excitation frequency.

The road surface velocity spectrum can be expressed as:

$$G_{\dot{q}}(\omega) = 4\pi^2 G_q(n_0) n_0^2 v \tag{3}$$

**3. SUSPENSION OPTIMAL DAMPING RATIO FOR IWMVS**

**3. 1. Mathematical Expression of the Occupant RMS Acceleration** The occupant RMS acceleration

is often used for comfort evaluation. In this section, based on the traveling vibration model of OVR for IWMVs, a mathematical expression of the occupant RMS acceleration is derived.

From Equation (1), the transfer matrix  $\mathbf{H}(j\omega)$  of the displacement vector  $\mathbf{z}$  to the road surface input  $q$  can be expressed as:

$$\mathbf{H}(j\omega) = [\mathbf{K} - \omega^2 \mathbf{M} + j\omega \mathbf{C}]^{-1} \mathbf{K}_1 \quad (4)$$

For the convenience in analyzing the vibration characteristics of IWMVs, the following variables are introduced:

$$r_2 = \frac{m_3}{m_2}, \quad r_1 = \frac{m_2}{m_1 + m_0}, \quad \omega_3 = \sqrt{\frac{K_3}{m_3}}, \quad \omega_2 = \sqrt{\frac{K_2}{m_2}},$$

$$\omega_1 = \sqrt{\frac{K_1}{m_1 + m_0}}; \quad \xi_3 = \frac{C_3}{2\sqrt{K_3 m_3}}, \quad \xi_2 = \frac{C_2}{2\sqrt{K_2 m_2}}.$$

where,  $\xi_2$  and  $\xi_3$  are the damping ratios of the suspension system and the occupant-cushion system, respectively.

For the sake of convenience, the following notations are given:

$$\begin{cases} \beta_1 = 2[\xi_2 \omega_2 (1 + r_1) + \xi_3 \omega_3 (1 + r_2)] \\ \beta_2 = \omega_1^2 + \omega_2^2 (1 + r_2) + \omega_3^2 (1 + r_1) + \\ 4\xi_2 \xi_3 \omega_2 \omega_3 (1 + r_1 + r_2) \\ \beta_3 = 2[\xi_2 \omega_1^2 \omega_2 + \xi_3 \omega_1^2 \omega_3 (1 + r_2) + \\ \xi_2 \omega_2 \omega_3^2 (1 + r_1 + r_2) + \xi_3 \omega_3^2 \omega_2 (1 + r_1 + r_2)] \\ \beta_4 = \omega_1^2 \omega_2^2 + \omega_2^2 \omega_3^2 (1 + r_2) + \\ \omega_2^2 \omega_3^2 (1 + r_1 + r_2) + 4\xi_2 \xi_3 \omega_1^2 \omega_2 \omega_3 \end{cases} \quad (5)$$

The transfer function of the occupant acceleration to the road excitation velocity can be expressed as:

$$H(j\omega) = \frac{\omega \omega_1^2 \omega_2 \omega_3 (\omega_2 \omega_3 - 4\xi_2 \xi_3 \omega^2) j - 2(\omega_2 \xi_3 + \omega_3 \xi_2) \omega}{-\omega^6 + \beta_1 \omega^5 j + \beta_2 \omega^4 - \beta_3 \omega^3 j - \beta_4 \omega^2 + 2\omega_1^2 \omega_2 \omega_3 (\omega_3 \xi_2 + \omega_2 \xi_3) \omega j + \omega_1^2 \omega_2^2 \omega_3^2} \quad (6)$$

According to reference [20], the occupant mean square acceleration can be expressed as:

$$\sigma_a^2 = \frac{1}{2\pi} \int_0^{+\infty} |H(j\omega)|^2 G_q(\omega) d\omega \quad (7)$$

Substituting Equation (3) into Equation (7), one obtains the following equation:

$$\sigma_a^2 = 2\pi G_q(n_0) n_0^2 v \int_0^{+\infty} |H(j\omega)|^2 d\omega \quad (8)$$

Substituting Equation (6) into Equation (8), then, Equation (8) can be further expressed as [21]:

$$\sigma_a = \pi n_0 \sqrt{\frac{G_q(n_0) v \lambda_2}{\lambda_1}} \quad (9)$$

where,  $\lambda_2 = h_2(\beta_1 \beta_3 \beta_6 - \beta_1 \beta_4 \beta_5 + \beta_5^2) + h_3(\beta_1^2 \beta_6 + \beta_3 \beta_5 - \beta_1 \beta_2 \beta_3) + h_4(\beta_1^2 \beta_4 - \beta_1 \beta_2 \beta_3 - \beta_1 \beta_5 + \beta_5^2)$

$$\lambda_1 = \beta_5^3 + 3\beta_1 \beta_3 \beta_5 \beta_6 - 2\beta_1 \beta_4 \beta_5^2 - \beta_2 \beta_3 \beta_5^2 - \beta_3^3 \beta_6 + \beta_3^2 \beta_4 \beta_5 + \beta_1 \beta_2 \beta_3^2 \beta_6 + \beta_1^3 \beta_6^2 - \beta_1^2 \beta_3 \beta_4 \beta_6 + \beta_1^2 \beta_4^2 \beta_5 - \beta_1 \beta_2 \beta_3 \beta_4 \beta_5 + \beta_1 \beta_2^2 \beta_5^2 - 2\beta_1^2 \beta_2 \beta_3 \beta_6$$

$$\beta_5 = 2\omega_1^2 \omega_2 \omega_3 (\omega_3 \xi_2 + \omega_2 \xi_3), \quad h_3 = 4(\omega_2^2 \xi_3^2 + \omega_3^2 \xi_2^2) \omega_1^4 \omega_2^2 \omega_3^2, \\ \beta_6 = \omega_1^2 \omega_2^2 \omega_3^2, \quad h_2 = 16\xi_2^2 \xi_3^2 \omega_1^4 \omega_2^2 \omega_3^2, \quad h_4 = \omega_1^4 \omega_2^4 \omega_3^4.$$

### 3. 2. Solution of suspension optimal damping ratio for IWMVs

Due to a large number of unknown parameters in the initial phase of vehicle design, designers often only consider the comfort index — the occupant RMS acceleration. Thus, in the following, based on Equation (9), the initial design method of the damping ratio of suspension system for IWMVs is discussed.

Taking the partial derivative of Equation (9) with respect to the damping ratio  $\xi_2$ , setting the derivative to zero:

$$\frac{\partial \sigma_a}{\partial \xi_2} = 0 \quad (10)$$

Substituting Equation (9) into Equation (10), one obtains the following equation:

$$\frac{\partial}{\partial \xi_2} \left( \pi n_0 \sqrt{\frac{G_q(n_0) v \lambda_2}{\lambda_1}} \right) = 0 \quad (11)$$

Through the identity transformation, Equation (11) can be expressed as:

$$\frac{\partial}{\partial \xi_2} \left( \frac{\lambda_2}{\lambda_1} \right) = 0 \quad (12)$$

According to Equation (12), one obtains the following equation about  $\xi_2$ :

$$P_8 \xi_2^8 + P_7 \xi_2^7 + P_6 \xi_2^6 + P_5 \xi_2^5 + P_4 \xi_2^4 + P_3 \xi_2^3 + P_2 \xi_2^2 + P_1 \xi_2 + P_0 = 0 \quad (13)$$

where,  $P_i$  is determined by vehicle parameters  $K_1, K_2, K_3, m_0, m_1, m_2, C_3$ , and the occupant mass  $m_3$ .

Equation (13) can be abbreviated as:

$$F(\xi_2) = 0 \quad (14)$$

The real root of Equation (14) is the optimal damping ratio of the suspension system. The optimal damping ratio  $\xi_{2op}$  can be expressed as:

$$\xi_2 = \xi_{2op}(K_1, K_2, K_3, m_0, m_1, m_2, m_3, C_3) \quad (15)$$

The optimal damping ratio  $\xi_{2op}$  of suspension system for IWMVs is determined by vehicle parameters  $K_1, K_2, K_3, m_0, m_1, m_2, C_3$ , and the occupant mass  $m_3$ .

For instance, the basic parameters of an IWMV are as follow:  $m_1=15.5$  kg,  $m_2=202.0$  kg,  $K_1=200.0$  kN/m,  $K_2=22.4$  kN/m,  $K_3=24.0$  kN/m,  $C_3=1000$  Ns/m;  $m_3=65.0$  kg. The most commonly running road condition of the vehicle is as follows:  $v=60.0$  km/h, B-class road. Under different values of the in-wheel motor mass  $m_0=0, 12, 24, 36,$  and  $48$  kg, the damping ratio  $\zeta_{2op}$  and the corresponding occupant RMS acceleration  $\sigma_a$  are shown in Table 1. Where,  $m_0=0$  represents the condition of the traditional car. Under different values of the in-wheel motor mass  $m_0=0, 12, 24, 36,$  and  $48$  kg, the relationship between the occupant RMS acceleration  $\sigma_a$  and the damping ratio  $\zeta_2$  is shown in Figure 2. From Figure 2, it can be seen that with the increase of the damping ratio  $\zeta_2$ , the occupant RMS acceleration  $\sigma_a$  decreases firstly and then increases. Moreover, when the damping ratio  $\zeta_2$  equals  $\zeta_{2op}$ ,  $\sigma_a$  reaches its minimum.

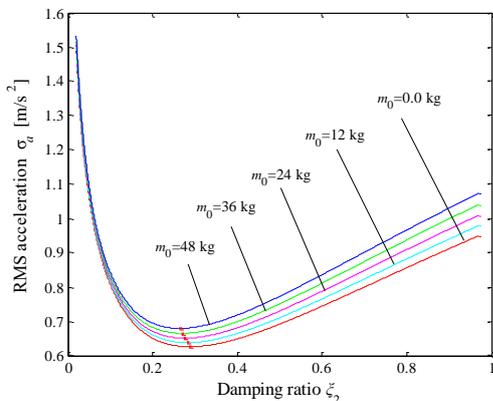
**4. FACTORS ANALYSIS OF SUSPENSION OPTIMAL DAMPING RATIO FOR IWMVS**

The thorough factors analysis of  $\zeta_{2op}$  was conducted. The basic parameters and the possible values range of an IWMV are shown in Table 2 [6].

**4. 1. Effect of Cushion Damping** The curve of the optimal damping ratio  $\zeta_{2op}$  versus cushion damping  $C_3$ , is shown in Figure 3.

**TABLE 1.** The optimal damping ratio  $\zeta_{2op}$  and the corresponding acceleration  $\sigma_a$

Motor mass $m_0$ (kg)	Optimal ratio $\zeta_{2op}$	Acceleration $\sigma_a$ (m/s <sup>2</sup> )
0	0.2895	0.6261
12	0.2834	0.6382
24	0.2772	0.6511
36	0.2712	0.6649
48	0.2653	0.6797



**Figure 2.** The occupant RMS acceleration  $\sigma_a$  versus the damping ratio  $\zeta_2$

**TABLE 2.** The basic parameters and the possible value ranges of an IWMV

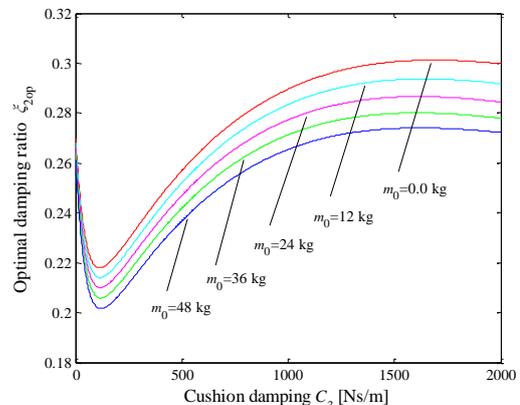
Vehicle parameter	Nominal value	Value range
$m_0$ (kg)	24.0	[0, 48]
$m_1$ (kg)	15.5	[10, 30]
$m_2$ (kg)	65.0	[50, 100]
$m_3$ (kg)	202.0	[150, 300]
$K_1$ (kN/m)	200.0	[100, 300]
$K_2$ (kN/m)	22.4	[10, 40]
$K_3$ (kN/m)	24.0	[10, 100]
$C_3$ (Ns/m)	1000	[0, 2000]

From Figure 3, it can be seen that with the increase of the cushion damping  $C_3$ , the optimal damping ratio  $\zeta_{2op}$  decreases firstly, then increases, finally trends to gentle. Moreover, the larger the value of the in-wheel motor mass  $m_0$  is, the smaller the design value of the optimal damping ratio  $\zeta_{2op}$  needs.

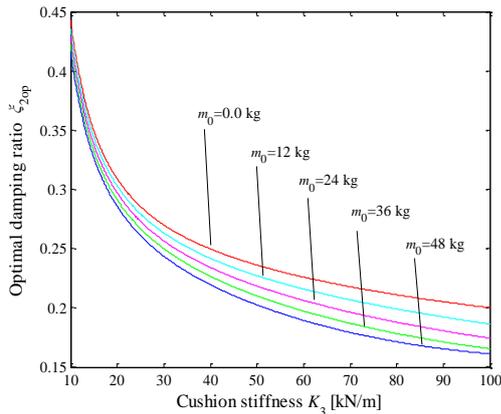
**4. 2. Effects of stiffness parameters** The stiffness parameters of the OVR model for IWMVs include the cushion stiffness  $K_3$ , the suspension stiffness  $K_2$ , and the tire stiffness  $K_1$ . Figures 4 to 6 depict the relationship of the optimal ratio  $\zeta_{2op}$  and the stiffness parameters.

From Figure 4, it can be seen that the optimal damping ratio  $\zeta_{2op}$  decreases with the increase of cushion stiffness  $K_3$ . When the cushion stiffness  $K_3$  belongs to the range [10, 100] kN/m, the range of the optimal damping ratio  $\zeta_{2op}$  is about [0.15, 0.45], rather than the damping ratio range [0.2, 0.4] for traditional cars.

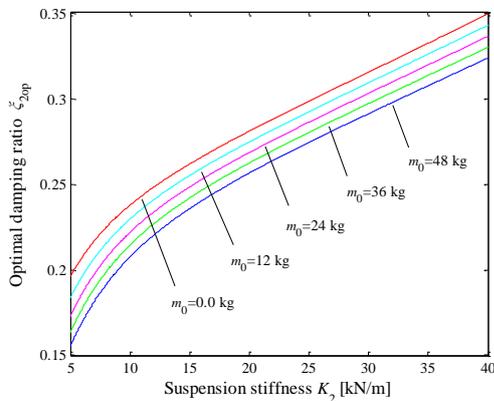
Figure 5 illustrates that the optimal damping ratio  $\zeta_{2op}$  increases with the increase of suspension stiffness  $K_2$ . Moreover, when the suspension stiffness  $K_2$  is less than 6.5 kN/m and the in-wheel motor mass  $m_0$  is larger than 12 kg, the optimal damping ratio  $\zeta_{2op}$  is less than 0.2.



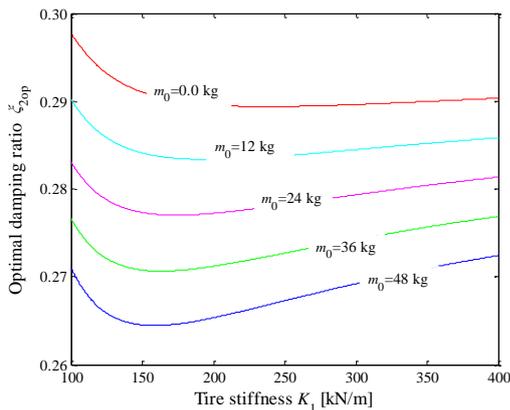
**Figure 3.** The optimal ratio  $\zeta_{2op}$  versus the cushion damping  $C_3$



**Figure 4.** The optimal damping ratio  $\zeta_{2op}$  versus cushion stiffness  $K_3$



**Figure 5.** The optimal damping ratio  $\zeta_{2op}$  versus suspension stiffness  $K_2$



**Figure 6.** The optimal damping ratio  $\zeta_{2op}$  versus tire stiffness  $K_1$

That is to say, the value of the optimal damping ratio  $\zeta_{2op}$  is out of [0.2, 0.4]. In such conditions, if the damping ratio range [0.2, 0.4] for the traditional cars is referred for IWMVs, it will be hard to realize optimal damping matching of suspension system. Figure 6

shows that the optimal damping ratio  $\zeta_{2op}$  decreases initially and then increases with the increase of tire stiffness  $K_1$ . The smaller the in-wheel motor mass  $m_0$  is, the more gentle the change trend becomes.

**4. 3. Effects of Mass Parameters**

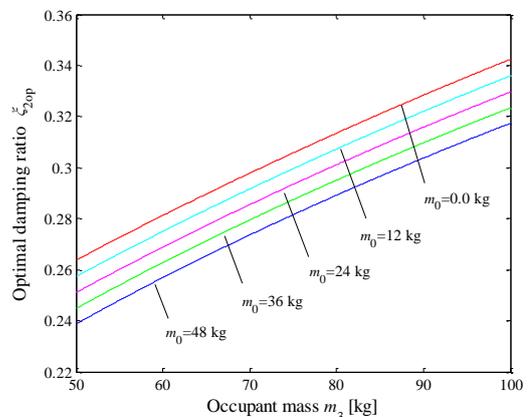
The mass parameters of the OVR model for IWMVs include the occupant mass  $m_3$ , the car body mass  $m_2$ , the tire system mass  $m_1$ , and the in-wheel motor mass  $m_0$ . Figures 7 to 9 depict the relationship of the optimal ratio  $\zeta_{2op}$  and the mass parameters.

Figure 7 depicts that the optimal damping ratio  $\zeta_{2op}$  increases approximately linearly as the occupant mass  $m_3$  increases. Figure 8 illustrates that the optimal damping ratio  $\zeta_{2op}$  is nonlinearly related to the car body mass  $m_2$ . The optimal damping ratio  $\zeta_{2op}$  decreases with the increase of the car body mass  $m_2$ . Figure 9 shows that there is almost a linearly descending relationship between the optimal damping ratio  $\zeta_{2op}$  and the tire system mass  $m_1$ . Figures 7 to 9 all prove that the larger the value of the in-wheel motor mass  $m_0$  is, the smaller the design value of the optimal damping ratio  $\zeta_{2op}$  needs.

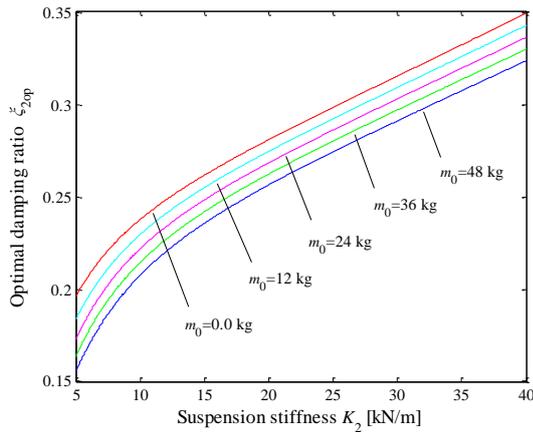
Totally, the occupant-cushion and the in-wheel motor system have important effects. The suspension optimal damping ratio  $\zeta_{2op}$  is irrelevant to driving road conditions. Moreover, the suspension optimal damping ratio  $\zeta_{2op}$  range [0.15, 0.45] for IWMVs is recommended, rather than the damping ratio range [0.2, 0.4] for traditional cars. The variation laws of the suspension optimal damping ratio  $\zeta_{2op}$  provide a theoretical basis for the initial design of suspension damping ratio  $\zeta_2$  for IWMVs.

**5. TEST VERIFICATION OF SUSPENSION OPTIMAL DAMPING RATIO FOR IWMVS**

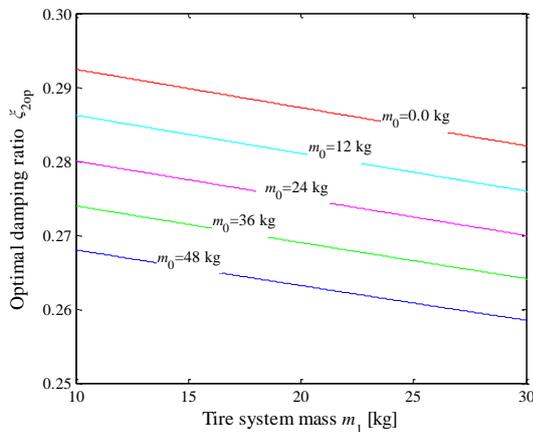
To verify the reliability of the suspension optimal damping ratio  $\zeta_{2op}$ , a comparison of the calculated  $\zeta_{2op}$  and the tested  $\zeta_2$  should be conducted.



**Figure 7.** The optimal damping ratio  $\zeta_{2op}$  versus occupant mass  $m_3$



**Figure 8.** The optimal damping ratio  $\xi_{2op}$  versus car body mass  $m_2$



**Figure 9.** The optimal damping ratio  $\xi_{2op}$  versus tire system mass  $m_1$

The calculated  $\xi_{2op}$  is regarded as the initial design value, while the tested  $\xi_2$  is regarded as the final precise design value of suspension damping ratio  $\xi_2$ .

**5. 1. Tested IWMV and Suspension Optimal Damping Ratio**

The driving mode of the tested vehicle is rear-wheel-drive using two in-wheel motors. The configuration and parameters of the tested IWMV are as follows:

Length×Width×Height (4501 mm×1704 mm×1469 mm), wheelbase 2604 mm, curb-weight 1115 kg; MacPherson front independent suspension, torsion beam rear suspension, two seats per row (two rows, no seat suspension), rigid polyurethane foam cushion with equivalent damping 1400 Ns/m and stiffness 28 kN/m. Under the full load condition, the front axle load 540 kg, the rear axle load 575 kg, ① for one front suspension system: the suspension vertical stiffness 12.3 kN/m, the tire vertical stiffness 189 kN/m, the tire system mass 24.0 kg; ② for one rear suspension system: the

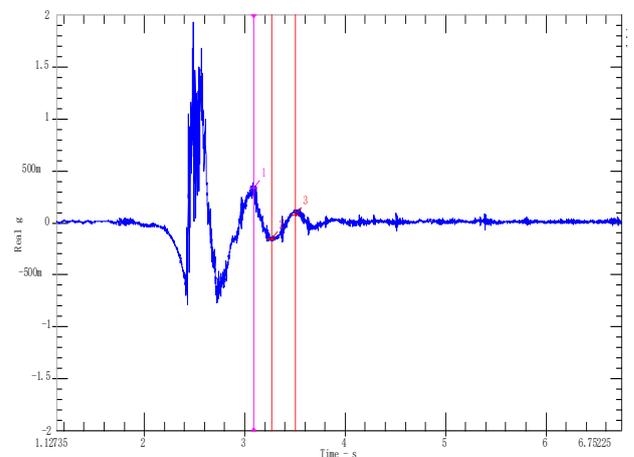
suspension vertical stiffness 19.3 kN/m, the tire vertical stiffness 210 kN/m, the tire system mass 20.5 kg, the in-wheel motor mass 17.0 kg.

Because the motor is attached to the rear driving wheel for the tested IWMV, we took the rear suspension system as an example to calculate the suspension optimal damping  $\xi_{2op}$ . According to the tested IWMV parameters, solving Equation (14), the calculated optimal damping  $\xi_{2op}$  is 0.25. The value 0.25 is regarded as the initial design value of the rear suspension damping ratio.

**5. 2. Measurement of Suspension Damping Ratio**

According to the Chinese standard GB/T 4783-1984, the measurement of the rear suspension damping ratio for the tested IWMV is carried out. The used equipment is German ICM data acquisition system.

According to the standard GB/T 4783-1984, the test is carried out under the full load condition. The specific practice is as follows: firstly, dismantled the seat cushion and two dampers in two front suspension systems; then, installed one DH105E acceleration sensor on the rear chassis frame; thirdly, put each rear wheel on a semi trapezoidal bump with height 60 mm, respectively; subsequently, pushed the vehicle off the bump and ensured that the two wheels were able to fall to the ground at the same time as much as possible; meanwhile, used the ICM data acquisition system to collect the free attenuation time history of the frame vertical vibration acceleration for 7 s. In order to ensure the reliability of the collected data, the test was repeated three times. Given space limitations and the similarity of each time history, the collected data for the first test is provided, as shown in Figure 10. According to the time histories of three tests, the adjacent three peak values and the corresponding time points for each test were extracted as shown in Table 3.



**Figure 10.** The free attenuation time history of the frame vertical acceleration for the first test

**TABLE 3.** The adjacent three peak values and the corresponding time points

Test		Point 1	Point 2	Point 3
First	Time (s)	$t_1=3.0970$	$t_2=3.2760$	$t_3=3.5120$
	Peak ( $10^{-3}g$ )	$A_1=337.27$	$A_2=-149.89$	$A_3=75.608$
Second	Time (s)	$t_1=3.0280$	$t_2=3.2070$	$t_3=3.4430$
	Peak ( $10^{-3}g$ )	$A_1=335.89$	$A_2=-148.72$	$A_3=75.218$
Third	Time (s)	$t_1=3.0460$	$t_2=3.2280$	$t_3=3.4640$
	Peak ( $10^{-3}g$ )	$A_1=338.02$	$A_2=-150.63$	$A_3=76.112$

From Table 3, using the time-history method, it can be obtained that the tested values of the rear suspension damping ratio for three tests are 0.238, 0.239, and 0.237, respectively. The mean value of the tested damping ratio is 0.24 which can be regarded as the final precise design value of suspension damping ratio  $\zeta_2$ , where, the formula of the damping ratio for the time-history method for our calculation is as follows:

$$\zeta = \frac{1}{\sqrt{1 + \frac{\pi^2}{\ln^2[(A_1 - A_2)/(A_3 - A_2)]}}} \quad (16)$$

**5. 3. Comparison of the Initial Value and the Precise Value for  $\zeta_2$**  A comparison of the damping ratio between initial design value (the calculated  $\zeta_{2op}$ ) and the final precise design value (the tested  $\zeta_2$ ) is shown in Table 4.

From Table 4, it can be seen that the calculated optimal damping ratio  $\zeta_{2op}$  is 0.25, the final precise design value is 0.24 and their absolute deviation and relative deviation are 0.01 and 5.4%, respectively.

The results show that the optimal damping ratio  $\zeta_{2op}$  is very close to the final precise design value. Thus, the optimal damping ratio  $\zeta_{2op}$  can provide a good initial design value for the accurate design stage of IWMVs. Compared with the traditional method, the proposed method can effectively avoid the blindness and subjectivity of the damping ratio selection among 0.2~0.4, so that it can reduce the workload and the design difficulty.

**TABLE 4.** A comparison of the calculated  $\zeta_{2op}$  and the final precise design value

Final precise design value	Initial design value	
	Optimal damping ratio $\zeta_{2op}$	Traditional method
0.24	0.25	0.2~0.4

## 6. CONCLUSION

A traveling vibration model of OVR for IWMVs is created, which considers the effects of the occupant-

cushion system and the in-wheel motor system. Based on the model, using a special form of infinite integral, a mathematical expression of the occupant RMS acceleration is derived. Using the RMS optimization criterion for ride comfort, an 8 order polynomial equation about the suspension optimal damping ratio is deduced. Finally, the reliability of the optimal damping ratio is validated. Some instructive conclusions and proposals are as follow:

(1) The optimal damping ratio  $\zeta_{2op}$  of suspension system is determined by vehicle parameters  $K_1, K_2, K_3, m_0, m_1, m_2, C_3$ , and occupant mass  $m_3$ .

(2) The optimal damping ratio  $\zeta_{2op}$  increases approximately linearly as the occupant mass  $m_3$  increases. The larger the value of the in-wheel motor mass  $m_0$  is, the smaller the design value of the optimal damping ratio  $\zeta_{2op}$  needs.

(3) The optimal damping ratio  $\zeta_{2op}$  decreases with the increase of cushion stiffness  $K_3$ . With the increase of the cushion damping  $C_3$ , the optimal damping ratio  $\zeta_{2op}$  decreases firstly, then increases, finally trends to gentle.

(4) The optimal damping ratio  $\zeta_{2op}$  range [0.15, 0.45] for IWMVs is recommended, rather than the damping ratio range [0.2, 0.4] for traditional cars. That is to say, the value of the optimal damping ratio  $\zeta_{2op}$  for IWMVs may be out of the range [0.2, 0.4].

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## Modelling and Test Verification of Suspension Optimal Damping Ratio for Electric Vehicles Considering Occupant-cushion and In-wheel Motor Effects

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نسبت میرایی تعلیق شاسی یک پارامتر کلیدی برای تطبیق میرایی موتورهای چرخ دنده ای (IWMV) می باشد. از آنجا که موتور به چرخ متصل است، روش طراحی اولیه نسبت میرایی اتومبیل های سنتی به طور کامل برای IWMV مناسب نیست. این مقاله روشی برای طراحی اولیه ابتکاری نسبت میرایی برای IWMV ها ارائه می دهد. در ابتدا، یک مدل ارتعاشات جابجایی وسیله نقلیه جاده ای (OVR) برای IWMV ایجاد شده است. مدل شامل ساکنان، کوسن، تعلیق، موتور درایو، جاده و سرعت حرکت است. ثانياً، بر اساس مدل، با استفاده از یک فرم خاص انتگرال بی نهایت، عبارت ریاضی متوسط مربعات ریشه (RMS) شتاب حاصل می شود. سوم، بر اساس معیار بهینه سازی RMS برای راحتی سواری، یک معادله چند جمله ای مرتبه ۸ در مورد نسبت خمش مطلوب تعلیق محاسبه می شود. پس از آن، از طریق تجزیه و تحلیل عوامل، اصول تغییر نسبت میرایی بهینه نسبت به پارامترهای خودرو نمایش داده می شود. در نهایت، قابلیت اطمینان نسبت میرایی مطلوب با آزمون تایید می شود. انحراف نسبی نسبت به محدوده مطلوب محاسبه شده و نسبت میرایی آزمایشی ۵/۴ درصد است. نتایج نشان می دهد که نسبت میرایی بهینه مطلوب می تواند راهنمای خوبی برای تطبیق میرایی تعلیق برای IWMV ها باشد.

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