IJE TRANSACTIONS A: Basics Vol. 28, No. 4, (April 2015) 599-607

International Journal of Engineering

Journal Homepage: www.ije.ir

Analysis of Magneto-hydrodynamics Jeffery-Hamel Flow with Nanoparticles by Hermite-Padé Approximation

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PAPER INFO

Paper history:
Received 05January 2014
Received in revised form 27 November2014
Accepted 13March2015

Keywords:
Jeffery-Hamel Flow
Magneto-hydrodynamic
Nanofluid
Dominating Singularity
Hermite- Pade Approximation

ABSTRACT

The combined effects of nanoparticles and magnetic field on the nonlinear Jeffery-Hamel flow are analyzed in the present study. The basic governing equations are solved into series solution using a semi-numerical analytical technique called Hermite- Padé approximation. The velocity profiles are presented in divergent channel for various values of nanoparticles solid volume fraction, Hartmann number, Reynolds number and channel angle. The dominating singularity behavior of the problem is analysed numerically and graphically. The critical relationship between the parameters is studied to observe the instability of the problem for nanofluid.

***doi**: 10.5829/idosi.ije.2015.28.04a.15*

1. INTRODUCTION

The study of flows in convergent-divergent channel is very important due to its industrial, aerospace, chemical, civil, environmental, mechanical and biomechanical engineering applications. Various applications of this type of mathematical model are to understand the flow of rivers and canals and the blood flow in the human body. Jeffery [1] and Hamel [2] first studied the two-dimensional steady motion of a viscous fluid through convergent-divergent channels which is called classical Jeffery-Hamel flow in fluid dynamics. Later, this problem has been extensively studied by various researchers. A survey of information on this problem can be found in [3]. The theory of Magneto-hydrodynamics (MHD) is inducing current in a moving conductive fluid in the presence of magnetic field; such induced current results force on ions of the conductive fluid. The theoretical study of MHD channel has been a subject of great interest due to its extensive applications in designing cooling systems with liquid metals, MHD generators, accelerators, pumps, and flow meters [4, 5].

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The small disturbance stability of MHD plane-Poiseuille flow was investigated by Makinde and Motsa [6] and Makinde [7]. Their results showed that magnetic field had stabilizing effects on the flow. Damping and controlling of electrically conducting fluid can be achieved by means of an electromagnetic body force (Lorentz force) produced by interaction of an applied magnetic field and an electric current that usually is externally supplied. Anwari et al. [8] studied the fundamental characteristics of linear Faraday MHD theoretically and numerically, for various loading configurations. Homsy et al. [9] emphasized on the idea that in such problems, the moving ions drag the bulk fluid with themselves, and such MHD system induces continued pumping of conductive fluid without any moving part.Apart from using numerical methods, the Jeffery-Hamel flow problem was solved by other techniques including the Homotopy analytical method (HAM),the Homotopy perturbation method (HPM), the Adomain decomposition method (ADM) and the spectral-Homotopy analysis method. Recently, the three analytical methods such as Homotopy analysis method, Homotopy perturbation method and Differential transformation method (DTM) were used by Joneidi et al. [10] to find the analytical

Please cite this article as: Md. S. Alam and M. A. H. Khan, Analysis of MHD Jeffery-Hamel Flow with Nanoparticles by Hermite-Pade Approximation, International Journal of Engineering (IJE), IJE TRANSACTIONS A: Basics Vol. 28, No. 4, (April 2015) 599-607

solution of Jeffery-Hamel flow. Moreover, the models on classical semi-analytical methods have experienced a revival, in connection with the scheme of new hybrid numerical-analytical techniques for nonlinear differential equations, such as Hermite– Padé approximation method, which demonstrated itself as a powerful benchmarking tool and a prospective substitute to traditional numerical techniques in various applications in science and engineering.

The classical Jeffery-Hamel problem was extended in Axford [11] to include the effects of external magnetic field on conducting fluid. Motsa et al. [12] found the solution of the nonlinear equation for the MHD Jeffery-Hamel problem by using novel hybrid spectral-homotopy analysis method. Moghimi et al. [13] also solved the Jeffery-Hamel flow problem by using the homotopy perturbation method. Taking into account the rising demands of modern technology, including chemical production, power station, and microelectronics, there is a need to develop new types of fluids that will be more effective in terms of heat exchange performance. The term 'nanofluid' was envisioned to describe a fluid in which nanometer-sized particles were suspended in conventional heat transfer basic fluids [14]. Rahmannezhad et al. [15] investigated the effects of a magnetic field on mixed convection of Al_2O_3-water nanofluid in a square lid-driven cavity. An experimental investigation was carried out to study mixed convection heat transfer from Al_2O_3-water nanofluid inside a vertical, W-shaped, copper-tube with uniform wall temperature in Rostamzadeh et al. [16]. The role of a convective surface in modelling with nanofluids has been investigated over a wedge by Rahman et al. [17]. They found that the surface convection significantly controls the rate of heat transfer in nanofluid and increasing volume fraction of nanoparticles to the base-fluid may not always increase the rate of heat transfer. Rahman et al. [18] investigated numerically the steady boundary layer flow and heat transfer characteristics of nanofluids using Buongiorno's model past a permeable exponentially shrinking/stretching surface with second order slip velocity.

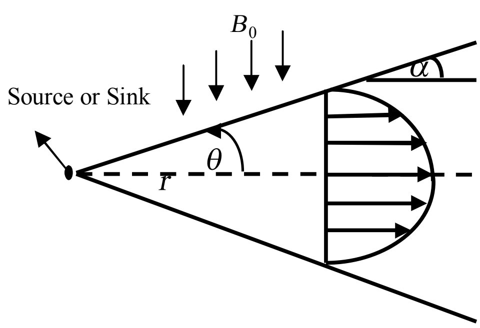


Figure 1. Geometry of the problem

Sheikholeslami et al. [19] studied the laminar nanofluid flow in a semi-porous channel using Homotopy Perturbation Method. Moreover, the effects of magnetic field and nanoparticles on the Jeffery-Hamel flow using a powerful analytical method called the Adomian decomposition method were studied by Sheikholeslami et al. [20].

The aim of this work is to apply the power series along with algebraic programming language MAPLE to find the approximate solutions into series of nonlinear differential equations governing the MHD Jeffery-Hamel flow with nanofluid. The series is analyzed to show a comparison between the Hermite–Padé approximation (HPA) and ADM results. The series is also investigated to show the velocity profiles with effect of nanoparticles volume fraction ϕ and Hartmann number *Ha*. The change in singularity graphs for channel angle α and flow Reynolds number *Re* by the effect of ϕ with the help of approximation method is an extension of Sheikholeslami et al. [20]. The critical relationship between the parameters in the flow using HPA is not addressed yet.

2. MATHEMATICAL FORMULATION

Consider a steady two-dimensional laminar incompressible flow of conducting viscous nanofluid from a source or sink between two channel walls intersect at an angle 2α in the axis of *z*. A cylindrical coordinate system (r,θ,z) is used and assume that the velocity is purely radial and depends on r and θ, so that there is no change in the flow parameter along the z-direction. It is presumed that there is a magnetic field acting in the vertical downward direction. The continuity equation, the Navier-Stokes equation and Maxwell's equation in reduced polar coordinates are [13].

$$\frac{\rho_{nf}}{r}\frac{\partial}{\partial r}(ru(r,\theta))=0, \tag{1}$$

$$\begin{aligned} u(r,\theta)\frac{\partial u(r,\theta)}{\partial r} &= -\frac{1}{\rho_{nf}}\frac{\partial P}{\partial r}+ \\ &\nu_{nf}\left(\frac{\partial^2 u(r,\theta)}{\partial r^2}+\frac{1}{r}\frac{\partial u(r,\theta)}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u(r,\theta)}{\partial \theta^2}-\frac{u(r,\theta)}{r^2}\right) \\ &-\frac{\sigma B_0^{\,2}}{\rho_{nf}r^2}u(r,\theta), \end{aligned} \tag{2}$$

$$\frac{1}{\rho_{nf}r}\frac{\partial P}{\partial \theta}-\frac{2\nu_{nf}}{r^2}\frac{\partial u(r,\theta)}{\partial \theta}=0, \tag{3}$$

where B_0 is the electromagnetic induction, σ the conductivity of the fluid, $u(r,\theta)$ the velocity along radial direction, P the fluid pressure. The effective density ρ_{nf}, the effective dynamic viscosity μ_{nf}, and the kinematic viscosity ν_{nf} of the nanofluid are given as [21]:

$$\rho_{nf} = \rho_f(1-\phi)+\rho_s\phi,\ \ \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}},\ \ \nu_{nf} = \frac{\mu_{nf}}{\rho_{nf}}, \tag{4}$$

Here, ϕ is the solid volume fraction. The boundary conditions are as follows:

At the centerline of the channel: $\dfrac{\partial u(r,\theta)}{\partial\theta} = 0.$

At the boundary of the channel: $u(r,\theta)=0.$

Considering purely radial flow, the continuity Equation (1) implies that

$$f(\theta) = ru(r,\theta) \tag{5}$$

The dimensionless form of the velocity parameter can be obtained according to [2]:

$$f(\eta) = \frac{f(\theta)}{f_{\max}},\ \text{where}\ \eta = \frac{\theta}{\alpha} \tag{6}$$

where α is the channel angle and θ is any angle.

Substituting (5) into (2) and (3) and eliminating the pressure term *P*, the nonlinear ordinary differential equation can be written as [20]:

$$\begin{aligned} &f'''(\eta) + 2\alpha\,\mathrm{Re}\,A^*(1-\phi)^{2.5}\,f(\eta)\,f'(\eta) + \\ &(4-(1-\phi)^{1.25}Ha)\alpha^2 f'(\eta) = 0, \end{aligned} \tag{7}$$

$$A^* = (1-\phi)+\frac{\rho_s}{\rho_f}\phi,\ \ Ha = \sqrt{\frac{\sigma B_0^2}{\rho_f \nu_f}}, \tag{8}$$

$$\begin{aligned} &\mathrm{Re} = \frac{f_{\max}\alpha}{\nu_f} = \frac{U_{\max} r\alpha}{\nu_f} \\ &\begin{pmatrix} \textit{divergent channel}: \alpha>0,\ f_{\max}>0 \\ \textit{convergent channel}: \alpha<0,\ f_{\max}<0 \end{pmatrix} \end{aligned} \tag{9}$$

where A^* is a parameter, *Ha* Hartmann number, *Re* Reynolds number and α channel angle. The boundary conditions are reduced to the following form:

$$f(0)=1,\ f'(0)=0,\ f(1)=0. \tag{10}$$

Physically, these boundary conditions mean that maximum values of velocity are observed at centerline $\eta=0$ as shown in Figure 1. Also, the rate of velocity is zero at $\eta=0$ and at the solid boundary, the no-slip condition is considered.

3. SERIES ANALYSIS

The following power series expansion is considered in terms of the parameter α as Equation (7) is non-linear for velocity field

$$f(\eta) = \sum_{i=0}^{\infty} f_i(\eta)\alpha^i, \tag{11}$$

We then find that $f(\eta)$ has a singularity at $\alpha=\alpha_c$ of the form $f(\eta)\sim C(\alpha_c-\alpha)^{\delta_c}$ with the critical exponent δ_c. The non-dimensional governing equation is then solved into series solution by substituting the series(11) into Equation (7) and the boundary conditions (10) and equating the coefficients of powers of α. With the help of MAPLE, we have computed the first 42 coefficients for the series of the velocity $f(\eta)$ in terms of α, *Ha*, *Re*, ϕ, A^*. The first few coefficients of the series for $f(\eta)$ are as follows:

$$\begin{aligned} &f(\eta,\alpha,Ha,\mathrm{Re},\phi,A^*) = 1-\eta^2-\frac{1}{30}\eta^2(\eta-1)(\eta+1) \\ &(\eta-2)(\eta+2)\,\mathrm{Re}\,A^*(1-\phi)^{2.5}\alpha+\frac{1}{12}\eta^2(\eta-1)(\eta+1) \\ &(4-(1-\phi)^{1.25}Ha)\alpha^2+\frac{1}{5040}\eta^2(\eta-1)(\eta+1)(5\eta^4-9\eta^2 \\ &-9)\,\mathrm{Re}(1-\phi)^{2.5}A^*(4-(1-\phi)^{1.25}Ha)\alpha^3+O(\alpha^4) \end{aligned} \tag{12}$$

Applying differential and algebraic approximate methods to the series, we determine the comparison between HPA and ADM solution and the convergence of critical values and the change in singularity graph for the channel angle and flow Reynolds number by the positive effect of nanoparticles volume fraction. The critical relationship between the parametersis also shown graphically using differential approximate method. The utility of the series solution has widened using Hermite-Pade' approximants method described below.

4. COMPUTATIONAL PROCEDURE

In the present analysis, we shall employ a very efficient solution method, known as Hermite-Padé approximants, which was first introduced by Padé [22] and Hermite [23]. We say that a function is an *approximant* for the series

$$S = \sum_{n=0}^{\infty} s_n\alpha^n \tag{13}$$

if it shares with S the same first few series coefficients at $|\alpha|<1$. Thus, the simplest approximants are the partial sums of the series S. When the series converges rapidly, such *polynomial* approximants can provide good approximations of the sum. Because of the continuation of analytical solution and dominating singularity behavior, the bifurcation study is performed using the partial sum of Equation (13).

TABLE 1. Comparison between ADM solution and HPA solution for velocity when $\phi = 0$, $\mathrm{Re} = 25$, and $\alpha = 5^\circ$

	$Ha = 0$			$Ha = 250$			$Ha = 500$		
η	**ADM**	**HPA**	**Difference**	**ADM**	**HPA**	**Difference**	**ADM**	**HPA**	**Difference**
0.0	1.000000	1.000000	0.000000	1.000000	1.000000	0.000000	1.000000	1.000000	0.000000
0.1	0.986637	0.987221	0.000584	0.990196	0.988723	0.001473	0.992695	0.990224	0.002471
0.2	0.947127	0.949319	0.002192	0.960841	0.955136	0.005705	0.970544	0.960953	0.009591
0.3	0.883146	0.887563	0.004417	0.912079	0.899943	0.012136	0.912273	0.912323	0.00005
0.4	0.797259	0.803959	0.0067	0.811225	0.824219	0.012994	0.832683	0.844478	0.011795
0.5	0.692638	0.701095	0.008457	0.713799	0.729267	0.015468	0.743421	0.757440	0.014019
0.6	0.572716	0.581931	0.009215	0.604866	0.616433	0.011567	0.643816	0.650935	0.007119
0.7	0.440850	0.449556	0.008706	0.476625	0.486859	0.010234	0.515303	0.524163	0.00886
0.8	0.300013	0.306907	0.006894	0.325834	0.341211	0.015377	0.361234	0.375515	0.014281
0.9	0.152552	0.156476	0.003924	0.171160	0.179355	0.008195	0.194730	0.202234	0.007504
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

The dominating behavior of the function $S(\alpha)$ represented by a series (13) may be written as

$$S(\alpha) \sim \begin{cases} B + A\left(1 - \dfrac{\alpha}{\alpha_c}\right)^{\delta_c} & \text{when } \delta_c \neq 0, 1, 2, \ldots\ldots, \\ B + A\left(1 - \dfrac{\alpha}{\alpha_c}\right)^{\delta_c} \ln\left|1 - \dfrac{\alpha}{\alpha_c}\right| & \text{when } \delta_c = 0, 1, 2, \ldots, \end{cases} \tag{14}$$

as $\alpha \to \alpha_c$, where A and B are some constants and α_c is the critical point with the critical exponent δ_c.

Drazin –Tourigney [24] Approximants is a particular kind of algebraic approximants and Khan [25] introduced High-order differential approximant (HODA) as a special type of differential approximants. More information about the above mentioned approximants can be found in the respective references.

5. RESULTS AND DISCUSSION

The main objective of the current work is to analyze the effect of nanoparticles and high magnetic field on Jeffery-Hamel flow of viscous incompressible conducting fluid by using Hermite-Pade' approximants. Although there are four parameters of interest in the present problem the effects of nanoparticles volume fraction ϕ , channel angle α , Reynolds number *Re* and Hartman number *Ha*. The series (12) is analyzed by differential approximation method to show the variations in the critical value α_c and Re_c with critical exponent β_c for various values of ϕ significantly.

The results of the numerical computations of velocity profiles for different values of the aforementioned parameters are displayed graphically in Figures. (2)- (5) and the comparison between ADM and HPA solutions is shown in table by analyzing the series in (12) using Hermite- Padé approximation (HPA) method. The values by HPA and ADM method are compared in Table 1 when $\phi = 0$, $\mathrm{Re} = 25$ and $\alpha = 5^\circ$. Table1 shows that there is a good agreement between the HPA and ADM results. Hence, such results confirm the accuracy of Hermite- Padé approximation method. In this table, difference is defined as follows:

$\text{Differece} = \left| f(\eta)_{\text{ADM}} - f(\eta)_{\text{HPA}} \right|$.

Table 2 displays the convergence of α_c up to 15 decimal places at $d = 7$ using $N = 42$ terms with $\mathrm{Re} = 286$, $Ha = 0$, $\phi = 0.2$ and the values of β_c confirm that $\alpha_c \approx 0.08730158 \approx 5^\circ$ is a simple pole using HODA.Moreover, it is seen from Table 3 that the critical channel semi-angle α_c decreases uniformly for the nanoparticles volume fraction ϕ with $Ha = 0$ at $d = 7$ taking $N = 42$. It is observed from Table 4 that the critical Reynolds number converges to 12 decimal places and Re_c is a pole verified by the values of β_c. Table 5 represents that Re_c almost identical for ϕ with $Ha = 0$ at $d = 7$ taking $N = 42$.

TABLE 2. Convergence of critical angles α_c and corresponding exponent δ_c at $\mathrm{Re} = 286$, $Ha = 0$, $\phi = 0.2$

d	**N**	α_c	δ_c
2	7	0.087791814807783	-0.81857308585
3	12	0.087301495423310	-1.00000278437
4	18	0.087301579020751	-1.00000000000
5	25	0.087301587301587	-1.00000000000
6	33	0.087301587301587	-1.00000000000
7	42	0.087301587301587	-1.00000000000

TABLE 3. Numerical values of critical angles α_c and corresponding exponent δ_c at Re = 286, Ha = 0.

ϕ	α_c	δ_c
0	0.129532298313616	-1.000000000000
0.05	0.105332830881605	-0.999999999999
0.1	0.093857368991825	-1.000000000000
0.15	0.088633460885129	-0.999999999999
0.2	0.087301587301587	-1.00000000000

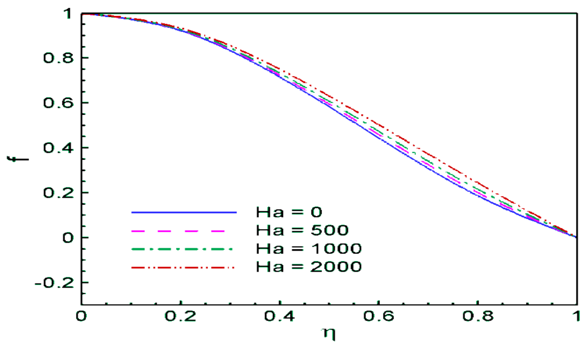
TABLE 4. Convergence of critical Reynolds number Re_c and corresponding exponent δ_c at $\alpha = 0.0873, Ha = 0, \phi = 0.2$.

d	N	Re_c	δ_c
2	7	288.957669152957	-0.6379425782386
3	12	286.941934247595	-0.9995797263022
4	18	286.939323918124	-0.9999999999996
5	25	286.939323918119	-1.0000000000000
6	33	286.939323918121	-1.0000000000000
7	42	286.939323918121	-1.0000000000000

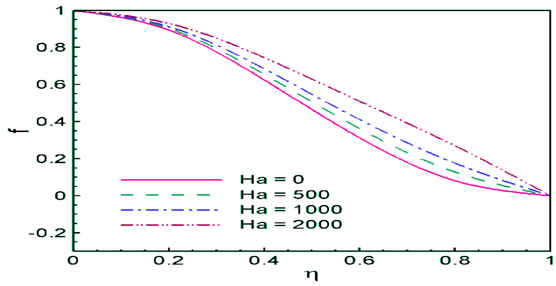
TABLE 5. Numerical values of critical Re_c and corresponding exponent δ_c at $\alpha = 0.0873$, $Ha = 0$.

ϕ	Re_c	δ_c
0	286.939323918122	-1.00000000000
0.05	286.939323918121	-1.00000000000
0.1	286.939323918120	-0.99999999999
0.15	286.939323918120	-0.99999999999
0.2	286.939323918121	-1.000000000000

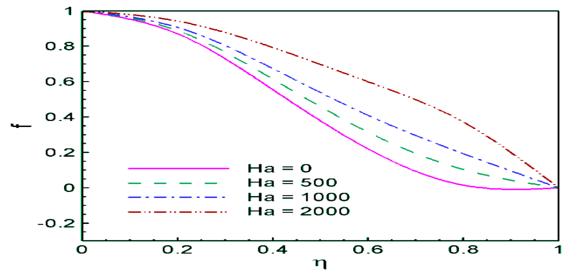
Figures 2 and 3 show the effect of magnetic field and channel angle on the velocity profiles for divergent channels. The velocity curves show that the rate of alteration is significantly reduced with increase of Hartmann number.The transverse magnetic field opposes the alteration phenomena clearly. Because the variation of Ha leads to the variation of the Lorentz force due to magnetic field and the Lorentz force produces more resistance to the alternation phenomena.



(a) $\alpha = 2.5^\circ, \mathrm{Re} = 200, \text{and}\, \phi = 0$

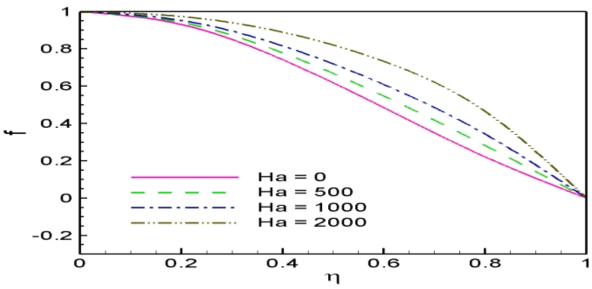


(b) $\alpha = 5^\circ, \mathrm{Re} = 200, \text{and}\, \phi = 0$,

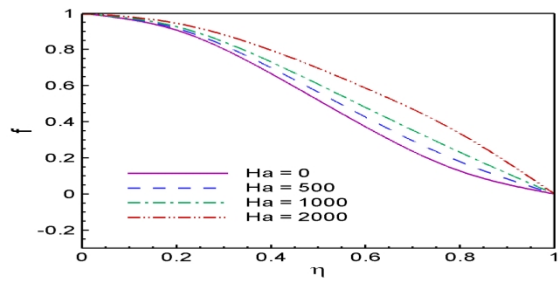


(c) $\alpha = 7.5^\circ, \mathrm{Re} = 200, \text{and}\, \phi = 0$

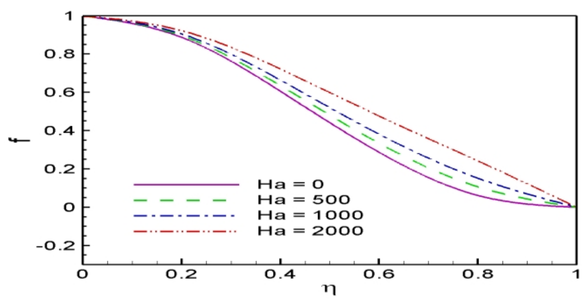
Figure 2. Velocity profiles in divergent channel with different values of Ha and α.



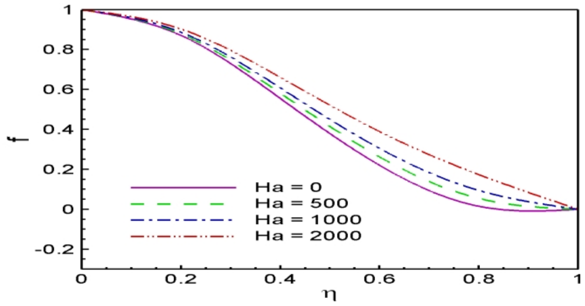
(a) $\alpha = 5^\circ, \mathrm{Re} = 75, \text{and}\, \phi = 0$



(b) $\alpha = 5^\circ, \mathrm{Re} = 150, \text{and}\, \phi = 0$,

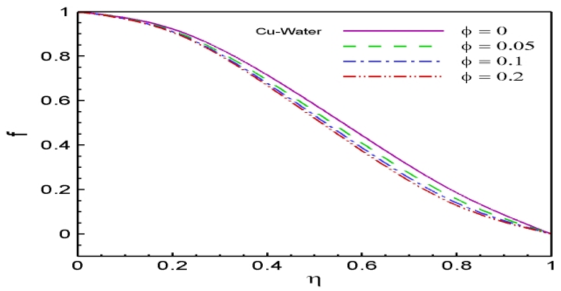


(c) $\alpha = 5^\circ, \mathrm{Re} = 225, \text{and}\, \phi = 0$

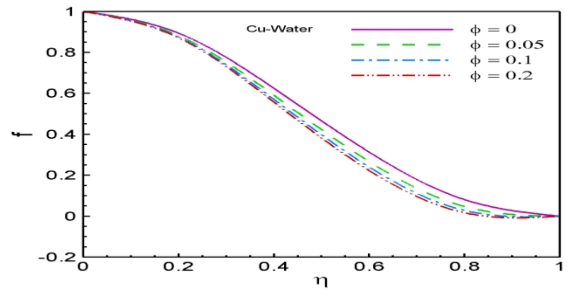


(d) $\alpha = 5^{\circ}, \mathrm{Re} = 300, \text{and} \phi = 0.$

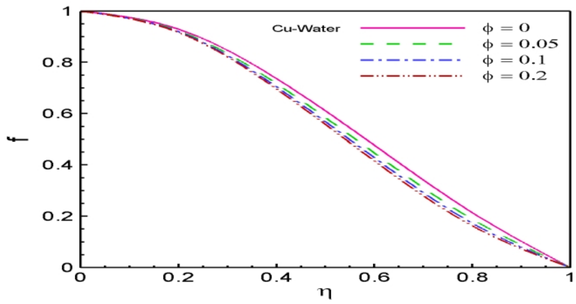
Figure 3. Velocity profiles in divergent channel with different values of *Ha* and *Re*.



(a) $\alpha = 5^{\circ}, \mathrm{Re} = 100, \text{and } Ha = 0$



(b) $\alpha = 5^{\circ}, \mathrm{Re} = 200, \text{and } Ha = 0,$



(c) $\alpha = 2^{\circ}, \mathrm{Re} = 200, \text{and } Ha = 0$

Figure 4. Effects of nanoparticle volume fraction on velocity profiles for Cu-Water.

It is seen from Figure 2 that the velocity increases moderately with rising *Ha* at a small angle $\alpha = 2.5^{\circ}$, but the differences between velocity profiles are more noticeable at larger angles. However, the backflow does not occur in converging channel but it is detected in diverging channel for higher values of α at a critical Reynolds number when $Ha = 0$. However, to diminish the backflow an increased *Ha* is essential. It can be concluded from Figure 2 that as channel angle increases, the variation of velocity is observed more due to *Ha*. Figure 3 represents the consequences of magnetic field on velocity profiles at $\alpha = 5^{\circ}$ with different Reynolds numbers. In Figure 3(a) at $\alpha = 5^{\circ}, \mathrm{Re} = 75$ as Hartmann number increases the velocity increases and no backflow is observed.

It can be seen from Figure 3(b) that at $\alpha = 5^{\circ}, \mathrm{Re} = 150$ backflow starts when magnetic field is absent, these properties are abolished with rising Hartmann number. The backflow enlarges at high Reynolds number, hence larger magnetic field is required to abolish it. Moreover, it is detected from Figure 3(c) at $\alpha = 5^{\circ}, \mathrm{Re} = 225$ that backflow is abolished at $Ha = 2000$, whereas there occurs backflow for each values of Hartmann number in Figure 3(d) at $\alpha = 5^{\circ}, \mathrm{Re} = 300$. Furthermore, a Cu-Water nanofluid flow is considered and the effect of nanoparticles volume fraction is analyzed. It is assumed that the base fluid and the nanoparticles are in thermal equilibrium and no slip occurs between them. The densities of water and Cu are $\rho_f = 998.1$ and $\rho_s = 8933$ respectively. Figure 4 implies that as volume fraction of nanoparticles increases, the boundary layer thickness increases. It is also observed that at higher values of Reynolds number and channel angle, backflow starts with rising values of nanoparticles volume fraction.

However, as the solid volume fraction increases, the velocity deceases which is consistent with physical phenomenon.Figure 5 predicts the combined effects of magnetic field and nanoparticles volume fraction on the velocity for divergent channel with fixed Reynolds number. The figure represents sensible increases in the velocity with rising Hartmann number for both viscous and nanofluid that coincide with those results of [20].
It is also observed that for all Hartmann numbers there is no backflow in the viscous fluid $\phi = 0$, nevertheless backflow starts for nanofluid with $Ha = 0$ at $\alpha = 5^{\circ}, \mathrm{Re} = 50$ and this phenomenon vanishes with the rising values of Hartmann number. Employing the algebraic approximation method to the series (12) we have obtained the singularity graphs of α and Re. Figure 6 shows the effect of nanoparticles on the singularity diagram of α. It is interesting to note that the curve turns at α_c, and as the values of ϕ increase, the singular points as simple poles change from $\alpha \approx 0.12953229 \approx 7.5^{\circ}$ to $\alpha \approx 0.09385736 \approx 5.5^{\circ}$ and

then to $\alpha \approx 0.08730158 \approx 5^{\circ}$ respectively. Moreover, from Figure 7 it is observed that the solution diagram of *Re* also turns at Re_c. The singular points remain almost similar and the three curves coincide for different values of ϕ. The conjecture of Figures 6 and 7 is consistent with the results shown in Tables 3 and 5 using differential approximation. The High-order Differential Approximant [25] is applied to the series (12) in order to determine the critical relationship between the parameters α and Re. Figure 8 displays the critical relation between the channel angular width α and Reynolds number Re for various values of ϕ. It is found that as α increases, then Re decreases and conversely Re increases when α decreases. This implies that both channel angle and Reynolds number are inversely proportional to each other which is in excellent agreement with classical Jeffery-Hamel flow when $\phi = 0$. There is a notable difference in the curves at $\phi = 0.1$ and $\phi = 0.2$ than in the curve at $\phi = 0$. Therefore, nanofluid has a significant impact on stability of Jeffery-Hamel flow.

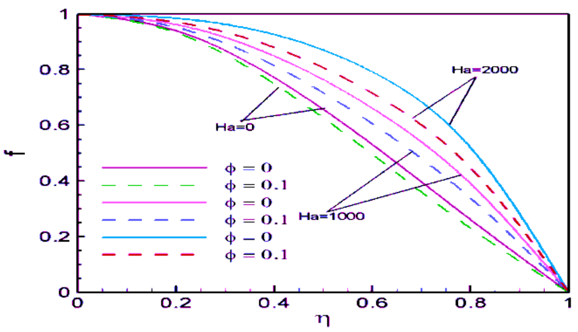


Figure 5. Velocity profiles for several values of Hartmann number and solid volume fraction at $\alpha = 5^{\circ}, \mathrm{Re} = 50$.

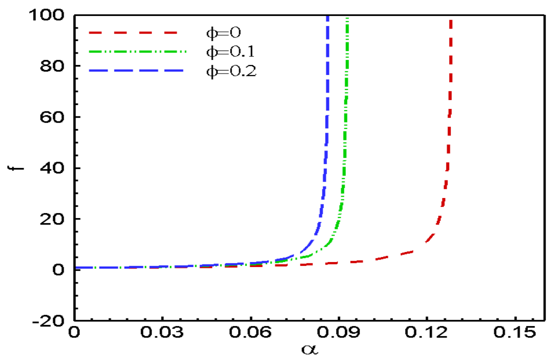


Figure 6. Approximate singularity diagram of α in the $(\alpha - f(\eta = 0.5))$ plane at $Ha = 0$, $\mathrm{Re} = 286.93$ with different ϕ obtained by Drazin-Tourigny method for $d = 7$

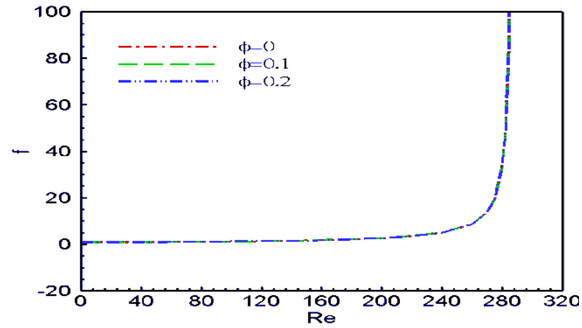


Figure 7. Approximate singularity diagram of *Re* in the $(\mathrm{Re} - f(\eta = 0.5))$ plane at $Ha = 0$, $\alpha = 0.0873$ with different ϕ obtained by Drazin-Tourigny method for $d = 7$.

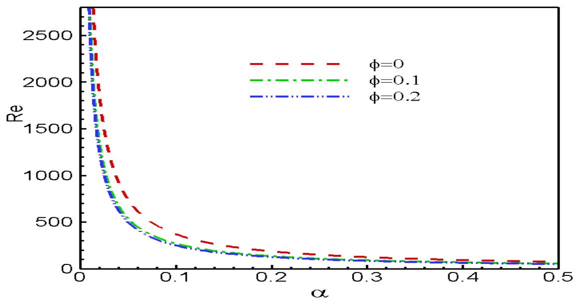


Figure 8. Critical relation between α and *Re* for different values of ϕ at $Ha = 1$ obtained by HODA for $d = 7$.

6. CONCLUSION

The magneto-hydrodynamic Jeffry-Hamel flow problem with nanofluid is investigated using a special type of Hermite-Padé approximation technique. A comparison is made between the available results obtained by Adomian decomposition method and the present approximate solutions.The accurate numerical approximation of the critical parameters of the flow is obtained. The numerical study indicates that HPA is a powerful approach for solving this problem. The influence of various physical parameters on the velocity field is discussed in detail. The basic conclusions are as follows:

- Increasing Reynolds number leads to backflow near the walls in the channel.
- Increasing Hartmann number produces to backflow reduction. High Hartmann number is required to decline of backflow in larger angles or Reynolds numbers.
- The velocity decreases as nanoparticles volume fraction increases.
- The dominating singularity behavior of the wall divergence semi-angle and flow Reynolds number is

analyzed with the effect of nanoparticles volume fraction. The critical relationship between the parameters with the effect of nanoparticles coincides with the conjecture of classical Jeffery-Hamel flow. Moreover, we provide a basis for guidance about new approximants idea for summing power series that should be chosen for many problems in fluid mechanics and similar subjects.

7. ACKNOWLEDGEMENTS

This work is done within the framework of the PhD program of the corresponding author under Department of Mathematics, Bangladesh University of Engineering and Technology, Dhaka. Financial support from the Bangabandhu Fellowship on Science and ICT project is acknowledged.

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Analysis of Magneto-hydrodynamics Jeffery-Hamel Flow with Nanoparticles by Hermite-Padé Approximation

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PAPER INFO

Paper history:
Received 05 January 2014
Received in revised form 27 November 2014
Accepted 13 March 2015

Keywords:
Jeffery-Hamel Flow
Magneto-hydrodynamic
Nanofluid
Dominating Singularity
Hermite- Pade Approximation

چکیده

اثرات ترکیبی نانوذرات و میدان مغناطیسی در رفتار غیر خطی جریان جفری-هامل در مطالعه حاضر تحلیل شده است. معادلات حاکم عمومی به راه حل سری با استفاده از یک روش تحلیلی نیمه عددی به نام تقریب Hermite- Padé حل شده است. پروفایل سرعت در کانال واگرا برای مقادیر مختلف کسر حجمی نانوذرات جامد، تعداد هارتمن، عدد رینولدز و زاویه کانال معرفی شده اند. رفتار یکه‌ی غالب مساله به صورت عددی و گرافیکی تحلیل شده است. ارتباط تعیین کننده بین پارامترها به منظور مطالعه‌ی ناپایداری نانو سیال بررسی شده است.

doi*: 10.5829/idosi.ije.2015.28.04a.15*