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A New Compromise Decision-making Model Based on TOPSIS and VIKOR for Solving Multi-objective Large-scale Programming Problems with a Block Angular Structure under Uncertainty

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ABSTRACT

This paper proposes a compromise model, based on a new method, to solve the multi-objective large-scale linear programming (MOLSLP) problems with block angular structure involving fuzzy parameters. The problem involves fuzzy parameters in the objective functions and constraints. In this compromise programming method, two concepts are considered simultaneously. First of them is that the optimal alternative is closer to fuzzy positive ideal solution (FPIS) and farther from fuzzy negative ideal solution (FNIS). Second of them is that the proposed method provides a maximum ''group utility'' for the ''majority'' and a minimum of an individual regret for the ''opponent''. In proposed method, the decomposition algorithm is utilized to reduce the large-dimensional objective space. A multi objective identical crisp linear programming is derived from the fuzzy linear model for solving the problem. Then, a compromise solution method is applied to solve each sub problem based on TOPSIS and VIKOR simultaneously. Finally, to illustrate the proposed method, an illustrative example is provided.

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1. INTRODUCTION

Decision makingcan be regarded as the mental processes encountered under various situations where a number of competing alternatives need to be chosen based on a set of criteria. Decision making problems are introduced in many fields such as management and engineering. MCDM is divided into multi-objective decision making (MODM) and multi-attribute decision making (MADM). MADM is applied to find suitable option among several alternatives versus multi-attribute but MODM studies decision problems in which the decision space is continuous. In other words, there are many decision problems with multi objective during decision making so they may conflict with each other [1-3]. The rate complexity is associated with proportional enhancements in number of variables. As the number of variables increases, the complexity of problem increases. In other words, there are various factors in the objective functions and constraints in these problems. Further, in large scale problems, there is so great scope to solve them by usual methods in a less time. Because optimization of large scale problems takes a long time, the original programming problems are decomposed into several smaller sub problems. Moreover, the complexity of optimization method is reduced. Furthermore, the decomposition methods are so great scope to solve the large scale problems by usual methods in a less time. Block angular structure problems are one of the most common large scale programming problems. Fortunately, these problems have special structures that can be decomposed easily [1, 4-6]. A suitable approach for treating large-scale problems is to reduce the size of the problem using model-reduction techniques. The block angular structure problems can be solved by a decomposition method. Dantzig-wolf and Benderz are two types of

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decomposition methods [4]. A Dantzig-wolf decomposing algorithm is introduced for parametric space in large scale linear optimization problems with fuzzy parameter [4, 7]. Then this method is applied on largescale linear programming problems with block angular structure [6, 8]. Recently, some compromise multi-criteria decision making (MCDM) methods are extended and applied to find the suitable solution for MOLSNLP problems. TOPSIS method is applied to solve multi-objective dynamics programming problems [9]. TOPSIS is applied for evaluating project risk response in fuzzy environment [10, 11]. An extended TOPSIS method is introduced for solving MODM problems [12]. TOPSIS is extended to solve MOLSNLP problems with block angular structure [1, 13]. VIKOR is another compromise MCDM method that is extended for solving MOLSNLP problems [6, 14].An integrated VIKOR methodology is proposed for plant location selection[15].

Moreover, much knowledge in the real world is uncertainty rather than crisp [16, 17]. Fuzzy set theory is valuable tool to describe this concept. Fuzzy set theory was proposed as a vagueness concept for decision-making problems with conflict of preferences involved in the selection process [16, 18]. Moreover, the fuzzy set concept and the MCDM method were manipulated to consider the fuzziness in the decision-making parameter and group decision-making process [19, 20]. The VIKOR method is applied for solving MOLSNLP problems where the formulation of objective functions and constraints is introduced with crisp data whereas coefficient of objective function and constraint may not be exact and complete.

This paper presents a model based on a novel compromised solution method to solve the multi-objective large-scale programming problems with block angular structure involving fuzzy coefficients. In this method, an aggregating function that is developed from TOPSIS and VIKOR is proposed based on the particular measure of ''closeness" to the ''ideal" solution. Because in a real case, the information of decision maker related to coefficient of objective function and constraint may not be exact and complete, we proposed a simple method which is applied to formulate the equivalent crisp model of the fuzzy optimization problem. The decomposition algorithm is utilized to reduce the large-dimensional objective space into a two-dimensional space. Because the feasiblespace of each sub problem is bigger than feasiblespace of original problem [1]. Then a multi-objective identical crisp programming is derived from each fuzzy linear model. In other words, a model with fuzzy coefficients in objective function will be transferred to crisp model. Then this method is applied for fuzzy constraints.Furthermore, two independent solution methods are proposed to solve convex problems. In the last step of proposed method, single objective programming problem is solved to find the final solution. In this paper, the advantages of TOPSIS and VIKOR are utilized simultaneously. The new proposed method is developed based on the strategy of the majority criteria and the individual regret in order to calculate the distance of alternatives from the ideal solutions. On the other hand side, unlike the traditional VIKOR method which did not consider both of relative distance from positive ideal solution (PIS) and negative ideal solution (NIS), this paper considers the distance of alternatives from the PIS and NIS simultaneously. Finally, to justify the proposed method, an illustrative example is provided. Then, the sensitivity analysis is described.

Some large scale problems are very hard to separate. Fortunately, some largescale programming problems contain special structure where the new proposed decomposition method is applied to solve largescale programming problem. Each of the independent sub problems can be treated separately. The problem also can be solved separately. Each of these features suggests problems composed solely of independent subsystems. In fact, the large scale programming problems represent the conditions of big company.

The remaining of this paper is organized as follows. The problem formulation is presented in the next section. In this section, the decomposed problem is introduced and then the parameters and variables are described. In section 3, the VIKOR solution method for fuzzy MOLSLP is introduced. In section 4, an example is proposed to illustrate the process of proposed method step by step. Then, the sensitivity analysis is described for each sub problem. The last section is devoted to conclusion.

2. PROBLEM STATEMENT

Consider the following fuzzy MOLSLP problem with the block angular structure:

$$P:$$
$$\text{Max (Min)} f_1(X, \tilde{U}_1)$$
$$\text{Max (Min)} f_2(X, \tilde{U}_2)$$
$$\vdots$$
$$\text{Max (Min)} f_L(X, \tilde{U}_L)$$

$$S.t. \quad FS = \begin{cases} \tilde{g}_m(x1) \le \tilde{B}_1 m = 1, 2, \dots, s_1 \\ \tilde{g}_m(x2) \le \tilde{B}_2 m = s_1 + 1, \dots, s_2 \\ \vdots \\ \tilde{g}_m(xN) \le \tilde{B}_N m = s_r + 1, \dots, s_N \\ \tilde{H}_i(X) = \sum_{j=1}^{N} \tilde{h}_{ij}(X_j) \le \tilde{B} i = 1, 2, \dots, \mathrm{w} \end{cases} \quad (1)$$

$$f_i(X, \tilde{U}_i) = \tilde{U}_i C_i X = \sum_{j=1}^{N} \tilde{U}_{ij} C_{ij} X_j i = 1, 2, \dots, \mathrm{L} \quad (2)$$

$\tilde{g}_m(xi) = V{\sim}_{mi} d_{mi} X_i$; $i = 1, 2, \dots, s_1$ are the inequality constraint functions and $\tilde{\mathrm{H}}_{\mathrm{i}}(\mathrm{X})$ are the common constraint functions on R^{n} which can be constrained as:

$\tilde{H_i}(X) = \sum_{j=1}^{N} O\sim_{ij} e_{ij} X_j i = 1, 2, ...,$

Where $V\sim_{mi} = (v_{m1}, v_{m2}, v_{m3})$

$O\sim_{ij} = (o_{ij1}, o_{ij2}, o_{ij3})$

$\tilde{B_m} = (b_{m1}, b_{m2}, b_{m3})$

$\tilde{B} = (r_i, s_i, t_i)$

Model parameters:

L the number of objective functions

q the number of sub problems

$N$the number of variables

$N_i$the set of variables of the i*th* sub problem, i= 1,2,. . . ,q

p_i *i*th sub problem

*R*the set of all real numbers

$C_i$anN-dimensional row vector of fuzzy parameters for the i*th* objective function

C_{ij}thecrisp coefficient for the j*th* variable of i*th* objective function

$d_{ij}$the crisp coefficient for the j*th* constraint of i*th* variable

$e_{ij}$the crisp coefficient for the i*th* common constraint for the j*th* variable

$\tilde{U_i}$an N-dimensional row vector of fuzzy parameters for the i*th* objective function

$\tilde{U_{ij}}$the fuzzy parameters for the j*th* variable of the i*th* objective function

$V\sim_{ij}$the fuzzy parameters for the i*th* constraint of the j*th* variable

$O_{ij}\sim$the fuzzy parameters for the j*th* variable of the i*th* common constraint

$W$the number of common constraints onR^N

$S_i$maximum amount of index for the constraints for the i*th* variable

$\tilde{B}$an w-dimensional column vector of right-hand sides of the common constraints

$\tilde{B_j}$anS_i-dimensional column vector of independent constraints right-hand sides whose elements are fuzzy parameters for the i*th* sub problem, i = 1,2,. . . ,q;

where$X = (x_1, x_2, .., x_N)$is the N-dimensional decision vector.$f_i(X, \tilde{U_i})$, i=1, 2… L is the objective function. It is assumed that the objective functions have an additively separable form. There are two types of constraints. First of them is separable form constraints which is applied in own sub problem. Second of them is common constrains which is applied in all of sub problems. Using Dantzig-Wolfe decomposition algorithm, the fuzzy MOLSLP problem can be decomposed into q sub-problems. The i*th* sub-problem for i = 1, . . , q is defined as:

$$P_i \begin{cases} \text{Max (Min)} f_1(X, \tilde{U_1}) = \sum_{j \in N_i} f_1(X_j, \tilde{U_1}) = \sum_{j \in N_i} \tilde{U_1} C_1 X_j \\ \text{Max (Min)} f_2(X, \tilde{U_2}) = \sum_{j \in N_i} f_2(X_j, \tilde{U_2}) = \sum_{j \in N_i} \tilde{U_2} C_2 X_j \\ \vdots \\ \text{Max (Min)} f_L(X, \tilde{U_L}) = \sum_{j \in N_i} f_L(X_j, \tilde{U_L}) = \sum_{j \in N_i} \tilde{U_L} C_L X_j \\ S.t. \quad FS_i = \begin{cases} \sum_{j \in N_i} \tilde{g_m}(X_j) \le \tilde{B_m} m = s_{j-1} + 1, \dots, s_j \\ \tilde{H_i}(X) = \sum_{j=1}^{N} \tilde{h_{ij}}(X_j) \le \tilde{B} i = 1, 2, \dots, w \end{cases} \end{cases} \quad (3)$$

As shown in problem (3), the i*th* sub problem consists of L objective functions. Moreover, $\tilde{h}_{ij}(X_j) = O\sim_{ij} e_{ij} X_{ij}$, where h_{ij} is the function of j*th* variable in i*th* common constraint and $\tilde{U}$ is the coefficient of the objective function and $\tilde{B}$is the coefficient of the right-hand side of constraints in problem (3). It is pointed out that all of the coefficients are presented as triangular fuzzy numbers.

3. NEW COMPROMISED SOLUTION METHOD FOR FUZZY MOLSLP

In this section, in the first attempt, the Dantzig-wolf decomposition method is successfully applied to decompose the original problem into the q independent linear sub problems. In other words, the L-dimensional problem space is reduced to a one-dimensional space by applying the Dantzig-Wolfe decomposition algorithm. Appling new compromised method; the objectives of each sub problem are aggregated. To this mean, the individual positive ideal solution (PIS) and negative ideal solution (NIS) are calculated for each objective. Applying PIS and NIS, the bi- objective problems are constructed for j*th* sub problem. Finally, the final single objective problem is solved to obtain final optimal solution. The proposed method has the following steps:

Step 1. Applying the Dantzig-wolf ecomposition method, decompose the primal problem into q independent sub problems for all objective functions and constraints to reduce the dimension of primal problem.

Step 2. Transfer each fuzzy programming problem into three crisp problems according to the following procedure. This method is proposed and extended to defusing some fuzzy problems [21-23].The coefficients of objective functions and constraints are considered as triangular fuzzy numbers.Therefore, there are three crisp objective functions for each fuzzy objective function. Moreover, each fuzzy constraint can be changed into three crisp constrains. The i*th* sub problem is transferred as:

$$P_{i1} : \begin{cases} \text{Min(Max)}(b_{i1} - a_{i1}) C_{i1} X_i \\ \text{Max (Min)}(b_{i1}) C_{i1} X_i \\ \text{Max (Min)}(c_{i1} - b_{i1}) C_{i1} X_i \end{cases} \quad (4)$$

$$Pi_2 : \begin{cases} \text{Min(Max)}(b_{i2} - a_{i2}) C_{i2} X_i \\ \text{Max (Min)}(b_{i2}) C_{i2} X_i \\ \text{Max (Min)}(c_{i2} - b_{i2}) C_{i2} X_i \end{cases} \quad (5)$$

.
.
.

$$P_{iL} : \begin{cases} \text{Min(Max)}(b_{iL} - a_{iL}) C_{iL} X_i \\ \text{Max (Min)}(b_{iL}) C_{iL} X_i \\ \text{Max (Min)}(c_{iL} - b_{iL}) C_{iL} X_i \end{cases} \quad (6)$$

$$S.t.\begin{cases}\begin{cases}v_{im1}d_{im}(xi) \le b_{im1} \\ v_{im2}d_{im}(xi) \le b_{im2}\; m = s_{i-1}+1,\dots,s_i \\ v_{im3}d_{im}(xi) \le b_{im3}\end{cases} \\ \begin{cases}\sum_{j=1}^{N} o_{ij1}e_{ij}X_j \le r_i \\ \sum_{j=1}^{N} o_{ij2}e_{ij}X_j \le s_i\; i = 1,\dots,w \\ \sum_{j=1}^{N} o_{ij3}e_{ij}X_j \le t_i\end{cases}\end{cases} \quad (7)$$

Step 3. Calculate the positive ideal solution (PIS) and the negative ideal solution (NIS) of each objective function with fuzzy coefficient under the given constraints. Note that the values of PIS and NIS are calculated through solving the multi-objective problem as a single objective using, each time, only one objective.

$$\text{PIS}: f_{bj}^{\ *} = \{\text{Max (Min)} f_{bj}(X_j)\ (f_{cj}(X_j),\ \forall b\ (\forall c)\} \quad (8)$$

$$\text{NIS}: f_{bj}^{\ -} = \{\text{Min (Max)} f_{bj}(X_j)\ (f_{cj}(X_j),\ \forall b\ (\forall c)\} \quad (9)$$

$f_{bj}(X_j)$Benefit objective for maximization

$f_{cj}(X_j)$Cost objective for maximization

Step 4. Applying PIS and NIS from the results of step 3, construct the functions ofs^{PIS}, R^{PIS}as a maximum ''group utility'' for the ''majority'' and a minimum of an individual regret for the ''opponent''.Furthermore, construct the values s^{NIS}andR^{NIS} to obtain a compromise solution,as shown follow:

$$S_i^{PIS} = \sum_{j\in B_i} w_j\left(\frac{f_{ij}^{\ *}-f_{ij}}{f_{ij}^{\ *}-f_{ij}^{\ -}}\right) + \sum_{j\in C_i} w_j\left(\frac{f_{ij}-f_{ij}^{\ *}}{f_{ij}^{\ -}-f_{ij}^{\ *}}\right) \quad (10)$$

$$R_i^{PIS} = \max_i\left(\frac{f_{ij}^{\ *}-f_{ij}}{f_{ij}^{\ *}-f_{ij}^{\ -}}\right) \quad (11)$$

$$S_i^{NIS} = \sum_{j\in B_i} w_j\left(\frac{f_{ij}-f_{ij}^{\ -}}{f_{ij}^{\ *}-f_{ij}^{\ -}}\right) + \sum_{j\in C_i} w_j\left(\frac{f_{ij}^{\ -}-f_{ij}}{f_{ij}^{\ -}-f_{ij}^{\ *}}\right) \quad (12)$$

$$R_i^{NIS} = min_i\left(\frac{f_{ij}^{\ *}-f_{ij}}{f_{ij}^{\ *}-f_{ij}^{\ -}}\right) \quad (13)$$

In order to obtain a compromise solution, the following bi-objective problem is introduced:

We can utilize a single objective instead of problem (13) based on a max-min decision making model. This method is proposed by Bellman and Zadeh and extended by Zimmermann [19, 23]. The steps of this model is shown in the following steps:

Step 5. Applying TOPSIS method, calculate the PIS and NIS of$S_i^{PIS}$and R_i^{PIS}. Furthermore, calculate the PIS and NIS ofS_i^{NIS} and R_i^{NIS}among all objective functions.

Step 6. Applying VIKOR method, construct the two objective function problems, relative closeness to the PIS and separation from NIS is as follow:

MinQ_i^{PIS}

MaxQ_i^{NIS}(14)$X \in FS_i$

where

$$Q_i^{PIS} = v(\sum_{j\in B_i} w_j\left(\frac{S_i^{PIS}-(S_i^{PIS})^*}{(S_i^{PIS})^- -(S_i^{PIS})^*}\right)) + (1-v)\left(\frac{R_i^{PIS}-(R_i^{PIS})^*}{(R_i^{PIS})^- -(R_i^{PIS})^*}\right) \quad (15)$$

$$Q_i^{NIS} v\left(\sum_{j\in B_i} w_j\left(\frac{(S_i^{NIS})^* - S_i^{NIS}}{(S_i^{NIS})^* - (S_i^{NIS})^-}\right)\right) + (1-v)\left(\frac{(R_i^{NIS})^* - R_i^{NIS}}{(R_i^{NIS})^* - (R_i^{NIS})^-}\right) \quad (16)$$

Step 7. Construct the two membership functions for $Q^{PIS}$and Q^{NIS}, respectively.

The linear membership function for the negative (orQ^{PIS}) objective can be defined as:

$$\mu_{i1}(x) = \frac{(Q_i^{PIS})^- - (Q_i^{PIS})}{(Q_i^{PIS})^- - (Q_i^{PIS})^*} \quad (17)$$

The linear membership function for the positive (orQ^{NIS}) objective can be defined as:

$$\mu_{i2}(x) = \frac{(Q_i^{NIS}) - (Q_i^{NIS})^-}{(Q_i^{NIS})^* - (Q_i^{NIS})^-} \quad (18)$$

Step 8. Construct the final single objective problem for each sub problem based on the membership functions. Then solve it to obtain the final optimal solution. Problem (14) is equivalent to the form of following problem as:

$$\max \lambda$$

$$S.t.\begin{cases}\frac{(Q_i^{PIS})^- - (Q_i^{PIS})}{(Q_i^{PIS})^- - (Q_i^{PIS})^*} \ge \lambda \\ \frac{(Q_i^{NIS}) - (Q_i^{NIS})^-}{(Q_i^{NIS})^* - (Q_i^{NIS})^-} \ge \lambda\end{cases} \quad (19)$$

$0 \le \lambda \le 1,\ X \in FS_1$

The final compromised solution and satisfactory level are obtained by solving problem (19).

4. ILLUSTRATIVE NUMERICAL EXAMPLE

In this section, we work out a numerical example to illustrate the new proposed method.This example has three objective functions. The objective functions and constrains are proposed as linear on R^3 where the coefficient of the objective functions and constraints are assumed as triangular fuzzy numbers. Moreover, the weights of objective functions are same for all sub problems. The linear programming example is proposed as:

P:

$$\max f_1(x) = (1,2,3)x_1 + (2,4,6)x_2 + (1,3,5)x_3$$

$$\max f_2(x) = (1,3,5)x_1 - (2,5,7)x_2 - (1,2,3)\,x_3 \quad (20)$$

$$\max f_3(x) = (2,4,6)x_1 + (1,3,5)x_2 - (3,6,9)\,x_3$$

Subject to:

$$FS = \begin{Bmatrix}(1,3,5)x_1 + (2,4,6)x_2 - (1,2,3)x_3 \le (4,8,12) \\ (0,0,0) \le (2,4,6)x_1 \le (5,10,15) \\ (0,0,0) \le (1,2,3)x_2 \le (2,5,8) \\ (0,0,0) \le (1,3,5)x_3 \le (1,5,9)\end{Bmatrix}$$

Then stepsof the problem are given below.

Step 1. Decompose the original master problem into three sub problems then solve the sub problems separately. The decomposed sub problems $\mathrm{P}_1, \mathrm{P}_2$ and P_3 are proposed as:

$$P_1:$$

$$\max f_1(x) = (1,2,3)x_1$$

$$\max f_2(x) = (1,3,5)x_1 \quad (21)$$

$$\max f_3(x) = (2,4,6)x_1$$

$$FS_1 = \begin{Bmatrix} x_1 + 2x_2 - 2x_3 \le 4 \\ 3x_1 + 4x_2 - 2x_3 \le 8 \\ 5x_1 + 6x_2 - 3x_3 \le 12 \\ 0 \le x_1 \le 2.5 \end{Bmatrix}$$

$$P_2:$$

$$\max f_1(x) = (2,4,6)x_2$$

$$\max f_2(x) = -(2,5,7)x_2 \quad (22)$$

$$\max f_3(x) = (1,3,5)x_2$$

$$FS_2 = \begin{Bmatrix} x_1 + 2x_2 - 2x_3 \le 4 \\ 3x_1 + 4x_2 - 2x_3 \le 8 \\ 5x_1 + 6x_2 - 3x_3 \le 14 \\ 0 \le x_2 \le 2 \end{Bmatrix}$$

$$P_3:$$

$$\max f_1(x) = (1,3,5)x_3$$

$$\max f_2(x) = -(1,2,3)\,x_3 \quad (23)$$

$$\max f_3(x) = -(3,6,9)\,x_3$$

$$FS_3 = \begin{Bmatrix} x_1 + 2x_2 - 2x_3 \le 4 \\ 3x_1 + 4x_2 - 2x_3 \le 8 \\ 5x_1 + 6x_2 - 3x_3 \le 14 \\ 0 \le x_3 \le 1 \end{Bmatrix}$$

Step 2. Using Equations (5)–(8), transfer each fuzzy programming problem P into three crisp sub problems.$\mathrm{P}_1, \mathrm{P}_2$ and $\mathrm{P}_3$are converted into three crisp objective functions programming problems. The sub problem P_1 can be transfer as follow:

$$P_1:$$

P_{11}:	P_{12}:	P_{13}:
$\min f_1(x) = x_1$	$\min f_1(x) = 2x_1$	$\min f_1(x) = 2\,x_1$
$\max f_2(x) = 2x_1$	$\max f_2(x) = 3x_1$	$\max f_2(x) = 4x_1$
$\max f_3(x) = x_1$	$\max f_3(x) = 2\,x_1$	$\max f_3(x) = 2\,x_1$
Subject to:	Subject to:	Subject to:
$X \in FS_1$ (24)	$X \in FS_1$ (25)	$X \in FS_1$ (26)

Like the first problem, the second problem P_2can be transferred into three crisp sub problems as:

P_2:

P_{21}:	P_{22}:	P_{23}:
$\min f_1(x) = 2\,x_2$	$\min f_1(x) = -3\,x_2$	$\min f_1(x) = 2\,x_2$
$\max f_2(x) = 4x_2$	$\max f_2(x) = -5x_2$	$\max f_2(x) = 3x_2$
$\max f_3(x) = 2\,x_2$	$\max f_3(x) = -2\,x_2$	$\max f_3(x) = 5\,x_2$
Subject to:	Subject to:	Subject to:
$X \in FS_2$ (27)	$X \in FS_2$ (28)	$X \in FS_2$ (29)

The third problem is similar to the other two problems as:

$$P_3:$$

P_{31}:	P_{32}:	P_{33}:
$\min f_1(x) = 2x_3$	$\min f_1(x) = -x_3$	$\min f_1(x) = -3x_3$
$\max f_2(x) = 3x_3$	$\max f_2(x) = -2\,x_3$	$\max f_2(x) = -6x_3$
$\max f_3(x) = 5x_3$	$\max f_3(x) = -3\,x_3$	$\max f_3(x) = -9x_3$
Subject to:	Subject to:	Subject to:
$X \in FS_3$ (30)	$X \in FS_3$ (31)	$X \in FS_3$ (32)

Step3. Calculate the individual PIS and NIS of each objective function for sub problems$\mathrm{P}_1, \mathrm{P}_2$ and P_3. The obtained PIS and NIS of sub problem $\mathrm{P}_1$are shown in Tables 1and 2.

PIS: $f_{11}^* = (f_1^*, f_2^*, f_3^*) = (0.0000, 5.0000, 2.5000)$.

$f_{12}^* = (f_1^*, f_2^*, f_3^*) = (0.0000, 7.5000, 5.0000)$

$f_{13}^* = (f_1^*, f_2^*, f_3^*) = (0.0000, 10.0000, 5.0000)$.

TABLE 1.PIS payoff table of (P_1)

		f_1	f_2	f_3	x_1
	$\min f_1$	0.0000*	0.0000	0.0000	0.0000
P_{11}	$\max f_2$	2.5000	5.0000*	2.5000	2.5000
	$\max f_3$	2.5000	5.0000	2.5000*	2.5000
	$\min f_1$	0.0000*	0.0000	0.0000	0.0000
P_{12}	$\max f_2$	5.0000	7.5000*	5.0000	2.5000
	$\max f_3$	5.0000	7.5000	5.0000*	2.5000
	$\min f_1$	0.0000*	0.0000	0.0000	0.0000
P_{13}	$\max f_2$	5.0000	10.0000*	5.0000	2.5000
	$\max f_3$	5.0000	10.0000	5.0000*	2.5000

TABLE 2.NIS payoff table of(P_1)

		f_1	f_2	f_3	x_1
	$\max f_1$	2.5000⁻	5.0000	2.5000	2.5555
P_{11}	$\min f_2$	0.0000	0.0000⁻	0.0000	0.0000
	$\min f_3$	0.0000	0.0000	0.0000⁻	0.0000
	$\max f_1$	5.000⁻	7.5000	5.0000	2.5000
P_{12}	$\min f_2$	0.0000	0.0000⁻	0.0000	0.0000
	$\min f_3$	0.0000	0.0000	0.0000⁻	0.0000
	$\max f_1$	5.0000⁻	10.0000	5.0000	2.5000
P_{13}	$\min f_2$	0.0000	0.0000⁻	0.0000	0.0000
	$\min f_3$	0.0000	0.0000	0.0000⁻	0.0000

NIS: $f_{11}^- = (f_1^-, f_2^-, f_3^-) = (2.5000, 0.0000, 0.0000)$.

$f_{12}^- = (f_1^-, f_2^-, f_3^-) = (5.0000, 0.0000, 0.0000) f_{13}^- = (f_1^-, f_2^-, f_3^-) = (5.0000, 0.0000, 0.0000)$

Steps 4 and 5. Applying PIS and NIS from the results of step 3, construct the functions of $\mathrm{S}^{\mathrm{PIS}}$and $\mathrm{R}^{\mathrm{PIS}}$as shorter distance from the PIS and $\mathrm{S}^{\mathrm{NIS}}$, $\mathrm{R}^{\mathrm{NIS}}$as farther distance from NIS for each sub problem. Then calculate the PIS and NIS of S_i^{PIS}, R_i^{PIS}, S_i^{NIS} and R_i^{NIS} as shown below:

$S_{PIS}{}^{*} = 0.4444 \quad R_{PIS}{}^{*} = 0.0000$

$S_{PIS}{}^{-} = 1.4444 \quad R_{PIS}{}^{-} = 0.3200$

$S_{NIS}{}^{*} = 1.8333 R_{NIS}{}^{*} = 0.3333$

$S_{NIS}{}^{-} = 0.8333 \quad R_{NIS}{}^{-} = 0.0133$

Step 6. Applying PIS and NIS from the results of step 5, construct the two objective functions (Q_1^{PIS}, Q_1^{NIS}) problem as follow:

$$Q_1{}^{PIS} = 0.0749x_1 + 0.3334 \quad (33)$$

$$Q_1{}^{NIS} = -0.2750x + 0.2 \quad (34)$$

$Q_{PIS}{}^{*} = 0.3334$

$Q_{PIS}{}^{-} = 0.5132$

$Q_{NIS}{}^{*} = 0.2$

$Q_{NIS}{}^{-} = -0.46$

Step 7. Construct the two membership functions for Q^{PIS} and Q^{NIS}, respectively.

$$\mu_1(x) = 1 - 0.4166x_1 \quad (35)$$

$$\mu_2(x) = -0.4166x_1 + 1 \quad (36)$$

Step 8. Final solution is obtained by solving the single problem (37) as:

$$\max \lambda$$

$$1 - 0.4166x_1 \geq \lambda$$

$$0 \leq \lambda \leq 1, \ X \in FS_1 \quad (37)$$

$$\lambda^* = 1.0000 x_1{}^* = 0.0000$$

λ^* is the maximum satisfactory level and $x_1{}^*$ is the final compromised solution for first sub problem. We want to obtain the ideal compromised solution. In other words, the objective function Q_1^{PIS} should be minimized whereas the function Q_1^{NIS} should be maximized. As shown in Figure 1, point $x_1{}^* = 0$ is optimum compromised solution. Moreover, the maximum level of λ is $\lambda^* = 1.0000$

Now, we solve the second sub problem similar to first sub problem. In this sub problem, Q_2^{PIS} and Q_2^{NIS} are optimized as shown in Figure 2. The maximum level of λ occurs at $\lambda^* = 0.5$. In other words, the more value of x_2 in this problem is better. But considering the constraint, the optimum compromised solution is $x_2{}^* = 1.000$.

Similar to sub problems P_1 and P_2, the problem P_3 is solved. The maximum satisfactory level ($\lambda^* = 0.5000$) is achieved for the compromised solution $x_3{}^* = 0.5000$. As shown in Figure 3, the best final solution of third sub problem is $x_3{}^* = 0.5000$. Moreover, the maximum satisfactory level is $\lambda^* = 0.5000$. In addition, the proposed method is applied for each sub problem independently. Therefore, this method allows utilizing the TOPSIS and VIKOR to obtain a compromise solution for each sub problems.

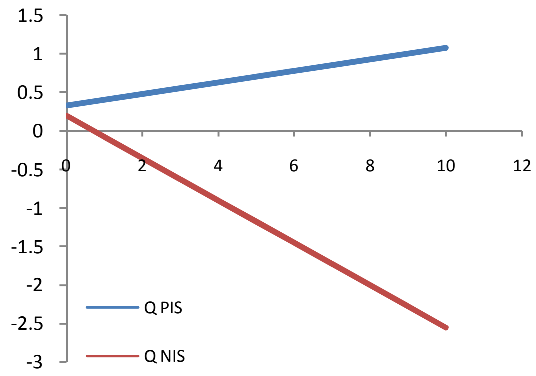

Figure1. The values of function Q_{ij} for problem P_1

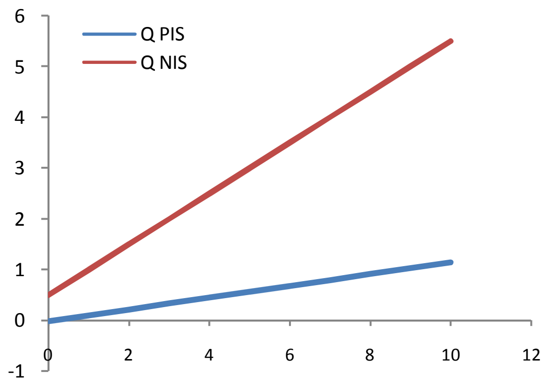

Figure2. The values of function Q_{ij} for problem P_2

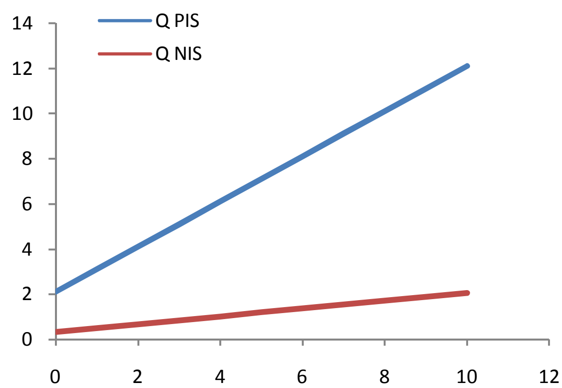

Figure3. The values of function Q_{ij} for problem

Moreover, the satisfactory level of each objective is determined.Applying VIKOR method, the result of sub problem 1 is solved and the optimum value of $x_1^* = 0.5000$and final solution of x_1^* in the proposed method is 0.5000. Moreover, the values of $x_2^*, x_3^*$are 1.0000, 0.5000 respectively, whereas the final solution of $x_2^*, x_3^*$are 4.0000, 1.0000 applying new proposed method.

5. CONCLUSION

In this paper, we focused on applying a new compromised approach to deal with MOLSLP problems with block angular structure.Our new method combines concepts from the TOPSIS and VIKOR methods. In other words, the proposed method, applies the advantages of TOPSIS and VIKOR methods simultaneously.Unlike the traditional VIKOR method which did not consider both relative distance from positive ideal solution (PIS) and negative ideal solution (NIS), this paper considered the distance of alternatives from the PIS and NIS simultaneously. Finally, to justify the proposed method, an illustrative example was provided. Then, the sensitivity analysis was described.Using TOPSIS and VIKOR, the new proposed method aggregated the fuzzy multi-objective into single objective based on new compromised logic. Because the uncertainty is the feature of real word decision making problems, the values of decision matrix can be presented with uncertainty. The proposed method assists experts to take data in the forms of linguistic terms in a programming problem. This leads to more realistic decision-making process in real situations. The Dantzig-Wolf decomposition method is utilized to reducean N-dimension problem into some q space sub problems. Therefore, the complexity of decision making problem is reduced. Then a useful method was applied to transfer each fuzzy sub problem to three crisp sub problems. Moreover, the fuzzy constraints were transferred into crisp constraints. Then, the proposed new compromised method was applied to obtain a suitable compromise solution. To obtain compromise solution of original problem, the individual positive ideal solution (PIS) and negative ideal solution (NIS) were calculated for each objective. Moreover, the VIKOR method is applied to calculate the amounts of ‘‘group utility’’ for the ‘‘majority’’ and the individual regret for the ‘‘opponent’’.The concept of membership function is introduced and applied to aggregate the objective functions in each sub problem. Therefore, this method can help the decision makers when the coefficient of objective functions and constraint is not crisp and the problem is large scale. Therefore, this method is applied in greater number of issues to deal with the real world problems. Finally, to justify the proposed method, an illustrative example was provided. The objective functions and constraints may be proposed as a fuzzy linear programming problem. Moreover, the programming problem can be proposed gray data. These subjects give a new opportunity for further research.

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A New Compromise Decision-making Model Based on TOPSIS and VIKOR for Solving Multi-objective Large-scale Programming Problems with a Block Angular Structure under Uncertainty

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چکیده

در این مقاله یک مدل سازشی بر اساس یک روش جدید به منظور حل مسائل برنامه ریزی چند هدفه مقیاس بزرگ با ساختار بلوکی زاویه دار شامل پارامترهای فازی ارائه می شود. مسئله مورد بررسی شامل پارامترهای فازی در توابع هدف و محدودیت ها می باشد.در روش های برنامه ریزی سازشی دو مفهوم به طور همزمان در تظر گرفته می شود. اولین مفهوم نزدیکی آلترناتیو ها به جواب ایده ال مثبت فازی و دوری از جواب ایده ال منفی می باشد. دومین مفهوم در نظر گرفته شده مبتنی بر مطلوبیت گروهی برای حداکثر و حداقل پشیمانی از نظرات می باشد. در روش ارائه شده یک الگوریتم تجزیه به منظور کاهش ابعاد فضای بزرگ تابع هدف مورد استفاده قرار گرفته است. به علاوه یک روش برنامه ریزی قطعی چند هدفه بر گرفته شده از مدل خطی فازی برای حل این مسئله ارائه شده است. سپس یک روش تصمیم گیری سازشی برمبنای روش های TOPSIS و VIKOR به صورت همزمان برای حل هر زیر مسئله بکار گرفته شده است. سرانجام به منظور تشریح مدل ارائه شده یک مثال توضیح داده شده است.

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