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Static Pull-in Analysis of Capacitive FGM Nanocantilevers Subjected to Thermal Moment using Eringen's Nonlocal Elasticity

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ABSTRACT

This paper aims to investigate the pull-in phenomenon of functionally graded (FG) capacitive nano-cantilevers subjected to an electrostatic force and thermal moment due to an applied voltage and thermal shock considering the intermolecular force within the framework of nonlocal elasticity theory to account for the small scale effect. The FG nano-beam is made of mixture of metal and ceramic which the material properties vary continuously through the thickness according to an exponential distribution law (E-FGM). The nonlocal elastic behavior is described by the differential constitutive model of Eringen which enables the present model to become effective in the analysis and design of nano-sensors. The nano-beam is modeled assuming the Euler–Bernoulli beam theory and the equations are derived using the equilibrium of an element. A Galerkin-based step by step linearization method has been used to solve the governing static deflection equation. The present solution is validated with existing results reported in previous studies. The effects of temperature change, Van der Waals (VdW) or Casimir force and small scale factor on the five types of FG nano-beams are discussed in detail. The results indicate that VdW/Casimir force and thermal moment reduce the pull-in voltage; however, on the contrary, small scale effect causes to slightly increase the amount of pull-in voltage.

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1. INTRODUCTION

Typically, a capacitive nano-beam device is constructed from two conducting electrodes where one is usually fixed and the other one is able to move in a manner that it is suspended by using a mechanical spring. By applying a voltage difference between two electrodes, the upper movable electrode displaces towards the ground electrode on account of the electrostatic force. At a certain voltage, the moving electrode becomes unstable and collapses or pulls-in to the ground plane. The voltage at this state is the so-called pull-in voltage[1].

Nano-structures are widely used in micro- and nano-scale devices and systems such as biosensors, atomic force micro-scopes, micro-electro-mechanical systems (MEMS) and nano-electro-mechanical systems (NEMS) result from their superior mechanical, chemical, and electronic properties[2]. As the dimensions of electromechanical systems reduce from micro- to nano-scale, intermolecular effects appear. The intermolecular force between two surfaces can be described by the Casimir interaction. Considering the ideal case, i.e. infinite parallel surfaces, perfect conductivity, etc, the Casimir interaction is proportional to the inverse fourth power of the separation. However, when separation is well below the plasma (for metals)/absorption (for dielectrics) wavelength of the surface material Casimir force should be corrected. In this case, the retardation is not significant and the intermolecular force between two surfaces varies with the inverse cube of the separation (Van der Waals force)[3-5].

Lin and Zhao applied an approximate analytical solution to study the Casimir force effect on the critical pull-in gap and pull-in voltage of nano-electro-mechanical switches[6]. Using cantilever beam with large deformation model, Wang et al.[7] investigated the pull-in instability of two nano-tubes under van der Waals force. They discussed the effect of some of the nano-tube parameters on the pull-in instability, as well. The influence of van der Waals (vdW) and Casimir

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forces on the stability of the electrostatic torsional NEMS actuators is analyzed by Guo and Zhao [8]. The nonlinear behavior of nano-scale electrostatic actuators considering the influence of Casimir force is carried out by Lin and Zhao[9]. They used models with one degree of freedom to obtain the bifurcation properties of the actuators. Abadyan et al.[10] used the homotopy perturbation method (HPM) in order to investigate the effect of the Casimir force on the pull-in instability of electrostatic actuators at nano-scale separations.

The small-size scales associated nanotechnologies are often sufficiently small to call the applicability of classical continuum models into question for nano-structures with very small dimensions. Classical or local continuum models do not admit intrinsic size dependence in the elastic solutions of inclusions and inhomogeneities. At micro- and nano-meter scales, however, size effect often become prominent, the cause of which need to be explicitly addressed with an increasing interest in the general area of nanotechnology [11]. These effects can be captured using size-dependent continuum mechanics such as strain gradient theory [12], modified couple stress theory [13], and nonlocal elasticity theory [14, 15]. It is worth pointing out that in the theories of strain gradient elasticity and modified couple stress, in addition to the classical stress components acting on elements of materials, the couple stress components, as higher-order stresses, are also available which tend to rotate the elements. These type of theories are able to predict the size effects with appearance of some higher-order material constants in the corresponding constitutive equations [16-18]. However, the nonlocal elasticity theory was proposed by Eringen to account for scale effect in elasticity by assuming the stress at a reference point to be a function of the strain field at every point in the body. In this way, the internal size scale could be considered in the constitutive equations simply as a material parameter.

In recent years, the applications of nonlocal elasticity, especially the nonlocal Euler–Bernoulli beam theory in micro- and nano-materials have turned into a hot research topic. The potential of applying the nonlocal Euler–Bernoulli beam theory to micro and nano-materials was indicated by Peddieson et al. [19] in which a nonlocal version of Euler–Bernoulli beam theory was formulated and applied to study a cantilever beam. Comparing the obtained results between MEMS and NEMS, they estimated the significance of nonlocal effects in nano-scale devices. Dequesnes et al. [20] studied the Pull-in phenomena and pull-in voltage of a carbon-based nano-electromechanical switch. They proposed a parametrized continuum model and compared the accuracy of their results with the reported experimental data. Reddy [21] derived theoretical formulations for nonlocal beams based on the Euler–Bernoulli, Timoshenko, Reddy and Levinson beam theories and brought out the effects of the nonlocal behavior on deflection, buckling load and natural frequencies. Wang et al. [22]presented analytical solutions for the free vibration of the nonlocal Timoshenko beams. The static pull-in instability of the beam-type nano-electromechanical systems has been measured theoretically by Beni et al.[23]. They investigated the instability considering the effect of Casimir attraction, elastic boundary conditions and size dependency. Thanks to the nonlocal elasticity theory, Yang et al. [24]studied the pull-in instability of the nano-switches subjected to the combination of electrostatic and intermolecular forces to account for the small scale effect. They found that the small scale effect contribute to the pull-in instability and freestanding behaviour of cantilever and fixed-fixed nano-beams in quite different ways. Peng et al.[25] reviewed the pull-in instability behavior of the nano scale actuators taking the nonlocal elasticity theory into account. Mousavi et al. [26]carried out a comprehensive study to determine the influence of nonlocal parameter on the pull-in instability characteristics of cantilever and clamped-clamped nano-beams. They observed that the cantilever and doubly clamped nano-beams behaved differently under small scale effect. That is, the nonlocal effect increases/decreases the pull-in voltage in cantilever/clamped-clamped nano-beams. Recently, by introducing a new formulation, Taghavi et al. [27]studied the pull-in instability of nano-switches under electrostatic and intermolecular forces. They used the hybrid nonlocal Euler-Bernoulli beam model and derived the governing equations of the beam-like movable electrodes of the cantilever and fixed-fixed nano-switches.

Since a temperature difference between FG MEMS/NEMS devices and operating environment causes a coupled behaviour, a full thermo-electro-mechanical analysis is required. Thus, the study of the FG structures under thermal loads certainly has been an active subject of research. Mohammadi-alasti et al. [28]studied the static behavior of the FG cantilever micro-beam subjected to a nonlinear electrostatic pressure and temperature changes. They derived nonlinear integral–differential thermo-electro-mechanical equation based on Euler–Bernoulli beam theory. Jafarsadeghi-pournaki et al. [29] and Zamanzadeh et al. [30] studied the static and dynamic instability of a FG micro-beam based on modified couple stress theory, respectively. In their studies, the micro-beam was subjected to both nonlinear electrostatic pressure and thermal changes.

The foregoing-mentioned articles reveal that a considerable amount of literature has been published on the static pull-in behavior of nano-beams using nonlocal elasticity theory. There are also a few published studies describing the effect of ceramic on the pull-in value of FG micro-beams under thermal moment. However, to the best of the authors' knowledge, by employing nonlocal elasticity theory, this is the first attempt on the static pull-in instability of FG nano-beam which is subjected to not only electrostatic pressure but also thermal load considering the effect of VdW/Casimir force. Using a Galerkin based step by step linearization method, nonlinear static deflection equation was solved and results are obtained in order to investigate the effects of small scale factor, VdW/Casimir force and ceramic constituent percentage on the static pull-in stability.

2. MECHANICAL MODEL AND GOVERNING EQUATIONS

Figure 1 shows a FG cantilever nano-beam of length L, width b, thickness h attached to an inertial reference frame $Oxyz$. The neutral axis and coordinates of the composite beam are shown in this figure, too.

In this study, the material properties of the FG nano-beam are assumed to vary through the thickness according to an exponential low function. Assuming $\bar{z}=z+h/2$, the exponential law is given by [28]:

$$P(z)=P_t e^{\delta_0 \bar{z}}=P_t e^{\delta_0 (z+h/2)}, \delta_0=\frac{1}{h}\ln\left(P_b/P_t\right) \tag{1}$$

$P(z)$ is a typical material property, P_t and P_b are the values of the properties at the top and bottom of the FG beam and δ_0 is defined as the dispersion of the ceramic into the metal. It is assumed that the top surface is made of pure metal and the bottom surface from a mixture of metal and ceramic. Also, it is assumed that ceramic content of the bottom surface varies from 0% to 100%. In order to determine material properties of the bottom surface (P_b), volume fraction of material is used [30]:

$$P_b = V_c P_c + V_m P_m \tag{2}$$

where, V is the volume fraction and subscripts "m" and "c" stand for the metal and ceramic, respectively. Therefore, V_m and V_c are the volume fractions of the metal and ceramic, respectively, and are related by: $V_m + V_c = 1$.

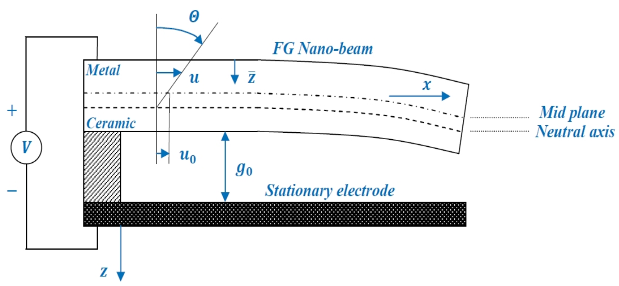


Figure 1. Geometry and coordinates of symmetric capacitive FG nano-beam. (Side and section view)

By changing the ceramic constituent percentage of the bottom surface, five different types of FG nano-beam are investigated. Using the above mentioned equations, parameter δ_0 for two material properties is specified as follow [30]:

$$\gamma=\frac{1}{h}\ln\left(E_b/E_m\right), \beta=\frac{1}{h}ln\left(\alpha_b/\alpha_m\right) \tag{3}$$

in which E and α denote the Young's modulus of elasticity and thermal expansion coefficient, respectively. Based on the Euler-Bernoulli beam theory, the displacement field (u, w) of an arbitrary point on the movable nano-beam can be expressed as [30]:

$$u=u_0 - z\frac{\partial w}{\partial x}, w=w_0(x) \tag{4}$$

where (u, w) are the axial and transverse displacements at any generic point and u_0 and w_0 are their counterparts calculated at the mid-plane. The only nonzero strain is given as[30]:

$$\varepsilon_x = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w}{\partial x^2} \tag{5}$$

In the classical elasticity, the stress state of any body at a point x is related to the strain state at the same point x. The constitutive equations of classical (macroscopic) elasticity are algebraic relationships between the stress and strain components. But, this theory is not in conflict with the atomic theory of lattice dynamics and experimental observations of phonon dispersion. As stated by Eringen [14, 15] the linear theory of nonlocal elasticity leads to a set of integropartial differential equations for the displacements field for homogeneous, isotropic bodies. In this theory, the fundamental equations involve spatial integrals which represent weighted averages of the contributions of related strain tensor at the related point in the body. Thus, the theory introduces the small length scale effect through a spatial integral constitutive relation.

As the structure is subjected to both temperature changes and mechanical load, the total strain can be decomposed into summation of mechanical and thermal components. According to Eringen's nonlocal elasticity theory, the one dimensional stress–strain relationship for the FG nano-beam is [14-30]:

$$\sigma_x = \mu\frac{\partial^2 \sigma_x}{\partial x^2} + E(z)\left(\varepsilon_x - \alpha(z)\theta\right);\ \ \mu=(e_0 a)^2 \tag{6}$$

where σ_x is axial stress, θ refers to temperature change measured with respect to an initial temperature T_∞, and $(e_0 a)$ is specified as the parameter showing the small scale effect on the response of the structure and may be determined from experiments or by matching dispersion curves of plane waves with those of atomic lattice dynamics [31]. By considering that the axial force, due to the free end of the beam, along the x axis is zero, it can be concluded that:

$$\int_A \sigma_x dA = 0 \tag{7}$$

Substituting Equation (6) into Equation (7):

$$\frac{\partial u_0}{\partial x} = \frac{1}{\int_A E(z)dA}\left(\int_A zE(z)dA\frac{\partial^2 w}{\partial x^2} + \int_A E(z)\alpha(z)dA\theta\right) \tag{8}$$

Multiplying Equation (7) by z, integrating over the cross-section area,the moment–curvature relation can be obtained as:

$$M(x) = \mu\frac{\partial^2 M}{\partial x^2} + (EI)_{eq}\frac{\partial^2 w}{\partial x^2} + \Omega\theta \tag{9}$$

For the purpose of this study $(EI)_{eq}$ and $\Omega\theta$ are taken to mean as the equivalent bending rigidity of FG nano-beam and thermal moment, respectively, where:

$$(EI)_{eq} = \int_A E(z)z^2 dA - \int_A \frac{\int_A zE(z)dA}{\int_A E(z)dA} E(z)zdA \tag{10}$$

$$\Omega = \int_A \frac{\int_A E(z)\alpha(z)dA}{\int_A E(z)dA} E(z)zdA - \int_A E(z)\alpha(z)zdA \tag{11}$$

For an Euler-Bernoulli nano-beam, shear deformation is ignored. The equilibrium equation of an element with respect to z-axis can be obtained as:

$$\left(V + \frac{\partial V}{\partial x}\right)dx - Vdx + Q(x)\,dx = 0 \tag{12}$$

where M is bending moment, V shear force and $Q(x)$ the distributed load. The equilibrium requirements of forces in the vertical direction and moments of an infinitesimal element of the beam give:

$$\frac{\partial V}{\partial x} = -Q(x) \Rightarrow \frac{\partial^2 M}{\partial x^2} = -Q(x) \tag{13}$$

Multiplying this equation by μ and substituting it into Equation (9), the governing equation for the static deformation of the FG nano-beam can be easily derived as:

$$(EI)_{eq}\frac{\partial^4 w}{\partial x^4} = Q(x) - \mu\frac{\partial^2 Q(x)}{\partial x^2} \tag{14}$$

Further, considering Equations (9) and (13), the expressions for the moment and shear force are expressed in terms of deflection, transverse load and thermal moment ($M_T = \Omega\theta$):

$$M(x) = -\mu Q(x) + (EI)_{eq}\frac{\partial^2 w}{\partial x^2} + M_T \tag{15}$$

$$V(x) = -\mu\frac{\partial Q(x)}{\partial x} + (EI)_{eq}\frac{\partial^3 w}{\partial x^3} \tag{16}$$

When the actuating voltage is applied between the nano-beam and substrate, the electrostatic force per unit length are computed using a standard capacitance model and is equal with [30]:

$$Q_{elect} = \frac{\varepsilon_0 bV^2}{2(g_0 - w)^2} \tag{17}$$

where $\varepsilon_0 = 8.85\times10^{-12}C^2N^{-1}m^{-2}$ is vacuum permittivity, b the width of the FG nano-beam, g_0 the initial gap between the nano-beam and the ground electrode, V the applied DC voltage, and w the flexural deflection. For the small/large separation regime, the dispersion force per unit length of the beam is defined considering the VdW/Casimir force [8]. The VdW(Q_3) and Casimir (Q_4) forces per unit length between nano-beam and substrate is given by:

$$Q_s = \frac{\Lambda_s}{(g_0 - w)^s};\quad s = 3,4\ \Lambda_3 = \frac{\mathcal{A}b}{6\pi}; \Lambda_4 = \frac{\pi^2\hbar cb}{240} \tag{18}$$

where $\mathcal{A} = (0.4-4)\times10^{-19}J$ is Hamaker's constant, $\hbar = 1.055\times10^{-34}Js$ is Planck's constant and $c = 2.998\times10^8$ is the speed of light [8]. Finally, $Q(x)$ is determined as the summation of the electrostatic force and the intermolecular force:

$$Q = Q_{elect} + Q_s \tag{19}$$

Thus, the governing static deflection of nonlocal Euler–Bernoulli beam can be derived by substituting Equation (19) into Equation (14):

$$(EI)_{eq}\frac{\partial^4 w}{\partial x^4} = \frac{\varepsilon_0 bV^2}{2(g_0 - w)^2} + \frac{\Lambda_s}{(g_0 - w)^s} \tag{20}$$

$$-\mu\left(\frac{3\varepsilon_0 bV^2}{(g_0-w)^4}+\frac{s(s+1)\Lambda_s}{(g_0-w)^{s+2}}\right)\left(\frac{\partial w}{\partial x}\right)^2$$
$$-\mu\left(\frac{\varepsilon_0 bV^2}{(g_0-w)^3}+\frac{s\Lambda_s}{(g_0-w)^{s+1}}\right)\left(\frac{\partial^2 w}{\partial^2 x}\right)$$

Besides, the moment and shear force are converted to the subsequent equations:

$$M(x)=-\mu\left(\frac{\varepsilon_0 bV^2}{2(g_0-w)^2}+\frac{\Lambda_s}{(g_0-w)^s}\right)+(EI)_{eq}\frac{\partial^2 w}{\partial^2 x}+M_T \tag{21}$$

$$V(x)=-\mu\left(\frac{\varepsilon_0 bV^2}{(g_0-w)^3}+\frac{s\Lambda_s}{(g_0-w)^{s+1}}\right)\frac{\partial w}{\partial x}+(EI)_{eq}\frac{\partial^3 w}{\partial^3 x} \tag{22}$$

It is easily seen from the above equations that the local Euler–Bernoulli beam theory is recovered when the parameter μ is set identically to zero.

Following dimensionless quantities are presented in order to rearrange the equations into a non-dimensional form:

$$\hat{w}=\frac{w}{g_0},\ \hat{x}=\frac{x}{L},\ \widehat{M}=\frac{M}{M^*},\ \widehat{M}_T=\frac{M_T}{M^*} \tag{23}$$

Therefore, the non-dimensional static deflection equation is:

$$\begin{aligned}\frac{\partial^4\hat{w}}{\partial\hat{x}^4}&=A_1\frac{V^2}{(1-\hat{w})^2}+A_2\frac{1}{(1-\hat{w})^s}\\&-\left[A_3\frac{V^2}{(1-\hat{w})^4}+A_4\frac{1}{(1-\hat{w})^{s+2}}\right]\frac{\partial^4\hat{w}}{\partial\hat{x}^4}\\&-\left[A_5\frac{V^2}{(1-\hat{w})^3}+A_6\frac{1}{(1-\hat{w})^{s+1}}\right]\left(\frac{\partial^2\hat{w}}{\partial\hat{x}^2}\right)\end{aligned} \tag{24}$$

where:

$$\begin{aligned}&A_1=\frac{L^4\varepsilon_0 b}{2(EI)_{eq}g_0^3},\ A_2=\frac{L^4\Lambda_s}{(EI)_{eq}g_0^{s+1}},\ A_3=\mu\frac{3\varepsilon_0 L^2 b}{(EI)_{eq}g_0^3}\\&A_4=\mu\frac{L^2 s(s+1)\Lambda_s}{(EI)_{eq}g_0^{s+1}},\ A_5=\mu\frac{\varepsilon_0 L^2 b}{(EI)_{eq}g_0^3},\ A_6=\mu\frac{L^2 s\Lambda_s}{(EI)_{eq}g_0^{s+1}}\end{aligned} \tag{25}$$

Moreover, the non-dimensional form of the Equations (21) and (22) can be obtained:

$$\widehat{M}=-B_1\frac{V^2}{(1-\hat{w})^2}-B_2\frac{1}{(1-\hat{w})^4}+\frac{\partial^2\hat{w}}{\partial\hat{x}^2}+\widehat{M}_T \tag{26}$$

$$\hat{V}=-\left(B_3\frac{V^2}{(1-\hat{w})^3}+B_4\frac{1}{(1-\hat{w})^{s+1}}\right)\frac{\partial\hat{w}}{\partial\hat{x}}+\frac{\partial^3\hat{w}}{\partial\hat{x}^3} \tag{27}$$

in which:

$$\begin{aligned}&B_1=\mu\frac{\varepsilon_0 bL^2}{2(EI)_{eq}g_0^3},\ B_2=\mu\frac{L^2\Lambda_s}{(EI)_{eq}g_0^{s+1}},\\&B_3=\mu\frac{\varepsilon_0 bL^2}{(EI)_{eq}g_0^3},\ B_4=\mu\frac{L^2 s\Lambda_s}{(EI)_{eq}g_0^{s+1}}\end{aligned} \tag{28}$$

Eventually, the boundary conditions for the FG cantilever nano-beam subjected to thermal moment and considering intermolecular force and taking nonlocal elasticity theory into account are introduced as:

$$\hat{x}=0\ \Rightarrow\ \hat{w}=0,\quad \frac{\partial\hat{w}}{\partial\hat{x}}=0$$

$$\hat{x}=1\Rightarrow\begin{cases}-B_1\dfrac{V^2}{(1-\hat{w})^2}-B_2\dfrac{1}{(1-\hat{w})^s}+\dfrac{\partial^2\hat{w}}{\partial\hat{x}^2}+\widehat{M}_T=0\\-\left(B_3\dfrac{V^2}{(1-\hat{w})^3}+B_4\dfrac{1}{(1-\hat{w})^{s+1}}\right)\dfrac{\partial\hat{w}}{\partial\hat{x}}+\dfrac{\partial^3\hat{w}}{\partial\hat{x}^3}=0\end{cases} \tag{29}$$

3. NUMERICAL APPROACH

Thanks to the nonlinear nature of the governing equations, the analytical solution methods cannot be used, therefore, the step by step linearization method (SSLM) [32] method has been used in order to linearize the equations. Afterwards, the obtained linearized differential equation is solved using a Galerkin based weighted residual method. Using this method, the smooth and continuous behavior of the beam can be approximated in each step and the amount of nonlinear forces in each step will be obtained from foregoing iterations. By using SSLM, the voltage applied to the nano-beam is increased from zero to its final value gradually. It is supposed that $\hat{w}_i$ is the displacement of the FG nano-beam dye to applied voltage V_i. With increasing the voltage and consequent virtual force variable (λ) the deflection of $(i+1)^{th}$ step may be obtained as [32]:

$$\begin{cases}V_{i+1}=V_i+\delta V\\ \lambda_{i+1}=\lambda_i+\delta\lambda\end{cases}\Rightarrow \hat{w}_{i+1}=\hat{w}_i+\delta\hat{w},\ \delta\hat{w}=\psi(\hat{x}) \tag{30}$$

in which δV, $\delta\lambda$ and $\delta\hat{w}$ are voltage variable, virtual force variable and deflection growth, respectively. So, the equation of the static deflection of the FG nano-beam can be rewritten at the step of (i+1) as follows:

$$\frac{\partial^4 \hat{w}_{i+1}}{\partial \hat{x}^4} = A_1 \frac{V_{i+1}^2}{\left(1-\hat{w}_{i+1}\right)^2} + A_2 \frac{\lambda_{i+1}}{\left(1-\hat{w}_{i+1}\right)^s} - \left[A_3 \frac{V_{i+1}^2}{\left(1-\hat{w}_{i+1}\right)^4} + A_4 \frac{\lambda_{i+1}}{\left(1-\hat{w}_{i+1}\right)^{s+2}}\right]\left(\frac{\partial \hat{w}_{i+1}}{\partial \hat{x}}\right)^2 - \left[A_5 \frac{V_{i+1}^2}{\left(1-\hat{w}_{i+1}\right)^3} + A_6 \frac{\lambda_{i+1}}{\left(1-\hat{w}_{i+1}\right)^{s+1}}\right]\left(\frac{\partial^2 \hat{w}_{i+1}}{\partial \hat{x}^2}\right) \tag{31}$$

By considering a small value of δV and $\delta\lambda$, the $\delta\hat{w}$ will be small enough; therefore, using Calculus of Variations theory and Taylor's series expansion about $\hat{w}_i$ and applying the truncation to first order for suitable values of δV and $\delta\lambda$, it is possible to obtain the desired accuracy. The linearized equation to calculate $\psi(\hat{x})$ can be concluded as:

$$\begin{aligned}
&\frac{\partial^4 \Psi}{\partial \hat{x}^4} - \left[A_1 \frac{2V_i^2}{\left(1-\hat{w}_i\right)^3} + A_2 \frac{s\lambda_i}{\left(1-\hat{w}_i\right)^{s+1}}\right]\Psi \\
&+ \left[A_3 \frac{4V_i^2}{\left(1-\hat{w}_i\right)^5} + A_4 \frac{(s+2)\lambda_i}{\left(1-\hat{w}_i\right)^{s+3}}\right]\left(\frac{\partial \hat{w}_i}{\partial \hat{x}}\right)^2 \Psi \\
&+ \left[A_3 \frac{V_i^2}{\left(1-\hat{w}_i\right)^4} + A_4 \frac{\lambda_i}{\left(1-\hat{w}_i\right)^{s+2}}\right] 2\left(\frac{\partial \hat{w}_i}{\partial \hat{x}}\right)\frac{\partial \Psi}{\partial \hat{x}} \\
&+ \left[A_5 \frac{3V_i^2}{\left(1-\hat{w}_i\right)^3} + A_6 \frac{s(s+1)\lambda_i}{\left(1-\hat{w}_i\right)^{s+2}}\right]\left(\frac{\partial^2 \hat{w}_i}{\partial \hat{x}^2}\right)\Psi \\
&+ \left[A_5 \frac{V_i^2}{\left(1-\hat{w}_i\right)^3} + A_6 \frac{\lambda_i}{\left(1-\hat{w}_i\right)^{s+2}}\right]\left(\frac{\partial^2 \Psi}{\partial \hat{x}^2}\right) \\
&= A_1 \frac{2V_i\delta V}{\left(1-\hat{w}_i\right)^2} + A_2 \frac{\delta\lambda}{\left(1-\hat{w}_i\right)^s} \\
&- \left[A_3 \frac{2V_i\delta V}{\left(1-\hat{w}_i\right)^4} + A_4 \frac{\delta\lambda}{\left(1-\hat{w}_i\right)^{s+2}}\right]\left(\frac{\partial \hat{w}_i}{\partial \hat{x}}\right)^2 \\
&- \left[A_5 \frac{2V_i\delta V}{\left(1-\hat{w}_i\right)^3} + A_6 \frac{\delta\lambda}{\left(1-\hat{w}_i\right)^{s+1}}\right]\left(\frac{\partial^2 \hat{w}_i}{\partial \hat{x}^2}\right)
\end{aligned} \tag{32}$$

An approximate solution for this linear ordinary differential equation based on the function spaces in terms of basic functions is:

$$\psi(\hat{x}) = \sum_{j=1}^{n} a_I \varphi_I(\hat{x}) \tag{33}$$

where $\varphi_i(\hat{x})$ is suitable shape function satisfying the corresponding nonlinear boundary conditions and a_i are constant coefficients calculated in each step by applying Galerkin weighted residual method.

4. NUMERICAL RESULTS

The considered nano-beam here is a wide beam, which has the geometric and material properties are listed in Tables 1 and 2.According to the different ceramic content of the bottom surface, five different types of FG nano-beam are investigated, the characteristics of which are shown in Table 3.To compare the results with those of the literature, the obtained pull-in voltages are validated using the results of References [28]. The parameters used in that micro-beam are presented in Table 4.

TABLE 1. Geometrical properties of FG nano-beam

Parameter	Length (L)	Width (b)	Thickness (h)	Initial gap (g_0)
Value	50 nm	10 nm	5 nm	7 nm

TABLE 2. Material properties of FG nano-beam [28]

Parameter	Value	
	Ceramic	metal
Material type	Siliconnitride (Si_3N_4)	Nickel (Ni)
Young's modulus (E)	310 GPa	204 GPa
Thermal expansion (α)	3.4×10^{-6} K^{-1}	13.2×10^{-6} K^{-1}

TABLE 3. Characteristics of five types of FG nano-beam [28]

Type	1	2	3	4	5
Ceramic percent of Bottom surface	**0% (Metal-rich)**	**25%**	**50%**	**75%**	**100%**
$E_b(Gpa)$	204	230.5	257	283.5	310
$\alpha_b\times10^{-6}(K^{-1})$	13.2	10.75	8.3	5.85	3.4
$\gamma\times10^7$	0	2.44	4.62	6.54	8.37
$\beta\times10^7$	0	-4.1	-9.28	-16.2	-27.1

TABLE 4. Geometrical properties of FG micro-beam [28]

Parameter	Length (L)	Width (*b*)	Thickness (*h*)	Initial gap (g_0)
Value	500 μm	90 μm	6 μm	2 μm

TABLE 5. Pull-in voltages for five types of FG micro-beam [28]

Type	1	2	3	4	5
Voltage (V)	13.48	13.75	14.19	14.52	14.85

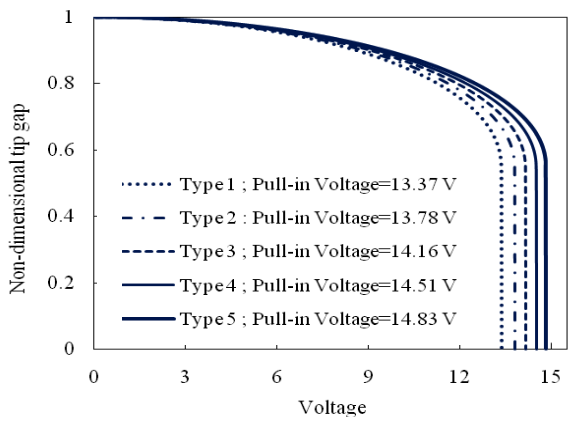


Figure 2. Tip gap versus applied voltage for five types of FG micro-beams ($\theta=0$).

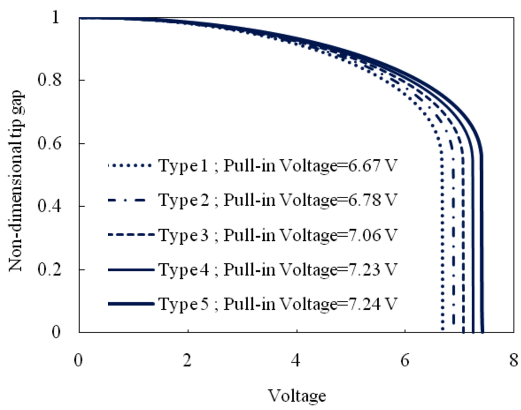


Figure 3. Tip gap versus applied voltage for five types of FG nano-beams ($\theta=0$).

Figure 2 illustrates the non-dimensional end deflection for five types of FG micro-beams versus voltage considering the geometrical properties reported in Table 4. Comparing the data of Table 5 with Figure 2 can clarify that the present results are quite close to those predicted by mohammadi-Alasti et al. [28].

Figure 3 shows the non-dimensional end deflection for five types of FG nano-beams versus voltage when temperature changes and VdW/Casimir force as well as small scale effect are neglected. It is apparent from this figure that enhancing the ceramic constituent of the FG nano-beam causes decreasing of the deflection, and as a result, the pull-in phenomenon happens in the higher voltages. Note that the bigger the ceramic constituent, the stiffer the FG nano-beam.

When the gap between substrate and movable beam is small enough, the movable beam might collapse onto the stationary electrode without applying voltage due to the Casimir or VdW force. Accordingly, the geometrical dimensions used in this study are determined such that with the purpose of ignoring this kind of collapse [24]. The effects of VdW (Figure 4a) and Casimir (Figure 4 b)force on the static pull-in voltages for five types of FG nano-beams is presented in Figure 4. The data would seem to suggest that considering VdW and Casimir forces causes a reduction in the amount of pull-in voltages. Moreover, the beam has initial deflection due to the presence of intermolecular force even when no voltage is applied. Compering the data of Figure 4 with that of Figure 3 results that the effect of Casimir force on the pull-in instability is more significant than the VdW force. Therefore, from now on, VdW force is neglected and the results are obtained by considering only Casimir force.

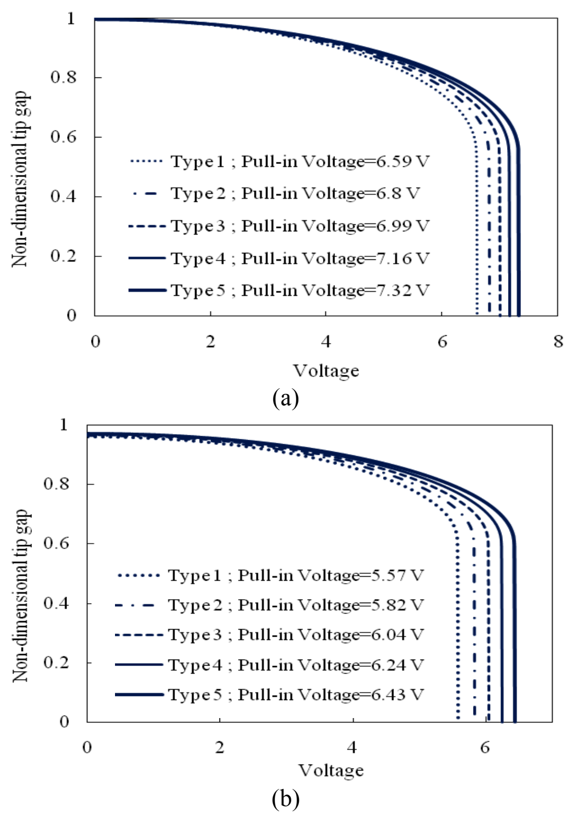
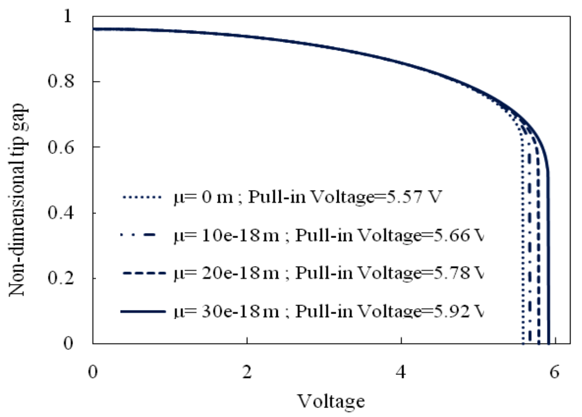
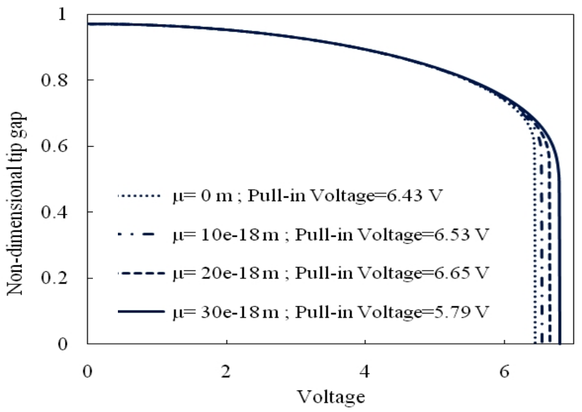


Figure 4. Tip gap versus applied voltage for five types of FG nano-beams considering, a) VdW force, b) Casimir force ($\theta=0$).

Figure 5 depicts the effect of small scale factor on the static pull-in voltages for homogenous (metal rich) (a) and FG (b) nano-beams. As expected, as the small scale factor increases, the pull-in voltages of the cantilever nano-beam increase.



(a)



(b)

Figure 5. Tip gap versus applied voltage, a) homogenous nano-beam, b) 5^{th} type of FG nano-beam ($\theta=0$).

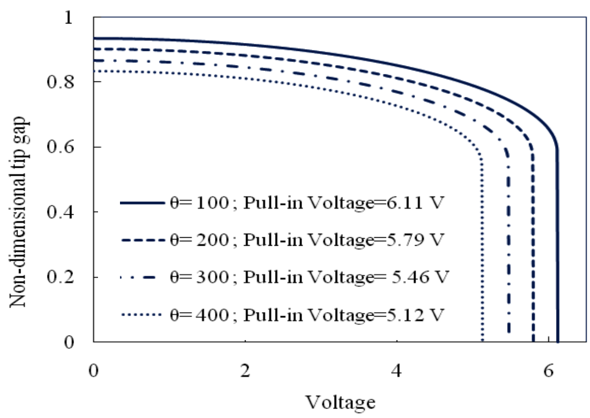


Figure 6. Tip gap versus applied voltage for 5th type of FG nano-beam with primal temperature changes considering Casimir force.

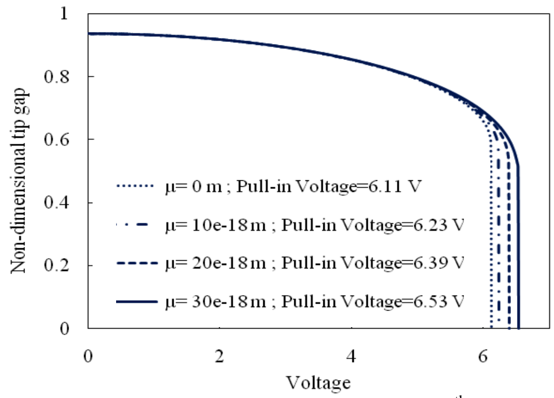


Figure 7. Tip gap versus applied voltage for 5^{th} type of FG nano-beam ($\theta=100$).

The considerable influence of the temperature changes on the pull-in voltage for 5th type of FG nano-beam is illustrated in Figure 6. What is interesting in this data is that, when the nano-beam heats up primarily, the pull-in phenomenon happens earlier than when there is no temperature. It means that the pull-in voltage will be decreased when the primal temperature is increased.

Figure 7presents the influence of small scale factor on the static pull-in instability for 5^{th} type of FG nano-beam when the beam is initially deflected by a thermal moment due to a temperature change of 100°C. In accordance with what was discussed so far, for the cantilever nano-beam, the inclusion of the small scale factor gives rise to a reduction of pull-in voltage values.

3. CONCLUSION

Coming to conclusion, this study set out in order to investigate the static pull-in instability of FG nano-cantilevers under nonlinear electrostatic and intermolecular forces subjected to a thermal moment based on Eringen's nonlocal elasticity. It was assumed that the upper surface was made of pure metal and the lower surface a mixture of metal and ceramic. Changing the ceramic constituent of the bottom surface, five different types of FG nano-beams were studied. Considering an exponential form to represent the continuous variation of material properties along the beam thickness, the nonlinear governing equations and boundary conditions based on Euler-Bernoulli beam theory in the framework of nonlocal elasticity theory was derived. The static instability of the FG nano-beam was studied through solving the equation of static deflection implementing SSLM and Galerkin method. This study produced results that corroborate the findings of a great number of the previous works in this field. It is found that the VdW/Casimir force and small scale factor regarding nonlocal continuum theory affect the pull-in behavior of capacitive nano-beams. As argued previously, for a cantilever nano-beam, considering the VdW/Casimir forceled the nano-beam to be deflected initially. Corresponding, based on the material properties and geometries assumed in this study, it was seen that the Casimir force has more influence than VdW force. Moreover, the results pointed out the significance of applying nonlocal continuum mechanics in nano-sized strictures. It is confirmed that for a cantilever-type nano-beams, an increase in the small scale factor leads to higher pull-in voltage. Another important result found was that temperature changes have considerable effect on the stability of the FG nano-beams. Indeed, applying voltage to the thermally deflected FG nano-beam can remarkably reduce the pull-in voltage. The high amount of the initially applied thermal moment can cause the low pull-in voltage value.

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Static Pull-in Analysis of Capacitive FGM Nanocantilevers Subjected to Thermal Moment using Eringen's Nonlocal Elasticity

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چکیده

هدف از این مقاله بررسی پدیده های ولتاژ ناپایداری استاتیکی نانوتیر یک‌سر گیردار خازنی ساخته شده از مواد متغیر تابعی می‌باشد. نانو تیر حاضر در معرض یک نیروی الکتروستاتیکی، لنگر حرارتی و نیروی بین مولکولی بوده و برای در نظر گرفتن اثرات اندازه از تئوری الاستیسیته غیرموضعی استفاده شده است. فرض شده است که مواد خواص‌متغیر تدریجی از مخلوط سرامیک و فلز ساخته شده و خصوصیات مواد در جهت ضخامت تیرتحت تابع نمایی تغییر کند. رفتار الاستیک غیر موضعی که توسط اِرینگن ارائه شده است، قادر می‌سازد که مدل حاضر در طراحی و آنالیز سنسورها موثرتر باشد. این نانوتیر با استفاده ازفرضیه تئوری تیر اویلر-برنولی مدل شده و معادلات با استفاده از روابط تعادل یک المان استخراج شده‌اند. با استفاده از روش خطی سازی گام به گام به همراه روش گلرکین، معادلات حاکم بر تغییر شکل استاتیکی حل شده‌اند. همچنین، حل حاضر با نتایج گزارش شده در کارهای قبلی تطبیق داده شده است. تاثیرات تغییرات دما، نیروی واندروالس یا کزیمیر و فاکتور اثر اندازه روی پنج نوع مختلف نانوتیر با خواص متغیر تدریجی به جزئیات بحث شده و نتایج نشان دادند که اعمال نیروی واندروالس یا کزیمیر و لنگر حرارتی باعث کاهش ولتاژهای ناپایداری و برعکس، اعمال تاثیرات اندازه باعث افزایش اندک ولتاژهای ناپایداری می‌شوند.

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