



## A New Nonlinear Multi-objective Redundancy Allocation Model with the Choice of Redundancy Strategy Solved by Compromise Programming Approach

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### ABSTRACT

One of the primary concerns in any system design problem is to prepare a highly reliable system with minimum cost. One way to increase the reliability of systems is to use redundancy in different forms such as active or standby. In this paper, a new nonlinear multi-objective integer programming model with the choice of redundancy strategy and component type is developed where standby strategy is of cold type. In the proposed model, system's reliability is maximized along with minimizing system's cost and weight. The proposed model contributes to the literature by determining the redundancy strategies concurrently with determining redundancy levels and component types. The multi-objective model is solved using the mathematical compromise programming technique for different  $L_p$  metrics and produces different Pareto solutions.

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## 1. INTRODUCTION

Reliability optimization is an important field in system engineering which has gained more attentions in recent years. One way to increase systems' reliabilities is to use redundant units using active or standby strategies. Standby electric generators in hospitals and other public facilities are a typical example of redundancy. As another normal example we can refer to use of electronic gyroscope and mechanical gyroscope in aircrafts [1]. Redundancy allocation is a field through which optimal number of redundant components is determined such that the system's reliability is maximized. In this paper the redundancy allocation problem (RAP) in systems with serial parallel structures and with choices of redundancy strategy and component type is studied where standby strategy is of cold type. Production system is an example of serial systems [2]. Space exploration and satellite systems achieve high reliability using cold standby redundancy for non-repairable systems [3]. Space inertial reference units are required to accurately monitor critical information for extended mission times without opportunities for repair.

Examples of non-repairable cold standby redundancy in spacecrafts can be found in [4, 5]. Many other systems use cold standby redundancy as an effective strategy to achieve high reliability including textile manufacturing systems [6], carbon recovery systems used in fertilizer plants [7] and active and passive safety systems in road vehicles [8], just to name a few. As an example for active strategy we can mention the serial parallel water desalination system which consists of filters, pumps, reverse osmosis membranes and power commutation equipment blocks in series for each of which there is an option to add redundancy in the form of parallel components.

To show the place of the proposed model, the literature is reviewed and classified into two main categories, where each one consists of single objective or multi-objective models. Note that the main focus is on models which were proposed for each category and less attention is paid to existing solution approaches.

In the area of single objective models with active strategy, Fyffe et al. [9] were the first who proposed a model for RAP where system's reliability is maximized subject to constraints on cost and weight. Ramirez-Marquez et al. [10] modeled the RAP using max-min approach, where the reliability of the subsystem with

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minimum reliability is maximized subject to constraints on cost and weight. Sun and Ruan [11] formulated the RAP such that system's cost is minimized subject to the requirement of meeting the minimum system's reliability. They presented an exact algorithm to solve the model.

In the field of multi-objective models with active strategy, Coit and Konak [12] considered a multi-objective RAP where the reliabilities of subsystems are maximized, simultaneously subject to constraints on cost and weight. They presented a multiple weighted objective heuristic to solve the model. Salazar et al. [13] studied three types of reliability optimization problems including redundancy allocation, reliability allocation and reliability-redundancy allocation. Their proposed multi-objective RAP maximizes system's reliability at the same time of minimizing system's cost and solved it through NSGAI. Taboada and Coit [14] considered a multi-objective model, which maximizes system's reliability concurrently with minimizing system's cost and weight and solved it using NSGA. Taboada and Coit [15] proposed a multiple objective evolutionary algorithm to solve a multi-objective redundancy allocation problem where the objectives were maximizing system's reliability and minimizing system's cost and weight. Wang et al. [16] considered a multi-objective RAP to maximize system's reliability and minimize system's cost with nonlinear cost and weight and solved the resulted model using NSGAI. Mahapatra [17] presented a bi-objective model, which simultaneously maximized the system's reliability and entropy considering nonlinear cost constraint. They solved the resulted model using global criterion method. Soylu and Ulusoy [18] considered the problem of maximizing the minimum subsystem reliability concurrently with minimizing the overall system cost and found the Pareto solutions of this problem by the augmented epsilon-constraint approach for small and moderate sized instances. Then they applied a well-known sorting procedure, UTADIS, to categorize the solutions into preference ordered classes. Khalili-Damghani and Amiri [19] considered an existing multi-objective RAP which involved maximizing system's reliability and minimizing system's cost and weight and solved it through a method based on epsilon-constraint and data envelopment analysis.

In the area of single objective RAP with cold standby strategy, Coit [20] studied cold standby redundancy optimization for non-repairable systems and developed a zero-one linear programming model to solve the problem. Coit [21] studied the same redundancy allocation problem where there were redundancy strategy choices for subsystems. In their approach the redundancy strategies were determined prior to redundancy levels. In application of meta-heuristics, Tavakkoli-Moghaddam et al. [22] developed a genetic algorithm to solve the same problem proposed

by Coit [21]. Later, bi or multi-objective version of the mentioned problem was studied by some scholars. Safari [23] and Chambari et al. [24] separately considered a bi-objective model for RAP to optimize reliability and cost of system with choice of redundancy strategy and solved the resulted model through NSGAI. Azizmohammadi et al. [25] considered a multi-objective RAP where the system reliability is maximized while minimizing the system's cost and volume. They proposed a hybrid multi-objective imperialist competition algorithm to solve the model.

In this paper, a new nonlinear multi-objective integer programming model for redundancy allocation with the choice of redundancy strategy is presented with three contradictory objectives including system's reliability, cost and weight. As mentioned previously, the first model for RAP with choice of redundancy strategy was proposed by Coit [20] and since then researchers have focused on solution approaches mostly in forms of meta-heuristics. In Coit's approach, redundancy levels are determined after deciding about redundancy strategies. In this paper, we present a new mathematical model to decide about redundancy strategies concurrently with redundancy levels and component type and it can be directly solved using typical optimization packages. In addition, the mathematical compromise programming approach is implemented for the first time to deal with the proposed multi-objective model.

The rest of the paper is organized as follows. In section 2 the studying problem is described and the proposed mathematical model is presented. Compromise programming technique as a solution procedure is presented in section 3. Experimental results are presented in section 4. Finally, conclusion is presented in section 5 along with some future research directions.

## 2. PROBLEM DEFINITION AND FORMULATION

The studying issue of this paper deals with designing highly reliable systems or products, which have serial parallel structure as illustrated in Figure 1. In this system, a number of subsystems are connected in series which each of its components work in parallel to enhance the system's reliability. The redundancy strategy of each subsystem can be in active or standby forms. In active redundancy, all components are simultaneously ready to use, whereas in standby redundancy just one component is in use and the others respectively begin to work only when the operating component fails. Standby redundancy is in three forms of cold, hot and warm standby. In this paper, we consider cold standby redundancy strategy along with active redundancy. Also, some subsystem may use no redundant component and just one component works therein.

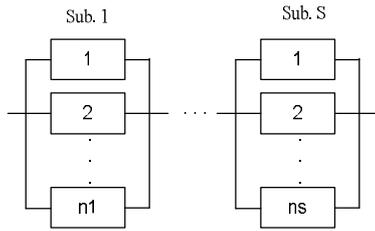


Figure 1. General serial parallel redundancy system

It is apparent that use of redundant component increases system's cost and weight which is not desirable. Therefore, designers try to minimize system's cost and weight together with maximizing systems' reliability. In this section, we propose a multi-objective model involving these objectives, which determines the suitable redundancy strategy, component type and redundancy level in each subsystem.

**Assumptions**

- The components are in two states of functioning or non-functioning, i.e. binary state.
- Components' time to failures follow Erlang distribution.
- The standby strategy is of cold type and the standby units do not fail before they are put into operation.
- The switch reliability to cold standby component is assumed to be imperfect.
- There are different component types with different specifications.
- Just one component type can be allocated in each subsystem.
- There is no repair or preventive maintenance.
- The replacement time is negligible.

**Decision Variables:**

- $n_{i,j,k}$  Number of components of type  $j$  used in subsystem  $i$  under strategy  $k$  ( $k \in A, S, N$ ).
- $Z_{i,j,k}$  A binary variable which is one if the component of type  $j$  is used in subsystem  $i$  under strategy  $k$  and zero otherwise.
- $X_{q,i,j}$  A binary variable which is one if  $q$  of the component of type  $j$  is used in subsystem  $i$  under standby strategy.

$$\begin{aligned}
 \text{Max } R(t) &= \prod_{i=1}^m (1 - \sum_{j=1}^{T_i} Z_{i,j,A} \times \prod_{j=1}^{T_i} (1 - r_{i,j}(t))^{n_{i,j,A}}) \times \\
 &\prod_{i=1}^m [ (1 - \sum_{j=1}^{T_i} Z_{i,j,S}) + \sum_{j=1}^{T_i} Z_{i,j,S} \times (r_{i,j}(t) + \delta_i(t) \times \exp(-\lambda_{i,j} t) \sum_{q=2}^{n_{\max}} X_{q,i,j} \sum_{l=k_{i,j}}^{k_{i,j} \times q - 1} \frac{(\lambda_{i,j} t)^l}{l!}) ] \times \prod_{i=1}^m \prod_{j=1}^{T_i} (r_{i,j}(t))^{Z_{i,j,N}}
 \end{aligned} \tag{1}$$

**Parameters**

- $\lambda_{i,j}, k_{i,j}$  Scale and shape parameters of Erlang distribution for component  $j$  in subsystem  $i$
- $r_{i,j}(t)$  Reliability of component  $j$  available for subsystem  $i$  at time  $t$
- $\delta_j(t)$  Switch reliability of component  $j$  at time  $t$  (imperfect switching)
- $C_{i,j}, W_{i,j}$  Cost and weight associated with component  $j$  available for subsystem  $i$

**The Proposed Mathematical Model**

$$\text{MinSystem's Cost} = \sum_{i=1}^m \sum_{j=1}^{T_i} \sum_{k \in A, S, N} C_{i,j} \times n_{i,j,k} \tag{2}$$

$$\text{MinSystem's weight} = \sum_{i=1}^m \sum_{j=1}^{T_i} \sum_{k \in A, S, N} W_{i,j} \times n_{i,j,k} \tag{3}$$

s.t.

$$r_{i,j}(t) = \exp(-\lambda_{i,j} t) \sum_{l=0}^{k_{i,j}-1} \frac{(\lambda_{i,j} t)^l}{l!} \quad i=1, \dots, m, \quad j=1, \dots, T_i \tag{4}$$

$$Z_{i,j,A} + Z_{i,j,S} + Z_{i,j,N} \leq 1 \quad i=1, \dots, m, \quad j=1, \dots, T_i \tag{5}$$

$$\sum_{j=1}^{T_i} Z_{i,j,A} + \sum_{j=1}^{T_i} Z_{i,j,S} + \sum_{j=1}^{T_i} Z_{i,j,N} = 1 \quad i=1, \dots, m \tag{6}$$

$$\sum_{q=2}^{n_{\max}} X_{q,i,j} = 1 \quad i=1, \dots, m, \quad j=1, \dots, T_i \tag{7}$$

$$n_{i,j,S} = Z_{i,j,S} \times \sum_{q=2}^{n_{\max}} q \times X_{q,i,j} \quad i=1, \dots, m, \quad j=1, \dots, T_i \tag{8}$$

$$2 \times Z_{i,j,A} \leq n_{i,j,A} \leq n_{\max} \times Z_{i,j,A} \quad i=1, \dots, m, \quad j=1, \dots, T_i \tag{9}$$

$$n_{i,j,N} = 1 \times (Z_{i,j,N}) \quad i=1, \dots, m, \quad j=1, \dots, T_i \tag{10}$$

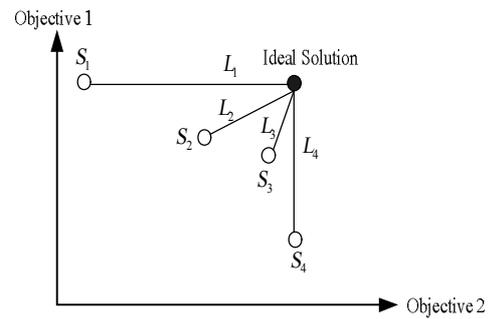
$$1 \leq \sum_{j=1}^{T_i} \sum_{k \in A, S, N} n_{i,j,k} \leq n_{\max} \tag{11}$$

Objective function (1) maximizes the reliability of the system, which consists of three terms. The first term multiplies the reliability of those subsystems whose components are in active redundancy. In cases where no component in any subsystem is in active redundancy, i.e. all  $Z_{i,j,A}$  are zero, this term is one, which is neutral in multiplication.

The second term multiplies the reliability of those subsystems whose components are in cold standby redundancy which itself consists of three parts. The first part is to ensure that in case no component is selected for standby redundancy in a subsystem, the multiplication is not zero. In other words, the value of this part is zero if a component type is selected in standby and it is one otherwise which is neutral in calculating the system reliability. The second and third parts are considered only if  $Z_{i,j,S}$  is one. The second part includes the reliability of the working component and the third part considers between one and  $n_{i,j,S} - 1$  failures regarding the reliability of the switch. The third term multiplies the reliability of those subsystems that choose no redundancy strategy. When all  $Z_{i,j,N}$  are zero this term is also one. Objective function (2) and objective function (3) minimizes the system's cost and weight, respectively. Constraint (4) calculates the reliability of components considering Erlang's parameters. Constraint set (5) ensures that for each subsystem at most one strategy is selected. Constraint set (6) states that only one component type and strategy is selected for each subsystem. Constraint set (7) declares that for cold standby strategy the number of components can only be a value between 2 and  $n_{max}$ . Constraint set (8) calculates the number of components in subsystems with cold standby strategy. This constraint is equivalent to  $2 \times Z_{i,j,S} \leq n_{i,j,S} \leq n_{max} \times Z_{i,j,S}$   $i=1, \dots, m, j=1, \dots, T$ . But, for the purpose of calculating the upper limit of the summation in the standby term of the objective function, constraint set (8) is considered in the model. Constraint set (9) indicates that for each subsystem, variable  $n_{i,j,A}$  gets value only when type  $j$  and strategy  $A$  are selected. Its value is at least 2 and at most  $n_{max}$ . Constraint set (10) ensures that in case of no redundancy the variable  $n_{i,j,N}$  gets one only when  $Z_{i,j,N}$  is one, and it is zero otherwise. Constraint set (11) indicates that the number of components in each subsystem is at least one and at most  $n_{max}$ .

### 3. COMPROMISE PROGRAMMING

Compromise programming is a mathematical programming technique which was originally developed by Zeleny [26, 27]. This method can be used for optimization of multi-objective problems to obtain the optimal



**Figure 2.** Graphical illustration of compromise programming for two objectives

solution and also for comparing the performance of alternatives in multi-criteria decision making analyses. As a matter of fact, the best compromise solution from a set of solutions is selected by a measure of distance called distance metric through which a discrete set of solutions is ranked according to their distance from an ideal solution. To understand how this measure works, consider a bi-objective problem whose both objectives are in maximization form. The ideal solution of this problem is the one which simultaneously maximizes the two objectives. In practical cases, this solution is infeasible due to conflict of objectives. Therefore a compromise must be sought. Figure 2 graphically illustrates the compromise programming. Where,  $S_1$  to  $S_4$  are four possible solutions and  $L_1 > L_4 > L_2 > L_3$  which are the distances between each solution and the ideal solution. Therefore, according to Figure 2,  $S_3$  is the best compromise solution where both objectives have equal weights.

Mathematically, compromise programming distance metric is presented in Equation (12).

$$L_p(w) = \left( \sum_{i=1}^n w_i \left[ \frac{f_i^+ - f_i^-}{f_i^+ - f_i^-} \right]^p \right)^{\frac{1}{p}} \quad (12)$$

where  $n$  is the number of objectives, in this paper  $n=3$ ,  $p$  is a parameter ( $p \in 1, 2, \infty$ ),  $w_i$  is the weight of the objective  $i$ ,  $f_i$  is the actual value of the objective function  $i$ ,  $f_i^+$  and  $f_i^-$  are respectively ideal and nadir solutions of the objective function  $i$ . For maximization problems, the former is achieved through maximizing each objective function subject to the constraints whilst the latter is determined by minimizing those objectives. This procedure is vice versa for minimization problems. The parameter  $p$  represents the importance of the maximal deviation from the ideal solution. If  $p=1$ , all deviations have equal importance. If  $p=2$ , the importance of deviations are in proportion to their

magnitude. As  $p$  increases, the importance of the deviations also increases. Similarly,  $w_i$ 's are the weights for various deviations which identify the relative importance of each objective. Apparently, for different values of  $p$  in  $L_p$  metrics and  $w_i$ , different compromise solutions can be obtained. For  $p = 1$ , the  $L_p$  metric, i.e.  $L_1$ , is called Manhattan metric.  $L_2$  is called the Euclidean metric and  $L_\infty$  is the chebychev metric. In all cases the corresponding metric needs to be minimized according to models 2, 3 and 4, respectively for  $L_1, L_2$  and  $L_\infty$ .

$$\min w_1 \left| \frac{f_1^+ - f_1^-}{f_1^+ - f_1^-} \right| + w_2 \left| \frac{f_2^+ - f_2^-}{f_2^+ - f_2^-} \right| + w_3 \left| \frac{f_3^+ - f_3^-}{f_3^+ - f_3^-} \right| \tag{13}$$

s.t.

Constraints of model(1)

$$\min \sqrt{w_1 \left[ \frac{f_1^+ - f_1^-}{f_1^+ - f_1^-} \right]^2 + w_2 \left[ \frac{f_2^+ - f_2^-}{f_2^+ - f_2^-} \right]^2 + w_3 \left[ \frac{f_3^+ - f_3^-}{f_3^+ - f_3^-} \right]^2} \tag{14}$$

s.t.

Constraints of model(1)

$$\min D_\infty$$

s.t.

$$\begin{aligned} w_1 \left[ \frac{f_1^+ - f_1^-}{f_1^+ - f_1^-} \right] &\leq D_\infty \\ w_2 \left[ \frac{f_2^+ - f_2^-}{f_2^+ - f_2^-} \right] &\leq D_\infty \\ w_3 \left[ \frac{f_3^+ - f_3^-}{f_3^+ - f_3^-} \right] &\leq D_\infty \end{aligned} \tag{15}$$

other constraints in model(1)

#### 4. COMPUTATIONAL RESULTS

To solve the proposed model using compromise programming, data taken from Taboada and Coit [26] are considered which represent a serial parallel system composed of three subsystems and four or five component choices. In that example the reliabilities of the components are reported as specific values between 0 and 1. To make the example compatible with our proposed model whose components' lifetimes follow Erlang distribution, the scale and shape parameters are determined such that those reliabilities are obtained. The scale and shape parameters are shown in Table 1 along with components' costs and weights. Maximum number of allowable components is 8, the reliability of switch equals 0.99 and the mission time is 100 unit of time. To start with compromise programming, ideal and nadir

solutions need to be calculated. From ideal solution, we mean that for a maximization problem the maximum value is achieved and for a minimization problem the minimum value is obtained. These values are obtained through maximizing the reliability objective and minimizing the cost and weight objectives. On the other side, nadir solutions can be obtained by minimizing the problem which is of maximization type and maximizing the problem which has a minimization nature. The nadir solutions are obtained by minimizing the reliability objective and maximizing the cost and weight objectives. The model is solved using GAMS (General Algebraic Modeling System) version 23.8.2 and the nadir and ideal results are presented in Table 2. Solving the proposed model using the compromise programming technique results in different Pareto solutions which depend on the norm of the  $L_p$  metric and the weights of the objectives. The results are presented in Table 3 and Table 4 and also depicted in Figure 3.

TABLE 1. Experimental data

Subsystem 1				
Component	k	$\lambda$	C	W
1	1	0.000619	9	9
2	2	0.00499	6	6
3	2	0.00564	6	4
4	1	0.00287	3	7
5	1	0.00328	2	8
Subsystem 2				
1	3	0.00665	12	5
2	3	0.01288	3	7
3	1	0.00356	2	3
4	1	0.00415	2	4
5	-	-	-	-
Subsystem 3				
1	1	0.000408	10	6
2	2	0.00564	6	8
3	1	0.00328	4	2
4	1	0.00342	3	4
5	1	0.004	2	4

TABLE 2. Ideal and Nadir local optimum solutions

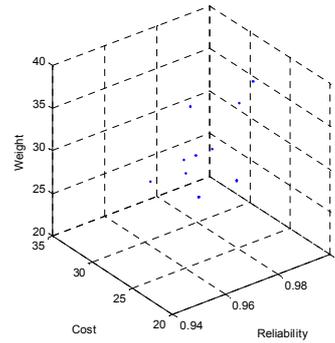
	Ideal solution	Nadir solution
Objective 1 (Reliability)	0.99999922	0.46907069
Objective 2(Cost)	6	248
Objective 3 (Weight)	9	192

**TABLE 3.** Experimental results with different  $L_p$  metrics and weights (R: Reliability, C: Cost, W: Weight, D: Distance)

Group	W1	W2	W3	$p$ Norm	R	C	W	D
1	0.5	0.3	0.2	$p=1$	0.9809	32	26	0.069
				$p=2$	0.9618	30	27	0.086
				$p=\infty$	0.9765	24	29	0.022
2	0.3	0.5	0.2	$p=1$	0.9765	24	29	0.072
				$p=2$	0.9774	24	38	0.091
				$p=\infty$	0.9477	21	35	0.031
3	0.4	0.3	0.3	$p=1$	0.9803	30	23	0.068
				$p=2$	0.9803	30	23	0.073
				$p=\infty$	0.9803	30	23	0.030
4	0.6	0.2	0.2	$p=1$	0.9854	32	26	0.057
				$p=2$	0.9803	30	23	0.063
				$p=\infty$	0.9854	30	28	0.021
5	0.7	0.2	0.1	$p=1$	0.9896	34	30	0.048
				$p=2$	0.9803	30	23	0.059
				$p=\infty$	0.9887	26	38	0.017

**TABLE 4.** The corresponding solutions for the results presented in Table 3

Group	$p$ Norm	Solution
1	$p=1$	$X_{1,3,S} = 2, X_{2,3,A} = 4, X_{3,3,S} = 3$
	$p=2$	$X_{1,2,A} = 2, X_{2,3,S} = 3, X_{3,3,A} = 3$
	$p=\infty$	$X_{1,3,S} = 2, X_{2,3,S} = 3, X_{3,5,S} = 3$
2	$p=1$	$X_{1,3,S} = 2, X_{2,3,S} = 3, X_{3,5,S} = 3$
	$p=2$	$X_{1,2,A} = 2, X_{2,2,S} = 2, X_{3,5,S} = 3$
	$p=\infty$	$X_{1,4,S} = 2, X_{2,3,S} = 3, X_{3,4,S} = 3$
3	$p=1$	$X_{1,3,S} = 2, X_{2,3,S} = 3, X_{3,3,S} = 3$
	$p=2$	$X_{1,3,S} = 2, X_{2,3,S} = 3, X_{3,3,S} = 3$
	$p=\infty$	$X_{1,3,S} = 2, X_{2,3,S} = 3, X_{3,3,S} = 3$
4	$p=1$	$X_{1,3,S} = 2, X_{2,3,S} = 4, X_{3,3,S} = 3$
	$p=2$	$X_{1,3,S} = 2, X_{2,3,S} = 3, X_{3,3,S} = 3$
	$p=\infty$	$X_{1,3,S} = 2, X_{2,2,S} = 2, X_{3,3,S} = 3$
5	$p=1$	$X_{1,3,S} = 2, X_{2,2,S} = 2, X_{3,3,S} = 4$
	$p=2$	$X_{1,3,S} = 2, X_{2,3,S} = 3, X_{3,3,S} = 3$
	$p=\infty$	$X_{1,3,S} = 2, X_{2,2,S} = 2, X_{3,5,S} = 4$



**Figure 3.** Pareto solutions

**TABLE 5.**  $L_2$  norm for Pareto solutions

Sol.	L2	Sol.	L2	Sol.	L2
1	1.177	6	0.8	11	1.041
2	0.814	7	1.041	12	1.183
3	0.829	8	1.041	13	1.489
4	0.829	9	1.041	14	1.041
5	1.248	10	1.252	15	1.451

To decide about the best compromise solution amongst Pareto solutions, first, the objective functions are normalized through Equation (16), where,  $f_i^{\min}(x)$  and  $f_i^{\max}(x)$  are the minimum and maximum values for  $f_i(x)$  in the Pareto optimal set on condition that all objectives are in minimization form. In other words, the reliability function is multiplied by -1 to be comparable with other objectives. The results for  $p=2$  are shown in Table 5. The results show that solution 6 is the best compromise solution with the lowest  $L_2$  norm. The resulted solution indicates that in order to design a highly reliable system with minimum cost and weight, the designer should set cold standby strategy as redundancy strategy for all subsystems. Redundancy levels should respectively be set to 2, 3, 3. And finally, redundancy types should respectively be considered as type 4, 3 and 4.

$$\frac{f_i(x) - f_i^{\min}(x)}{f_i^{\max}(x) - f_i^{\min}(x)}, \forall i = 1, \dots, n \quad (16)$$

### 5. CONCLUSION

This paper presents a new nonlinear multi-objective model which maximizes system's reliability concurrently with minimizing system's cost and weight. The model involves choices of component types and redundancy strategies which can be in the forms of

active, cold-standby or no redundancy strategy. The main advantage of the proposed model is that the redundancy strategies and redundancy levels can be determined simultaneously. The multi-objective model has been dealt using compromise programming technique with different  $L_p$  metrics and results in Pareto solutions indicating redundancy levels, redundancy strategies and component types. For future research, other mathematical programming technique can be implemented to deal with the proposed model. Furthermore, heuristic and meta-heuristic approaches can be employed to solve large-sized problems. Also, the model can be extended to allow component mixing, i.e. different component types can be allowed in a subsystem [28]. Considering the failure mode and effects analysis for detecting the most trouble making components or subsystems and incorporating it in the redundancy allocation problem is another research area which interested readers can follow [29].

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# A New Nonlinear Multi-objective Redundancy Allocation Model with the Choice of Redundancy Strategy Solved by Compromise Programming Approach

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یکی از مسائل مهم در طراحی سیستم، طراحی سیستمی با قابلیت اطمینان بالا و با کمترین هزینه است. یکی از روش های افزایش قابلیت اطمینان سیستم ها استفاده از سیاست جزء مازاد در حالت های فعال یا آماده به کار است. در این مقاله یک مدل جدید چند هدفه و غیرخطی عدد صحیح با انتخاب استراتژی جزء مازاد و با انتخاب نوع اجزاء توسعه داده شده است به طوری که استراتژی آماده بکار از نوع آماده بکار سرد است. مدل چند هدفه پیشنهادی، قابلیت اطمینان سیستم را ماکزیمم و وزن و هزینه سیستم را مینی موم می کند. همچنین در این مدل به طور همزمان سطوح افزونگی، استراتژی افزونگی و نوع اجزاء مشخص می شود. مدل چند هدفه حاصل با روش ریاضی برنامه ریزی تعاملی با نرم های مختلف حل شده و بهترین جواب از بین جواب های پارتو ارائه شده است.

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