IJE TRANSACTIONS C: Aspects Vol. 26, No. 9, (September 2013) 933-942

International Journal of Engineering

Journal Homepage: www.ije.ir

Monitoring and Change Point Estimation of AR(1) Autocorrelated Polynomial Profiles

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PAPER INFO

Paper history:
Received 06 August 2012
Received in revised form 12 March 2013
Accepted 18 April 2013

Keywords:
Polynomial Profiles
Phase II Monitoring
Autocorrelation
Average Run Length (ARL)
General Linear Test (GLT)
Change Point

ABSTRACT

In this paper, a remedial measure is first proposed to eliminate the effect of autocorrelation in phase II monitoring of autocorrelated polynomial profiles, where there is a first order autoregressive (AR(1)) relation between the error terms in each profile. Then, a control chart based on the generalized linear test (GLT) is proposed to monitor the coefficients of polynomial profiles and an R-chart is used to monitor the error variance, the combination of which is called GLT/R chart. The performance of the proposed GLT/R chart is evaluated by comparing it to those obtained from prevalent methods including multivariate T^2, EWMA/R and T^2 residual control charts, in terms of the average run length (ARL) criterion. Furthermore, an estimator based on the likelihood ratio approach is proposed to estimate the change point in the parameters of autocorrelated polynomial profiles. The results of extensive simulation experiments show good performance of the proposed estimator. Finally, the applicability of the proposed method is illustrated using a real data example.

doi: *10.5829/idosi.ije.2013.26.09c.01*

1. INTRODUCTION

Sometimes, quality characteristic(s) of a process follows a univariate or multivariate distribution and consequently, statistical control charts are used to monitor them. However, in many situations, the quality of a process or a product can be better characterized by a relationship between a response and one or more independent variables, where this relationship is referred to a profile. Many researchers including Stover and Brill [1], Kang and Albin [2], Mahmoud and Woodall [3], Woodall et al. [4], Wang and Tsung [5], and Woodall [6] discussed practical applications of profiles. Authors, including Kang and Albin [2], Kim et al. [7], Mahmoud et al. [8], Mahmoud and Woodall [3], Mestek et al. [9], Stover and Brill [1], Gupta et al. [10], Kang and Albin [2], Kim et al. [7], Noorossana et al. [11], Zou et al. [12], Niaki et al. [13], and Saghaei et al. [14] studied monitoring of simple linear profiles in phases I and II. The purpose of the phase I analysis is to evaluate the stability of a process and to estimate process parameters while, in phase II analysis, one is interested in detecting shifts in the process parameters as quickly as possible. In some cases, models that are more complicated are needed to represent profiles. Kazemzadeh et al. [15] extended three phase I methods in polynomial profile monitoring. Zou et al. [16] proposed a multivariate exponentially weighted moving average (MEWMA) control chart for monitoring general linear profiles in phase II. Kazemzadeh et al. [17] transformed polynomial regression to an orthogonal polynomial regression model and proposed a method based on using exponentially weighted moving average (EWMA) control charts to monitor the parameters of the orthogonal polynomial model in phase II.

In all aforementioned research works, it is assumed that the error terms of the models are independent and identically distributed (iid) normal random variables. However, in some cases these assumptions are violated. Noorossana et al. [18] investigated the effect of non-normality of the error terms on the performance of the EWMA/R method proposed by Kang and Albin [2].

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Jensen et al. [19] developed a linear mixed model (LMM) to account for the autocorrelation within a linear profile. Jensen and Brich [20] showed that use of mixed models have significant advantages when there is autocorrelation within nonlinear regression models. Noorossana et al. [21] considered linear profiles and modeled autocorrelation between profiles as a first order autoregressive AR(1) process. Kazemzadeh et al. [17, 22] considered polynomial profiles in the presence of profile autocorrelation modeled by AR(1).

Soleimani et al. [23] investigated the effect of within profile autocorrelation in simple linear profiles and proposed a transformation technique to eliminate the effect of autocorrelation. Niaki et al. [13] addressed the problem of monitoring autocorrelated polynomial profiles and proposed three control schemes including, and EWMA/R control charts based on a transformation method for monitoring purposes in phase II.

In this paper, the problem of monitoring autocorrelated polynomial profiles is addressed in which the relationship between a response and a single explanatory variable is defined by a k^{th} order polynomial regression, where it is assumed that the error terms within each profile are correlated based on a first order autoregressive model. Moreover, we assume there is no correlation between polynomial profiles. Niaki et al. [13] employed the concept of generalized linear test to design a control chart for monitoring linear regression profiles and applied an R-chart, simultaneously, to detect shifts in the error variance. Accordingly, we first extend the method of Soleimani et al. [23] to autocorrelated polynomial profiles and then develop a GLT/R control chart based on the work of Niaki et al. [13] to monitor polynomial profiles. In addition, three other existing methods that were extended to monitor autocorrelated polynomial profiles are compared with the GLT/R method by simulation studies via the average run length (ARL) criterion. Finally, a change point estimator based on the likelihood ratio approach is proposed to determine the location of a change in the profile.

The rest of the paper is organized as follows. In the next section, the problem formulation as well as assumptions is given. The transformation technique and application of the general linear test to monitor polynomial profiles are presented in section 3. Three existing monitoring methods are presented and a change point estimator is developed to determine the location of a step change in the parameters of the profile in sections 4 and 5, respectively. The effect of autocorrelation on the performance of GLT/R control chart along with the performances of the proposed methods is investigated in section 6 and 7. In section 8, a real data example is provided to illustrate the applicability of the proposed method. Finally, the paper is concluded in section 9.

2. AUTOCORRELATED POLYNOMIAL PROFILE MODELING

Considering the j^{th} sample of the process being monitored is collected over time for a single explanatory variable x, the observations are $\left(x_i, x_i^2, \ldots, x_i^k, y_{ij}\right);\ i=1,2,\ldots n$ In other words, the subscript i shows the i^{th} observation within each profile, and subscript j shows the j^{th} profile collected over time. When the process is in statistical control, the autocorrelated polynomial profile is modeled as:

$$\begin{aligned} y_{ij} &= A_0 + A_1 x_i + A_2 x_i^2 + \ldots + A_k x_i^k + \varepsilon_{ij} \\ \varepsilon_{ij} &= \Phi \varepsilon_{(i-1)j} + a_{ij} \end{aligned} \tag{1}$$

where ε_{ij}'s are the correlated error terms, a_{ij}'s are independent and identically distributed (iid) normal random variables with mean zero and variance σ^2, $A_0, A_1, \ldots, A_k$ are model parameters and $-1 < \Phi < 1$ is the autocorrelation coefficient. Moreover, it is assumed that x-values are known constants from profile to profile and that since phase II profile monitoring is at aimed the in-control values of Φ, $A_0, A_1, \ldots, A_k$, σ^2 are assumed known.

It can be easily shown that the existing autoregressive structure between the error terms, defined in Equation (1), leads to autocorrelation between observations at different values of x in each profile. It means that, the observations in each profile can be expressed by:

$$y_{ij} = A_0 + A_1 x_i + A_2 x_i^2 + \ldots + A_k x_i^k + \varepsilon_{ij}$$

and

$$y_{(i-1)j} = A_0 + A_1 x_{(i-1)} + A_2 x_{(i-1)}^2 + \ldots + A_k x_{(i-1)}^k + \varepsilon_{(i-1)j}$$

leading to:

$$\begin{aligned} & y_{ij} - \left(A_0 + A_1 x_i + A_2 x_i^2 + \ldots + A_k x_i^k\right) = \\ & \Phi\left[y_{(i-1)j} - \left(A_0 + A_1 x_{(i-1)} + A_2 x_{(i-1)}^2 + \ldots + A_k x_{(i-1)}^k + a_{ij}\right)\right] \end{aligned} \tag{2}$$

3. THE PROPOSED METHOD

In this section, a transformation technique first proposed by Soleimani et al. [23] for simple linear profiles is first extended for autocorrelated AR(1) polynomial profiles to eliminate within-profile autocorrelation. Then, a control chart based on the general linear statistical test as well as three existing monitoring schemes are derived to monitor the coefficients of the transformed polynomial regression model.

3. 1. Transformation In the proposed transformation technique, all observations on the response variable are transformed via the following equation:

$$Y'_{ij} = Y_{ij} - \Phi\, Y_{(i-1)j} \tag{3}$$

If observations Y_{ij} and $Y_{(i-1)j}$ in Equation (3) are replaced by their equivalents in the regression model (1), a polynomial regression model with independent error terms is obtained as:

$$\begin{aligned} Y'_{ij} = {} & A_0(1-\Phi) + A_1\left(X_i - \Phi X_{i-1}\right) + A_2\left(X_i^2 - \Phi X_{i-1}^2\right) + \ldots + \\ & A_k\left(X_i^k - \Phi X_{i-1}^k\right) + \left(\varepsilon_{ij} - \Phi \varepsilon_{(i-1)j}\right);\ i = 1,2,\ldots,n \end{aligned} \tag{4}$$

that results in

$$Y'_{ij} = A'_0 + A'_1 X'_i + A'_2 X'^2_i + \ldots + A'_k X'^k_i + a_{ij} \tag{5}$$

where

$$Y'_{ij} = Y_{ij} - \Phi\, Y_{(i-1)j}$$

$$X'_i = X_i - \Phi X_{i-1},\ X'^2_i = X_i^2 - \Phi X_{i-1}^2, \ldots, X'^k_i = X_i^k - \Phi X_{i-1}^k$$

and

$$A'_0 = A_0(1-\Phi), A'_1 = A_1, A'_2 = A_2, \ldots, A'_k = A_k$$

It should be noted that the transformed model contains one observation less than the original model. Now, the control charts are developed to monitor the parameters of the polynomial profiles in Equation (5). This is demonstrated in the next subsection.

3. 2. GLT/R Chart In this method, the general linear test is used to monitor the coefficients of the transformed polynomial regression model. At first, a sample of size n (n>4) is collected from process periodically at time j and the regression parameters $(A_0, A_1, A_2, \ldots, A_k)$ are estimated by the least squares method. Then, the F-statistic given in Equation (6) is employed in order to monitor the coefficients:

$$F_j^* = \frac{\frac{1}{k+1}\left[\begin{array}{l}\sum_{i=1}^{n-1}\left(Y'_{ij} - A'_0 - A'_1 X'_i - A'_2 X'^2_i - \ldots - A'_k X'^k_i\right)^2 - \\ \sum_{i=1}^{n-1}\left(Y'_{ij} - \hat{A}'_0 - \hat{A}'_1 X'_i - \hat{A}'_2 X'^2_i - \ldots - \hat{A}'_k X'^k_i\right)\end{array}\right]}{\frac{1}{n-k-1}\left[\sum_{i=1}^{n-1}\left(Y'_{ij} - \hat{A}'_0 - \hat{A}'_1 X'_i - \hat{A}'_2 X'^2_i - \ldots - \hat{A}'_k X'^k_i\right)\right]} \tag{6}$$

All the coefficients of the polynomial profile in Equation (5) are simultaneously in-control when $F_j^* < F_{\left(1-\alpha;\, df_{R_j} - df_{F_j};\, df_{F_j}\right)}$, where df_{F_j} and df_{R_j} are the degrees of freedom for the j^{th} full and reduced (under the null hypothesis that the profile is in-control) models, respectively.

Note that by the aforementioned approach only the process mean is monitored. Therefore, an R-chart may be used simultaneously in order to detect shifts in the error variance. The R control chart statistic denoted by R_j is calculated by $R_j = max(e_{ij}) - min(e_{ij})$, in which the residuals of the transformed model (e_{ij}) is obtained by:

$$e_{ij} = y'_{ij} - \left(A'_0 + A'_1 X'_i + A'_2 X'^2_i + \ldots + A'_k X'^k_i\right);\ i = 2,3,\ldots,n \tag{7}$$

The lower and upper control limits for the R control chart are:

$$LCL = \sigma\left(d_2 - Ld_3\right) \text{ and } UCL = \sigma\left(d_2 + Ld_3\right) \tag{8}$$

respectively, where $L(>0)$ is chosen to give a specified in-control ARL, d_2 and d_3 are constants that depend on the sample size.

4. EXISTING MONITORING SCHEMES

In order to compare the performances of the proposed monitoring scheme, in this section, three existing approaches including T^2, $T^2_{residual}$, and EWMA/R control chart are extended to monitor the transformed polynomial profile.

4. 1. T^2 Chart The first method is a modified version of the T^2 control chart proposed by Kang and Albin [2]. To reduce the effect of existing autocorrelation between the error terms in each profile, all the parameters of the original model, $(A_0, A_1, A_2, \ldots, A_k)$, are replaced by their transformed ones. This method is used when the number of parameters (k) is not very large. The modified T^2 statistic is obtained by:

$$\begin{aligned} T_j^2 = {} & \left[\left(\hat{A}'_{0j}, \hat{A}'_{1j}, \hat{A}'_{2j}, \ldots, \hat{A}'_{kj}\right) - \left(A'_{0j}, A'_{1j}, A'_{2j}, \ldots, A'_{kj}\right)\right]^T \Sigma^{-1} \\ & \left[\left(\hat{A}'_{0j}, \hat{A}'_{1j}, \hat{A}'_{2j}, \ldots, \hat{A}'_{kj}\right) - \left(A'_{0j}, A'_{1j}, A'_{2j}, \ldots, A'_{kj}\right)\right] \end{aligned} \tag{9}$$

where

$$\Sigma = \left[\sigma^2\left(X^T X\right)^{-1}\right] \tag{10}$$

4. 2. Residual-based T^2 Chart In the second method, the residuals of the transformed model is used, where the residuals are obtained as:

$$e_{ij} = y'_{ij} - \left(A'_0 + A'_1 X'_i + A'_2 X'^2_i + \ldots + A'_k X'^k_i\right);\ i = 2,3,\ldots,n \tag{11}$$

The T^2 statistics and the upper control limit for the residual-based T^2 control chart, $T^2_{residual}$ thereafter, are determined using the following equations, respectively:

$$T_j^2 = \left(\underline{e}_j - \underline{0}\right)\Sigma^{-1}_{\underline{e}_j}\left(\underline{e}_j - \underline{0}\right)^T \tag{12}$$

$$UCL = \chi^2_{\alpha;n-1} \tag{13}$$

where,

$\underline{e}_j = \left(e_{2j}, e_{3j}, \dots, e_{nj}\right)^T$, $\Sigma_{\underline{e}_j} = \sigma^2 I$,

I, is the identity matrix, $\underline{0}$ is the zero vector, n is the number of x values and $\chi^2_{\alpha;n-1}$ is the 100(1-α) percentile of the chi-square distribution with n-1 degrees of freedom.

4. 3. EWMA/R Chart An EWMA control chart in combination with an R-chart is employed as a third method to monitor not only the average value of the residuals, but also to detect shifts in the process variance. These charts are the same as the ones proposed by Kang and Albin [2], where the residuals are obtained using Equation (11) and the average value of the residuals for the j^{th} profile are obtained by:

$$\bar{e}_j = \sum_{i=2}^{n} e_{ij} / (n-1)$$

The EWMA control chart statistic, denoted by z_j for $j = 1,2,\dots,$ is given by

$$z_j = \theta\,\bar{e}_j + (1-\theta)\,z_{j-1} \tag{14}$$

where $\theta, (0 < \theta \le 1),$ is a smoothing constant and $z_0 = 0$. The lower and the upper control limits for the EWMA control chart are:

$$LCL = -L\sigma\sqrt{\theta/(2-\theta)(n-1)}$$
$$UCL = L\sigma\sqrt{\theta/(2-\theta)(n-1)} \tag{15}$$

respectively, where $L(>0)$ is selected to give a specified in-control ARL. The R control chart statistic in this method is similar to the statistic introduced in the GLT/R method.

5. CHANGE POINT ESTIMATOR

Following an out-of-control signal from each of the proposed control charts, process engineers initiate a search to identify the root causes of variation. Knowledge on the exact time of the change would simplify the search to identify and remove the root causes. Therefore, in this section, we develop a change point estimator to determine the location of a step change in phase II monitoring of autocorrelated polynomial profiles. The logarithm of the likelihood function for in-control process is:

$$l_0 = S\,(n-1)\,ln\left(\frac{1}{\sigma_0\sqrt{2\pi}}\right) - \frac{1}{2}\sum_{j=1}^{S}\sum_{i=2}^{n}\frac{\left(y'_{ij} - A'_{00} - A'_{10}X'_i - A'_{20}X'^2_i - \dots - A'_{k0}X'^k_i\right)^2}{\sigma_0^2} \tag{16}$$

Assuming a change occurring at time t with a signal receiving at time S, the logarithm of the likelihood function will be:

$$l_a = t(n-1)ln\left(\frac{1}{\sigma_0\sqrt{2\pi}}\right) - \frac{1}{2}\sum_{j=1}^{t}\sum_{i=2}^{n}\frac{\left(y'_{ij} - A'_{00} - A'_{10}X'_i - A'_{20}X'^2_i - \dots - A'_{k0}X'^k_i\right)^2}{\sigma_0^2}$$
$$+ (S-t)(n-1)ln\left(\frac{1}{\sigma_1\sqrt{2\pi}}\right) - \frac{1}{2}\sum_{j=t+1}^{S}\sum_{i=2}^{n}\frac{\left(y'_{ij} - A'_{01} - A'_{11}X'_i - A'_{21}X'^2_i - \dots - A'_{k1}X'^k_i\right)^2}{\sigma_1^2} \tag{17}$$

where $(A_{01}, A_{11}, A_{21}, \dots, A_{k1})$ and σ_1^2 are unknown out-of-control parameters of the process and should be estimated. The maximum likelihood estimator of these parameters is given in the following equations:

$$\hat{\boldsymbol{A}}_1 = \left(\boldsymbol{X}'_{t,S}\,\boldsymbol{X}_{t,S}\right)^{-1}\boldsymbol{X}'_{t,S}\,\boldsymbol{Y}_{t,S} \tag{18}$$

$$\hat{\sigma}_1^2 = \frac{\sum_{j=t+1}^{S}\sum_{i=2}^{n}\left(y'_{ij} - A'_{01} - A'_{11}X'_i - A'_{21}X'^2_i - \dots - A'_{k1}X'^k_i\right)^2}{(S-t)(n-1)} \tag{19}$$

in which $X_{t,S}$ and $Y_{t,S}$ are $(S-t)(n-1)\times(k+1)$ and $(S-t)(n-1)\times 1$ matrices, respectively. Now, the likelihood ratio statistic is:

$$lr_{t,S} = -2\,(l_0 - l_a) = -(S-t)(n-1)$$
$$-2(S-t)(n-1)\,ln\left(\frac{1}{\sigma_0\sqrt{2\pi}}\right) + 2(S-t)(n-1)\,ln\left(\frac{1}{\sigma_1\sqrt{2\pi}}\right)$$
$$+ \sum_{j=t+1}^{S}\sum_{i=2}^{n}\frac{\left(y'_{ij} - A'_{00} - A'_{10}X'_i - A'_{20}X'^2_i - \dots - A'_{k0}X'^k_i\right)^2}{\sigma_0^2} \tag{20}$$

Finally, the estimator is calculated based on the following equation:

$$\hat{t} = arg\,max\left(lr_{t,S}\right);\ 1 \le t < S \tag{21}$$

The performances of the proposed control charts and the change point estimator are evaluated in the following sections.

6. SIMULATION EXPERIMENTS

In this section, the performance of the GLT/R control chart for monitoring polynomial profiles when within-profile autocorrelation is present and the proposed transformation method is not utilized, is first evaluated. Then, applying the proposed transformation technique in order to eliminate within profile autocorrelations, the performance of the GLT/R control chart is compared to the performance of the T^2, $T^2_{residual}$, EWMA/R methods under both weak and strong autocorrelation coefficients of 0.1 and 0.9, respectively. The following example used by Kazemzadeh et al. [17, 22, 24] is used in this study:

$$y_{ij} = 3 + 2x_i + x_i^2 + \varepsilon_{ij}$$
$$\varepsilon_{ij} = \Phi\varepsilon_{(i-1)j} + a_{ij} \tag{22}$$

where a_{ij} follows a normal distribution with mean zero and variance one and *x*-values are 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10. In the simulation experiments, the effect of different autocorrelation coefficients Φ on the performances of the GLT/R control chart under different shifts in the intercept λ, in the second parameter β, in the third parameter δ and in error standard deviation γ based on the in-control ARL criterion is studied. The results of 10,000 simulation runs are summarized in Table 1. In this table, λ, β, δ and γ are measured in multiples and the in-control ARL is considered 200.

As shown in Table 1, when the transformation technique is not used, the in-control ARLs of the GLT/R control chart decrease in the presence of autocorrelation within profiles, leading to its poor performance. Moreover, this effect is more considerable when the autocorrelation coefficient gets larger.

When the proposed transformation method is used however, the performances of T^2, $T^2_{residual}$, EWMA/R and GLT/R methods are then compared employing the same example introduced earlier in Equation (22). Two autocorrelation coefficients $\Phi=0.1$ (weak) and $\Phi=0.9$ (strong) are considered where all control-charting methods are designed to have an overall in-control ARL of 200. To achieve this, the smoothing constant θ in the EWMA control chart is set 0.2. Furthermore, in the EWMA/R and GLT/R control chart, we set the values of L equal to 2.973 and 3.08, respectively for both Φ=0.1 and Φ=0.9 autocorrelation coefficients. For T^2 and $T^2_{residual}$ charts, UCLs are set 12.84 and 23.59, respectively. Finally, for GLT/R chart UCL is set 12.916. We used 10,000 simulation runs to study the out-of-control ARL under different shifts in the intercept, in the second, and in the third parameters along with the error standard deviation. The results are summarized in Tables 2 to 5.

The results in Table 2 show that under the intercept shifts from A_0 to $A_0+\lambda\alpha$ in both weak and strong autocorrelations (Φ=0.1 and Φ=0.9), while the EWMA/R chart uniformly performs better than the other three, the $T^2_{residual}$ chart has the worst performance. Further, it can be seen that the out-of-control ARLs for the strong autocorrelation are larger than the ones in the weak autocorrelation.

The results in Table 3 show that under the shifts in the second parameter from A_1 to $A_1+\beta\sigma$, the GLT/R chart performs uniformly better than the other three charts in weak autocorrelation case, while for strong autocorrelation, EWMA/R performs better than the other methods. However, the T^2 chart has the worst performance.

As shown in Table 4, under the weak autocorrelation, GLT/R and EWMA/R methods are roughly the same. However, under the strong autocorrelation, EWMA/R method performs uniformly better than the other methods. T^2 method dose not perform well in this situation either.

Finally, the results in Table 5 show that under the standard deviation shifts from σ to $\sigma\gamma$ in both weak and strong autocorrelation situations, the $T^2_{residual}$ control chart performs uniformly better than the other three charts. In addition, similar performances are obtained for both weak and strong autocorrelations. This means that the autocorrelation coefficient does not affect the out-of-control ARL under a standard deviation shift.

7. PERFORMANCE EVALUATION

The model in Equation (22) is used once more to evaluate the performance of the proposed method to estimate a step-change point in the parameters of the model. Applying the EWMA/R control chart to monitor the process and to detect a shift, the true change point is simulated to occur at t = 25. When a shift is detected by the control chart, the estimator in Equation (21) is used to estimate the change point. The averages (AVE) and the standard deviations (STD) of the estimates of a step change in the second parameter and the standard deviation of the model are summarized in Table 6, based on 10,000 simulation runs. Also, the precision performances of the estimated change points are reported in Table 6, where the probabilities $P(|\hat{t}-t|=0)$, $P(|\hat{t}-t|\le 1)$, $P(|\hat{t}-t|\le 2)$, $P(|\hat{t}-t|\le 3)$, $P(|\hat{t}-t|\le 4)$ and $P(|\hat{t}-t|\le 5)$ are denoted by P0, P1, P2, P3, P4 and P5, respectively.

According to the results in Table 6, the proposed change point estimator accurately and precisely estimates the change point for different values of shifts in the parameters of the model. However, the performance of the proposed change point estimator deteriorates as the value of the autocorrelation coefficient increases. In other words, the estimator displays a better performance for weak rather than strong autocorrelation.

8. A CASE STUDY

In this section, the application of the proposed method is illustrated by a real data example adopted from Amiri et al. [25]. In this example, the relationship between torque produced by an automobile engine and its speed is a key quality characteristic which should be monitored over time. According to Amiri et al. [25], this relationship can be modeled by a second order polynomial profile in which there is an AR(1) autocorrelation structure within each profile. Considering a total of 26 engines, each engine is first run at different speed values of 1500, 2000, 2500, 2660, 2800, 2940, 3500, 4000, 4500, 5000, 5225, 5500, 5775, and 6000 revolutions per minute (PRM) and then the corresponding torque values for each engine are measured.

TABLE 1. The effect of autocorrelation coefficient on in-control ARL of GLT/R control chart under different shifts in intercept, second parameter, third parameter and error standard deviation without utilizing the proposed transformation method

λ **(Shift in the intercept)**

		0	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
Autocorrelation coefficients (Φ)	**0**	200.1	197.6	185.2	159.2	124.7	95.6	69.2	48.5	34.7	24.3	18.0
	0.1	181.3	178.8	150.6	123.9	95.9	72.5	51.2	36.5	26.3	19.1	14.1
	0.3	81.3	75.4	66.7	53.1	41.3	32.9	23.8	18.3	13.7	10.7	8.6
	0.5	24.5	23.9	21.9	18.6	15.7	13.3	11.1	9.1	7.4	6.2	5.1
	0.7	7.2	7.1	6.7	6.4	5.8	5.5	4.9	4.5	4.1	3.6	3.3
	0.9	2.6	2.5	2.6	2.5	2.5	2.3	2.3	2.2	2.1	2.1	2.0

β **(Shift in the second parameter)**

		0	**0.025**	**0.05**	**0.075**	**0.1**	**0.125**	**0.15**	**0.175**	**0.2**	**0.225**	**0.25**
Autocorrelation coefficients (Φ)	**0**	200	195.4	191.8	190.9	170.2	149.3	130.4	107.6	89.3	71.5	57.9
	0.1	182.3	178.2	171.2	161.1	140.4	121.1	101.2	83.7	67.0	53.8	43.6
	0.3	79.8	80.8	74.2	68.2	60.1	52.4	45.1	38.6	31.9	26.9	22.1
	0.5	24.6	24.3	23.6	22.4	20.6	18.6	16.8	14.9	13.1	11.7	10.2
	0.7	7.1	7.0	7.0	6.7	6.5	6.3	5.8	5.5	5.2	4.9	4.5
	0.9	2.6	2.6	2.5	2.4	2.4	2.3	2.3	2.2	2.2	2.1	2.1

δ **(shift in the third parameter)**

		0	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**	**0.1**
Autocorrelation coefficients (Φ)	**0**	200.5	199.2	188.4	166.1	135.7	108.3	79.9	58.1	41.5	30.5	21.9
	0.1	185.1	177.2	156.8	136.3	111.1	83.2	60.9	44.1	32.2	23.4	17.3
	0.3	81.5	78.6	69.1	58.7	48.1	37.4	29.3	22.7	17.1	13.2	10.5
	0.5	24.6	24.4	22.5	20.1	17.3	14.8	12.7	10.5	8.7	7.2	5.9
	0.7	7.2	7.1	6.9	6.5	6.2	5.6	5.2	4.6	4.2	3.7	3.3
	0.9	2.6	2.6	2.5	2.5	2.4	2.4	2.3	2.2	2.1	2.1	1.9

γ **(shift in the standard deviation)**

		1	**1.1**	**1.2**	**1.3**	**1.4**	**1.5**	**1.6**	**1.7**	**1.8**	**1.9**	**2**
Autocorrelation coefficients (Φ)	**0**	200.7	95.3	40.0	19.5	10.7	4.7	3.5	2.8	2.3	2.3	2.1
	0.1	186.7	108.1	50.2	24.3	13.2	8.1	5.6	4.1	3.2	2.5	2.1
	0.3	80.1	71.4	47.8	28.2	16.6	10.4	6.9	5.1	3.8	3.1	2.5
	0.5	24.0	23.7	22.2	18.1	13.8	9.8	7.4	5.4	4.3	3.4	2.8
	0.7	7.1	7.1	6.9	6.7	6.2	5.5	4.8	4.1	3.5	3.1	2.6
	0.9	2.7	2.6	2.6	2.6	2.5	2.5	2.3	2.3	2.1	2.1	1.9

TABLE 2. Out-of-control ARL comparisons under shifts from A_0 to $A_0 + \lambda\sigma$ with $\Phi = 0.1$ and $\Phi = 0.9$

$\Phi = 0.1$	λ										
Methods	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
$T^2_{residual}$	198.5	188.5	152.9	112.8	78.5	53.2	34.7	23.1	14.9	10.3	6.9
T^2	200	173.7	122.3	76.2	44.6	26.9	16.6	10.4	7.8	4.7	3.5
EWMA/R	200	112.8	38.7	17.4	10.1	6.9	5.2	4.2	3.6	3.1	2.7
GLT/R	199.4	162.3	102.4	56.8	29.7	16.2	9.5	6.1	3.8	2.8	2.2
$\Phi = 0.9$	λ										
Methods	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
$T^2_{residual}$	199.8	197.6	199.3	197.6	198.6	196.6	195.1	190.2	188.4	186.5	184.1
T^2	200.1	198.4	196.7	196.2	195.2	193.0	187.3	183.0	180.6	172.2	169.8
EWMA/R	200	200	192.1	187.1	179.2	164.5	152.0	137.6	125.2	112.5	102.1
GLT/R	200.5	192.2	199.2	200.5	191.4	191.7	186.8	173.7	163.6	160.8	157.6

TABLE 3. Out-of-control ARL comparisons under shifts from A_1 to $A_1 + \beta\sigma$ with $\Phi = 0.1$ and $\Phi = 0.9$

$\Phi = 0.1$	β										
Methods	**0**	**0.025**	**0.05**	**0.075**	**0.1**	**0.125**	**0.15**	**0.175**	**0.2**	**0.225**	**0.25**
$T^2_{residual}$	200.3	164.1	103.1	53.4	26.7	13.2	7.1	4.2	2.6	1.8	1.4
T^2	200.1	190.6	158.9	122.2	90.4	64.1	45.5	32.2	22.8	16.7	12.4
EWMA/R	200.7	63.4	16.7	7.9	5.1	3.8	3.0	2.5	2.5	2.0	1.8
GLT/R	198.4	137.2	65.1	27.4	12.7	6.5	3.7	2.4	1.7	1.3	1.1
$\Phi = 0.9$	β										
Methods	**0**	**0.025**	**0.05**	**0.075**	**0.1**	**0.125**	**0.15**	**0.175**	**0.2**	**0.225**	**0.25**
$T^2_{residual}$	197.8	197.1	186.7	179.6	165.2	149.3	131.8	114.3	99.4	86.3	73.1
T^2	198.4	196.5	192.2	180.4	169.5	152.9	141.1	126.8	112.8	98.5	85.1
EWMA/R	197.3	181.90	133.8	85.3	55.1	35.7	25.1	18.5	14.1	11.3	9.4
GLT/R	198.2	192.2	181.2	168.9	148.4	127.9	106.2	94.8	74.3	58.9	52.4

TABLE 4. Out-of-control *ARL* comparisons under shifts from A_2 to $A_2 + \delta\sigma$ with $\Phi = 0.1$ and $\Phi = 0.9$

$\Phi = 0.1$	δ										
Methods	**0**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**	**0.1**
$T^2_{residual}$	200.7	43.6	5.2	1.5	1.1	1.1	1.0	1.0	1.0	1.0	1.0
T^2	199.2	174.6	127.5	80.1	49.7	29.4	18.5	11.9	7.7	5.4	3.9
EWMA/R	200.3	8.4	3.1	2.1	1.5	1.1	1.1	1.0	1.0	1.0	1.0
GLT/R	198.9	28.1	3.7	1.3	1.1	1.0	1.0	1.0	1.0	1.0	1.0
$\Phi = 0.9$	δ										
Methods	**0**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**	**0.1**
$T^2_{residual}$	197.1	159.1	92.7	45.2	20.8	10.1	5.3	3.1	2.1	1.5	1.2
T^2	200.4	191.6	167.8	138.4	109.7	84.2	63.6	47.6	35.7	27.4	20.3
EWMA/R	200.2	60.6	15.7	7.5	4.9	3.6	2.9	2.4	2.1	1.9	1.8
GLT/R	197.4	155.8	71.9	33.5	16.5	8.8	4.9	3.1	2.1	1.6	1.3

TABLE 5. Out-of-control *ARL* comparisons under standard deviation shifts from σ to $\gamma\sigma$ with $\Phi = 0.1$ and $\Phi = 0.9$

$\Phi = 0.1$	γ										
Methods	**1**	**1.1**	**1.2**	**1.3**	**1.4**	**1.5**	**1.6**	**1.7**	**1.8**	**1.9**	**2**
$T^2_{residual}$	200.5	47.2	16.6	8.1	4.7	3.2	2.4	1.9	1.6	1.4	1.3
T^2	197.4	72.1	32.4	17.9	11.5	8.0	5.9	4.6	3.7	3.2	2.7
EWMA/R	199.3	61.7	24.7	12.4	7.2	4.7	3.4	2.6	2.1	1.8	1.6
GLT/R	196.6	175.2	125.2	76.1	41.1	23.7	14.2	9.3	6.7	4.9	3.9
$\Phi = 0.9$	γ										
Methods	**1**	**1.1**	**1.2**	**1.3**	**1.4**	**1.5**	**1.6**	**1.7**	**1.8**	**1.9**	**2**
$T^2_{residual}$	199.3	47.1	17.2	7.9	4.6	3.1	2.3	1.9	1.6	1.4	1.3
T^2	200.6	70.4	32.8	18.1	11.3	7.9	5.8	4.6	3.7	3.2	2.7
EWMA/R	199.4	61.2	24.8	7.1	4.7	3.4	2.6	2.2	2.2	1.8	1.6
GLT/R	198.3	174.2	125.5	75.3	41.2	23.6	14.4	9.4	6.7	5.1	3.9

TABLE 6. The average, standard deviation and precision performances of the change point estimator under shifts in the second parameter and standard deviation of the model with $\Phi = 0.1$ and $\Phi = 0.9$

shifts from A_2 to $A_2 + \delta\sigma$ with $\Phi = 0.1$								
δ	***AVE***	***STD***	***P0***	***P1***	***P2***	***P3***	***P4***	***P5***
0.01	24.9	3.9	0.38	0.61	0.73	0.81	0.86	0.89
0.02	24.9	1.2	0.78	0.93	0.97	0.98	0.99	0.99
0.04	25.0	0.1	0.98	0.99	1.00	1.00	1.00	1.00
0.06	25.0	0.02	0.99	1.00	1.00	1.00	1.00	1.00
0.08	25.0	0.01	0.99	1.00	1.00	1.00	1.00	1.00
shifts from A_2 to $A_2 + \delta\sigma$ with $\Phi = 0.9$								
δ	***AVE***	***STD***	***P0***	***P1***	***P2***	***P3***	***P4***	***P5***
0.01	35.9	21.9	0.06	0.14	0.21	0.26	0.31	0.36
0.02	26.2	6.9	0.19	0.38	0.49	0.58	0.65	0.71
0.04	24.7	2.9	0.52	0.76	0.86	0.91	0.94	0.96
0.06	24.8	1.4	0.77	0.93	0.97	0.98	0.99	0.99
0.08	24.9	0.7	0.90	0.98	0.99	0.99	1.00	1.00
shifts from σ to $\gamma\sigma$ with $\Phi = 0.1$								
γ	***AVE***	***STD***	***P0***	***P1***	***P2***	***P3***	***P4***	***P5***
1.2	27.6	9.1	0.15	0.31	0.42	0.51	0.57	0.63
1.4	24.9	4.2	0.37	0.61	0.74	0.82	0.86	0.89
1.6	24.6	3.1	0.55	0.78	0.87	0.92	0.94	0.95
1.8	24.6	2.5	0.67	0.87	0.93	0.95	0.96	0.97
2	24.6	2.5	0.76	0.91	0.95	0.96	0.97	0.98
shifts from σ to $\gamma\sigma$ with $\Phi = 0.9$								
γ	***AVE***	***STD***	***P0***	***P1***	***P2***	***P3***	***P4***	***P5***
1.2	27.6	9.1	0.15	0.31	0.41	0.50	0.57	0.62
1.4	25.1	3.9	0.38	0.62	0.75	0.82	0.87	0.90
1.6	24.6	3.2	0.55	0.78	0.87	0.91	0.94	0.95
1.8	24.6	2.7	0.68	0.87	0.92	0.95	0.96	0.97
2	24.7	2.2	0.76	0.91	0.95	0.96	0.97	0.98

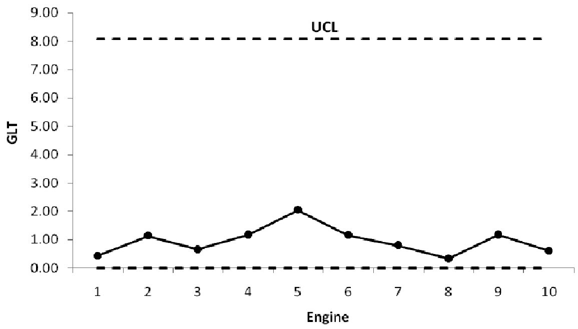

Figure 1. GLT control chart for real data example

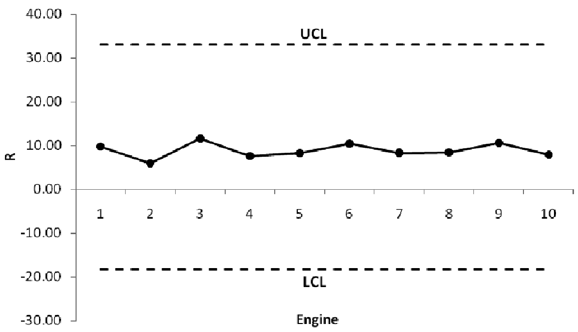

Figure 2. R control chart for real data example

Based on a retrospective analysis of a set of 16 profiles as historical data, the following model is obtained for relationship between the response and explanatory variable:

$$y_{ij} = 111.1977 - 0.006115x_i - 0.0000049x_i^2 + \varepsilon_{ij} \quad (23)$$

in which the standard deviation and correlation coefficient are 2.184 and 0.75, respectively. Applying the proposed transformation technique, this model can be used to construct the GLT/R control chart for phase II studies. Based on 10,000 simulation runs, the parameter of the R-chart and UCL of the GLT chart are set equal to L=15.4 and UCL=8.0807 under Φ=0.75 in order to obtain an overall in-control ARL of 200. The values of d_2 and d_3 are 3.407 and 0.763, respectively. Now, the remaining 10 profiles are used to monitor the underlying process in phase II. The results are shown in Figures 1 and 2.

9. CONCLUSION

In this paper, the effect of within-profile autocorrelation on the performance of a GLT/R chart designed to monitor polynomial profiles under independency of the error terms was first investigated. The results showed that autocorrelation leads to poor performance of the chart. Then, a transformation technique was employed for polynomial profiles. Finally, the performances of T^2, $T^2_{residual}$, EWMA/R and GLT/R control charts using the transformation technique were compared in terms of out-of-control ARL. The results showed that the GLT/R scheme performs better than the other charts under the shifts in the second regression parameter. However, the $T^2_{residual}$ method had better performance in comparison with the other three methods under the shifts in the standard deviation. Furthermore, a change point estimator was proposed to estimate the location of a step change in the parameters of autocorrelated polynomial profiles. The results of a performance evaluation study showed that the proposed method estimates the change point accurately and precisely for various values of shifts. In addition, a real data example was used to illustrate the applicability of the proposed method .

Applying the linear mixed model (LMM) and comparing it with the transformation method proposed in this paper would be an interesting subject which is recommended for future research.

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Monitoring and Change Point Estimation of AR(1) Autocorrelated Polynomial Profiles

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PAPER INFO

Paper history:
Received 06 August 2012
Received in revised form 12 March 2013
Accepted 18 April 2013

Keywords:
Polynomial Profiles
Phase II Monitoring
Autocorrelation
Average Run Length (ARL)
General Linear Test (GLT)
Change Point

چکیده

در این مقاله، ابتدا روشی برای حذف اثر خودهمبستگی در فاز ۲ پایش پروفایلهای چند جمله ای پیشنهاد می شود که ارتباط بین عناصر خطا در هر پروفایل توسط یک مدل AR(1) بیان می شود. سپس یک نمودار کنترل بر اساس آزمون عمومی خطی برای پایش ضرایب پروفایل به همراه یک نمودار R برای پایش واریانس خطا پیشنهاد می شود. آنگاه، عملکرد نمودار پیشنهادی با عملکرد روشهای مطرح T^2، EWMA/R، و T^2 باقیمانده ها بر اساس متوسط طول دنباله به طریق شبیه سازی مقایسه می شود. در پایان، یک تخمینگر نسبت درستنمایی برای تخمین نقطهٔ تغییر در پارامترهای پروفایلهای خود همبسته به دست می آید. نتایج یک مطالعهٔ شبیه سازی حاکی از عملکرد خوب تخمینگر پیشنهادی است. در پایان، کاربردی بودن روش پیشنهادی با استفاده از داده های واقعی نشان داده می شود

doi: *10.5829/idosi.ije.2013.26.09c.01*