



Boundary Shear Stress in a Trapezoidal Channel

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ABSTRACT

This paper focuses on a hydraulic radius separation approach used to calculate the boundary shear stress in terms of bed and wall shear stress proposed in a trapezoidal channel. The average bed and sidewall shear stress in smooth trapezoidal open channels are derived after using Guo and Julien's early equations, taking a part of an investigation to cover both rectangular and trapezoidal channels. On the basis of the conformal mapping procedure, a numerical solution is obtained for a case of constant eddy viscosity without concerning secondary currents effect. In comparison with laboratory measurements data, the first approximation for a rectangular channel overestimates the average bed shear stress measurement by about 4.8% and by decreasing a sidewall slope, overestimation increases. It, however; underestimates the average sidewall shear stress by about 6.88%; in this case, underestimation increases while sidewall slope is decreasing. A second approximation is then presented by introducing two lumped empirical correction factors for taking into account the effects of secondary currents, variable eddy viscosity and else. Using experimental data, in terms of average bed shear stress, the second approximation agrees very well (at least $R^2 > 0.99$, and an average relative error less than 5.35%) over a wide range of width to depth ratios. In terms of average wall shear stress, the second approximation returns acceptable results despite the scatter data with an average relative error less than 5.95% and by $R^2 > 0.93$ where it seems to be reasonable.

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1. INTRODUCTION

The distribution of boundary shear stress around the wetted perimeter in open channels is known to depend upon the shape of the cross section [1-9], the longitudinal variation in planform geometry [10], the boundary roughness distribution [11-14] and the structure of secondary flows [15-19]. The importance of understanding boundary shear stress distributions is demonstrated by the use of local or mean boundary shear stress in many hydraulic equations concerning resistance, sediment, and dispersion or cavitation problems. Furthermore, to estimate the bed-load transport in open channel flows, one must separate the bed shear stress from the total shear stress. Similarly, one must know about the sidewall shear stress to study channel migration or to prevent bank erosion. Moreover, a sidewall correction procedure is often needed in laboratory flume studies of velocity profiles, bed form resistance and sediment transport [120-22].

However, accurate computation of the local or mean shear stress is a difficult task even using sophisticated turbulence models. As an alternative, various empirical, analytical or simplified computational methods were developed [15, 23-33].

Some of them relies on splitting the channel cross-section into sub-regions inside which, the weight of fluid is balanced by shear force acting along the corresponding wall sections for computation of the local, mean wall, and the mean bed shear stress in straight and prismatic channels [34]. The idea of mapping the channel cross-section and defining those sub-regions by the way of bounding orthogonals to the isovels is attributed to Leighly [35].

Those orthogonals were assumed to be 'surfaces of zero shear' [34], meaning that the weight of water flowing within the sub-region volumes had to be exactly balanced by the shear forces acting on the corresponding parts of the channel wall. In an appendix Leighly [35] and Einstein [36] proposed the hydraulic radius separation method that permits dividing a cross-sectional area into two areas A_b and A_w , as it is shown

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in Figure 1. The division of the channel into 3 regions is achieved by computing the balance between shear force exerted on each wall section and the adjacent weight of water.

However, although the separation concept embraced by many, the calculation of the division lines is difficult; Keulegan [37] proposed a geometrical method based on the bisector of the base angle and Chiu and Chiu [38] suggested a mathematical model for the isovels, permitting the application of Leighly's idea.

Interestingly, Einstein [36] neither provides a sketch nor offers details to how such division lines would be computed.

In a subsequent discussion, Haywood plotted straight lines obtained iteratively, so that (a) there was no friction along the lines, and (b) the weight of fluid matched the wall shear force. This is different from the first sketch reported by Yang and Lim [31] which was based upon the assumption that an absence of friction at the division interface, uses a piece-wise linear curve approach obtained from energy dissipation considerations. Subsequently, Guo and Julien [25] proposed an analytical approach by analyzing the continuity and momentum equations to establish the bed and wall shear stress.

This paper points towards using a method of Guo and Julien [25] as a part of an investigation for the calculation of average bed and wall shear stress in straight prismatic trapezoidal channels.

Although their method provided some good results, it was only presented for rectangular cross section channels.

Starting with the Guo and Julien's [25] equations for a steady uniform flow, the objective of this paper is first to achieve a theoretical basis for the boundary shear stress in trapezoidal open channels. Then, by assuming various sidewall slopes, as a first approximation, a numerical solution is possible to calculate the boundary shear stress using conformal mapping procedure, after neglecting secondary currents and assuming a constant value for eddy viscosity term. A second approximation is then presented by introducing two lumped empirical correction factors for the effects of secondary currents, variable eddy viscosity and other possible effects. Finally, another numerical study is achieved to generalize the concluded average bed and wall equations to cover sidewall slopes of trapezoidal channels from 30° to 90° sidewall angles. The experimental data documented by Javid [39] are used for validation purposes of equations obtained.

2. THEORETICAL ANALYSIS

Guo and Julien's [25] method starts to consider control volume BCHGB [A_b] in Figure 1, that has a unit length

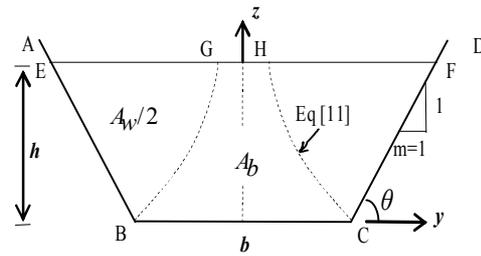


Figure 1. Comparison partition of cross-sectional area for bed shear stress and sidewall shear stress

in the flow direction x and assumes delimitations of BG and CH that are symmetric with respect to the axis z and $\theta = 90^\circ$. The flow direction defined the axis x , and the cross-sectional plane $y-z$ as it is shown in Figure 2. In addition, the main flow velocity in the axis x denoted as u , and the secondary currents in the plane $y-z$ are v and w , respectively. They analyzed the continuity and the momentum equations to demonstrate that the bed shear stress is comprised of three terms, namely the gravitational (I), the secondary flows (II), and the interfacial shear stress (III) in the following equation:

$$\bar{\tau}_b = \frac{\rho g S A_b}{b} - \frac{2}{b} \int_L \rho u (vdz - wdy) + \frac{2}{b} \int_L (\tau_{yx} dz - \tau_{zx} dy) \quad (1)$$

For the case of steady uniform flow in a rectangular open channel; in which g =gravitational acceleration; ρ =mass density of water; S =channel bed slope; b is the channel width; and τ_{yx} and τ_{zx} = shear stress in the flow direction x applied on the $z-x$ plane and the $y-x$ plane, respectively. One can prove that although Equation (1) was derived for a smooth rectangular channel, they are valid for all types of cross sections as long as BG and CH are symmetric [25, 40]. Similarly, in case of trapezoidal channels, the average sidewall shear stress, $\bar{\tau}_w$, can also be derived from the force balance in an arbitrary control volume, $BGEB$ or $CFHC$ in Figure 1. However, a short way to derive the average sidewall shear stress is to consider the overall force balance in the flow direction. That is

$$2h\sqrt{1+m^2}\bar{\tau}_w + b\bar{\tau}_b = \rho g S (b + mh)h \quad (2)$$

in which the first term on the left-hand side is shear force on the two side walls, the second term is shear force on the channel bed, and the right-hand side is the component of water gravity in the flow direction.

Applying Equation (2) in Equation (1) gives the average wall shear stress as:

$$\bar{\tau}_w = \frac{\rho g S (b + mh - A_b/h)}{2\sqrt{1+m^2}} + \frac{\int_L \rho u (vdz - wdy)}{h\sqrt{1+m^2}} - \frac{\int_L (\tau_{yx} dz - \tau_{zx} dy)}{h\sqrt{1+m^2}} \quad (3)$$

where, h is the flow depth and $m = \cot\theta$.

3. FIRST APPROXIMATION WITHOUT SECONDARY CURRENTS

To estimate the boundary shear stress, using Equations (1) and (3), one must know the main velocity u and secondary currents, v and w , shear stress τ_{yx} and τ_{zx} , and the integration path CH. On the other hand, to solve for velocity field, one must know the boundary shear stress. This interaction between velocity and shear stress makes a solution for boundary shear stress or velocity profiles very complicated, as shown by Chiu and Chiou [38]. As a first approximation, one may neglect the effects of secondary currents and the fluid shear stress. Thus, Equation (1) becomes

$$\overline{\tau_b} = \frac{\rho g S A_b}{b} \quad \text{or} \quad (4a)$$

$$\frac{\overline{\tau_b}}{\rho g h S} = \frac{A_b}{bh} \quad (4b)$$

and Equation (3) becomes

$$\overline{\tau_w} = \frac{\rho g S (b + mh - A_b/h)}{2\sqrt{1+m^2}} \quad \text{or} \quad (5a)$$

$$\frac{\overline{\tau_w}}{\rho g h S} = \frac{m + (b/h)(1 - \overline{\tau_b}/\rho g h S)}{2\sqrt{1+m^2}} \quad (5b)$$

The remaining problem is to find the area, A_b , which is equivalent to finding the delimitations BG and CH, in Figure 2.

3. 1. Delimitations BG and CH One can show that the corresponding momentum equation in the flow direction x is

$$\rho \left(v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = \rho g S + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \quad (6)$$

The convective accelerations on the left-hand side of Equation (6) accounts for secondary currents. The first term on the right-hand side is the gravity component in the flow direction, and the other two are net shear stress applied on a differential element of fluid. The first approximation assumes that: (1) secondary currents are neglected; and (2) the eddy viscosity ν_t is constant. Applying these two assumptions to Equation (6) gives:

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = -\frac{gS}{\nu + \nu_t} = const \quad (7)$$

in which, $\tau_{yx} = \rho(\nu + \nu_t)\partial u/\partial y$; $\tau_{zx} = \rho(\nu + \nu_t)\partial u/\partial z$; and ν = water kinematic viscosity. The above equation is called as the Poisson equation and can be solved by a conformal mapping procedure [25, 41]. That is, the orthogonals of the velocity contours used to delineate BG and CH in Figure 2. Although a solution for Equation (7) gives a laminar velocity profile, and the

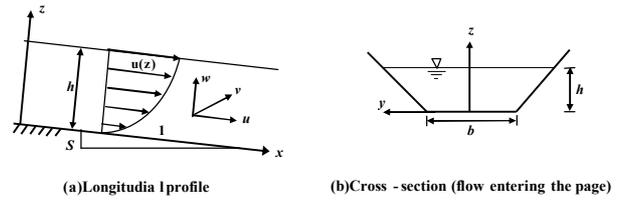


Figure 2. Coordinate system in open-channel flows

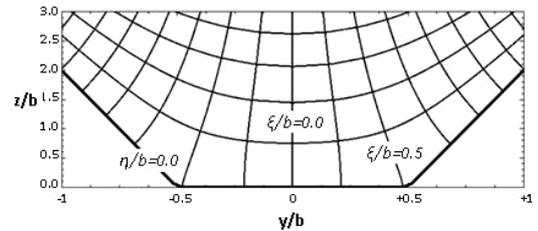


Figure 3. Estimated isovels and orthogonals pattern

orthogonals provide a first approximation of the boundary shear stress.

To delimitate the potential lines and the streamlines, Schwarz–Christoffel transformation [42, 43] used an assumption of a trapezoidal cross-section channel by width b and flow depth h from Figure 2 and a channel sidewall aspect ratio (1:1). The related transformation between physical flow domain (ω -plane) and a half-upper plane (ζ -plane) is:

$$\frac{d\omega}{d\zeta} = k_1(\zeta + b/2)^{-1/4}(\zeta - b/2)^{-1/4} \quad (8)$$

in which, $\omega = y + iz$ and $\zeta = \xi + i\eta$; k_1 is transformation constant; $-b/2$ and $+b/2$ are arbitrary values for left and right toes of channel cross-section in transformation, respectively. In other words, the values of streamline crossing from corners equals to $-b/2$ and $+b/2$. Applying the theorem of integration to the above gives:

$$\omega = k_2 + \frac{2k_1(-b + 2\zeta)^{3/4}(2 + 4\zeta)^{1/4}}{3(b + 2\zeta)^{1/4}} \times HyperGeometric\ 2F1\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{\zeta}{b}\right] \quad (9)$$

in which, $HyperGeometric\ 2F1$ is known as Equation (10) and k_2 is constant of integration.

$$HyperGeometric\ 2F1[c, d, e, x] = 1 + \frac{cd}{e}x + \frac{c(c+1)d(d+1)}{e(e+1)}x^2 + \dots \quad (10)$$

It is notable that k_1, k_2 can be specified by applying the channel toes condition in Equation (9). In order to obtain the isovels and orthogonals pattern, over than 100,000 various points for each parameter b, ξ and η in Equation (10) were applied to use the

Mathematica Software Package. Figure 3 shows the results of this numerical analysis; in which η/b and ξ/b are dimensionless values for isovels and orthogonals, respectively. Likewise, Equation (11) estimates over than 1000 points by applying the streamline condition which crosses from right toe of the channel ($\xi = b/2$ and various values for η) in Equation (9) to delimitate division line CH as:

$$y/b = 0.5 \text{Exp}(-0.57z/b) \quad (11)$$

as shown in Figure 1.

3. 2. Average Bed and Sidewall Shear Stress With reference to Figure 1, after considering symmetry with respect to the channel centerline, the area A_b can be estimated as follows:

$$A_b = 2 \int_0^h y dz = 1.7544 b^2 [1 - \text{Exp}(-0.57h/b)] \quad (12)$$

Substituting Equation (11) into Equation (4b) gives the average bed shear stress as:

$$\frac{\bar{\tau}_b}{\rho ghS} = 1.7544 \frac{b}{h} [1 - \text{Exp}(-0.57h/b)] \quad (13)$$

So, the average wall shear stress may be calculated by substituting Equation (13) into Equation (5b).

4. SECOND APPROXIMATION WITH CORRECTION FACTORS

The first approximation also implies that the maximum velocity occurs at the water surface. However, studies by Javid [39] and Javid and Mohammadi [44] illustrate that the secondary flow cells' specific performance in trapezoidal channels change the pattern of flow lines and potential lines, especially in the corners and surface water. So, the second approximation aims at improving upon the first approximation by introducing two lumped empirical correction factors in the first approximation. Substituting Equation (12) into Equation (4b) gives:

$$\frac{\bar{\tau}_b}{\rho ghS} = \frac{1}{bh} \int_0^h \text{Exp}(-0.57z/b) \quad (14)$$

Applying the theorem of integration by parts and then the mean value theorem for integrals to the above gives:

$$\frac{\bar{\tau}_b}{\rho ghS} = \text{Exp}(-0.57h/b) - \frac{0.57\lambda h}{b} \text{Exp}(-0.57\lambda h/b) \quad (15)$$

To include the effects of secondary currents, variable eddy viscosity and other possible effects, two lumped empirical correction factors λ_1 and λ_2 are introduced in Equation (15). In other words, using Equations (14) and (15) one can assume:

$$\frac{\bar{\tau}_b}{\rho ghS} = \text{Exp}(-0.57h/b) - \lambda_1 \frac{h}{b} \text{Exp}(-0.57\lambda_2 h/b) \quad (16)$$

Substituting Equation (16) into Equation (4b) gives the second approximation for the average sidewall shear stress. For as much as $m=1$ for trapezoidal channel with sidewall aspect ratio (1:1), Equation (5b) reduces to Equation (17):

$$\frac{\bar{\tau}_w}{\rho ghS} = \frac{\sqrt{2}}{4} \left(1 + \frac{b}{h} [1 - \frac{\bar{\tau}_b}{\rho ghS}] \right) \quad (17)$$

in which $\frac{\bar{\tau}_b}{\rho ghS}$ is estimated by Equation (16). To ensure the validation of the above two equations in both narrow and wide channels, Equations (16) and (17) were analyzed together with experimental measurements from Yuen [22] by SPSS Software Package. Analysis indicates that the first empirical correction factor, λ_1 , is not sensitive to b/h ratio, and changes of average bed and sidewall shear stress are negligible by various b/h ratios, like what was found in Guo and Julien's [25] work for rectangular cross-section channels. However, analysis of the results yields:

$$\lambda_1 \cong 0.33 \quad (18)$$

But second empirical correction factor, λ_2 , is very sensitive to b/h ratio value and λ_2 decreases by increasing b/h . By comparison between the value of λ_2 and empirical data, Equation (19) is estimated:

$$\lambda_2 = 4.25 + 3.04 \text{Ln}(h/b) \quad (19)$$

This could be due to the influence of the wall distance variation in height along the channel symmetry axis which is fixed in rectangular channels with vertical walls, and thus results in a constant value [25]. Finally, with Equations (18) and (19), the second approximation of the average bed shear stress, Equation (16) reduces to:

$$\frac{\bar{\tau}_b}{\rho ghS} = \text{Exp}\left(-0.57\frac{h}{b}\right) - 0.33\frac{h}{b} \text{Exp}\left(-0.57\frac{h}{b}\left(4.25 + 3.04\text{Ln}\left(\frac{h}{b}\right)\right)\right) \quad (20)$$

and the sidewall shear stress can be calculated by substituting Equation (20) in Equation (17). It is also noticeable that Equation (20) is based on extensive range in trapezoidal channel for a given geometrical shape and covers both subcritical and supercritical flows, and may also be extended to deal with non smooth channels, although that is not needed here. One can demonstrate that for a very wide channel where $b/h \rightarrow \infty$, Equation (20) reduces to:

$$\frac{\bar{\tau}_b}{\rho ghS} = 1 \quad (21)$$

This coincides with the result in a two-dimensional flow. However, the average sidewall shear stress for large width–depth ratios is not zero, which can be clearly seen by incorporating the condition $b/h \rightarrow \infty$ in Equation (17) which yields:

$$\frac{\bar{\tau}_w}{\rho ghS} = 0.6549 \quad (22)$$

TABLE 1. Coefficients and calculated correction factors

Sidewall Slope	Experimental data source	α	λ_1	β	κ
30°	Javid [26, 44, 39]	-0.2374	0.0545	10.850	5.2680
45°	Yuen [22]	-0.5696	0.3273	4.2433	3.0333
68°	Flintham and Carling [11]	-1.4376	-0.0974	-0.5679	0.9251
90°	Knight et al. [4, 45]	-2.7877	-1.4355	0.6000	-0.1845

TABLE 2. Average error and correlation coefficients between estimated equations compared with results of analysis

	α	λ_1	β	κ
Error	2.5%	3.8%	1.0%	5.9%
R^2	>0.999			

5. TRAPEZOIDAL CHANNEL WITH SIDEWALL SLOPES 30°, 68°, 90°

Equations for determining the average bed and wall shear stress in trapezoidal channel with sidewall slopes 30°, 68° and rectangular channel obtained by performing the described procedures for the channel with sidewall slope of 45° which represented in the prior part. Consider Equation (11) as:

$$y = 0.5b \cdot \text{Exp}(\alpha \cdot z / b) \quad (23)$$

then, corresponding average bed shear stress as a first approximation is:

$$\frac{\bar{\tau}_b}{\rho g h S} = -\frac{b}{h} \frac{1}{\alpha} [1 - \text{Exp}(\alpha \cdot h / b)] \quad (24)$$

and for second approximation consider Equation (20) as:

$$\frac{\bar{\tau}_b}{\rho g h S} = \text{Exp}\left(\alpha \frac{h}{b}\right) - \lambda_1 \frac{h}{b} \text{Exp}\left(\alpha \frac{h}{b} \left[\beta + \kappa \cdot \text{Ln}\left(\frac{h}{b}\right)\right]\right) \quad (25)$$

thus, experimental data used to determine the correction factors λ_1 , β , κ and coefficients of Equations (24) and (25) which change by variation of sidewall slope, α , are summarized in Table 1 for various sidewall slopes of trapezoidal channels.

Then, the experimental data is not available for every trapezoidal channels sidewall slope, therefore, to generalize the calculated coefficients, Equations (26) to (29) estimate the values of α , λ_1 , β and κ , respectively.

$$\alpha = -7.667 + 7.482 \text{Cos}(0.0121\theta - 0.235) \quad (26)$$

$$\lambda_1 = (-172 + 8.7\theta - 0.093\theta^2) / 100 \quad (27)$$

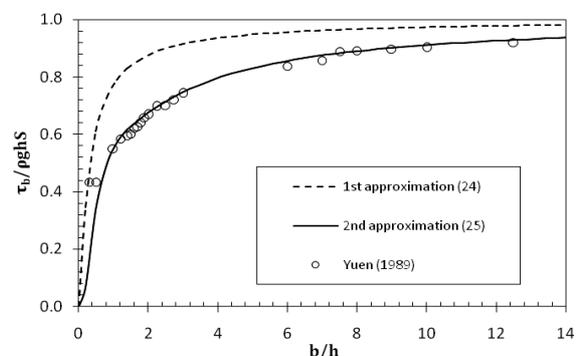
$$\beta = (3149 - 86.78\theta + 0.583\theta^2) / 100 \quad (28)$$

$$\kappa = (996 - 19.52\theta + 0.092\theta^2) / 100 \quad (29)$$

in which θ is in degree. If one define the relative error as

$$\text{Error} = \left| \frac{\text{calculated} - \text{measured}}{\text{measured}} \right| \quad (30)$$

The average relative error and corresponding correlation coefficient for Equations (26) to (29) and the respective data are listed in Table 2. Therefore, by the above estimated relations, the average bed and the sidewall shear stress can be obtained for a trapezoidal channel with sidewall slopes of 30° to 90°. For example, to obtain the average bed and sidewall shear stress in a trapezoidal channel with a sidewall slope of 66°, at first α , λ_1 , β and κ should be calculated from Equations (26) to (29). Then by substituting the calculated parameters in Equations (24) and (25), the first and second approximation to determine the average bed shear stress for arbitrary values of the fluid flow and slope of the given channel geometry can be obtained, respectively. The related sidewall shear stress can also be calculated by substituting the calculated average bed shear stress from Equation (24) or (25) in Equation (5b). The authors believe that the presented relations are valid for trapezoidal channels with sidewall slope of less than 30°, may be additional laboratory studies are needed, but for channels with sidewall slopes 30° to 90° many numerical studies have been conducted which all prove the validation of presented relations.

**Figure 4.** Comparison of first and second approximation for average bed shear stress with the measurements ($\theta=45^\circ$)

6. COMPARISON WITH EXPERIMENTAL DATA

The existing experimental data in smooth trapezoidal and rectangular open channels have been well documented by Javid [39]. The data set includes those of Cruff [46], Ghosh and Roy [1], Kartha and Leutheusser [47], Myers [48], Knight and Macdonald [49], Noutsopoulos and Hadjipanous [50], Knight et al. [45], Flintham and Carling [11] and Yuen [22] and also the results of some laboratory studies on trapezoidal channels with sidewall slope of 30° provided by Javid [39]. In order to validate the proposed method, one can choose the rectangular and trapezoidal channel with sidewall slope of 45° to peruse.

6. 1. Trapezoidal Channel with 45° Sidewall Slope

Presented model herein has been applied to the data taken for smooth trapezoidal channel with a sidewall slope of 45° provided by Yuen [22]. Mean bed and mean sidewall shear stress computed by first and second approximation are shown in Figures 4 and 5 respectively, together with the experimental measurements. In terms of the average bed shear stress, one can see that the first approximation Equation (24) overestimates the average bed shear stress about 24.8% as shown in Figure 4. This result reveals that except for gravity, the effects of secondary currents and interface shear stress should be considered at least empirically. The second approximation denoted by the solid line, agrees very well with the experimental data. The relative error between Equation (25) and the data is about 5.2%. If the two largest relative errors are excluded, the average relative error then reduces to 1.57% and correlation coefficient will be greater than 0.997. The odd wave is obtained only with second approximation in narrow channels emphasizing the influence of the secondary currents and the Reynolds shear stress. In terms of the sidewall shear stress, unlike those of the average bed shear stress, the first approximation denoted by a dashed line, is 33% less than the experimental data when the width–depth ratio becomes large. This finding shows that the first approximation does not agree with the average sidewall shear stress. However, the second approximation improves the first approximation greatly, as it is denoted by the solid line in Figure 5.

6. 2. Rectangular Channel The average bed and wall shear stress predicted by first and second approximation in a rectangular channel was compared with the experimental data taken by Knight et al. [45]. In terms of the mean bed shear stress, the first approximation provides noticeably the same prediction likewise the second approximation. In this case, the average relative error by excluding the three largest

relative errors are, about 4.8 and 3.5%, respectively, as shown in Figure 6. In terms of the average wall shear stress, one can see in Figure 7, the first approximation denoted by dashed line, predicts reasonably well results with an average relative error of about 7.37%. It has a little perceptible difference in comparing with the results obtained by using the second approximation denoted by solid line. However, the second approximation agrees very well with the experimental data by an average relative error of about 5.5% without excluding any measurement data used here. To summarize the study results, relative calculated average error and correlation coefficients for different sidewall slopes by first and second approximations are listed in Table 3.

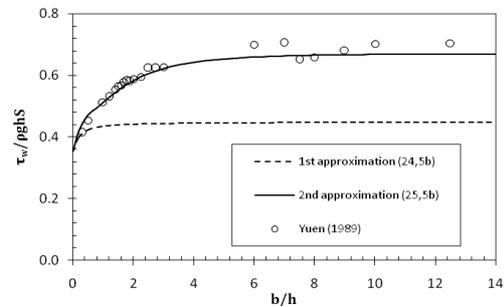


Figure 5. Comparison of first and second approximation for average wall shear stress with the measurements ($\theta=45^\circ$)

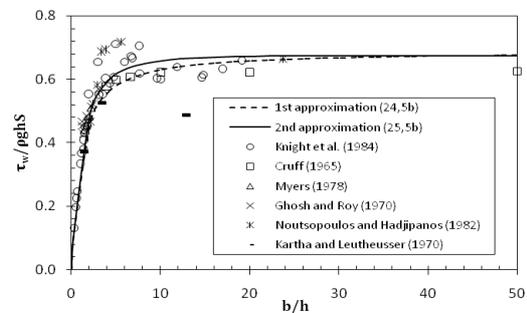


Figure 7. Comparison of first and second approximation for average wall shear stress with the measurements ($\theta=90^\circ$)

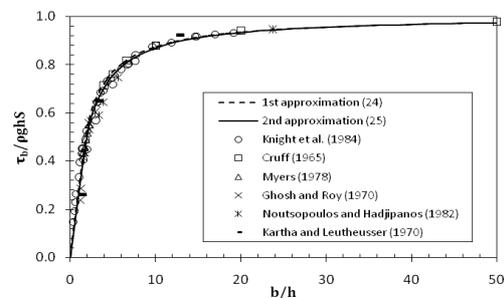


Figure 6. Comparison of first and second approximation for average bed shear stress with the measurements ($\theta=90^\circ$)

TABLE 3. Summary of relative average error and correlation coefficients between calculated first and second approximations compared with experimental data

		First Approximation				Second Approximation			
		30°	45°	68°	90°	30°	45°	68°	90°
Average Bed Shear Stress	Err (%)	17.0	24.88	14.49	4.80	2.46	5.25	2.81	5.35
	R ² (%)	99.1	97.7	99.3	99.1	99.2	99.7	99.4	99.0
Averse Wall Shear Stress	Err (%)	4.82	25.46	21.77	6.88	1.61	2.48	4.96	5.95
	R ² (%)	96.1	97.0	96.7	91.9	99.9	97.3	98.8	93.3

7. CONCLUSIONS

In this research, Guo and Julien's [25] method was used as a part of investigation for calculating the average bed and wall shear stress in straight prismatic trapezoidal channels. A numerical solution is possible for the case where the eddy viscosity is a constant value and secondary flow cells are negligible. Exerting the Schwarz–Christoffel transformation, a numerical solution is obtained. Using the Mathematica Software Package, related values of isovels and rays extracted. This leads to Equation (24) presented for calculating the average bed shear stress as a first approximation. In addition, Equation (5b) derived to calculate the average wall shear stress for each of two approximations. Comparing with the experimental measurements listed in Table 3, this first approximation by decreasing the sidewall slope of a trapezoidal channel, the model overestimates the measured average bed shear stress. However, in case of average wall shear stress by decreasing the sidewall slope, the first approximation underestimates the measurements. A second approximation is then proposed after introducing two empirical coefficients. The second approximation [Equation (25)] yields a better agreement with the experimental measurements (by $R^2 > 0.93$) and an average relative error is less than 5.95% for the average bed shear stress despite the scatter of data. Finally, herein the second approximation is, therefore, recommended for practical purposes.

8. REFERENCES

- Ghosh, S.N. and Roy, N., "Boundary shear distribution in open channel flow", *Journal of the Hydraulics Division, ASCE*, Vol. 96, No. 4 (1970), 967–994.
- Houjou, K., Shimizu, Y. and Ishii, C., "Calculation of Shear stress in smooth rectangular open-channel flows", *Journal of Hydraulic Engineering, ASCE*, Vol. 131, No. 1 (1990), 30–37.
- Knight, D.W., Yuen, K.W.H. and Alhamid, A.A.I., "Boundary shear stress distributions in open channel flow", In: K. Beven, P. Chatwin, J. Millbank (eds.), *Physical Mechanisms of Mixing and Transport in the Environment*, Wiley, New York, (1994), 51–87.
- Knight, D.W. and Hamed, M.E., "Boundary shear in symmetrical compound channels", *Journal of Hydraulic Engineering, ASCE*, Vol. 109, No. 10 (1984), 1412–1427.
- Lane, E.W., "Progress report on studies on the design of stable channels by the bureau of reclamation", *Proc. Journal of the Hydraulics Division, ASCE*, Vol. 79 No. 280 (1953), 1–30.
- Mohammadi, M., "Boundary shear stress in a partly-filled pipe channels", *Proceedings Journal of Eng., Faculty of Eng.*, Tabriz University, Vol. 54, August, (2009), Tabriz, Iran. (In Farsi)
- Mohammadi, M., "Shape effects on boundary shear stress in open channels", *Proceeding Journal of Eng., Faculty of Eng.*, Tabriz University, August, (2004), Tabriz, Iran. (In Farsi)
- Mohammadi, M. and Knight, D.W., "Boundary shear stress distribution in a V-shaped channel", *Proceeding 1st International Conference on: Hydraulics of Dams & River Structures (HDRS)*, (2004), 401-410, 26-28 April, Esteghlal Hotel, Tehran, Iran. <http://hdrs.pwit.ac.ir/>
- Yang, S-Q., and Lim, S.Y., "Boundary shear stress distributions in trapezoidal channels", *Journal of Hydraulic Research*, 43(1), (2005), 98–102.
- Pizzuto, J.E., "A numerical model for calculating the distributions of velocity and boundary shear stress across irregular straight open channels", *Water Resources Research*, AGU Vol. 27, No. 9 (1991), 2457–2466.
- Flintham, T.P. and Carling, P.A., "The prediction of mean bed and wall boundary shear in uniform and compositely rough channels", In: W.R. White (ed.), *Proceedings International Conference on River Regime*, Wiley, New York, (1988), 267–286.
- Ghosh, S.N. and Mehta, P.J., "Boundary shear distribution in compound channel with varying roughness distribution", *Proceedings - Institution of Civil Engineers*, Vol. 57, (1974), 159–164.
- Knight, D.W., Alhamid, A.A.I. and Yuen, K.W.H., "Boundary shear in differentially roughened trapezoidal channels". In: R. A. Falconer, K. Shiono, R.G.S. Matthew (eds.), *Hydraulic and Environmental Modelling: Estuarine and River Waters*, Ashgate, Aldershot, (1992), 3–14.
- Yang, S-Q., "Depth-averaged shear stress and velocity in open-channel flows", *Journal of Hydraulic Engineering, ASCE*, Vol. 136, No. 11 November, (2010), 952–958.
- Berlamont, J.E., Trouw, K. and Luyckx, G., "Shear stress distribution in partially filled pipes", *Journal of Hydraulic Engineering*, Vol. 129, No. 9 (2003), 697–705.

16. Knight, D.W., Omran, M. and Tang, X., "Modeling depth averaged velocity and boundary shear in trapezoidal channels with secondary flows", *Journal of Hydraulic Engineering, ASCE*, Vol. 133, No. 1 (2007), 39–47.
17. Muste, M. and Patel, V.C., "Velocity profiles for particles and liquid in open-channel flow with suspended sediment", *Journal of Hydraulic Engineering, ASCE*, Vol. 123, No. 9 (1997), 742–751.
18. Nezu, I. and Nakagawa, H., "Turbulence in Open Channel Flows", *Balkema*, Rotterdam, Netherlands, (1993).
19. Tominaga, A., Nezu, I., Ezaki, K. and Nakagawa, H., "Three dimensional turbulent structure in straight open channel flows", *Journal of Hydraulic Research*, Vol. 27, No. 11 (1989), 149–173.
20. Alhamid, A.A.I., "Boundary shear stress and velocity distributions in differentially roughened trapezoidal open channels", *PhD Thesis, University of Birmingham*, Birmingham, England, (1991).
21. Mohammadi, M., "Resistance to flow and the influence of boundary shear stress on sediment transport in smooth rigid boundary channels", *PhD Thesis, University of Birmingham*, England, (1998).
22. Yuen, K.W.H., "A study of boundary shear stress, flow resistance and momentum transfer in open channels with simple and compound trapezoidal cross sections", *Ph.D. Thesis, University of Birmingham*, Birmingham, England, (1989).
23. Christensen, B. and Fredsoe, J., "Bed shear stress distribution in straight channels with arbitrary cross section", *Progress Rep.* 77, (1998), Dept. of Hydrodynamics and Water Resources, TU Denmark, Lyngby.
24. Galip, S., Neslihan, S. and Recep, Y., "Boundary shear stress analysis in smooth rectangular channels", *Canadian Journal of Civil Engineering*, Vol. 33 No. 3 (2006), 336–342.
25. Guo, J. and Julien, P.Y., "Shear stress in smooth rectangular open-channel flows", *Journal of Hydraulic Engineering, ASCE*, Vol. 131, No. 1 (2005), 30–37.
26. Javid, S. and Mohammadi, M., "New formulas to determine the wall shear force in partially filled pipes", *Proc. 10th Iranian Conference on Hydraulics*, Vol. 1, (2011b), Rasht, Iran. (In Farsi)
27. Khodashenas, S.R. and Paquier, A., "A geometrical method for computing the distribution of boundary shear stress across irregular straight open channels", *Journal of Hydraulic Research*, Vol. 37, No. 3 (1999), 381–388.
28. Lundgren, H. and Jonsson, I.G., "Shear and velocity distribution in shallow channels", *Journal of the Hydraulics Division, ASCE*, Vol. 90, No. 1 (1964), 1–21.
29. Mohammadi, M. and Javid, S., "Evaluation of shear stress in rectangular channels", *Proc. 6th Iranian National Congress on Civil Eng.*, Vol. 2, (2010), Semnan University, Semnan, Iran. (In Farsi)
30. Ramana Prasad, B.V. and Russell Manson, J., "Discussion of a geometrical method for computing the distribution of boundary shear stress across irregular straight open channels", *Journal of Hydraulic Research*, Vol. 40, No. 4 (2002), 537–539.
31. Yang, S-Q., and Lim, S.Y., "Mechanism of energy transportation and turbulent flow in a 3D channel", *Journal of Hydraulic Engineering, ASCE*, Vol. 123, No. 8 (1997), 684–692.
32. Yang, S-Q., and Lim, S.Y., "Discussion on: a geometrical method for computing the distribution of boundary shear stress across irregular straight open channels", *Journal of Hydraulic Research*, 40(3), (2002), 535–542.
33. Yang, S-Q., Yu, J.X. and Wang, Y.Z., "Estimation of diffusion coefficients, lateral shear stress and velocity in open channels with complex geometry", *Water Resources Research*, AGU Vol. 40, No. 5 (2004), W05202.
34. Graf, W., "Hydraulics of sediment transport", *McGraw-Hill, New York*, (1971).
35. Leighly, J.B., "Toward a theory of the morphologic significance of turbulence in the flow of water in streams", University of California Publ. *Geography*, Vol. 6, No. 1 (1932), 1–22.
36. Einstein, H.A. "Formulas for the transportation of bed-load", *Transactions of the American Society of Civil Engineers*, Vol. 107, (1942), 561–597.
37. Keulegan, G.H., "Laws of turbulent flow in open-channels", *Natl. Bur. Stand. Circ. (U. S.)*, Vol. 21, (1938), 709–741.
38. Chiu, C. L. and Chiou, J., "DStructure of 3-D flow in rectangular open-channels", *Journal of Hydraulic Engineering*, Vol. 112, No. 11, (1986), 1050–1068.
39. Javid, S., On the effect of cross sectional shape on shear stress distribution in open channel flow., *MSc. Thesis, Islamic Azad University, Mahabad Branch*, Mahabad, Iran, (2011). (In Farsi)
40. Yang, S-Q. and Lim, S.Y., Discussion of "Shear stress in smooth rectangular open-channel flows" by Junke Guo & Pierre Y. Julien, *Journal of Hydraulic Engineering, ASCE*, Vol. 132, No. 6 (2006), 629–632.
41. White, F.M., *Viscous Fluid Flow*, McGraw-Hill, New York, (1991).
42. Driscoll, T.A. and Trefethen, L.N., "Schwarz-Christoffel Mapping (Cambridge Monographs on Applied and Computational Mathematics)", *Cambridge University Press, Cambridge*, UK, (2002).
43. Spiegel, M.R., *Complex Variables*, McGraw-Hill, New York, (1993).
44. Javid, S. and Mohammadi, M., "Estimation of shear stress in smooth trapezoidal open channels using conformal mapping", *Journal of Soil and Water Science*, Vol. 22, No. 2 (2011a), 37, Tabriz University, Tabriz Iran. (In Farsi)
45. Knight, D.W., Demetriou, J.D. and Hamed, M.E., "Boundary shear in smooth rectangular channels", *Journal of Hydraulic Engineering, ASCE*, Vol. 110, No. 4 (1984), 405–422.
46. Cruff, R.W., "Cross-channel transfer of linear momentum in smooth rectangular channels", *Water Supply Paper*, 1592-B, U.S. Geological Survey, Center, Miss., B1–B26, (1965).
47. Kartha, V.C. and Leutheusser, H.J., "Distribution of tractive force in open-channels", *Journal of the Hydraulics Division, ASCE*, Vol. 96, No. 7 (1970), 1469–1483.
48. Myers, W.R.C., "Momentum transfer in a compound channel", *Journal of Hydraulic Research*, Vol. 16, No. 2 (1978), 139–150.
49. Knight, D.W. and MacDonald, J.A., "Open-channel flow with varying bed roughness", *Journal of the Hydraulics Division, ASCE*, Vol. 105, No. 9 (1979), 1167–1183.
50. Noutsopoulos, G. C. and Hadjipanous, P. A., "Discussion of boundary shear in smooth and rough channels", by D.W. Knight., *Journal of the Hydraulics Division*, Vol. 108, No. 6 (1982), 809–812.

Boundary Shear Stress in a Trapezoidal Channel

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در این تحقیق با استفاده از نظریه تفکیک شعاع هیدرولیکی به محاسبه تنش‌های برشی متوسط بستر و دیواره‌ها در مقاطع دوزنقه‌ای پرداخته می‌شود. تنش‌های برشی متوسط بستر و دیواره‌ها در مقاطع دوزنقه‌ای رو باز با جداره صاف با تکیه بر معادلات اولیه گو و جولیان برای بسط آن‌ها به مقاطع مستطیلی و دوزنقه‌ای، محاسبه می‌شوند. با استفاده از نگاهت هم‌مدیس و با کمک تحلیل‌های عددی مقادیر تنش‌های برشی متوسط به‌عنوان فرض اول با صرف نظر از اثرات جریانات ثانویه و تغییرات ویسکوزیته دینامیکی محاسبه می‌شوند. در مقایسه با داده‌های آزمایشگاهی، روابط به‌دست آمده از فرض اول برای مقاطع مستطیلی، مقادیر تنش‌های برشی متوسط بستر را در حدود $6/8\%$ زیاد بر آورد می‌کند و با کاهش شیب دیواره‌ها این خطا افزایش می‌یابد و تنش‌های برشی متوسط دیواره را در حدود $6/8\%$ کم بر آورد می‌کند که با کاهش شیب دیواره‌ها این خطا افزایش می‌یابد. در ادامه با در نظر گرفتن دو ضریب اصلاحی تجربی برای در نظر گرفتن اثرات جریانات ثانویه و سایر عوامل ممکنه به‌عنوان فرض دوم تنش‌های برشی متوسط محاسبه و تعیین می‌گردند. مقایسه داده‌های آزمایشگاهی با نتایج حاصل از روابط به‌دست آمده از فرض دوم برای تنش‌های برشی متوسط بستر حاکی از مطابقت این روابط با خطایی کمتر از $5/35\%$ و ضریب همبستگی بزرگتر از 99% با نتایج آزمایشگاهی دارد. همچنین در مورد نتایج مقایسه تنش‌های برشی متوسط دیواره‌ها، مطالعات متعدد نشان می‌دهد علی‌رغم پراکندگی داده‌های آزمایشگاهی، فرض دوم نتایج قابل قبولی با خطایی در حدود کمتر از $5/95\%$ و ضریب همبستگی حداقل بزرگتر از 93% ارائه می‌دهد.

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