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Accurate Determination of Pull-in Voltage for MEMS Capacitive Devices with Clamped Square Diaphragm

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ABSTRACT

Accurate determination of the pull-in, or the collapse voltage is critical in the design process. In this paper an analytical method is presented that provides a more accurate determination of the pull-in voltage for MEMS capacitive devices with clamped square diaphragm. The method incorporates both the linearized modle of the electrostatic force and the nonlinear deflection model of a clamped square diaphragm. The capacitor structure has been designed using a low stress doped poly silicon diaphragm with a proposed thickness of 0.8 μm and an area of 2.4 mm^2, an air gap of 3.0 μm, and a 1.0 μm thick back plate. The value of pull-in voltage calculated using equation is about 6.85V and the finite element analysis (FEA) results show that the pull-in occurs at 6.75V. The resulting pull-in voltage and deflection profile of the diaphragm are in close agreement with finite element analysis results.

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1. INTRODUCTION

MEMS capacitive devices have been studied by many researchers because of their superior performances, e.g. high sensitivity, low power consumption, flat frequency response in wide bandwidth, low noise level, stability and reliability [1]. To operate such devices, they need to be biased with a DC voltage (to form a surface charge) [2, 3]. The electrostatic force associated with bias voltage is nonlinear due to its inverse square relationship with the air gap thickness between the capacitor electrodes. This gives rise to a phenomenon known as 'pull-in' that reduces the dynamic range of the diaphragm displacement. If the bias voltage exceeds this pull-in limit, the dia- phragm will collapse.

Accurate determination of the pull-in, or the collapse voltage, is critical in the design process. Unfortunately, the commonly used parallel-plate approximation method of pull-in voltage determination introduces significant errors if the diaphragm is fully clamped. In addition, for a clamped diaphragm, the small deflection model of diaphragm deformation [4] does not account for nonlinearities associated with the presence of in-built residual stress in the diaphragm and predicts unrealistic- ally high deformation values. Thus, it is necessary to formulate a method to determine the pull-in voltage that accounts for the nonlinear nature of the design parameters.

In this paper an analytical solution has been described to calculate the pull-in voltage and diaphragm deflection for a clamped square diaphragm under electostatic actuation. The method incorporates both the linearized model of the electrostatic force and the nonlinear deflection model of a clamped square diaphragm.

2. THE PARALLEL-PLATE CAPACITOR

A parallel plate approximation is first considered to highlight the major aspects of the functional analysis of the device. In this analysis, any fringe field capacitance associated with the capacitor electrodes is neglected and the capacitor electrodes and contacts are assumed to be perfect. It is assumed that the capacitor structure is situated in a vacuum environment to ensure zero external mechanical loading of the top electrode. It is also assumed that the restoring force of the diaphragm (spring force) is a linear function of its displacement.

A lumped element model of a movable plate capacitor is shown in Figure 1a, and an equivalent circuit in Figure 1b. Neglecting the damping effect (as the structure is assumed to be in vacuum), the equation of

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motion of the movable plate due to the electrostatic attraction force F_E caused by a constant supply voltage V can be expressed as:

$$m\frac{d^2x}{dt^2}+kx=F_E \tag{1}$$

where x represents the displacement, *m[kg]* is mass of the diaphragm, *k[N/m]* is the effective spring constant of the bending membrane. The electrostatic attraction force F_E between the plates due to the charges on the plates can be found by differentiating the stored energy of the capacitor with respect to the position of the movable plate and is expressed as:

$$F_E=-\frac{d}{dx}\left(\frac{1}{2}CV^2\right)=\frac{\varepsilon_0 AV^2}{2(d_0-x)^2} \tag{2}$$

where ε_0*[F/m]* is the permittivity of free space, C the capacitance, $A[m^2]$ the area of a capacitor plate and d_0*[m]* the thickness of the air gap. In equilibrium, we have:

$$m\frac{d^2x}{dt^2}+kx=\frac{\varepsilon_0 AV^2}{2(d_0-x)^2} \tag{3}$$

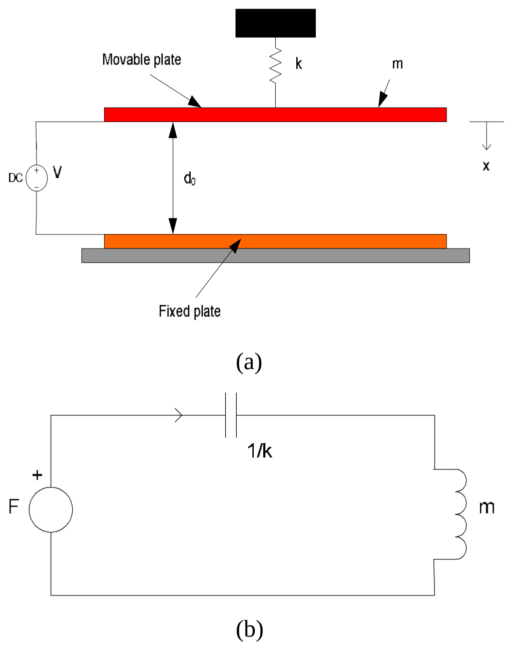


Figure 1. Lumped parameter model of a movable plate capacitor (a), equivalent circuit (b)

By expanding Equation (2) using a Taylor series approximat- ion about a minimal distance, x_0, we get:

$$F_E=\frac{\varepsilon_0 AV^2}{2(d_0-x)^2}= \frac{\varepsilon_0 AV^2}{2(d_0-x)^2}\Big|_{x=x_0}+\frac{\varepsilon_0 AV^2(-2)(-1)}{2(d_0-x)^3}\Big|_{x=x_0}(x-x_0)+\ldots \tag{4}$$

Substituting F_E from Equation (4) in Equation (3) and after rearrangement, we get:

$$m\frac{d^2x}{dt^2}+\left(k-\frac{\varepsilon_0 AV^2}{2(d_0-x_0)^3}\right)x=\frac{\varepsilon_0 AV^2}{2(d_0-x_0)^2}\left[1-2\frac{x_0}{d_0-x_0}+\ldots\right] \tag{5}$$

3. FULLY CLAMPED SQUARE DIAPHRAGM

Due to the presence of residual stress and a significantly large deflection of the diaphragm compared to its thickness, the developed strain energy in the middle of the diaphragm causes a stretch of the middle surface. Consequently, the deflection of the middle surface no longer depends solely on the external forces and the rigidity of the diaphragm increases with the deflection. This deflection dependent nonlinear behavior of a fully clamped diaphragm is known as spring hardening and a large deflection model of analysis must be applied to determine the deflection. Thus, the analytical solution for diaphragm deflection from electrostatic forces must account for this spring hardening effect in addition to the nonlinear and non-uniform electrostatic forces. Following the large deflection model, for a fully clamp- ed square diaphragm with built in residual stress, the deflection of the midpoint of the diaphragm under a uniform pressure load P can be expressed as [5,6]:

$$P(h_0)=C_1\frac{t\sigma}{a^2}h_0+C_2(v)\frac{tE}{a^4}h_0^3 \tag{6}$$

where P is the applied uniform pressure, t, the diaphrag- m thickness, E, the Young's modulus, v, the in-plane Poisson's ratio, σ, the residual stress, a, the half of the diaphragm side length and h_0 , the deflection of the diaphragm midpoint. The quantities, C_1 and C_2 are numerical parameters and their values are given by:

$$C_1=3.45,\ \&\ C_2=1.994(1-0.271v)/(1-v) \tag{7}$$

In a literature [7], it was shown that for clamped diaphragms, a fringe field correction is required for the Young's mod- ulus as given by:

$$\hat{E}=E/(1-v^2) \tag{7}$$

where $\hat{E}$ is the effective Young's modulus.

4. PULL-IN VOLTAGE

The analysis carried out in section 2 for a parallel plate capacitor structure can now be extended to the case of a fully clamped square diaphragm separated from a rigid back plate by a small air gap (Figure 2). The deflection of the diaphragm is due to the resultant effect of the electrostatic, restoring elastic and air damping forces acting simultaneously.

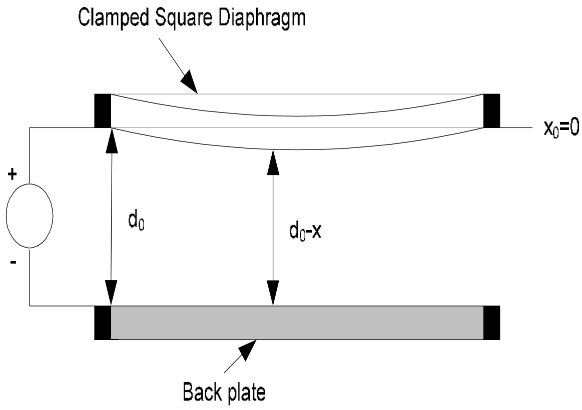


Figure 2. Linearization of the electrostatic force about zero displacement

At equilibrium, the damping force can be neglected. For a parallel plate configuration, the nonlinear electrostatic force is always uniform. However, for a rigidly clamped diaphragm under electrostatic force, the latter becomes non-uniform depending on the deformation profile of the rigidly clamped diaphragm and results in a lower deflection [8]. Thus, to evaluate the deflection of a clamped diaphragm by an electrostatic force, it is necessary to obtain a uniform, linear model of the electrostatic force to apply in the load deflection Equation (6).

A uniform, linearized model of the electrostatic force can be obtained from Equation (5) by linearizing the force about the zero deflection point, i.e. $x_0=0$ as shown in Figure 2. Since, prior to any deflection the diaphragm is flat applied without any significant error if the air gap thickness is very small compared to the lateral dimentions of the diaphragm. Thus, linearizing Equation (5) about the point, $x_0=0$, we get:

$$m\frac{d^2x}{dt^2}+\left(k-\frac{\varepsilon_0 AV^2}{d_0^{\ 3}}\right)x=\frac{\varepsilon_0 AV^2}{2d_0^2} \tag{9}$$

Rearranging and neglecting the time dependent term:

$$\frac{\varepsilon_0 AV^2}{2d_0^{\ 2}}+\frac{\varepsilon_0 AV^2}{d_0^3}x=kx \tag{10}$$

The left hand side of Equation (10) describes an approximate, uniform, linear electrostatic force that can be used to evaluate the pull-in voltage. From Equation (10), the effective linearized uniform electrostatic pressure on the diaphra- gm due to an applied bias voltage can be evaluated as:

$$P_{eff}=\varepsilon_0 V^2\left(\frac{1}{2d_0^2}+\frac{x}{d_0^3}\right) \tag{11}$$

According to the experimental results [9-12] and finite element analysis of the capacitive structure with fully clamped diaphragm using Intellisuite MEMS tool, the system becomes unstable at where the diaphragm is di- splaced about one-third of the original gap (Figure 4). Therefore, the distance where the pull-in occurs is:

$$x_{PI}=\frac{d_0}{3} \tag{12}$$

and the pull-in gap is:

$$d_{PI}=\frac{2d_0}{3} \tag{13}$$

Substituting x in Equation (11) by the pull-in deflection from Equation (12), the effective pull-in pressure $P_{eff\ -PI}$ can be evaluat- ed as:

$$P_{eff\ -PI}=\frac{5}{6}\frac{\varepsilon_0 V_{PI}^2}{d_0^2} \tag{14}$$

Next, substituting the pull-in deflection (1/3) d_0 for h_0, replacing $P(h_0)$ by $P_{eff\ -PI}$, and using the effective Young's modulus $\hat{E}$ in (6), we obtain:

$$\frac{5}{6}\frac{\varepsilon_0 V_{PI}^2}{d_0^2}=C_1\frac{t\sigma}{a^2}\left(\frac{d_0}{3}\right)+C_2(\nu)\frac{t\hat{E}}{a^4}\left(\frac{d_0}{3}\right)^3 \tag{15}$$

The above equation can now be solved for the pull-in voltage V_{PI} as:

$$V_{PI}=\sqrt{\frac{6d_0^{\ 2}}{5\varepsilon_0}\left[C_1\frac{t\sigma}{a^2}\left(\frac{d_0}{3}\right)+C_2(\nu)\frac{t\hat{E}}{a^4}\left(\frac{d_0}{3}\right)^3\right]} \tag{16}$$

Equation (16) provides the desired approximation of the pull-in voltage for a clamped square diaphragm. If the voltage is increased beyond this pull-in voltage, the resulting electrostatic force will cause the movable plate to collapse onto the fixed plate and the capacitor will be short-circuited.

5. FINITE ELEMENT ANALYSIS

The analysis was done using the MEMS design and simulation tool. The objectives of analysis are first, to verify the deformation of the diaphragm due to the electrostatic attraction force between the diaphragm and the back plate and the mechanically applied force. Second, to verify the capacitance between the diphragm and the back plate. The analysis options are nonlinear analysis, accuracy of convergence is 0.001 micron, maximum mesh size is 2.4% of X-Y dimension.

5. 1. Simulation Setup of Clamped Square Figure 3 shows the simulation setup of the MEMS capacitive structure with clamped low stress doped poly silicon diaphragm. The diaphragm has a proposed thickness of 0.8 μm, an area of 2.4 mm^2, an air gap of 3.0 μm and a 1.0 μm thick poly silicon back plate. A DC bias voltage is provided between the diaphragm and the back plate. The back plate and 4 lateral faces of diaphragm are fixed.

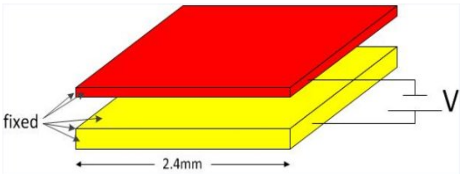


Figure 3. Schematic simulation setup for capacitive structure with clamped diaphragm.

6. RESULTS AND DISCUSSION

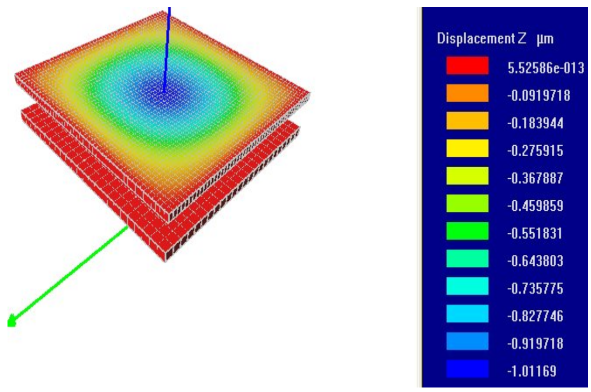
To illustrate the above model of pull-in voltage evaluation, a capacitive structure having design parameters as given in Table 1 has been used.

Figure 4a shows diaphragm deformation in the Z-axis at a distance of one-third of the air gap. The Z-axis deformation is -1.01micron at a bias voltage of 6.75 V. Above this critical bias voltage the diaphragm collapses on the back plate (Figure 4b). Thus, the finite element analysis results show the structure becomes unstable at one-third of the original gap and diaphragm collapse due to pull-in phenomenon.

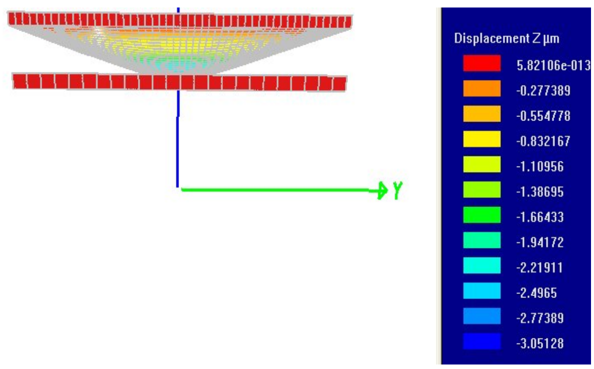
At a distance of one-third of the air gap (1μm) the pull-in occurs. The pull-in voltage calculated using Equation (16) is 6.85V. Results from 3-D finite element analysis and the developed analytical model for the diaphragm deflection for different bias voltages are plotted in Figure 5. This curve can be naturally divided into four regions. In the first linear region for small bias voltages, the electrostatic forces are insignificant compared with the pressure load. For intermediate voltage, the electros- tatic forces have a notable influence. In the region close to collapse of the structure, the influence becomes significant enough system becomes unstable at the pull-in voltage, where the diaphragm of the structure is displaced about one-third of the original gap. Above the critical bias voltage the structure collapses. The FEA results show that the pull-in occurs at 6.75V and this is in close agreement with the analytical value. The capacitance versus voltage characteristics using finite element analysis are shown in Figure 6. FEA computed capacitance is 17.08 pF and calculated value is 17 pF in zero bias, corresponding to an air gap of about 3 μm. The pull-in voltage can also be obtained from the capacitance-voltage (C-V) plot. From Figure 6, it can be seen that the pull-in occurs at 6.75V and above this critical bias voltage the structure collapses and capacitance value increases rapidly. This voltage is an important parameter for evaluating the mechanical properties of the diaphragm.

TABLE 1. Design Parameters

Parameter	Value
Diaphragm thickness, *t*	0.8 μm
Diaphragm side length, *a*	2.4 mm
Air gap thickness, d_0	3.0 μm
Young's modulus, *E*	160 GPa
Poisson's ratio, *v*	0.22
Diaphragm residual stress, *σ*	20 MPa
Diaphragm material	Low stress doped Poly Si
Back plate thickness, *h*	1 μm
Back plate material	Poly Si



(a) Diaphragm deformation in the Z- axis



(b) Diaphragm collapse due to pull-in phenomenon

Figure 4. Finite element analysis of the capacitive structure

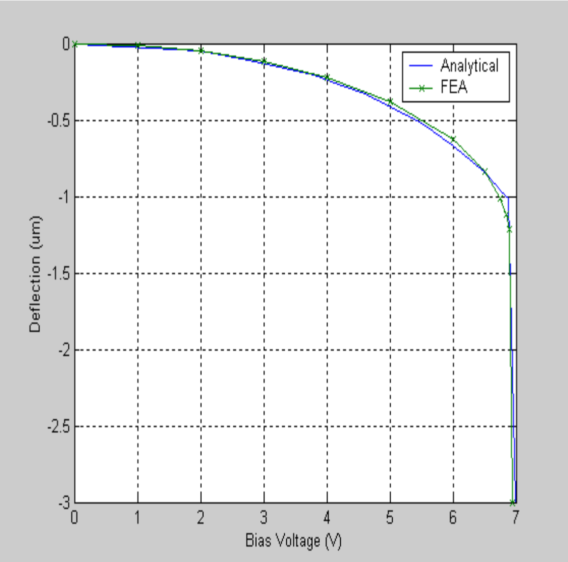


Figure 5. Comparison of diaphragm deflection calculated using the analytical model and finite element analysis for different bias voltages.

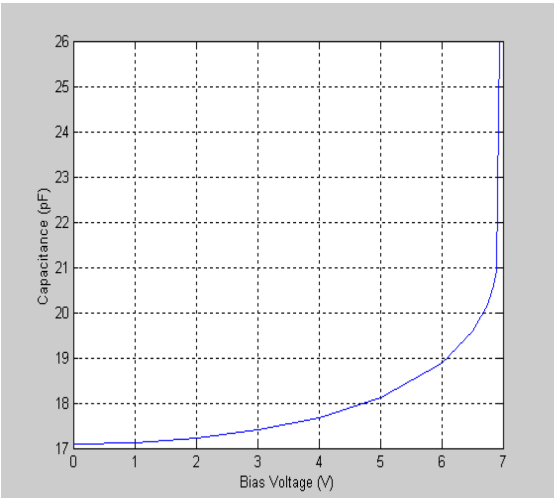


Figure 6. Capacitance vs. bias voltage characteristics

7. CONCLUTIONS

An analytical method is presented to determine more accurate pull-in voltage and diaphragm deflection of a MEMS capacitive device having a clamped square diaphragm. The method incorporates the linearized and uniform nature of the electrostatic force associated with a clamped diaphragm deformation. The resulting analytical model provides a more accurate approximation of the pull-in voltage. The value of pull-in voltage calculated using Equation (16) is about 6.85V and the finite element analysis (FEA) results show that the pull-in occurs at 6.75V. The results from the analytical model are in close agreement with finite element analysis results. The method can be extended to determine the pull-in voltage for other microstructures utilizing electrostatic actuation if the air gap thickness is very small compared to lateral dimensions.

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چکیده

در طراحی المان‌های الکترواستاتیک تعیین دقیق ولتاژ شکست حیاتی می‌باشد. در این مقاله یک حل عددی برای تعیین دقیق‌تر ولتاژ شکست برای ادوات ممزی خازنی با دیافراگم مربعی از اطراف بسته، ارائه شده است. در این روش از ترکیب نیروی الکترواستاتیک خطی شده و خمیدگی غیرخطی دیافراگم استفاده می‌شود. ساختار خازنی با استفاده از دیافراگم پلی سیلیکونی کم‌استرس با ضخامت ۰٫۸ میکرومتر، سطح ۲٫٤ میلی‌متر مربع، فاصله‌ی هوایی ۳ میکرومتر و صفحه زیرین با ضخامت یک میکرومتر طراحی شده است. با استفاده از معادله‌ی ریاضی مقدار ولتاژ شکست ٦٫٨٥ ولت به دست آمده است. از آنالیز اجزای محدود مقدار ٦٫٧٥ ولت برای ولتاژ شکست حاصل شده است. بنابراین مقدار به دست آمده از معادله‌ی ارائه شده بسیار نزدیک به نتایج آنالیز اجزای محدود می‌باشد.

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