# RELIABILITY MEASURES AND SENSITIVITY ANALYSIS OF A COMPLEX MATRIX SYSTEM INCLUDING POWER FAILURE

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**Abstract** This paper investigates the reliability characteristics of a complex system having nine subsystems arranged in the form of 3x3 matrix in which each row contains three subsystems. The configuration of the row is of the type 2-out-of-3: F. Each subsystem has *n* units connected in series. The system fails if any one row containing three subsystems fails. The considered system analyzed incorporating different types of power failure which also leads to failure of the system. With the help of Supplementary variable technique, Laplace transformations and copula methodology, the transition state probabilities, asymptotic behavior, availability, reliability, M.T.T.F., busy period, sensitivity analysis and cost effectiveness of the system have been evaluated. Finally, some particular cases and numerical examples have been taken to describe the model.

**Keywords** Availability; Reliability; M.T.T.F.; Busy period; Sensitivity analysis; Cost effectiveness; Gumbel-Hougaard copula.

**چکیده** این مقاله اعتبار ویژگی‌های یک سیستم پیچیده با ۹ زیرسیستم که در شکل ماتریس ۳ ۳ مرتب شده است را بررسی می‌کند در این ماتریس هر سطر شامل ۳ زیرسیستم است. پیکربندی ردیف از نوع ۲ از۳: F می‌باشد. هر زیرسیستم دارای n واحد مرتبط به هم از نوع سری است. اگر هر یک از سطرهای شامل ۳ زیرسیستم با شکست مواجه شود، سیستم از کار می‌افتد. سیستم مورد نظر ترکیب انواع مختلفی از قطعی‌های قدرت را که منجر به از کار افتادن سیستم می‌شود را تجزیه و تحلیل می‌کند. با کمک روش متغیر متمم ، تبدیل لاپلاس و روش کوپلا احتمال حالت گذار، رفتار مجانبی، در دسترس بودن، قابلیت اطمینان، MTTF، دوره شلوغی، آنالیز حساسیت وتاثیر هزینه سیستم مورد بررسی قرار گرفته است. در پایان تعدادی از موارد خاص و مثال‌های عددی برای توصیف مدل به کار گرفته شده است.

## 1. INTRODUCTION

In real life we can see many instances in which the system stops working due to the power failure. This is one of the features of some under - developed and developing countries. Power cut may be due to power shortage or natural disasters which may occur anywhere in the world. So, the power failure may be one of the causes for the system to become non operational. This is a situation in which system as such is functional but not in operation due to power cut. This may result in productivity loss, or affect the cost effectiveness of the system. Computer systems and other electronic devices containing logic circuitry are susceptible to data loss due to the sudden loss of power. When the power comes back, the system restarts. The time between the system stop and restart is said to be reboot delay. These can include video projectors, alarm systems as well as computers. There are many causes of power failures in an electricity network. Examples of these causes include: faults at power stations, damage to electric transmission lines, substations or other parts of the distribution system, a short circuit, or overloading of electricity mains etc. A very common situation of power loss due to natural calamities is that Tree limbs create a short circuit in electrical lines during a storm. This will typically result in a power outage to the area supplied by these lines. Power failures are categorized into following three phenomena, relating to the duration and effect of the failure.

**Transient fault-** A transient fault is a momentary loss of power typically caused by temporary fault on a power line. Power is automatically restored once the fault is cleared.

**Brownout-**A Brownout or sag is a drop in voltage in an electrical power supply. The term brownout comes from the dimming experienced by lighting when the voltage sags.

**Blackout-**A Blackout refers to the total loss of power to an area, and is the most severe form of power failure that can occur. Blackouts which result from or result in power stations tripping are particularly difficult to recover quickly. Failures may last from a few minutes to a few weeks depending on the nature of the blackout and the configuration of the electrical network. Power failures are particularly critical at sites where the environment and public safety are at risk. Institutions such as hospitals, sewage treatment plants, mines etc., will usually have backup power sources, such as standby generators, which will automatically / manually start up when electrical power is lost. Other critical systems, such as telecommunications, are also required to have emergency power backups. Telephone exchange rooms usually have arrays of lead – acid batteries for backup, and also a socket for connecting a generator during extended periods of power failure. Hence, while analyzing the system, power failure should also be considered which leads to non-operational state of the system.

Many researchers have analyzed different types of systems with different failures. Gupta and Agrawal [1] considered a parallel redundant complex system with two types of failure under preemptive-repeat repair discipline. Jain et al. [2] provides a research note on finite queue with two types of failures and preemptive priority. Jain et al. [3] examined maintenance cost analysis for replacement model with perfect/minimal repair. Jain et al. [4] and Kontoleon and Kontoleon [5] discussed the reliability analysis of a system subject to partial, degraded and catastrophic failures. Pandey and Jacob [7] studied the cost analysis, availability and MTTF of a three state standby complex system under common cause and human failures. Ram and Singh [8] analyzed availability and cost analysis of a parallel redundant complex system with two types of failure under preemptive-resume repair discipline using Gumbel-Hougaard family of copula in repair. In most of the studies the system is investigated with different types of failures such as partial failure, catastrophic failure, common cause failure and human failure but no thought has been given to power failures while analyzing the systems. From this discussion, it seems that power failures can be included while analyzing the system because it is one of the causes which can affect the performance of the system. Zuo et al. [12] have taken a real industrial application in which a petro-chemical company in Canada produces crude oil has been described. It uses several methane reformer furnaces to produce hydrogen for hydro-treating. These furnaces have hundreds of tubes which are filled with catalyst. The tubes in a furnace are arranged vertically. They have taken many failure scenarios for the furnace. For example: (i) if at least one row of tubes has at least a given number of consecutive tubes that are failed, (ii) whenever there is at least one cluster of size $r \times s$ of failed tubes. By the above description it is clear that in industrial area there are some systems in which systems configuration can be considered in the form of a matrix. The linear connected-($r$, $s$)-out-of-($m$, $n$): F is such type of system that has $m \times n$ components and consists of $n$ columns and m rows. The system fails if and only if a connected-($r$, $s$)-matrix of components fail. This type of system can be used for modeling engineering systems such as temperature feeler systems, supervision systems etc. [11].

Keeping above facts in mind, in the present paper we analyze a complex system with three types of power failures, namely transient, brownout and blackout. We considered a system has nine subsystems arranged in the form of 3x3 matrix in which each row is having three subsystems. Configuration of each subsystem is 1-out-of-$n$: F, i.e. if any one unit out of total $n$ units fails then the subsystem fails. It is to be mentioned here that the more the number of units, the greater the chances of failures. The whole system fails if any one row containing three subsystems fails. The row is 2-out-of-3: F, i.e. if any two subsystems out of three fail then the row will fail which lead to complete failure of the system. Here, it is also assumed that the system can stop working due to any of the three types of power failures. Whenever system stops due to any of the power failures, a backup power source, for example standby generator starts automatically and the system starts working; but the system stops working if the backup power source also fails. Furthermore, it is also assumed that the backup power source is not as efficient as main power source, and cannot be operable for a very long period of time. For the failures, the repairs are done perfectly so after repair each subsystem is as good as new. Furthermore, in real life problems we can have situations in which a system can be repaired in two different ways. This feature is also incorporated in this study. We have used Gumbel-Hougaard family of copula [6, 9 and 10] to find joint distribution of repairs whenever power failure and standby power supply are being repaired with different repair rates. Failure rates are assumed to be constant in general, whereas the repairs follow general distribution. With the help of Supplementary variable technique, Laplace transformation and copula methodology, following reliability measures of the system have been evaluated:

1. Transition state probabilities of the system.
2. Asymptotic behaviour of the system.
3. Various measures such as availability, reliability, M.T.T.F., busy period, sensitivity analysis and cost effectiveness of the system.

Some numerical examples are also presented to illustrate the model mathematically.

## 2. ASSUMPTIONS

1. Initially the system is in perfectly good state, i.e. all the units are functioning perfectly.
2. At $t = 0$ all the components are perfect, and $t > 0$ they start operating.
3. We considered the system having nine subsystems arranged in the form of 3x3 matrix in which each row contains three subsystems. The configuration of each row is 2-out-of-3-F, i.e. if any two subsystems fails then the row will fail. The whole system fails if any one row fails.
4. Each subsystem has $n$ units connected in series (1-out-of-$n$: F).
5. The system is analyzed with three types of power failures.
6. When the system stops working due to the power failure, a backup power source starts automatically and the system restarts,. Here we assume the backup power source is not very efficient and cannot operate the system for a long period.
7. The repair of the failed subsystem is perfect. After repair each subsystem is as good as new.
8. The joint probability distribution of repairs when both the power failures and the backup power source are under repair is computed by Gumbel - Hougaard family of copula.

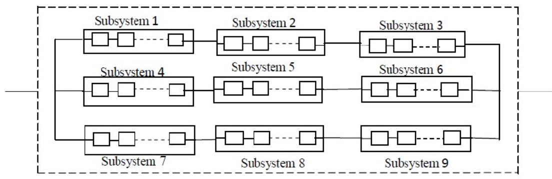


**Figure 1.** Diagram of investigated system

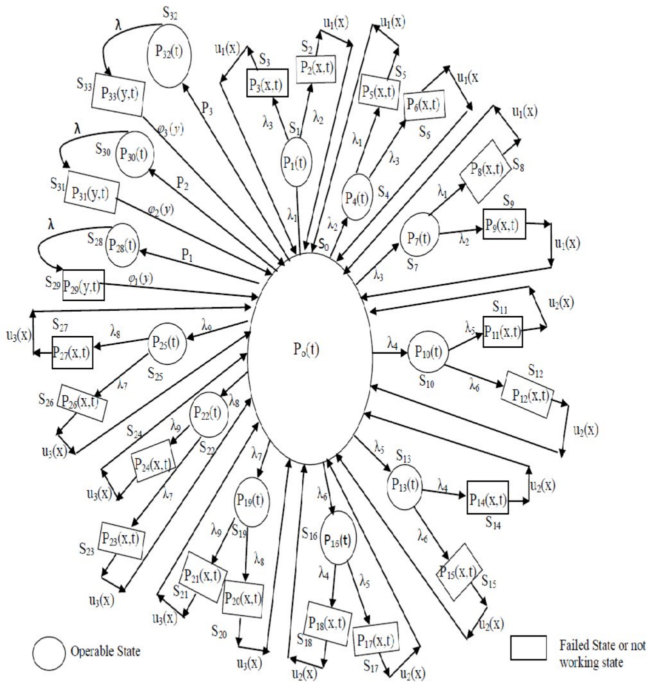


**Figure 2**. Transition diagram

**TABLE 1.** State specification

| States | System State | States | System State | States | System state |
|---|---|---|---|---|---|
| $S_0$ | W | $S_{12}$ | $F_R$ | $S_{24}$ | $F_R$ |
| $S_1$ | W | $S_{13}$ | W | $S_{25}$ | W |
| $S_2$ | $F_R$ | $S_{14}$ | $F_R$ | $S_{26}$ | $F_R$ |
| $S_3$ | $F_R$ | $S_{15}$ | $F_R$ | $S_{27}$ | $F_R$ |
| $S_4$ | W | $S_{16}$ | W | $S_{28}$ | $W_P$ |
| $S_5$ | $F_R$ | $S_{17}$ | $F_R$ | $S_{29}$ | $W_N$ |
| $S_6$ | $F_R$ | $S_{18}$ | $F_R$ | $S_{30}$ | $W_P$ |
| $S_7$ | W | $S_{19}$ | W | $S_{31}$ | $W_N$ |
| $S_8$ | $F_R$ | $S_{20}$ | $F_R$ | $S_{32}$ | $W_P$ |
| $S_9$ | $F_R$ | $S_{21}$ | $F_R$ | $S_{33}$ | $W_N$ |
| $S_{10}$ | W | $S_{22}$ | W | | |
| $S_{11}$ | $F_R$ | $S_{23}$ | $F_R$ | | |

W: Working state, $W_N$: System is not working as Power failure and standby Power supply under repair, $F_R$: Failed under repair, $W_P$: Working due to standby power supply

## 3. NOTATIONS

$\lambda_1 / \lambda_2 / \lambda_3$: Failure rate of subsystem 1 / subsystem 2 / subsystem 3, where

$$\lambda_1 = \sum_{i=1}^{n} \lambda_i,\ \lambda_2 = \sum_{j=1}^{n} \lambda_j \text{ and } \lambda_3 = \sum_{k=1}^{n} \lambda_k$$

$\lambda_4 / \lambda_5 / \lambda_6$: Failure rate of subsystem 4 / subsystem 5 / subsystem 6, where

$$\lambda_4 = \sum_{p=1}^{n} \lambda_p,\ \lambda_5 = \sum_{q=1}^{n} \lambda_q \text{ and } \lambda_6 = \sum_{r=1}^{n} \lambda_r$$

$\lambda_7 / \lambda_8 / \lambda_9$: Failure rate of subsystem 7 / subsystem 8 / subsystem 9, where

$$\lambda_7 = \sum_{l=1}^{n} \lambda_l,\ \lambda_8 = \sum_{m=1}^{n} \lambda_m \text{ and } \lambda_9 = \sum_{g=1}^{n} \lambda_g,$$

where $n$ is the total number of units in each subsystems

$P_1$: Failure rate of blackout.

$P_2$: Failure rate of transient fault.

$P_3$: Failure rate of Brownout.

$\lambda$: Failure rate of standby power supply.

$u_1(x)$: Repair rate of subsystem 1, subsystem 2 and subsystem 3.

$u_2(x)$: Repair rate of subsystem 4, subsystem 5 and subsystem 6.

$u_3(x)$: Repair rate of subsystem 7, subsystem 8 and subsystem 9.

$\psi_1(y)$: Repair rate of blackout.

$\psi_2(y)$: Repair rate of transient fault.

$\psi_3(y)$: Repair rate of brownout.

$v(y)$: Repair rate of standby power supply.

$x$: Elapsed repair time.

$y$: Elapsed repair time for power failure and standby power supply failure.

$P_i(t)$: Probability that the system is in $S_i$ state at instant $t$ for $i = 1$ to $i = 33$.

$\overline{P}_i(s)$: Laplace transform of $P_i(t)$.

$P_i(x, t)$: Probability density function that at time $t$ the system is in failed state $S_i$ and the system is under repair, elapsed repair time is $x$.

$P_j(y,t)$: Probability density function that at time $t$ the system is not in working state $S_j$ (j = 29, 31 and 33) and the main power source and standby power source failure are under repair, elapsed repair time is $y$.

$E_p(t)$: Expected profit during the interval (0, $t$].

$K_1$, $K_2$: Revenue per unit time and service cost per unit time respectively.

$\overline{S}_\eta(x)$: Laplace transform of

$$S_\eta(x) = \int_0^\infty \eta(x) \exp(-sx - \int_0^x \eta(x))dx$$

If $u_1 = v(y)$, $u_2 = \psi_1(y), \psi_2(y), \psi_3(y)$ then the expression for the joint probability according to Gumbel-Hougaard family of copula is given by

$$\varphi_1(y) = \exp[-\{(-\log v(y))^\theta + (-\log \psi_1(y))^\theta\}^{1/\theta}]$$

$$\varphi_2(y) = \exp[-\{(-\log v(y))^\theta + (-\log \psi_2(y))^\theta\}^{1/\theta}]$$

$$\varphi_3(y) = \exp[-\{(-\log v(y))^\theta + (-\log \psi_3(y))^\theta\}^{1/\theta}]$$

where θ is the parameter which may take all values in the interval [1, ∞).

## 4. FORMULATION OF MATHEMATICAL MODEL

By probability consideration and continuity arguments the following difference-differential equations governing the behavior of the system seems to be good.

$$\begin{bmatrix} \frac{d}{dt} + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \\ \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + P_1 + P_2 + P_3 \end{bmatrix} P_0(t) = \int_0^\infty u_1(x) P_3(x,t)dx$$
$$+ \int_0^\infty u_1(x) P_2(x,t)dx + \int_0^\infty u_1(x) P_5(x,t)dx + \int_0^\infty u_1(x) P_6(x,t)dx$$
$$+ \int_0^\infty u_1(x) P_8(x,t)dx + \int_0^\infty u_1(x) P_9(x,t)dx + \int_0^\infty u_2(x) P_{11}(x,t)dx$$
$$+ \int_0^\infty u_2(x) P_{12}(x,t)dx + \int_0^\infty u_2(x) P_{14}(x,t)dx + \int_0^\infty u_2(x) P_{15}(x,t)dx$$
$$+ \int_0^\infty u_2(x) P_{17}(x,t)dx + \int_0^\infty u_2(x) P_{18}(x,t)dx + \int_0^\infty u_3(x) P_{20}(x,t)dx$$
$$+ \int_0^\infty u_3(x) P_{21}(x,t)dx + \int_0^\infty u_3(x) P_{23}(x,t)dx + \int_0^\infty u_3(x) P_{24}(x,t)dx$$
$$+ \int_0^\infty u_3(x) P_{26}(x,t)dx + \int_0^\infty u_3(x) P_{27}(x,t)dx + \int_0^\infty \varphi_1(x) P_{29}(x,t)dx$$
$$+ \int_0^\infty \varphi_2(x) P_{31}(x,t)dx + \int_0^\infty \varphi_3(x) P_{33}(x,t)dx \quad (1)$$

$$\left[\frac{d}{dt}+\lambda_3+\lambda_2\right]P_1(t)=\lambda_1 P_0(t) \tag{2}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+u_1(x)\right]P_2(x,t)=0 \tag{3}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+u_1(x)\right]P_3(x,t)=0 \tag{4}$$

$$\left[\frac{d}{dt}+\lambda_3+\lambda_1\right]P_4(t)=\lambda_2 P_0(t) \tag{5}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+u_1(x)\right]P_5(x,t)=0 \tag{6}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+u_1(x)\right]P_6(x,t)=0 \tag{7}$$

$$\left[\frac{d}{dt}+\lambda_1+\lambda_2\right]P_7(t)=\lambda_3 P_0(t) \tag{8}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+u_1(x)\right]P_8(x,t)=0 \tag{9}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+u_1(x)\right]P_9(x,t)=0 \tag{10}$$

$$\left[\frac{d}{dt}+\lambda_5+\lambda_6\right]P_{10}(t)=\lambda_4 P_0(t) \tag{11}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+u_2(x)\right]P_{11}(x,t)=0 \tag{12}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+u_2(x)\right]P_{12}(x,t)=0 \tag{13}$$

$$\left[\frac{d}{dt}+\lambda_4+\lambda_6\right]P_{13}(t)=\lambda_5 P_0(t) \tag{14}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+u_2(x)\right]P_{14}(x,t)=0 \tag{15}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+u_2(x)\right]P_{15}(x,t)=0 \tag{16}$$

$$\left[\frac{d}{dt}+\lambda_5+\lambda_4\right]P_{16}(t)=\lambda_6 P_0(t) \tag{17}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+u_2(x)\right]P_{17}(x,t)=0 \tag{18}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+u_2(x)\right]P_{18}(x,t)=0 \tag{19}$$

$$\left[\frac{d}{dt}+\lambda_8+\lambda_9\right]P_{19}(t)=\lambda_7 P_0(t) \tag{20}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+u_3(x)\right]P_{20}(x,t)=0 \tag{21}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+u_3(x)\right]P_{21}(x,t)=0 \tag{22}$$

$$\left[\frac{d}{dt}+\lambda_7+\lambda_9\right]P_{22}(t)=\lambda_8 P_0(t) \tag{23}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+u_3(x)\right]P_{23}(x,t)=0 \tag{24}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+u_3(x)\right]P_{24}(x,t)=0 \tag{25}$$

$$\left[\frac{d}{dt}+\lambda_7+\lambda_8\right]P_{25}(t)=\lambda_9 P_0(t) \tag{26}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+u_3(x)\right]P_{26}(x,t)=0 \tag{27}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+u_3(x)\right]P_{27}(x,t)=0 \tag{28}$$

$$\left[\frac{d}{dt}+\lambda\right]P_{28}(t)=P_1 P_0(t) \tag{29}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\varphi_1(y)\right]P_{29}(y,t)=0 \tag{30}$$

$$\left[\frac{d}{dt}+\lambda\right]P_{30}(t)=P_2 P_0(t) \tag{31}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\varphi_2(y)\right]P_{31}(y,t)=0 \tag{32}$$

$$\left[\frac{d}{dt}+\lambda\right]P_{32}(t)=P_3 P_0(t) \tag{33}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\varphi_3(y)\right]P_{33}(y,t)=0 \tag{34}$$

Boundary conditions:

$$P_2(0,t) = \lambda_2 P_1(t) \quad (35)$$

$$P_3(0,t) = \lambda_3 P_1(t) \quad (36)$$

$$P_5(0,t) = \lambda_1 P_4(t) \quad (37)$$

$$P_6(0,t) = \lambda_3 P_4(t) \quad (38)$$

$$P_8(0,t) = \lambda_1 P_7(t) \quad (39)$$

$$P_9(0,t) = \lambda_2 P_7(t) \quad (40)$$

$$P_{11}(0,t) = \lambda_5 P_{10}(t) \quad (41)$$

$$P_{12}(0,t) = \lambda_6 P_{10}(t) \quad (42)$$

$$P_{14}(0,t) = \lambda_4 P_{13}(t) \quad (43)$$

$$P_{15}(0,t) = \lambda_6 P_{13}(t) \quad (44)$$

$$P_{17}(0,t) = \lambda_5 P_{16}(t) \quad (45)$$

$$P_{18}(0,t) = \lambda_4 P_{16}(t) \quad (46)$$

$$P_{20}(0,t) = \lambda_8 P_{19}(t) \quad (47)$$

$$P_{21}(0,t) = \lambda_9 P_{19}(t) \quad (48)$$

$$P_{23}(0,t) = \lambda_7 P_{22}(t) \quad (49)$$

$$P_{24}(0,t) = \lambda_9 P_{22}(t) \quad (50)$$

$$P_{26}(0,t) = \lambda_7 P_{25}(t) \quad (51)$$

$$P_{27}(0,t) = \lambda_8 P_{25}(t) \quad (52)$$

$$P_{29}(0,t) = \lambda P_{28}(t) \quad (53)$$

$$P_{31}(0,t) = \lambda P_{30}(t) \quad (54)$$

$$P_{33}(0,t) = \lambda P_{32}(t) \quad (55)$$

Initial conditions:

$P_0(0) = 1$, and other state probabilities are zero at $t$=0. (56)

## 5. SOLUTION OF THE MODEL

Taking Laplace transformation of (1) to (56) and on further simplification, one can obtain transition state probabilities of the system as:

$$\overline{P}_0(s) = 1/A(s) \quad (57)$$

$$\overline{P}_1(s) = \frac{\lambda_1}{s+\lambda_3+\lambda_2} \times \overline{P}_0(s) \quad (58)$$

$$\overline{P}_2(s) = \frac{\lambda_2\lambda_1}{s+\lambda_3+\lambda_2} \times \overline{P}_0(s) \times \left[\frac{1-\overline{S}_{u_1}(s)}{s}\right] \quad (59)$$

$$\overline{P}_3(s) = \frac{\lambda_3\lambda_1}{s+\lambda_3+\lambda_2} \times \overline{P}_0(s) \times \left[\frac{1-\overline{S}_{u_1}(s)}{s}\right] \quad (60)$$

$$\overline{P}_4(s) = \frac{\lambda_2}{s+\lambda_3+\lambda_1} \times \overline{P}_0(s) \quad (61)$$

$$\overline{P}_5(s) = \frac{\lambda_1\lambda_2}{s+\lambda_3+\lambda_1} \times \overline{P}_0(s) \times \left[\frac{1-\overline{S}_{u_1}(s)}{s}\right] \quad (62)$$

$$\overline{P}_6(s) = \frac{\lambda_3\lambda_2}{s+\lambda_3+\lambda_1} \times \overline{P}_0(s) \times \left[\frac{1-\overline{S}_{u_1}(s)}{s}\right] \quad (63)$$

$$\overline{P}_7(s) = \frac{\lambda_3}{s+\lambda_1+\lambda_2} \times \overline{P}_0(s) \quad (64)$$

$$\overline{P}_8(s) = \frac{\lambda_1\lambda_3}{s+\lambda_1+\lambda_2} \times \overline{P}_0(s) \times \left[\frac{1-\overline{S}_{u_1}(s)}{s}\right] \quad (65)$$

$$\overline{P}_9(s) = \frac{\lambda_2\lambda_3}{s+\lambda_1+\lambda_2} \times \overline{P}_0(s) \times \left[\frac{1-\overline{S}_{u_1}(s)}{s}\right] \quad (66)$$

$$\overline{P}_{10}(s) = \frac{\lambda_4}{s+\lambda_5+\lambda_6} \times \overline{P}_0(s) \quad (67)$$

$$\overline{P}_{11}(s) = \frac{\lambda_5\lambda_4}{s+\lambda_5+\lambda_6} \times \overline{P}_0(s) \times \left[\frac{1-\overline{S}_{u_2}(s)}{s}\right] \quad (68)$$

$$\overline{P}_{12}(s) = \frac{\lambda_6\lambda_4}{s+\lambda_5+\lambda_6} \times \overline{P}_0(s) \times \left[\frac{1-\overline{S}_{u_2}(s)}{s}\right] \quad (69)$$

$$\overline{P}_{13}(s) = \frac{\lambda_5}{s+\lambda_4+\lambda_6} \times \overline{P}_0(s) \quad (70)$$

$$\overline{P}_{14}(s) = \frac{\lambda_4\lambda_5}{s+\lambda_4+\lambda_6} \times \overline{P}_0(s) \times \left[\frac{1-\overline{S}_{u_2}(s)}{s}\right] \quad (71)$$

$$\overline{P}_{15}(s) = \frac{\lambda_6\lambda_5}{s+\lambda_4+\lambda_6} \times \overline{P}_0(s) \times \left[\frac{1-\overline{S}_{u_2}(s)}{s}\right] \quad (72)$$

$$\overline{P}_{16}(s) = \frac{\lambda_6}{s+\lambda_5+\lambda_4} \times \overline{P}_0(s) \quad (73)$$

$$\overline{P}_{17}(s)=\frac{\lambda_5\lambda_6}{s+\lambda_5+\lambda_4}\times\overline{P}_0(s)\times\left[\frac{1-\overline{S}_{u_2}(s)}{s}\right] \tag{74}$$

$$\overline{P}_{18}(s)=\frac{\lambda_4\lambda_6}{s+\lambda_5+\lambda_4}\times\overline{P}_0(s)\times\left[\frac{1-\overline{S}_{u_2}(s)}{s}\right] \tag{75}$$

$$\overline{P}_{19}(s)=\frac{\lambda_7}{s+\lambda_8+\lambda_9}\times\overline{P}_0(s) \tag{76}$$

$$\overline{P}_{20}(s)=\frac{\lambda_7\lambda_8}{s+\lambda_8+\lambda_9}\times\overline{P}_0(s)\times\left[\frac{1-\overline{S}_{u_3}(s)}{s}\right] \tag{77}$$

$$\overline{P}_{21}(s)=\frac{\lambda_7\lambda_9}{s+\lambda_8+\lambda_9}\times\overline{P}_0(s)\times\left[\frac{1-\overline{S}_{u_3}(s)}{s}\right] \tag{78}$$

$$\overline{P}_{22}(s)=\frac{\lambda_8}{s+\lambda_7+\lambda_9}\times\overline{P}_0(s) \tag{79}$$

$$\overline{P}_{23}(s)=\frac{\lambda_7\lambda_8}{s+\lambda_7+\lambda_9}\times\overline{P}_0(s)\times\left[\frac{1-\overline{S}_{u_3}(s)}{s}\right] \tag{80}$$

$$\overline{P}_{24}(s)=\frac{\lambda_9\lambda_8}{s+\lambda_7+\lambda_9}\times\overline{P}_0(s)\times\left[\frac{1-\overline{S}_{u_3}(s)}{s}\right] \tag{81}$$

$$\overline{P}_{25}(s)=\frac{\lambda_9}{s+\lambda_7+\lambda_8}\times\overline{P}_0(s) \tag{82}$$

$$\overline{P}_{26}(s)=\frac{\lambda_7\lambda_9}{s+\lambda_7+\lambda_8}\times\overline{P}_0(s)\times\left[\frac{1-\overline{S}_{u_3}(s)}{s}\right] \tag{83}$$

$$\overline{P}_{27}(s)=\frac{\lambda_8\lambda_9}{s+\lambda_7+\lambda_8}\times\overline{P}_0(s)\times\left[\frac{1-\overline{S}_{u_3}(s)}{s}\right] \tag{84}$$

$$\overline{P}_{28}(s)=\frac{P_1}{s+\lambda}\times\overline{P}_0(s) \tag{85}$$

$$\overline{P}_{29}(s)=\frac{\lambda P_1}{s+\lambda}\times\overline{P}_0(s)\times\left[\frac{1-\overline{S}_{\varphi_1}(s)}{s}\right] \tag{86}$$

$$\overline{P}_{30}(s)=\frac{P_2}{s+\lambda}\times\overline{P}_0(s) \tag{87}$$

$$\overline{P}_{31}(s)=\frac{\lambda P_2}{s+\lambda}\times\overline{P}_0(s)\times\left[\frac{1-\overline{S}_{\varphi_2}(s)}{s}\right] \tag{88}$$

$$\overline{P}_{32}(s)=\frac{P_3}{s+\lambda}\times\overline{P}_0(s) \tag{89}$$

$$\overline{P}_{33}(s)=\frac{\lambda P_3}{s+\lambda}\times\overline{P}_0(s)\times\left[\frac{1-\overline{S}_{\varphi_3}(s)}{s}\right] \tag{90}$$

where

$$\begin{aligned}
A(s)=&(s+\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5+\lambda_6+\lambda_7+\lambda_8+\lambda_9\\
&+P_1+P_2+P_3)-\frac{\lambda_3\lambda_1}{s+\lambda_3+\lambda_2}\times\overline{S}_{u_1}(s)-\\
&\frac{\lambda_2\lambda_1}{s+\lambda_3+\lambda_2}\times\overline{S}_{u_1}(s)-\frac{\lambda_1\lambda_2}{s+\lambda_3+\lambda_1}\times\overline{S}_{u_1}(s)\\
&-\frac{\lambda_3\lambda_2}{s+\lambda_3+\lambda_1}\times\overline{S}_{u_1}(s)-\frac{\lambda_1\lambda_3}{s+\lambda_1+\lambda_2}\times\overline{S}_{u_1}(s)\\
&-\frac{\lambda_2\lambda_3}{s+\lambda_1+\lambda_2}\times\overline{S}_{u_1}(s)-\overline{S}_{u_2}(s)\times\frac{\lambda_5\lambda_4}{s+\lambda_5+\lambda_6}\\
&-\frac{\lambda_6\lambda_4}{s+\lambda_5+\lambda_6}\times\overline{S}_{u_2}(s)-\frac{\lambda_4\lambda_5}{s+\lambda_4+\lambda_6}\times\overline{S}_{u_2}(s)\\
&-\frac{\lambda_6\lambda_5}{s+\lambda_4+\lambda_6}\times\overline{S}_{u_2}(s)-\frac{\lambda_5\lambda_6}{s+\lambda_5+\lambda_4}\times\overline{S}_{u_2}(s)\\
&-\frac{\lambda_4\lambda_6}{s+\lambda_5+\lambda_4}\times\overline{S}_{u_2}(s)-\frac{\lambda_7\lambda_8}{s+\lambda_8+\lambda_9}\times\overline{S}_{u_3}(s)\\
&-\frac{\lambda_7\lambda_9}{s+\lambda_8+\lambda_9}\times\overline{S}_{u_3}(s)-\frac{\lambda_7\lambda_8}{s+\lambda_7+\lambda_9}\times\overline{S}_{u_3}(s)\\
&-\frac{\lambda_9\lambda_8}{s+\lambda_7+\lambda_9}\times\overline{S}_{u_3}(s)-\frac{\lambda_7\lambda_9}{s+\lambda_7+\lambda_8}\times\overline{S}_{u_3}(s)\\
&-\frac{\lambda_8\lambda_9}{s+\lambda_7+\lambda_8}\times\overline{S}_{u_3}(s)-\frac{\lambda P_1}{s+\lambda}\times\overline{S}_{\varphi_1}(s)-\frac{\lambda P_2}{s+\lambda}\\
&\times\overline{S}_{\varphi_2}(s)-\frac{\lambda P_3}{s+\lambda}\times\overline{S}_{\varphi_3}(s)
\end{aligned} \tag{91}$$

Also up and down state probabilities of the system are given by

$$\begin{aligned}
\overline{P}_{up}(s)=&\overline{P}_0(s)+\overline{P}_1(s)+\overline{P}_4(s)+\overline{P}_7(s)+\overline{P}_{10}(s)+\overline{P}_{13}(s)\\
&+\overline{P}_{16}(s)+\overline{P}_{19}(s)+\overline{P}_{22}(s)+\overline{P}_{25}(s)+\overline{P}_{28}(s)\\
&+\overline{P}_{30}(s)+\overline{P}_{32}(s)
\end{aligned} \tag{92}$$

$$\begin{aligned}
\overline{P}_{down}(s)=&\overline{P}_2(s)+\overline{P}_3(s)+\overline{P}_5(s)+\overline{P}_6(s)+\overline{P}_8(s)+\overline{P}_9(s)+\\
&\overline{P}_{11}(s)+\overline{P}_{12}(s)+\overline{P}_{14}(s)+\overline{P}_{15}(s)+\overline{P}_{17}(s)\\
&+\overline{P}_{18}(s)+\overline{P}_{20}(s)+\overline{P}_{21}(s)+\overline{P}_{23}(s)+\overline{P}_{24}(s)\\
&+\overline{P}_{26}(s)+\overline{P}_{27}(s)+\overline{P}_{29}(s)+\overline{P}_{31}(s)+\overline{P}_{33}(s)
\end{aligned} \tag{93}$$

From equations (92) and (93), we have

$\overline{P}_{\text{up}}(s)+\overline{P}_{\text{down}}(s)=1/s.$

## 6. ASYMPTOTIC BEHAVIOUR

Using Able's lemma

$$\lim_{s \to 0} \{s\overline{F}(s)\} = \lim_{t \to \infty} F(t)$$

in equations (92) and (93), one can obtain the following time independent probabilities.

$$P_{up} = \frac{1}{A(0)} + \frac{\lambda_1}{\lambda_3+\lambda_2} \times \frac{1}{A(0)} + \frac{\lambda_2}{\lambda_3+\lambda_1} \times \frac{1}{A(0)} + \frac{\lambda_3}{\lambda_1+\lambda_2} \times \frac{1}{A(0)} + \frac{\lambda_4}{\lambda_5+\lambda_6} \times \frac{1}{A(0)} + \frac{\lambda_5}{\lambda_4+\lambda_6} \times \frac{1}{A(0)} + \frac{\lambda_6}{\lambda_5+\lambda_4} \times \frac{1}{A(0)} + \frac{\lambda_7}{\lambda_8+\lambda_9} \times \frac{1}{A(0)} + \frac{\lambda_8}{\lambda_7+\lambda_9} \times \frac{1}{A(0)} + \frac{\lambda_9}{\lambda_7+\lambda_8} \times \frac{1}{A(0)} + \frac{P_1}{\lambda} \times \frac{1}{A(0)} + \frac{P_2}{\lambda} \times \frac{1}{A(0)} + \frac{P_3}{\lambda} \times \frac{1}{A(0)}$$

$$\begin{aligned} P_{down} = & \frac{\lambda_2\lambda_1}{\lambda_3+\lambda_2} \times \frac{1}{A(0)} \times \overline{M}_{u_1} + \frac{\lambda_3\lambda_1}{\lambda_3+\lambda_2} \times \frac{1}{A(0)} \times \overline{M}_{u_1} \\ & + \frac{\lambda_1\lambda_2}{\lambda_3+\lambda_1} \times \frac{1}{A(0)} \times \overline{M}_{u_1} + \frac{\lambda_3\lambda_2}{\lambda_3+\lambda_1} \times \frac{1}{A(0)} \times \overline{M}_{u_1} \\ & + \frac{\lambda_1\lambda_3}{\lambda_1+\lambda_2} \times \frac{1}{A(0)} \times \overline{M}_{u_1} + \frac{\lambda_2\lambda_3}{\lambda_1+\lambda_2} \times \frac{1}{A(0)} \times \overline{M}_{u_1} \\ & + \frac{\lambda_5\lambda_4}{\lambda_5+\lambda_6} \times \frac{1}{A(0)} \times \overline{M}_{u_2} + \frac{\lambda_6\lambda_4}{\lambda_5+\lambda_6} \times \frac{1}{A(0)} \times \overline{M}_{u_2} \\ & + \frac{\lambda_4\lambda_5}{\lambda_4+\lambda_6} \times \frac{1}{A(0)} \times \overline{M}_{u_2} + \frac{\lambda_6\lambda_5}{\lambda_4+\lambda_6} \times \frac{1}{A(0)} \times \overline{M}_{u_2} \\ & + \frac{\lambda_5\lambda_6}{\lambda_5+\lambda_4} \times \frac{1}{A(0)} \times \overline{M}_{u_2} + \frac{\lambda_4\lambda_6}{\lambda_5+\lambda_4} \times \frac{1}{A(0)} \times \overline{M}_{u_2} \\ & + \frac{\lambda_7\lambda_8}{\lambda_8+\lambda_9} \times \frac{1}{A(0)} \times \overline{M}_{u_3} + \frac{\lambda_7\lambda_9}{\lambda_8+\lambda_9} \times \frac{1}{A(0)} \times \overline{M}_{u_3} \\ & + \frac{\lambda_7\lambda_8}{\lambda_7+\lambda_9} \times \frac{1}{A(0)} \times \overline{M}_{u_3} + \frac{\lambda_9\lambda_8}{\lambda_7+\lambda_9} \times \frac{1}{A(0)} \times \overline{M}_{u_3} \\ & + \frac{\lambda_7\lambda_9}{\lambda_7+\lambda_8} \times \frac{1}{A(0)} \times \overline{M}_{u_3} + \frac{\lambda_8\lambda_9}{\lambda_7+\lambda_8} \times \frac{1}{A(0)} \times \overline{M}_{u_3} \\ & + \frac{\lambda P_1}{\lambda} \times \frac{1}{A(0)} \times \overline{M}_{\varphi_1} + \frac{\lambda P_2}{\lambda} \times \frac{1}{A(0)} \times \overline{M}_{\varphi_2} + \frac{\lambda P_3}{\lambda} \times \frac{1}{A(0)} \times \overline{M}_{\varphi_3} \end{aligned}$$

where

$$A(0) = \lim_{s\to 0} A(s)$$

$$\overline{M}_{u_1} = \lim_{s\to 0}\left\{\frac{1-\overline{S}_{u_1}(s)}{s}\right\}, \overline{M}_{u_2} = \lim_{s\to 0}\left\{\frac{1-\overline{S}_{u_2}(s)}{s}\right\},$$

$$\overline{M}_{u_3} = \lim_{s\to 0}\left\{\frac{1-\overline{S}_{u_3}(s)}{s}\right\}, \overline{M}_{\varphi_1} = \lim_{s\to 0}\left\{\frac{1-\overline{S}_{\varphi_1}(s)}{s}\right\},$$

$$\overline{M}_{\varphi_2} = \lim_{s\to 0}\left\{\frac{1-\overline{S}_{\varphi_2}(s)}{s}\right\}, \overline{M}_{\varphi_3} = \lim_{s\to 0}\left\{\frac{1-\overline{S}_{\varphi_3}(s)}{s}\right\}$$

## 7. PARTICULAR CASES

**7.1. When Repair Follows Exponential Distribution** In this case the results can be derived by putting

$$\begin{aligned} &\overline{S}_{u_1}(s) = \frac{u_1(x)}{s+u_1(x)}, \ \overline{S}_{u_2}(s) = \frac{u_2(x)}{s+u_2(x)}, \\ &\overline{S}_{u_3}(s) = \frac{u_3(x)}{s+u_3(x)}, \ \overline{S}_{\varphi_1}(s) = \frac{\varphi_1(y)}{s+\varphi_1(y)}, \\ &\overline{S}_{\varphi_2}(s) = \frac{\varphi_2(y)}{s+\varphi_2(y)}, \ \overline{S}_{\varphi_3}(s) = \frac{\varphi_3(y)}{s+\varphi_3(y)} \end{aligned} \tag{94}$$

in equations (92) and (93), which yield

$$\begin{aligned} \overline{P}_{up}(s) = & \frac{1}{B(s)} + \frac{\lambda_1}{s+\lambda_3+\lambda_2} \times \frac{1}{B(s)} + \frac{\lambda_2}{s+\lambda_3+\lambda_1} \times \frac{1}{B(s)} + \frac{\lambda_3}{s+\lambda_1+\lambda_2} \times \frac{1}{B(s)} + \frac{\lambda_4}{s+\lambda_5+\lambda_6} \times \\ & \frac{1}{B(s)} + \frac{1}{B(s)} \times \frac{\lambda_5}{s+\lambda_4+\lambda_6} + \frac{\lambda_6}{s+\lambda_5+\lambda_4} \times \frac{1}{B(s)} + \frac{\lambda_7}{s+\lambda_8+\lambda_9} \times \frac{1}{B(s)} + \frac{\lambda_8}{s+\lambda_7+\lambda_9} \times \\ & \frac{1}{B(s)} + \frac{\lambda_9}{s+\lambda_7+\lambda_8} \times \frac{1}{B(s)} + \frac{P_1}{s+\lambda} \times \frac{1}{B(s)} + \frac{P_2}{s+\lambda} \times \frac{1}{B(s)} + \frac{P_3}{s+\lambda} \times \frac{1}{B(s)} \end{aligned}$$

$$\begin{aligned} \overline{P}_{down}(s) = & \frac{\lambda_2\lambda_1}{s+\lambda_3+\lambda_2} \times \frac{1}{B(s)} \times \left[\frac{1}{s+u_1(x)}\right] + \frac{\lambda_3\lambda_1}{s+\lambda_3+\lambda_2} \times \frac{1}{B(s)} \times \left[\frac{1}{s+u_1(x)}\right] + \frac{\lambda_1\lambda_2}{s+\lambda_3+\lambda_1} \times \frac{1}{B(s)} \times \\ & \left[\frac{1}{s+u_1(x)}\right] + \frac{\lambda_3\lambda_2}{s+\lambda_3+\lambda_1} \times \left[\frac{1}{s+u_1(x)}\right] \times \frac{1}{B(s)} + \frac{\lambda_1\lambda_3}{s+\lambda_1+\lambda_2} \times \frac{1}{B(s)} \times \left[\frac{1}{s+u_1(x)}\right] \end{aligned}$$

$$
+\frac{\lambda_2\lambda_3}{s+\lambda_1+\lambda_2}\times\frac{1}{B(s)}\times\left[\frac{1}{s+u_1(x)}\right]+\frac{1}{B(s)}
$$

$$
\times\frac{\lambda_5\lambda_4}{s+\lambda_5+\lambda_6}\times\left[\frac{1}{s+u_2(x)}\right]+\frac{\lambda_6\lambda_4}{s+\lambda_5+\lambda_6}\times
$$

$$
\frac{1}{B(s)}\times\left[\frac{1}{s+u_2(x)}\right]+\frac{\lambda_4\lambda_5}{s+\lambda_4+\lambda_6}\times\frac{1}{B(s)}
$$

$$
\times\left[\frac{1}{s+u_2(x)}\right]+\frac{\lambda_6\lambda_5}{s+\lambda_4+\lambda_6}\times\frac{1}{B(s)}\times\left[\frac{1}{s+u_2(x)}\right]
$$

$$
+\frac{\lambda_5\lambda_6}{s+\lambda_5+\lambda_4}\times\frac{1}{B(s)}\times\left[\frac{1}{s+u_2(x)}\right]+\frac{\lambda_4\lambda_6}{s+\lambda_5+\lambda_4}
$$

$$
\times\frac{1}{B(s)}\times\left[\frac{1}{s+u_2(x)}\right]+\frac{\lambda_7\lambda_8}{s+\lambda_8+\lambda_9}\times\frac{1}{B(s)}\times
$$

$$
\left[\frac{1}{s+u_3(x)}\right]+\frac{\lambda_7\lambda_9}{s+\lambda_8+\lambda_9}\times\left[\frac{1}{s+u_3(x)}\right]
$$

$$
\times\frac{1}{B(s)}+\frac{\lambda_7\lambda_8}{s+\lambda_7+\lambda_9}\times\frac{1}{B(s)}\times\left[\frac{1}{s+u_3(x)}\right]
$$

$$
+\frac{\lambda_9\lambda_8}{s+\lambda_7+\lambda_9}\times\frac{1}{B(s)}\times\left[\frac{1}{s+u_3(x)}\right]+\frac{1}{B(s)}
$$

$$
\times\frac{\lambda_7\lambda_9}{s+\lambda_7+\lambda_8}\times\left[\frac{1}{s+u_3(x)}\right]+\frac{\lambda_8\lambda_9}{s+\lambda_7+\lambda_8}\times\frac{1}{B(s)}
$$

$$
\times\left[\frac{1}{s+u_3(x)}\right]+\frac{\lambda P_1}{s+\lambda}\times\frac{1}{B(s)}\times\left[\frac{1}{s+\phi_1(y)}\right]+\frac{\lambda P_2}{s+\lambda}
$$

$$
\times\frac{1}{B(s)}\times\left[\frac{1}{s+\phi_2(y)}\right]+\frac{\lambda P_3}{s+\lambda}\times\frac{1}{B(s)}\times\left[\frac{1}{s+\phi_3(y)}\right]
$$

where

$$
B(s)=(s+\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5+\lambda_6+\lambda_7+\lambda_8+\lambda_9
$$

$$
+P_1+P_2+P_3)-\frac{\lambda_3\lambda_1}{s+\lambda_3+\lambda_2}\times\frac{u_1(x)}{s+u_1(x)}-
$$

$$
\frac{\lambda_2\lambda_1}{s+\lambda_3+\lambda_2}\times\frac{u_1(x)}{s+u_1(x)}-\frac{\lambda_1\lambda_2}{s+\lambda_3+\lambda_1}\times\frac{u_1(x)}{s+u_1(x)}
$$

$$
-\frac{\lambda_3\lambda_2}{s+\lambda_3+\lambda_1}\times\frac{u_1(x)}{s+u_1(x)}-\frac{\lambda_1\lambda_3}{s+\lambda_1+\lambda_2}\times\frac{u_1(x)}{s+u_1(x)}
$$

$$
-\frac{\lambda_2\lambda_3}{s+\lambda_1+\lambda_2}\times\frac{u_1(x)}{s+u_1(x)}-\frac{\lambda_5\lambda_4}{s+\lambda_5+\lambda_6}\times\frac{u_2(x)}{s+u_2(x)}-
$$

$$
\frac{\lambda_6\lambda_4}{s+\lambda_5+\lambda_6}\times\frac{u_2(x)}{s+u_2(x)}-\frac{\lambda_4\lambda_5}{s+\lambda_4+\lambda_6}\times\frac{u_2(x)}{s+u_2(x)}
$$

$$
-\frac{\lambda_6\lambda_5}{s+\lambda_4+\lambda_6}\times\frac{u_2(x)}{s+u_2(x)}-\frac{\lambda_5\lambda_6}{s+\lambda_5+\lambda_4}\times\frac{u_2(x)}{s+u_2(x)}
$$

$$
-\frac{\lambda_4\lambda_6}{s+\lambda_5+\lambda_4}\times\frac{u_2(x)}{s+u_2(x)}-\frac{\lambda_7\lambda_8}{s+\lambda_8+\lambda_9}\times\frac{u_3(x)}{s+u_3(x)}
$$

$$
-\frac{\lambda_7\lambda_9}{s+\lambda_8+\lambda_9}\times\frac{u_3(x)}{s+u_3(x)}-\frac{\lambda_7\lambda_8}{s+\lambda_7+\lambda_9}\times\frac{u_3(x)}{s+u_3(x)}
$$

$$
-\frac{\lambda_9\lambda_8}{s+\lambda_7+\lambda_9}\times\frac{u_3(x)}{s+u_3(x)}-\frac{\lambda_7\lambda_9}{s+\lambda_7+\lambda_8}\times\frac{u_3(x)}{s+u_3(x)}
$$

$$
-\frac{\lambda_8\lambda_9}{s+\lambda_7+\lambda_8}\times\frac{u_3(x)}{s+u_3(x)}-\frac{\lambda P_1}{s+\lambda}\times\frac{\phi_1(y)}{s+\phi_1(y)}-\frac{\lambda P_2}{s+\lambda}
$$

$$
\times\frac{\phi_2(y)}{s+\phi_2(y)}-\frac{\lambda P_3}{s+\lambda}\times\frac{\phi_3(y)}{s+\phi_3(y)}
$$

**7.2. If Blackout (Total Loss of Power) Occurs Only** Up and down state probabilities in this case can be derived by putting $P_2 = P_3 = 0$ in Equations (92) and (93), which are given by:

$$
\overline{P}_{up}(s)=\frac{1}{C(s)}+\frac{\lambda_1}{s+\lambda_3+\lambda_2}\times\frac{1}{C(s)}+\frac{\lambda_2}{s+\lambda_3+\lambda_1}\times\frac{1}{C(s)}
$$

$$
+\frac{\lambda_3}{s+\lambda_1+\lambda_2}\times\frac{1}{C(s)}+\frac{\lambda_4}{s+\lambda_5+\lambda_6}\times\frac{1}{C(s)}+
$$

$$
\frac{\lambda_5}{s+\lambda_4+\lambda_6}\times\frac{1}{C(s)}+\frac{\lambda_6}{s+\lambda_5+\lambda_4}\times\frac{1}{C(s)}+
$$

$$
\frac{\lambda_7}{s+\lambda_8+\lambda_9}\times\frac{1}{C(s)}+\frac{\lambda_8}{s+\lambda_7+\lambda_9}\times\frac{1}{C(s)}+
$$

$$
\frac{\lambda_9}{s+\lambda_7+\lambda_8}\times\frac{1}{C(s)}+\frac{P_1}{s+\lambda}\times\frac{1}{C(s)}
$$

$$
\overline{P}_{down}(s)=\frac{\lambda_2\lambda_1}{s+\lambda_3+\lambda_2}\times\frac{1}{C(s)}\times\left[\frac{1-\overline{S}_{u_1}(s)}{s}\right]+\frac{1}{C(s)}
$$

$$
\times\frac{\lambda_3\lambda_1}{s+\lambda_3+\lambda_2}\times\left[\frac{1-\overline{S}_{u_1}(s)}{s}\right]+\frac{\lambda_1\lambda_2}{s+\lambda_3+\lambda_1}\times\frac{1}{C(s)}
$$

$$
\times\left[\frac{1-\overline{S}_{u_1}(s)}{s}\right]+\frac{\lambda_3\lambda_2}{s+\lambda_3+\lambda_1}\times\frac{1}{C(s)}\times\left[\frac{1-\overline{S}_{u_1}(s)}{s}\right]
$$

$$
+\frac{\lambda_1\lambda_3}{s+\lambda_1+\lambda_2}\times\left[\frac{1-\overline{S}_{u_1}(s)}{s}\right]\times\frac{1}{C(s)}+\frac{\lambda_2\lambda_3}{s+\lambda_1+\lambda_2}
$$

$$
\times\frac{1}{C(s)}\times\left[\frac{1-\overline{S}_{u_1}(s)}{s}\right]+\frac{\lambda_5\lambda_4}{s+\lambda_5+\lambda_6}\times\frac{1}{C(s)}\times\left[\frac{1-\overline{S}_{u_2}(s)}{s}\right]
$$

$$
+\frac{1}{C(s)}\times\frac{\lambda_6\lambda_4}{s+\lambda_5+\lambda_6}\times\left[\frac{1-\overline{S}_{u_2}(s)}{s}\right]+\frac{1}{C(s)}\times\frac{\lambda_4\lambda_5}{s+\lambda_4+\lambda_6}
$$

$$
\times\left[\frac{1-\overline{S}_{u_2}(s)}{s}\right]+\frac{\lambda_6\lambda_5}{s+\lambda_4+\lambda_6}\times\frac{1}{C(s)}\times\left[\frac{1-\overline{S}_{u_2}(s)}{s}\right]
$$

$$
+\frac{\lambda_5\lambda_6}{s+\lambda_5+\lambda_4}\times\frac{1}{C(s)}\times\left[\frac{1-\overline{S}_{u_2}(s)}{s}\right]+\frac{\lambda_4\lambda_6}{s+\lambda_5+\lambda_4}
$$

$$\times\frac{1}{C(s)}\times\left[\frac{1-\bar{S}_{u_2}(s)}{s}\right]+\frac{\lambda_7\lambda_8}{s+\lambda_8+\lambda_9}\times\left[\frac{1-\bar{S}_{u_3}(s)}{s}\right]\times\frac{1}{C(s)}$$
$$+\frac{\lambda_7\lambda_9}{s+\lambda_8+\lambda_9}\times\frac{1}{C(s)}\times\left[\frac{1-\bar{S}_{u_3}(s)}{s}\right]+\frac{\lambda_7\lambda_8}{s+\lambda_7+\lambda_9}\times\frac{1}{C(s)}$$
$$\times\left[\frac{1-\bar{S}_{u_3}(s)}{s}\right]+\frac{1}{C(s)}\times\frac{\lambda_9\lambda_8}{s+\lambda_7+\lambda_9}\times\left[\frac{1-\bar{S}_{u_3}(s)}{s}\right]$$
$$+\frac{1}{C(s)}\times\frac{\lambda_7\lambda_9}{s+\lambda_7+\lambda_8}\times\left[\frac{1-\bar{S}_{u_3}(s)}{s}\right]+\frac{1}{C(s)}\times\frac{\lambda_8\lambda_9}{s+\lambda_7+\lambda_8}$$
$$\times\left[\frac{1-\bar{S}_{u_3}(s)}{s}\right]+\frac{\lambda P_1}{s+\lambda}\times\frac{1}{C(s)}\times\left[\frac{1-\bar{S}_{\phi_1}(s)}{s}\right]$$

where

$$C(s)=(s+\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5+\lambda_6+\lambda_7+\lambda_8+\lambda_9+$$
$$P_1)-\frac{\lambda_3\lambda_1}{s+\lambda_3+\lambda_2}\times\bar{S}_{u_1}(s)-\frac{\lambda_2\lambda_1}{s+\lambda_3+\lambda_2}\times\bar{S}_{u_1}(s)$$
$$-\frac{\lambda_1\lambda_2}{s+\lambda_3+\lambda_1}\times\bar{S}_{u_1}(s)-\frac{\lambda_3\lambda_2}{s+\lambda_3+\lambda_1}\times\bar{S}_{u_1}(s)-$$
$$\frac{\lambda_1\lambda_3}{s+\lambda_1+\lambda_2}\times\bar{S}_{u_1}(s)-\frac{\lambda_2\lambda_3}{s+\lambda_1+\lambda_2}\times\bar{S}_{u_1}(s)-$$
$$\frac{\lambda_5\lambda_4}{s+\lambda_5+\lambda_6}\times\bar{S}_{u_2}(s)-\frac{\lambda_6\lambda_4}{s+\lambda_5+\lambda_6}\times\bar{S}_{u_2}(s)-$$
$$\frac{\lambda_4\lambda_5}{s+\lambda_4+\lambda_6}\times\bar{S}_{u_2}(s)-\frac{\lambda_6\lambda_5}{s+\lambda_4+\lambda_6}\times\bar{S}_{u_2}(s)-$$
$$\frac{\lambda_5\lambda_6}{s+\lambda_5+\lambda_4}\times\bar{S}_{u_2}(s)-\frac{\lambda_4\lambda_6}{s+\lambda_5+\lambda_4}\times\bar{S}_{u_2}(s)-$$
$$\frac{\lambda_7\lambda_8}{s+\lambda_8+\lambda_9}\times\bar{S}_{u_3}(s)-\frac{\lambda_7\lambda_9}{s+\lambda_8+\lambda_9}\times\bar{S}_{u_3}(s)-$$
$$\frac{\lambda_7\lambda_8}{s+\lambda_7+\lambda_9}\times\bar{S}_{u_3}(s)-\frac{\lambda_9\lambda_8}{s+\lambda_7+\lambda_9}\times\bar{S}_{u_3}(s)-$$
$$\frac{\lambda_7\lambda_9}{s+\lambda_7+\lambda_8}\times\bar{S}_{u_3}(s)-\frac{\lambda_8\lambda_9}{s+\lambda_7+\lambda_8}\times\bar{S}_{u_3}(s)-$$
$$\frac{\lambda P_1}{s+\lambda}\times\bar{S}_{\varphi_1}(s)$$

**7.3. If Transient Failure (Momentary Loss of Power) Occurs Only** Up and down state probabilities in this case can be derived by putting $P_1$=$P_3$=0 in Equations (92) and (93).

**7.4. If Brownout (Drop in Voltage in an Electrical Power Supply) Occurs Only** Up and down state probabilities in this case can be derived by putting $P_1$=$P_2$=0 in Equations (92) and (93).

## 8. NUMERICAL COMPUTATION

The Maple software has been used to analyze availability, reliability, M.T.T.F, busy period, cost effectiveness and sensitivity of the system.

**8.1. Availability Analysis** Take $\lambda_1$ = 0.1, $\lambda_2$ = 0.1, $\lambda_3$ = 0.1, $\lambda_4$ = 0.2, $\lambda_5$ = 0.2, $\lambda_6$ = 0.2, $\lambda_7$ = 0.3, $\lambda_8$ = 0.3, $\lambda_9$ = 0.3, $P_1$ = 0.4, $P_2$ = 0.5, $P_3$ = 0.6, $\lambda$ = 0.7, $u_1$ = $u_2$ = $u_3$ = $\psi_1$ = $\psi_2$ = $\psi_3$ = $\varphi_1$ = $\varphi_2$ = $\varphi_3$ = $v$ = 1, $\theta$ = 1, $x$ = 1 and $y$ = 1. Also, if the repair follows exponential distribution i.e. Equation (94) holds, then putting all these values in Equation (92) and taking inverse Laplace transformation, we get

$$P_{up}(t)=0.6910925727\,e^{(-0.009071706019t)}+1.140884736$$
$$e^{(-2.038776412t)}-0.365780518610^{-5}\,e^{(-0.9984127284t)}$$
$$-0.00169749090\,e^{(-0.6366204841t)}-0.02276566549$$
$$e^{(-0.4320684754t)}+0.00662544791\,e^{(-0.2318488763t)}$$
$$-0.8141359426\,e^{(-2.853201318t)} \qquad (95)$$

Now, varying $t$ from 0 to 10 in Equation (95), we obtain Table 2 and correspondingly Figure 3 representing the behavior of availability of the system with respect to time.

**TABLE 2.** Time vs. Availability

| Time | Availibility |
|---|---|
| 0 | 1 |
| 1 | 0.776014607 |
| 2 | 0.689394547 |
| 3 | 0.671724623 |
| 4 | 0.665228159 |
| 5 | 0.65987168 |
| 6 | 0.65439497 |
| 7 | 0.648753542 |
| 8 | 0.643022658 |
| 9 | 0.637260688 |
| 10 | 0.631504915 |

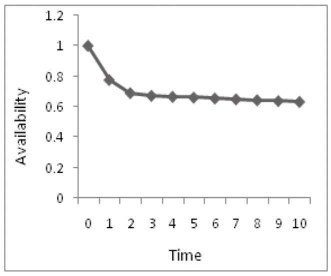


**Figure 3.** Time vs. Availability

**8.2. Reliability Analysis** Let us fix failure rates as $\lambda_1 = 0.1$, $\lambda_2 = 0.1$, $\lambda_3 = 0.1$, $\lambda_4 = 0.2$, $\lambda_5 = 0.2$, $\lambda_6 = 0.2$, $\lambda_7 = 0.3$, $\lambda_8 = 0.3$, $\lambda_9 = 0.3$, $P_1 = 0.4$, $P_2 = 0.5$, $P_3 = 0.6$, $\lambda = 0.7$, repair rates $u_1 = u_2 = u_3 = \psi_1 = \psi_2 = \psi_3 = \varphi_1 = \varphi_2 = \varphi_3 = v = 0$, $\theta = 1$, $x = 1$ and $y = 1$. Also, let the repair follows exponential distribution. Now, by putting all these values in Equation (92), using Equation (94) and setting $t$ = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, one can obtain Table 3 and Figure 4 which represent how reliability varies as the time increases.

**TABLE 3.** Time vs. Reliability

| Time | Reliability |
|---|---|
| 0 | 1 |
| 1 | 0.679457337 |
| 2 | 0.400208826 |
| 3 | 0.241163682 |
| 4 | 0.150576792 |
| 5 | 0.097618882 |
| 6 | 0.06567626 |
| 7 | 0.045740246 |
| 8 | 0.032848564 |
| 9 | 0.024214784 |
| 10 | 0.018238742 |

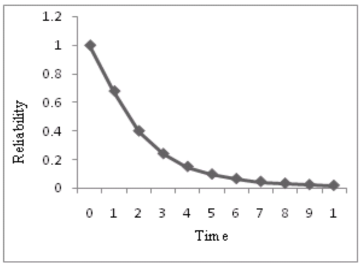


**Figure 4.** Time vs. Reliability

**8.3. M.T.T.F. Analysis** Let us suppose that repair follows exponential distribution then using equation (94) and from

$$\text{M.T.T.F.} = \lim_{s \to 0} \overline{P}_{\text{up}}(s)$$

we have the following four cases:

1. Fixing $\lambda_1 = 0.1$, $\lambda_2 = 0.1$, $\lambda_3 = 0.1$, $\lambda_4 = 0.2$, $\lambda_5 = 0.2$, $\lambda_6 = 0.2$, $\lambda_7 = 0.3$, $\lambda_8 = 0.3$, $\lambda_9 = 0.3$, $P_2 = 0.5$, $P_3 = 0.6$, $\lambda = 0.7$, $u_1 = u_2 = u_3 = \psi_1 = \psi_2 = \psi_3 = \varphi_1 = \varphi_2 = \varphi_3 = v = 0$, $\theta = 1$, $x = 1$ and $y = 1$ and varying the value of $P_1$ as 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, one can obtain variation of M.T.T.F. with respect to $P_1$.
2. Let us set $\lambda_1 = 0.1$, $\lambda_2 = 0.1$, $\lambda_3 = 0.1$, $\lambda_4 = 0.2$, $\lambda_5 = 0.2$, $\lambda_6 = 0.2$, $\lambda_7 = 0.3$, $\lambda_8 = 0.3$, $\lambda_9 = 0.3$, $P_1 = 0.4$, $P_3 = 0.6$, $\lambda = 0.7$, $u_1 = u_2 = u_3 = \psi_1 = \psi_2 = \psi_3 = \varphi_1 = \varphi_2 = \varphi_3 = v = 0$, $\theta = 1$, $x = 1$ and $y = 1$ and varying $P_2$ as 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, one can obtain change of M.T.T.F. with respect to $P_2$.
3. By taking $\lambda_1 = 0.1$, $\lambda_2 = 0.1$, $\lambda_3 = 0.1$, $\lambda_4 = 0.2$, $\lambda_5 = 0.2$, $\lambda_6 = 0.2$, $\lambda_7 = 0.3$, $\lambda_8 = 0.3$, $\lambda_9 = 0.3$, $P_1 = 0.4$, $P_2 = 0.5$, $\lambda = 0.7$, $u_1 = u_2 = u_3 = \psi_1 = \psi_2 = \psi_3 = \varphi_1 = \varphi_2 = \varphi_3 = v = 0$, $\theta = 1$, $x = 1$ and $y = 1$ and varying $P_3$ as 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, one can obtain variation of M.T.T.F. with respect to $P_3$.

All the three cases described above are depicted by Table 4 and Figure 5.

**TABLE 4.** Power supply failure ($P_i$) vs. M.T.T.F. where i = 1 (Blackout), i = 2 (Transient) and i = 3 (Brownout)

| $P_1$ | M.T.T.F. | $P_2$ | M.T.T.F. | $P_2$ | M.T.T.F. |
|---|---|---|---|---|---|
| 0.1 | 2.404761905 | 0.1 | 2.438423645 | 0.1 | 2.474489796 |
| 0.2 | 2.373271889 | 0.2 | 2.404761904 | 0.2 | 2.438423645 |
| 0.3 | 2.34375 | 0.3 | 2.373271889 | 0.3 | 2.404761904 |
| 0.4 | 2.316017316 | 0.4 | 2.34375 | 0.4 | 2.373271889 |
| 0.5 | 2.289915966 | 0.5 | 2.316017316 | 0.5 | 2.34375 |
| 0.6 | 2.265306122 | 0.6 | 2.289915966 | 0.6 | 2.316017316 |
| 0.7 | 2.242063492 | 0.7 | 2.265306122 | 0.7 | 2.289915966 |
| 0.8 | 2.22007722 | 0.8 | 2.242063492 | 0.8 | 2.265306122 |
| 0.9 | 2.19924812 | 0.9 | 2.2007722 | 0.9 | 2.242063492 |
| 1 | 2.179487179 | 1 | 2.19924812 | 1 | 2.2007722 |

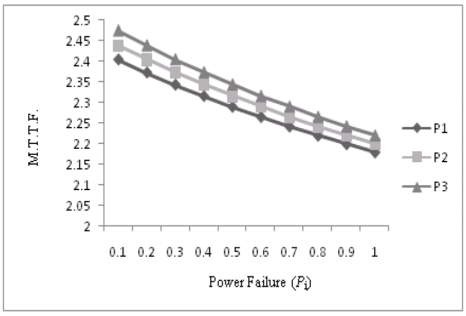


**Figure 5.** Power supply failure ($P_i$) vs. M.T.T.F. where i = 1 (Blackout), i = 2 (Transient) and i = 3 (Brownout )

4. Assume $\lambda_1 = 0.1$, $\lambda_2 = 0.1$, $\lambda_3 = 0.1$, $\lambda_4 = 0.2$, $\lambda_5 = 0.2$, $\lambda_6 = 0.2$, $\lambda_7 = 0.3$, $\lambda_8 = 0.3$, $\lambda_9 = 0.3$, $P_1 = 0.4$, $P_2 = 0.5$, $P_3 = 0.6$, repairs rates be $u_1 = u_2 = u_3 = \psi_1 = \psi_2 = \psi_3 = \varphi_1 = \varphi_2 = \varphi_3 = v = 0$, $\theta = 1$, $x = 1$ and $y = 1$ and increase the value of $\lambda$ from 0.1 to 1.0, we obtain Table 5 and Figure 6 which represents the manner in which M.T.T.F. varies with respect to $\lambda$.

**TABLE 5.** Standby power supply failure (λ) vs. M.T.T.F.

| λ | M.T.T.F |
|---|---|
| 0.1 | 6.212121212 |
| 0.2 | 3.93939393939 |
| 0.3 | 3.181818182 |
| 0.4 | 2.803030303 |
| 0.5 | 2.575757576 |
| 0.6 | 2.424242424 |
| 0.7 | 2.316017316 |
| 0.8 | 2.234848485 |
| 0.9 | 2.171717171 |
| 1 | 2.121212121 |

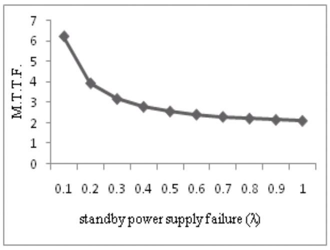


**Figure 6.** Standby power supply failure (λ) vs. MTTF

**8.4. Busy Period Analysis** Let the equation (94) holds then Mean time to repair (M.T.T.R.) of the system is given by

$$M.T.T.R. = \lim_{s \to 0} \overline{P}_{down}(s)$$

1. Letting $\lambda_1 = 0.1$, $\lambda_2 = 0.1$, $\lambda_3 = 0.1$, $\lambda_4 = 0.2$, $\lambda_5 = 0.2$, $\lambda_6 = 0.2$, $\lambda_7 = 0.3$, $\lambda_8 = 0.3$, $\lambda_9 = 0.3$, $P_2 = 0.5$, $P_3 = 0.6$, $\lambda = 0.7$, $u_1 = u_2 = u_3 = \psi_1 = \psi_2 = \psi_3 = \varphi_1 = \varphi_2 = \varphi_3 = v = 1$, $\theta = 1$, $x = 1$ and $y = 1$ and varying $P_1$ as 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.1, we obtain the changes of busy period with respect to $P_1$.
2. Taking the values $\lambda_1 = 0.1$, $\lambda_2 = 0.1$, $\lambda_3 = 0.1$, $\lambda_4 = 0.2$, $\lambda_5 = 0.2$, $\lambda_6 = 0.2$, $\lambda_7 = 0.3$, $\lambda_8 = 0.3$, $\lambda_9 = 0.3$, $P_1 = 0.4$, $P_3 = 0.6$, $\lambda = 0.7$, $u_1 = u_2 = u_3 = \psi_1 = \psi_2 = \psi_3 = \varphi_1 = \varphi_2 = \varphi_3 = v = 1$, $\theta = 1$, $x = 1$, $y = 1$ and varying $P_2 = 0.1$, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, one can observe how the busy period changes with respect to $P_2$.
3. Varying $P_3 = 0.1$, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, and keeping other parameter fixed at $\lambda_1 = 0.1$, $\lambda_2 = 0.1$, $\lambda_3 = 0.1$, $\lambda_4 = 0.2$, $\lambda_5 = 0.2$, $\lambda_6 = 0.2$, $\lambda_7 = 0.3$, $\lambda_8 = 0.3$, $\lambda_9 = 0.3$, $P_1 = 0.4$, $P_2 = 0.5$, $\lambda = 0.7$, $u_1 = u_2 = u_3 = \psi_1 = \psi_2 = \psi_3 = \varphi_1 = \varphi_2 = \varphi_3 = v = 1$, $\theta = 1$, $x = 1$, $y = 1$, one can get the variation of busy period with respect to $P_3$.

   Variations of busy period with respect to $P_1$, $P_2$ and $P_3$ in the cases (1), (2) and (3) have been shown by Table 6 and Figure 7.

**TABLE 6.** Power failure ($P_i$) vs. Busy Period where i = 1(Blackout), i = 2 (Transient) and i = 3 (Brownout)

| | Busy Period w.r.t. | | |
|---|---|---|---|
| $P_i$ | Blackout ($P_1$) | Transient ($P_2$) | Brownout ($P_3$) |
| 0.1 | 29.1 | 28.1 | 27.1 |
| 0.2 | 29.2 | 28.2 | 27.2 |
| 0.3 | 29.3 | 28.3 | 27.3 |
| 0.4 | 29.4 | 28.4 | 27.4 |
| 0.5 | 29.5 | 28.5 | 27.5 |
| 0.6 | 29.6 | 28.6 | 27.6 |
| 0.7 | 29.7 | 28.7 | 27.7 |
| 0.8 | 29.8 | 28.8 | 27.8 |
| 0.9 | 29.9 | 28.9 | 27.9 |
| 1 | 30 | 29 | 28 |

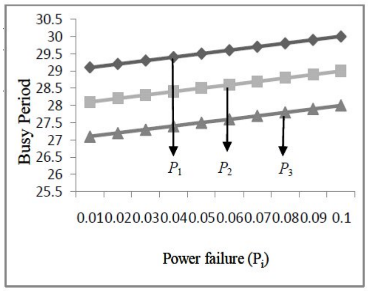


**Figure 7.** Power failure ($P_i$) vs. Busy Period where i = 1(Blackout), i = 2 (Transient) and i = 3 (Brownout)

**8.5. Sensitivity Analysis** Assuming that Equation (94) holds, we first perform a sensitivity analysis for changes in R(t) resulting from changes in system parameters $P_1$, $P_2$, $P_3$ and λ.

Putting $\lambda_1 = 0.1$, $\lambda_2 = 0.1$, $\lambda_3 = 0.1$, $\lambda_4 = 0.2$, $\lambda_5 = 0.2$, $\lambda_6 = 0.2$, $\lambda_7 = 0.3$, $\lambda_8 = 0.3$, $\lambda_9 = 0.3$, $P_2 = 0.5$, $P_3 = 0.6$, $\lambda = 0.7$, $u_1 = u_2 = u_3 = \psi_1 = \psi_2 = \psi_3 = \varphi_1 = \varphi_2 = \varphi_3 = v = 0$, $\theta = 1$, $x = 1$ and $y = 1$ in Equation (92), and then differentiating with respect to $P_1$, we get:

$$\frac{\partial R(s)}{\partial P_1} = -\frac{1}{(s+2.9+P_1)^2} - \frac{.3}{(s+.2)(s+2.9+P_1)^2} - \frac{.6}{(s+.4)(s+2.9+P_1)^2} - \frac{.9}{(s+.6)(s+2.9+P_1)^2} + \frac{1}{(s+.7)(s+2.9+P_1)} - \frac{P_1}{(s+.7)(s+2.9+P_1)^2} - \frac{1.1}{(s+.7)(s+2.9+P_1)^2}$$

Taking inverse Laplace transformation gives:

$$\frac{\partial R(t)}{\partial P_1} = 27.50000000 \frac{e^{(-0.700000000\,0\,t)}}{(11+5P_1)^2} - 2.400000$$

$$000\frac{e^{(-0.4000000000\ t)}}{(5+2P_1)^2} - 30\frac{e^{(-0.2000000000\ t)}}{(27+10P_1)^2} +$$

$$(0.100\ 00000\ 00(5+2P_1)(11+5P_1)(23+10$$

$$P_1)(27+10P_1)(7000P_1^3+48300P_1^2+11100$$

$$90P_1+82797)t+\ 700000P_1^6+0.9660000$$

$$10\ ^7 P_1^5+0.5523100010\ ^8 P_1^4+0.1673316$$

$$00\ 10\ ^9 P_1^3+0.28304245010^9 P_1^2+0.253$$

$$119174\ 10^9 P_1+0.933389589010^8)$$

$$e^{(-0.1000000000\,(29+10P_1)t)} /((11+5P_1)^2$$

$$(23+10P_1)^2(5+2P_1)^2(27+10P_1)^2)$$

$$-\frac{90e^{(-0.6000000000\ t)}}{(23+10P_1)^2}$$

Using the same procedure described above, we can get $\frac{\partial R(t)}{\partial P_2}, \frac{\partial R(t)}{\partial P_3}$ and $\frac{\partial R(t)}{\partial \lambda}$.

Now, we perform a sensitivity analysis of changes in M.T.T.F. with respect to $P_1$, $P_2$, $P_3$ and $\lambda$. Setting $\lambda_1 = 0.1$, $\lambda_2 = 0.1$, $\lambda_3 = 0.1$, $\lambda_4 = 0.2$, $\lambda_5 = 0.2$, $\lambda_6 = 0.2$, $\lambda_7 = 0.3$, $\lambda_8 = 0.3$, $\lambda_9 = 0.3$, $P_2 = 0.5$, $P_3 = 0.6$, $\lambda = 0.7$, $u_1 = u_2 = u_3 = \psi_1 = \psi_2 = \psi_3 = \varphi_1 = \varphi_2 = \varphi_3 = v = 0$, $\theta = 1$, $x = 1$ and $y = 1$ in Equation (92) then using Equation (94), we get:

$$M.T.T.F. = 0.7142857143\,\frac{99+20P_1}{29+10P_1}$$

Differentiating it with respect to $P_1$, we have:

$$\frac{\partial M.T.T.F.}{\partial P_1} = 14.28571429\frac{1}{29+10P_1} - \frac{7.142857143(99+20P_1)}{(29+10P_1)^2}$$

Using the same procedure $\frac{\partial M.T.T.F.}{\partial P_2}$, $\frac{\partial M.T.T.F.}{\partial P_3}$ and $\frac{\partial M.T.T.F.}{\partial \lambda}$ can be obtained. Numerical results of the sensitivity analysis for the system reliability and the M.T.T.F. are presented in Figures 8 - 11 and Tables 7-10.

**TABLE 7.** Sensitivity of system reliability w. r. t. $P_i$ where i=1 (Blackout), i=2 (Transient) and i=3 (Brownout)

| Time | $\frac{\partial R(t)}{\partial P_1}$ | $\frac{\partial R(t)}{\partial P_2}$ | $\frac{\partial R(t)}{\partial P_3}$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | -0.036564414 | -0.035039093 | -0.028560154 |
| 2 | -0.033135699 | -0.0314714 | -0.024744067 |
| 3 | -0.025842503 | -0.024514741 | -0.019190421 |
| 4 | -0.019602452 | -0.018589659 | -0.014535014 |
| 5 | -0.01473436 | -0.013971798 | -0.010920474 |
| 6 | -0.011069977 | -0.010497208 | -0.008205377 |
| 7 | -0.008353782 | -0.007922136 | -0.006194561 |
| 8 | -0.006348868 | -0.006021452 | -0.004710522 |
| 9 | -0.004865326 | -0.004614962 | -0.003612083 |
| 10 | -0.003760494 | -0.003567407 | -0.0027936 |

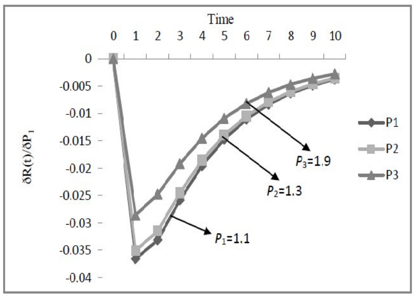


**Figure 8.** Sensitivity of system reliability w. r. t. $P_i$ where i = 1 (Blackout), i = 2 (Transient) and i = 3 (Brownout)

**TABLE 8.** Sensitivity of system reliability w.r.t. standby power supply failure ($\lambda$)

| | $\partial R(t)/\partial\lambda$ | |
|---|---|---|
| Time | λ=0.2 | λ=0.5 |
| 0 | 0 | 0 |
| 1 | -0.274123478 | -0.215938461 |
| 2 | -0.544280636 | -0.324031722 |
| 3 | -0.711007298 | -0.315920924 |
| 4 | -0.799534658 | -0.26411302 |
| 5 | -0.832609568 | -0.204165507 |
| 6 | -0.827422185 | -0.150504276 |
| 7 | -0.7967571 | -0.107462627 |
| 8 | -0.750021404 | -0.074991328 |
| 9 | -0.694048922 | -0.051435788 |
| 10 | -0.633724011 | -0.034806997 |

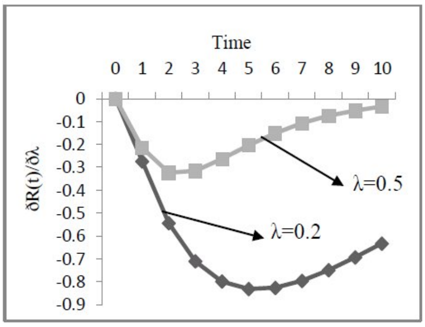


**Figure 9.** Sensitivity of system reliability w.r.t. standby power supply failure ($\lambda$)

**TABLE 9.** Sensitivity of M.T.T.F. w. r. t. $P_i$ where i=1 (Blackout), i=2 (Transient) and i=3 (Brownout)

| $P_i$ i=1, 2, 3 | $\frac{\partial M.T.T.F.}{\partial P_1}$ | $\frac{\partial M.T.T.F.}{\partial P_2}$ | $\frac{\partial M.T.T.F.}{\partial P_3}$ |
|---|---|---|---|
| 0.1 | -0.325396825 | -0.348224903 | -0.373542274 |
| 0.2 | -0.304742084 | -0.325396826 | -0.348224902 |
| 0.3 | -0.285993303 | -0.304742084 | -0.325396825 |
| 0.4 | -0.268922996 | -0.285993304 | -0.304742084 |
| 0.5 | -0.253336629 | -0.268922996 | -0.285993304 |
| 0.6 | -0.239067055 | -0.253336629 | -0.268922996 |
| 0.7 | -0.225970017 | -0.239067056 | -0.253336629 |
| 0.8 | -0.213920484 | -0.225970018 | -0.239067056 |
| 0.9 | -0.202809656 | -0.213920485 | -0.225970018 |
| 1 | -0.1925425 | -0.202809656 | -0.213920485 |

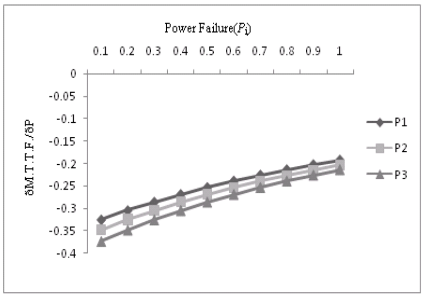


**Figure 10.** Sensitivity of MTTF w. r. t. $P_i$ where i =1 (Blackout), i = 2 (Transient) and i = 3 (Brownout)

**TABLE 10.** Sensitivity of M.T.T.F. w. r. t. standby power supply failure (λ)

| λ | $\partial M.T.T.F./\partial \lambda$ |
|---|---|
| 0.1 | -45.45454546 |
| 0.2 | -11.36363637 |
| 0.3 | -5.050505047 |
| 0.4 | -2.840909092 |
| 0.5 | -1.818181819 |
| 0.6 | -1.262626264 |
| 0.7 | -0.927643784 |
| 0.8 | -0.710227274 |
| 0.9 | -0.561167229 |
| 1 | -0.454545455 |

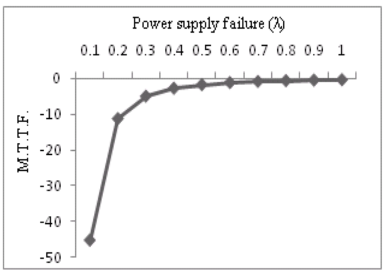


**Figure11.** Sensitivity of M.T.T.F. w. r. t. standby power supply failure (λ)

**8.6. Cost Analysis** Let us assume that $\lambda_1 = 0.1$, $\lambda_2 = 0.1$, $\lambda_3 = 0.1$, $\lambda_4 = 0.2$, $\lambda_5 = 0.2$, $\lambda_6 = 0.2$, $\lambda_7 = 0.3$, $\lambda_8 = 0.3$, $\lambda_9 = 0.3$, $P_1 = 0.4$, $P_2 = 0.5$, $P_3 = 0.6$, $\lambda = 0.7$, repairs rates be $u_1 = u_2 = u_3 = \psi_1 = \psi_2 = \psi_3 = \varphi_1 = \varphi_2 = \varphi_3 = v = 1$, $\theta = 1$, $x = 1$ and $y = 1$. Moreover, if the repair follows exponential distribution then using Equation (94), we can obtain Equation (95). If the service facility is always available, then expected profit during the interval (0, $t$] is given by:

$$E_P(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t$$

where $K_1$ and $K_2$ are the revenue per unit time and service cost per unit time respectively, then:

$$\begin{aligned} E_p(t) = K_1 (&-76.18110323\, e^{(-0.009071706019t)} - 0.5595928662 \\ & e^{(-2.038776412t)} + 0.3663620347\ 10^{-5} \\ & e^{(-0.9984127284t)} + 0.0026664094\ 89 e^{-0.6366204841t)} \\ & + 0.0526899479\ 7 e^{-0.4320684754t)} - 0.028576579 \\ & 78 e^{-0.2318488763t)} + 0.2853412192\ e^{-2.853201318t)} \\ & + 76.42857144\ ) - t K_2 \end{aligned} \qquad (96)$$

Keeping $K_1 = 1$ and varying $K_2$ at 0.1, 0.2, 0.3, 0.4, 0.5 in equation (96), one can obtain Table 11 which is depicted by Figure 12.

**TABLE 11.** Time vs. Expected Profit

| Time | $E_p(t)$ | | | | |
|---|---|---|---|---|---|
| | $K_2$=0.1 | $K_2$=0.2 | $K_2$=0.3 | $K_2$=0.4 | $K_2$=0.5 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0.79198937 | 0.69198937 | 0.59198937 | 0.49198937 | 0.39198937 |
| 2 | 1.41363213 | 1.21363213 | 1.01363213 | 0.81363213 | 0.61363213 |
| 3 | 1.99216313 | 1.69216313 | 1.39216313 | 1.09216313 | 0.79216313 |
| 4 | 2.56038882 | 2.16038882 | 1.76038882 | 1.36038882 | 0.96038882 |
| 5 | 3.12293435 | 2.62293435 | 2.12293435 | 1.62293435 | 1.12293435 |
| 6 | 3.68008367 | 3.08008367 | 2.48008367 | 1.88008367 | 1.28008367 |
| 7 | 4.23166831 | 3.53166831 | 2.83166831 | 2.13166831 | 1.43166831 |
| 8 | 4.77756107 | 3.97756107 | 3.17756107 | 2.37756107 | 1.57756107 |
| 9 | 5.31770351 | 4.41770351 | 3.51770351 | 2.61770351 | 1.71770351 |
| 10 | 5.85208468 | 4.85208468 | 3.85208468 | 2.85208468 | 1.85208468 |

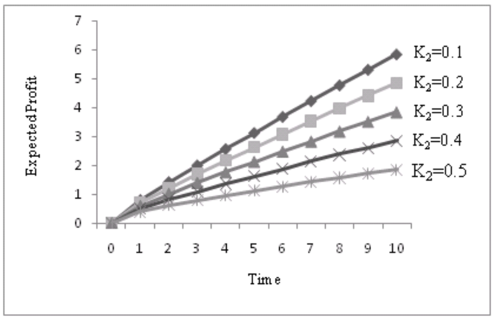


**Figure 12.** Time vs. Expected Profit

## 9. CONCLUSIONS

In this paper, we analyzed the availability, reliability, MTTF, busy period, sensitivity and cost effectiveness of the complex system incorporating power failures. To numerically examine the behaviour of availability, reliability, M.T.T.F. and cost effectiveness of the system, the various parameters are fixed as $\lambda_1 = 0.1$, $\lambda_2 = 0.1$, $\lambda_3 = 0.1$, $\lambda_4 = 0.2$, $\lambda_5 = 0.2$, $\lambda_6 = 0.2$, $\lambda_7 = 0.3$, $\lambda_8 = 0.3$, $\lambda_9 = 0.3$, $P_1 = 0.4$, $P_2 = 0.5$, $P_3 = 0.6$. One can easily conclude from Figure 3 that the availability of the system decreases with the increment in time and later on it stabilizes at value 0.6. Figure 4 represents the variation of reliability of the system. It shows that the reliability of the system decreases rapidly as the time increases and it attains a value of 0.01 after a long period of time.

By critically examining the Figures 5 and 6 one can conclude that M.T.T.F. of the system decreases from 2.4047 to 2.1794, from 2.4384 to 2.1992 and from 2.4744 to 2.2200 with respect to $P_1$, $P_2$ and $P_3$ respectively in a same manner for the considered values but the M.T.T.F. of the system with respect to $\lambda$ varies from 6.2121 to 2.1212. M.T.T.F. of the system has been obtained in the order: M.T.T.F. w. r. t. $\lambda$ > M.T.T.F. w. r. t. $P_3$ > M.T.T.F. w. r. t. $P_2$ > M.T.T.F. w. r. t. $P_1$. So M.T.T.F. of the system is highest with respect to $\lambda$ and lowest with respect to $P_1$.

Figure 7 is the graph of power failures, namely blackout ($P_1$), transient ($P_2$) and brownout ($P_3$) vs. busy period and its values have given in Table 6. Observation of Figure 7 reveals that Busy period increases as the value of $P_1$, $P_2$ and $P_3$ increases. The curve indicate that Busy period with respect to $P_1$ > Busy period with respect to $P_2$ > Busy period with respect to $P_3$. So, the busy period is highest in case of $P_1$ and lowest in case of $P_3$.

The sensitivities of the system reliability with respect to $P_1$, $P_2$, $P_3$ and $\lambda$ are shown in Figures 8 and 9. Figure 8 shows the sensitivity of system reliability with respect to $P_1$ at 1.1, $P_2$ at 0.1 and $P_3$ at 1.9. It reveals that the sensitivity initially decreases and then tends to increase as time passes and attain a value -0.0037, -0.0035 and -0.0027 at t = 10 with respect to $P_1$, $P_2$ and $P_3$ respectively. It is clear from the graph that system reliability is more sensitive w. r. t. $P_3$. The sensitivity of $\lambda$ on the system reliability is shown in Figure 9. It is clear from this figure that the sensitivity of the system reliability initially decreases and then increases with respect to $\lambda$ at 0.2 and 0.5. It is interesting to note that the system becomes more sensitive with the increase in failure rate. So, we can conclude that the system can be made less sensitive by controlling its failure rates. Moreover, Figures 10 and 11 show the sensitivity of M.T.T.F. with respect to $P_1$, $P_2$, $P_3$ and $\lambda$ which show that it increases from -0.325396825 to -0.1925425, -0.348224903 to -0.202809656, -0.373542274 to -0.213920484 and -45.45454546 to -0.454545455 as $P_1$, $P_2$, $P_3$ and $\lambda$ increase from 0.1 to 1. Critical observation of these graphs points out that M.T.T.F. of the system is more sensitive with respect to $P_1$.

Keeping revenue cost per unit time at 1 and varying service cost from 0.1 to 0.5, one can obtain Figure 12. It is very clear that the profit decreases as the service cost increases. The highest and lowest values of expected profit are obtained to be 5.85 and 0.39 respectively for the considered values.

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