A CLOSED FORM SOLUTION FOR FREE VIBRATION ANALYSIS OF TUBE-IN-TUBE SYSTEMS IN TALL BUILDINGS

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Abstract In this paper the dynamic response of tube-in-tube systems for tall building structures is investigated. Inner and outer tubes are modeled using equivalent continuous orthotropic membranes; in which, each tube is individually modeled by a cantilever box beam. By applying the compatibility conditions on deformation of the two tubes, the governing dynamic equations of the tube-in-tube structure and their associated boundary conditions are derived using the variational principle of virtual work. Appling differential calculus with some simplifications, and deriving non-trivial solution of these equations, a closed form solution is presented to obtain natural frequency and mode shape of tube-in-tube structures. A quick estimate of these quantities is of particular importance at the early stages of tube-in-tube systems design prior to a full dynamic analysis. In order to illustrate the efficiency of the proposed model a symmetrical building is analyzed and comparisons are made with more accurate results obtained by three dimensional computer dynamic analysis and previous published methods.

Keywords Tall buildings; Tube-in-tube; Free vibration; Variational principle; Natural frequency; Mode Shape.

چکیده در این مقاله پاسخ دینامیکی سیستم قاب محیطی تو در تو در ساختمانهای بلند مورد بررسی قرار گرفته است. قاب‌های داخلی و خارجی با استفاده از صفحات اورتوتروپیک پیوسته معادل‌سازی شده‌اند که در آن هر قاب به طور جداگانه با یک تیر طره‌ای جایگزین شده است. با در نظر گرفتن سازگاری تغییرشکل‌های دو قاب داخلی و خارجی و به‌کارگیری اصول حساب تغییرات و کار مجازی معادلات حاکم بر حرکت سازه و شرایط مرزی بدست آمده‌اند. با اعمال برخی ساده‌سازی‌ها بر ارتعاش آزاد سیستم قاب محیطی تو در تو معادلات حاکم بر فرکانس طبیعی و مود شکل متناظر حاصل شده است. تخمین این مشخصات ارتعاشی در آنالیز اولیه سیستم قاب محیطی تو در تو و برآورد اولیه مفید می‌باشد. صحت و کارایی نتایج حاصله از روش پیشنهادی برای ساختمان بلندی با سیستم قاب محیطی تو در تو آنالیز و نتایج حاصله با آنالیز کامپیوتری و نتایج حاصله از روش‌های قبلی موجود مقایسه شده است.

Nomenclature

A	Sum of the cross sectional area of exterior columns
C_i	Arbitrary coefficients of general solution
d_c	Dimension of column
E	Modulus of elasticity
EI	Flexural rigidity of a tall building
G	Shear modulus
h	Height of story
H	Total height of the building
j	An integer representing mode number
$P(x,t)$	Distributed external force
s_f	Span spaces of flange frame
s_w	Span spaces of web frame
S	Shear rigidity of a tall building
t	Thickness of equivalent membrane
u	Natural mode shape function
$2W_f$	Width of flange frame
$2W_w$	Width of web frame
W_{ext}	Virtual work done by external forces
x	Spatial position of any material along the height of the building
$y(x,t)$	Lateral displacement of the building
δ	Variation operator
ν	Poisson ratio
ρ	Mass density
ω	Circular natural frequency
ζ	Nondimensional coordinate for x

1. INTRODUCTION

Tall building structures are affected by lateral loads due to wind or earthquake actions to an extent that they play an important role in structural design. Over the last decades, various forms of tall building structures have been developed to be efficient in resisting lateral loadings [1]. In general, framed tube structures are widely accepted as an economical system in high rise buildings, over a wide range of building heights. Framed tube system, in its simplest form consists of closely spaced exterior columns tied at each floor level by relatively deep spandrels [2]. In order to increase the efficiency of the framed tube under the lateral loads, framed tube commonly is utilized with central core. A framed tube under lateral loads acts like a cantilevered box beam to resist the overturning moment; and the central core acting like second tube within the outside tube. The central core may be designed not only for gravity load but also to resist lateral loads. When lateral sway is critical and starts controlling the design, the ''framed tube'' can be supplemented by a tube instead of the central core to create ''tube-in-tube'' system [1]. In recent years, due to developments in design technology and material qualities in civil engineering, the structures have become lighter and more slender. These will cause the structure to vibrate when they are located in environments where earthquake or high winds exist.

These vibrations may lead to serious structural damage with potential for structural failure. A great number of researches have studied the natural modes of tall buildings over the past decades. Youlin [3] has analyzed tube-in-tube structures. In this work, the outer framed tube was considered as an equivalent closed tube. Based on compatibility of deformation between the two tubes, a differential equation was developed and formulas for calculating horizontal displacements of tube-in-tube structure and load distribution on the two tubes were derived. Then, the natural period of the first mode of tube-in-tube structure and the seismic loads on it were obtained by the Top-Displacement method. Wang [4] obtained a formula for calculating the natural frequencies of tube-in-tube structures in tall buildings directly from the fourth-order Sturm-Liouville differential equations. In another study, numerical solutions of eigenvalues for free vibration of tube-in-tube structures by using modified ODE solver for eigenvalue problem was presented by him based on an existing Ordinary Differential Equation (ODE) solver [5]. Lee [6] proposed a simple mathematical model for approximate analysis of framed tube structures with multiple internal tubes using the minimum potential energy principle in conjunction with the variational approach. Lee [7] presented an approximate solution that was formulated for free vibration analysis of tube-in-tube tall buildings. The governing partial differential equation of motion has been reduced to an ordinary differential equation with variable coefficients on the assumption that the transverse vibration is harmonic. A power-series solution was used to obtain mode shape functions for the tube-in-tube structures.

In the present study, a mathematical model is proposed for accurate prediction of the fundamental natural frequencies and corresponding mode shapes of tube-in-tube systems in tall building structures. Tube-in-tube system is analyzed using an orthotropic box beam analogy approach in which each tube is individually modeled by a box beam thus accounting for flexural as well as the shear deformations [6, 8-10]. Appling compatibility of deformation for the two tubes, the governing dynamic equations of the tube-in-tube structure and their associated boundary conditions are derived using variational principle of virtual work. Appling differential calculus with some simplifications, and deriving non-trivial solution of these equations, a closed form solution has been presented to obtain the fundamental natural frequencies and mode shapes of tube-in-tube structures. The method is simple and accurate enough. It can greatly reduce the computational work as compared to finite element approach, and yet a solution of acceptable precision can be obtained.

2. DERIVATION OF THE BOUNDARY AND EIGENVALUE PROBLEMS

It is well known that highly accurate results for vibrational analyses of tall building structures may be obtained using appropriate finite element based computer softwares. However, the analytical technique based on continuum modeling, not only provides a simple and convenient means for free vibration analysis, it also permits one to visualize directly the dynamic performance of tall building structures, and hence to understand qualitatively how the natural frequencies and corresponding mode shapes are related to the structural parameters by carrying out simple parametric studies. The analytical techniques based on continuum modeling can clearly reveal the static and dynamic characteristics of tall building structures. Moreover, information provided by the analytic technique, can also be used to develop a basis for analysis of tall building structures using approximate methods. Furthermore, to investigate the influence of particular structural parameters on static and dynamic characteristics of a given structure, it is very convenient if analytical methods are utilized. The advantages of both analytical and approximate methods based on continuum modeling for tall building structures are not realized by the highly time consuming modeling in finite element analyses. In contrast, the finite element approach requires one to solve thousands of linear simultaneous equations to obtain quantitative results in detail. So it is a powerful tool for analysis and design at the detailed and final design stages of tall buildings. It takes much more time hence expensive, for the modeling of a given structure using finite elements.

In this section an approximate mathematical model is presented to obtain natural frequencies and corresponding mode shapes of tube-in-tube structures in tall buildings. A discrete tube-in-tube structure can be modeled as an assemblage of equivalent orthotropic plate panels. Consequently, a framed-tube structure may be analyzed as a continuum. Figure 1 shows a typical framed-tube structure with internal tube. Each of the tubes is composed of four equivalent orthotropic plate panels. All framed tubes under consideration consist of an assemblage of such plate panels of uniform thickness in vertical planes [10]. Each framed tube structure can be modeled as cantilever beam with hollow section. Thus, the tube-in-tube model can represent a coupled system consisting of two tubes. Since floor slabs is essentially rigid within their own plane, the relative lateral displacements between the two tubes can be assumed negligible at each floor level. The outside tube and the internal tube, each described by a set of differential equations, are forced to have compatible lateral deflection at the floor level.

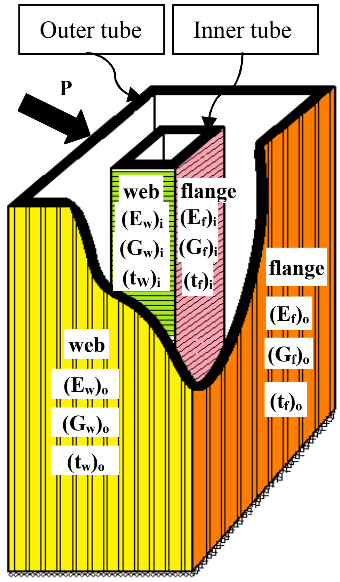

Figure 1. Equivalent tube-in-tube structure.

The following assumptions are adopted in this paper for structural analysis:

a) Structure's behavior is linear elastic.

b) Floor slabs are considered to be rigid diaphragms within their own plane.

c) The spacing of beams and columns are uniform through the building height.

d) Sectional properties for beams and columns are uniform throughout the height of building.

Analytical analysis for natural frequencies and corresponding mode shapes is carried out using the Hamilton's principle. By considering a distributed system defined over the closed domains $0 \le x \le H$, where x is the spatial position of any material point of the system, H is total height of the structure and lateral displacement of cantilevered beams are denoted by $y(x,t)$ (see Figures 2 and 3).

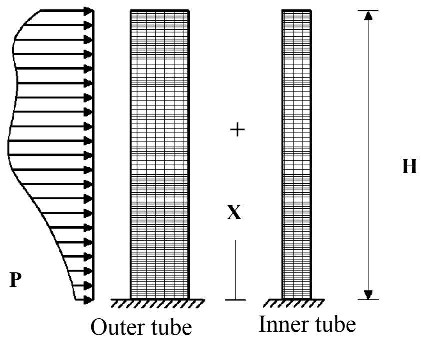

Figure 2. Behavior of tube-in-tube structure.

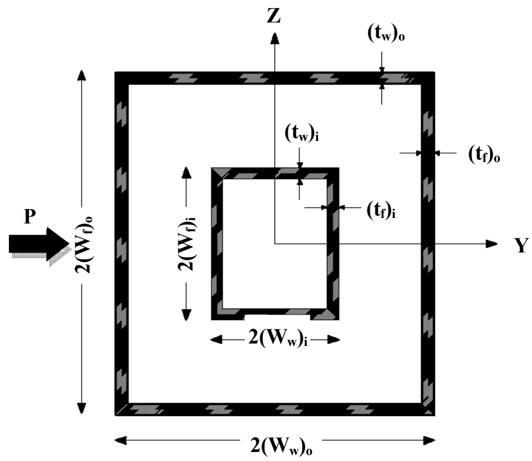

Figure 3. Structural plan of the tube-in-tube structure.

Hamilton's principle [11-14] states that:

$$\int_{t_1}^{t_2} \delta L \; dt + \int_{t_1}^{t_2} \delta W_{ext} \; dt = \int_{t_1}^{t_2} \delta (T - V) \; dt + \int_{t_1}^{t_2} \delta W_{ext} \; dt = 0 \quad (1)$$

where δ denote variation with respect to field variable y, L is the Lagrangian, W_{ext} is the work done by external forces, T and V are the kinetic and potential energy of the tube-in-tube structure, respectively.

Assuming that lateral deflection, y, is identical in both tubes, the potential energy and kinetic energy are written as follows:
potential energy [11]:

$$V(t) = \frac{1}{2}\int_0^H EI(x)[\frac{\partial^2 y(x,t)}{\partial x^2}]^2 dx + \frac{1}{2}\int_0^H S(x)[\frac{\partial y(x,t)}{\partial x}]^2 dx \quad (2)$$

kinetic energy [11]:

$$T(t) = \frac{1}{2}\int_0^H \rho \; A(x)[\frac{\partial y(x,t)}{\partial t}]^2 dx \quad (3)$$

Lagrangian of tube-in-tube structure can be obtained as follows [11]:

$$L = -\frac{1}{2}\int_0^H EI(x)[\frac{\partial^2 y(x,t)}{\partial x^2}]^2 dx - \frac{1}{2}\int_0^H S(x)[\frac{\partial y(x,t)}{\partial x}]^2 dx + \frac{1}{2}\int_0^H \rho \; A(x)[\frac{\partial y(x,t)}{\partial t}]^2 dx \quad (4)$$

work done by external forces:

$$W_{ext} = \int_0^H P(x,t) \; y(x,t) \; dx \quad (5)$$

where EI is the sum of flexural rigidities for inner tube and outer tube, S is the sum of shear rigidities for inner and outer tubes, ρ is the mass density, $P(x,t)$ is a distributed external force and A is sum of the cross-sectional areas for framed tube structures. Substituting Equations (4) and (5) in Equation (1) and integrating by parts, the following basic equation of motion for the tube-in-tube system is obtained:

$$\frac{\partial}{\partial x}(S \; y') - \frac{\partial^2}{\partial x^2}(EI \; y'') - \rho \; A \; \ddot{y} + P = 0, \quad 0 < x < H \quad (6)$$

with boundary conditions:

$$[- S \; y' + \frac{\partial}{\partial x}(EI \; y'')]\Big|_{x=H} = 0 \quad (7)$$

and

$$[EI \; y'']\big|_{x=H} = 0 \quad (8)$$

where a dot over a variable indicates the differential with respect to time and a prime indicates the partial differential with respect to x. Equations (7) and (8) state that shear force and moment are zero at the free top of the structure. As there are zero deflection and zero rotation at the fixed base of the structure the following boundary conditions are obtained:

$$y(0,t) = 0 \quad (9)$$

and

$$y'(0,t) = 0 \quad (10)$$

It is considered that the free vibration motion at any point of the structural height x is harmonic and the deflected shapes are independent of time t [13-14]. The displacement vector can then be written, in a separable form of variables x and t, as follows:

$$y(x,t) = u(\zeta) \; \sin \; (\omega \; t) \quad (11)$$

where $\zeta = x/H$ is the relative height of structure, ω is the circular frequency, and u is the mode shape function. In free vibration analysis of any structure, the applied external force $P(x,t)$ is equaled to zero. Substituting Equation (11) into Equations (6-10) and carrying out the necessary differentiation, the eigenvalue equation of the symmetric frame structures and its boundary conditions are:

governing equation:

$$\frac{d^2u_{(\zeta)}}{d\zeta^2}(EI_{(\zeta)}\ \frac{d^2u_{(\zeta)}}{d\zeta^2}) - H^2\frac{d}{d\zeta}(S_{(\zeta)}\ \frac{du_{(\zeta)}}{d\zeta}) \\ -\rho\ A_{(\zeta)}\ \omega^2\ H^4\ u_{(\zeta)} = 0, \quad 0 < \zeta < 1.0 \tag{12}$$

boundary conditions:

$$u = 0 \qquad \text{at} \quad \zeta = 0 \tag{13}$$

$$\frac{du}{d\zeta} = 0 \qquad \text{at} \quad \zeta = 0 \tag{14}$$

$$\frac{d^2u}{d\zeta^2} = 0 \qquad \text{at} \quad \zeta = 1.0 \tag{15}$$

$$H^2\frac{S}{EI}\frac{du}{d\zeta} - \frac{d}{d\zeta}(\frac{d^2u}{d\zeta^2}) = 0 \qquad \text{at} \quad \zeta = 1.0 \tag{16}$$

3. SOLUTION METHOD

3.1. Theoretical Method of Solution When values of ρ, EI and S are constants in the length of structure, to obtain a theoretical solution to the free vibration of tube-in-tube structures, eigen problem given in Equation (12) may be written as:

$$\frac{d^4u_{(\zeta)}}{d\zeta^4} - \alpha^2\frac{d^2u_{(\zeta)}}{d\zeta^2} - \beta^2\omega^2\ u_{(\zeta)} = 0, \quad 0 < \zeta < 1.0 \tag{17}$$

with boundary conditions of the eigenvalue equation as follows:

$$u = 0 \qquad \text{at} \quad \zeta = 0 \tag{18}$$

$$\frac{du}{d\zeta} = 0 \qquad \text{at} \quad \zeta = 0 \tag{19}$$

$$\frac{d^2u}{d\zeta^2} = 0 \qquad \text{at} \quad \zeta = 1.0 \tag{20}$$

$$\alpha^2\frac{du}{d\zeta} - \frac{d^3u}{d\zeta^3} = 0 \qquad \text{at} \quad \zeta = 1.0 \tag{21}$$

in which the structural parameters are as follows:

$$\alpha^2 = \frac{S}{EI}H^2 \tag{22}$$

$$\beta^2 = \frac{\rho\ A}{EI}H^4 \tag{23}$$

Substituting $u(\zeta) = C\ e^{\mu\zeta}$ into Equation (17) gives:

$$\mu^4 - \alpha^2\mu^2 - \beta^2\omega^2 = 0 \tag{24}$$

Hence

$$p_{1,2} = \pm\mu_1 \tag{25}$$

and

$$p_{3,4} = \pm i\mu_2 \tag{26}$$

where

$$\mu_1 = \sqrt{\sqrt{\beta^2\omega^2 + \frac{\alpha^4}{4}} + \frac{\alpha^2}{2}} \tag{27}$$

$$\mu_2 = \sqrt{\sqrt{\beta^2\omega^2 + \frac{\alpha^4}{4}} - \frac{\alpha^2}{2}} \tag{28}$$

Thus, for a uniform tube structure the general solution of Equation (17) which is a fourth-order differential equation with constant coefficients, the general solution is:

$$u = C_1\cos\mu_1\zeta + C_2\sin\mu_1\zeta + C_3\sinh\mu_2\zeta + C_4\cosh\mu_2\zeta \tag{29}$$

Solution of Equation (24), $u(\zeta)$ and its related derivatives are:

$$\begin{bmatrix} u_{(\zeta)} \\ u'_{(\zeta)} \\ u''_{(\zeta)} \\ u'''_{(\zeta)} - \alpha^2\ u'_{(\zeta)} \end{bmatrix} = \\ \begin{bmatrix} \cosh\mu_1\zeta & \sinh\mu_1\zeta \\ \mu_1\sinh\mu_1\zeta & \mu_1\cosh\mu_1\zeta \\ \mu_1^2\cosh\mu_1\zeta & \mu_1^2\sinh\mu_1\zeta \\ \mu_1\mu_2^2\sinh\mu_1\zeta & \mu_1\mu_2^2\cosh\mu_1\zeta \end{bmatrix} \\ \begin{matrix} \cos\mu_2\zeta & \sin\mu_2\zeta \\ -\mu_2\sin\mu_2\zeta & \mu_2\cos\mu_2\zeta \\ -\mu_2^2\cos\mu_2\zeta & -\mu_2^2\sin\mu_2\zeta \\ \mu_2\mu_1^2\sin\mu_2\zeta & -\mu_2\mu_1^2\cos\mu_2\zeta \end{matrix}\Bigg] \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} \tag{30}$$

Applying boundary conditions to Equation (30) leads to:

$$\begin{bmatrix} u_{(0)} \\ u'_{(0)} \\ u''_{(1)} \\ u'''_{(1)} - \alpha^2 \; u'_{(1)} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & \mu_1 & 0 & \mu_2 \\ \mu_1{}^2 \cosh \mu_1 & \mu_1{}^2 \sinh \mu_1 & -\mu_2{}^2 \cos \mu_2 & -\mu_2{}^2 \sin \mu_2 \\ \mu_1 \mu_2{}^2 \sinh \mu_1 & \mu_1 \mu_2{}^2 \cosh \mu_1 & \mu_2 \mu_1{}^2 \sin \mu_2 & -\mu_2 \mu_1{}^2 \cos \mu_2 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (31)$$

Solution of Equation (31) consists of two parts. The first part is the eigenvector which corresponds to the mode shape, while the second part is the eigenvalue, which corresponds to the frequency of free vibration of the tube-in-tube structure. A closed form solution has been derived for determining the natural frequencies and corresponding mode shapes of symmetric tube-in-tube structures in tall buildings.

3.2. Vibrational Frequencies and Corresponding Mode Shapes

Mathematically, the non-trivial solution of Equation (31) can only be obtained when the determinant of the coefficients is equal to zero, i.e.

$$\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & \mu_1 & 0 & \mu_2 \\ \mu_1{}^2 \cosh \mu_1 & \mu_1{}^2 \sinh \mu_1 & -\mu_2{}^2 \cos \mu_2 & -\mu_2{}^2 \sin \mu_2 \\ \mu_1 \mu_2{}^2 \sinh \mu_1 & \mu_1 \mu_2{}^2 \cosh \mu_1 & \mu_2 \mu_1{}^2 \sin \mu_2 & -\mu_2 \mu_1{}^2 \cos \mu_2 \end{vmatrix} = 0 \quad (32)$$

The solution of Equation (32) is then obtained by Mathematica 7.0.0 software [15] as follows:

$$1 + \left(1 + \frac{\alpha^4}{2\beta^2 \; \omega^2}\right) \cosh \mu_1 \; \cos \mu_2 + \frac{\alpha^2}{2\beta \; \omega} \sinh \mu_1 \; \sin \mu_2 = 0 \quad (33)$$

From Equation (31), coefficients in Equation (29) are obtained as follows:

$$C_3 = -C_1; C_4 = -\frac{\mu_1}{\mu_2} C_2 \quad (34)$$

where

$$C_2 = -\frac{\mu_1^2 \cosh \mu_1 + \mu_2^2 \cos \mu_2}{\mu_1^2 \sinh \mu_1 + \beta\omega \; \sin \mu_2} C_1 \quad (35)$$

Hence, the corresponding natural mode shapes can be expressed as follows [11]:

$$u^{(j)}(\zeta) = A^{(j)} \; w^{(j)} \quad (36)$$

where $A^{(j)}$ are unknown constants, and $w^{(j)}$ the non-normalized mode shape functions, given by:

$$\begin{aligned} w^{(j)} &= \cosh \mu_1^{(j)} \zeta - \cos \mu_2^{(j)} \zeta \\ &- \left(\frac{\mu_1^{2(j)} \cosh \mu_1^{(j)} + \mu_2^{2(j)} \cos \mu_2^{(j)}}{\mu_1^{2(j)} \sinh \mu_1^{(j)} + \beta\omega \; \sin \mu_2^{(j)}} \right) \\ &\times (\sinh \mu_1^{(j)} \zeta - \frac{\mu_1^{(j)}}{\mu_2^{(j)}} \sin \mu_2^{(j)} \zeta) \end{aligned} \quad (37)$$

The natural modes $u^{(j)}(\zeta)$ (*j=1,2,…*) constitute a complete set of orthogonal modes. Hence, free vibration frequencies and associated mode shapes of a symmetric-plan tube-in-tube structure can be determined by using Equation (33) and Equations (36-37) respectively.

4. NUMERICAL INVESTIGATION

To demonstrate the simplicity and accuracy of the proposed method a tube in tube tall building structure which studied by Lee [7] is analyzed. Geometric parameters of the building are listed in Table 1. The plan and sectional views of the structure are shown in Figure 4. The flexural rigidity of the outer tube is $(EI)_o = 3.52872\times10^9$ $KN.m^2$, the flexural rigidity of the inner tube is $(EI)_i = 7.5538\times10^9$ $KN.m^2$, the shear rigidity $S = 3.9888\times10^7$ $KN.m^2$, mass per unit length is $m = 325.828$ *t/m*, the total height of the building is $H = 75.9$ *m*.

TABLE 1. Specifications of the 25-story building.

Outer tube dimensions		Inner tube dimensions		Center to center spacing of columns		Height of building
$2(W_f)_o$ (m)	$2(W_w)_o$ (m)	$2(W_f)_i$ (m)	$2(W_w)_i$ (m)	s_w (m)	s_f (m)	H (m)
30	26	7.65	10.40	2.0	2.0	75.9

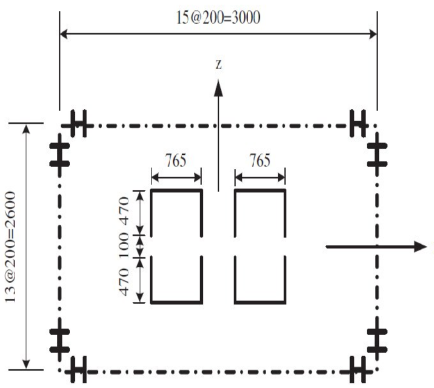

Figure 4. Typical floor plan of tube-in-tube tall building.

The first two natural frequencies are compared with those obtained by Top Displacement Method (TDM) [3], Mode Superposition Method (MSM) [3], Finite Element Method (FEM) (COSMOS/M1988) [16], Variational Method (VM) [17], Sturm-Liouville equation (SL) [4], ODE solver (ODE) [5] and Power-Series Solution (PSS) [7]. The capabilities of the proposed method and those of the previous published approximate methods are compared in Table 2.

TABLE 2. Comparison of natural frequencies of tube-in-tube tall building.

Methods	ω_1(rad/s)	ω_2(rad/s)
Proposed method	3.705	16.127
TDM	3.157	-
MSM	3.279	17.921
FEM	3.715	21.200
VM	3.462	21.200
SL	3.462	21.525
ODE	3.461	19.239
PSS	3.518	20.763

The results of approximate proposed method are in good agreement with results available in the published literature.

5. DISCUSSIONS

It can be seen that the results calculated by the proposed method agree well with those obtained from a detailed finite element analysis and previous published methods. The difference of the natural frequency and corresponding mode shape of tube-in-tube tall building in the frequencies computed by the proposed method and reference methods are within acceptable ranges. The main sources of error between the proposed approximate method and other methods are as follows [19-20]:

(a) All closely spaced perimeter columns tied at each floor level by deep spandrel beams considered to form a tubular structure.
(b) Modeling the frame panels as equivalent orthotropic membranes, so it can be analyzed as a continuous structure.
(c) The approximation to derive *EI* and *GA* parameters.
(d) The effect of shear lag in the external and internal tubes has been neglected in assessing the global behavior of the tube-in-tube structures in approximate method.

The numerical example demonstrates the accuracy and simplicity of the proposed method as compared to previous published methods.

6. CONCLUSION

Based on Hamilton's principle in conjunction with the variational approach and differential calculus, a simple mathematical model is presented for the free vibration analysis of tube-in-tube structures. Each tube through the height of the structure is modelled as cantilever beam. Also the tube structures are modelled as a box assemblage of orthotropic plate panels. By simplifying assumption related to behavior of tube-in-tube structures and using Hamilton's principle, the complex free vibration analysis is reduced to a linear differential equation with four boundary conditions. By obtaining the trivial and non-trivial solutions of these equations a closed form solution is obtained for calculating the natural frequencies of free vibration and associated mode shapes of tube-in-tube tall building structures. The accuracy, simplicity and reliability of the proposed method are verified for a tube-in-tube tall building. Differences between natural frequencies of proposed method and previous published works are small. Accuracy and economy of the proposed method is confirmed. The proposed method is simple, accurate, economical and reliable, and especially suitable for use at preliminary design stages where a large number of structures with different features are required to be analyzed repeatedly.

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