influencing parameters, such as radiation-conduction parameter (RC) on thermal behavior of the system. As a main result, it was found that the rise in the value of RC and emissivity and decrease in optical thickness and scattering albedo increase the rate of radiative transfer. Varady and Fedorov [4] studied the conduction-radiation heat transfer in a semitransparent glass foam layer. It was assumed that the foam layer is thin with uniform thickness, which is bounded by hot combustion gases on top and glass melt on bottom. They used Schuster-Schwarzchild approximation for solving the radiative transfer equation and presented a method to obtain the effective thermal conductivity of the foam layer. Regarding the combustion process in furnaces, some results are presented in literature. Modeling and simulation of an industrial furnace under the conventional combustion as well as under the highly preheated and diluted air combustion (HPDAC) conditions was done by Hamidi and Rahimi [5].

It is clear that in different types of furnaces, the heat sources have a major role in thermal performance of these systems. Based on the information available in literature, and to the best of author's knowledge, the two-dimensional radiative-conductive heat transfer problem for obtaining the thermal behavior of furnaces with considering the effects of heat source in these systems has not been yet carried out. Thereby, the objective of the present work is the analysis of combined conduction and radiation heat transfer in a furnace considering the effects of two heat sources.

2. ANALYSIS

The physical model along with coordinate system is depicted in Fig.1, in which a square enclosure with isothermal and gray surfaces and two heat sources with rectangular shape are considered.

Modest [6] presented a general equation for emitting-absorbing and scattering gray medium at any direction and location as follow:

$$\frac{dI}{ds} = \hat{s}.\nabla I(r,\hat{s}) + \beta(r) I(r,\hat{s}) = \sigma_a(r) I_b(r) + \frac{\sigma_s(r)}{4\pi}\int_{4\pi} I(r,\hat{s}')\phi(r,\hat{s}',\hat{s})d\Omega' \quad (1)$$

where I, β, σ_a, I_b, σ_s, ϕ and Ω are radiation

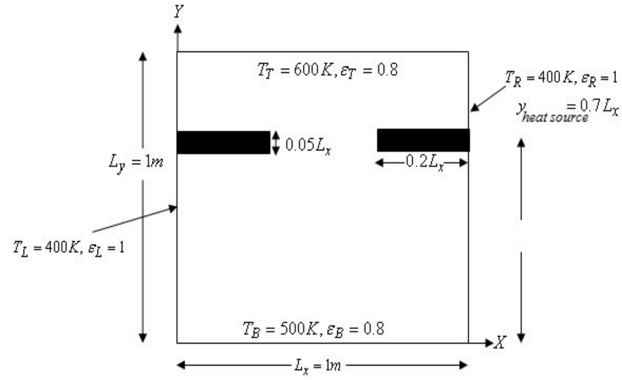


Figure 1. Computational test domain

intensity, extinction coefficient, absorption coefficient, black body radiation intensity, scattering coefficient, scattering phase function and solid angle, respectively. For the diffusely reflecting walls, the radiative transfer equation (RTE) is subject to the following boundary conditions:

$$I(r_w,\hat{s}_i) = \varepsilon(r_w) I_b(r_w) + \frac{\rho(r_w)}{\pi}\int_{n.s_j<0} I(r_w,\hat{s}_j)\left|\hat{n}.\hat{s}_j\right| d\Omega'. \quad (2)$$

where ε and ρ are wall emissivity and reflectivity of the surface, respectively.

It is obvious that the integro-differantial equation (1) does not have analytical solution, so the researchers represented different approximate solutions for solving this equation. Amongst the many numerical methods of solving RTE, the discrete ordinate method or S_N method is widey used by investigators. DOM is a wonderful and applicable tool in transforming the RTE into a set of ordinary differential equations for n different directions, $\hat{s}_i = 1,2,...,n$. Besides, by numerical quadrature definition, the integrals over direction are substituted as follows:

$$\int_{4\pi} f(\hat{s})d\Omega \cong \sum_{i=1}^{n} w_i f(\hat{s}_i) \quad (3)$$

where w_i are quadrature weight associated with in any directions s_i. Thus, RTE is approximated by a set of ordinary differential equations, as follows:

$$\frac{dI}{ds} = \hat{s}_i.\nabla I(r,\hat{s}_i) + \beta(r) I(r,\hat{s}_i)$$
$$= \sigma_a(r) I_b(r) + \frac{\sigma_s(r)}{4\pi} \sum_{j=1}^{n} w_j I(r,\hat{s}_j)\phi(r,\hat{s}_i,\hat{s}_j) \tag{4}$$

with the following boundary conditions:

$$I(r_w,\hat{s}_i) = \varepsilon(r_w) I_b(r_w) + \frac{\rho(r_w)}{\pi} \sum_{\hat{n}.\hat{s}<0} w_j I(r_w,\hat{s}_j) \left|\hat{n}.\hat{s}_j\right| \tag{5}$$

for : $\hat{n}.\hat{s}_i > 0$

By determination of the intensities in the desired directions, integrated quantities can be computed. Consequently, the radiative heat flux inside the medium may be calculated as follows:

$$q(r) = \int_{4\pi} I(r,\hat{s})\hat{s}d\Omega \approx \sum_{i=1}^{n} w_i I_i(r)\hat{s}_i \tag{6}$$

where q is the heat flux.

For two dimensional problems, RTE is stated as:

$$\xi_i \frac{\partial I_i}{\partial x} + \eta_i \frac{\partial I_i}{\partial y} + \beta I_i = \beta S_i \tag{7}$$

where ξ and η are direction cosines with regard to X and Y directions. The source term is obtained by:

$$S_i = (1-\omega) I_b + \frac{\omega}{4\pi} \sum_{j=1}^{n} w_j \phi_{ij} I_j \tag{8}$$

where ω is scattering albedo (σ_S / β).

The equation for determination of radiation intensity employing DOM and upon discreditizing RTE with finite volume method appears as [6]:

$$I_{p_i}^m = \frac{I_x^m + I_y^m + k_a I_b + S'^m}{\beta + x^m sign(x^m) + y^m sign(y^m)} \tag{9}$$

The various terms in the above equation are as follows:

$$I_x^m = x^m [u_0(x^m) I_{x-1,y}^m - u_0(-x^m) I_{x+1,y}^m]$$
$$x^m = \frac{\xi^m}{\Delta x} \quad y^m = \frac{\eta^m}{\Delta y} \tag{10}$$

$$I_y^m = y^m [u_0(y^m) I_{x,y-1}^m - u_0(-y^m) I_{x,y+1}^m] \tag{11}$$

$$u_0(x) = \begin{cases} 1 & x>0 \\ 0 & x<0 \end{cases} \quad sign(x) = \begin{cases} 1 & x>0 \\ -1 & x<0 \end{cases} \tag{12}$$

$$S'^m = \frac{\sigma_s}{4\pi} \sum_{m'} w^{m'} I^{m'} \tag{13}$$

where superscripts m and m' denote the direction which is surveyed and the other directions, respectively, and subscript i is control volume counter. On the other hand, for two-dimensional combined radiation-conduction problems, the energy equation is given by:

$$k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = \sigma_a(4\pi I_b - G) + q_{heatsource} \tag{14}$$

where T is the absolute temperature and G is the irradiation that can be computed by the following equation:

$$G = \sum_m w^m I_p^m \tag{15}$$

In equation (14), $q_{heatsource}$ is a source term denoting the rate of heat generation inside the heat sources. It should be noted that the last term in the right hand side of equation (14) must be omitted for the grid points outside the heat source domain.

3. RESULTS AND DISCUSSION

In the present study, the radiative transfer equation along with the gas energy equation are solved numerically for obtaining the thermal behavior of a rectangular furnace while the effect of heat source is also considered into account. Besides, attempt is made to investigate the effect of radiation-conduction parameter on thermal characteristics of the system. The radiation-conduction parameter is calculated by the following equation:

$$RC = \sigma T_H^3 H / k \tag{8}$$

where σ, T_H, H and k are Stefan-Boltzmann's constant [$5.67\times10^{-8} W/m^2/K^4$], hot wall temperature, characteristic length and thermal conductivity.

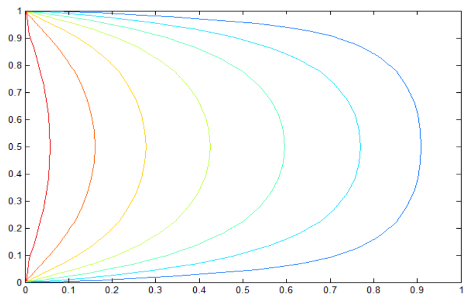
Before going to the analysis, the grid independence test has been conducted and presented in Table 1 which contains the amount of averaged heat flux (W/m^2) on the walls. Results show that mesh size of 51×51 is optimum as the grid size. This grid size is used in all subsequent simulations.

TABLE1. Grid independent test

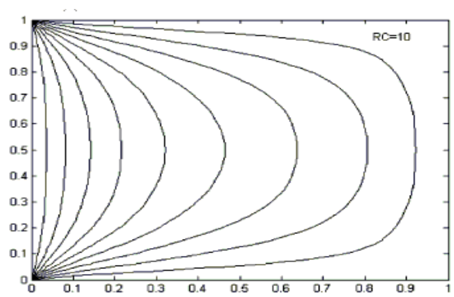
Mesh size	q_T (−)	q_B(−)	q_L(−)	q_R(−)
38×38	19117.566	27329.131	39726.466	39726.466
45×45	19117.567	27329.132	39726.467	39726.467
51×51	19117.567	27329.132	39726.467	39726.467
63×63	19117.567	27329.132	39726.467	39726.467

($T_T = 600K, T_B = 500K, T_L = 400K, T_R = 400K$,
$RC = 10, \omega = 0.2, \varepsilon_T = 0.8, \varepsilon_B = 0.8, \varepsilon_L = 1, \varepsilon_R = 1, \sigma_a = 7$)

In order to validate the present numerical method, in a test case with the same values of parameters as reported in the literature [3], the isothermal lines of a participating media inside a two-dimensional rectangular enclosure are calculated and plotted in Fig. 2 for radiation-conduction parameter (RC) equal to 10. In the numerical analysis, the radiation-conduction parameter has been varied keeping other influencing parameters constant . It is clearly seen that there is a good consistency between these theoretical results.



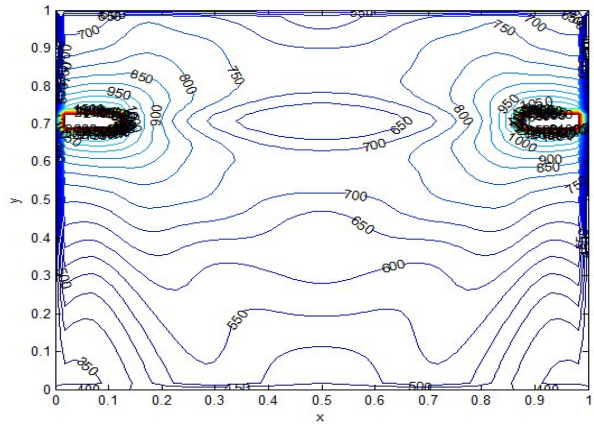
(a)



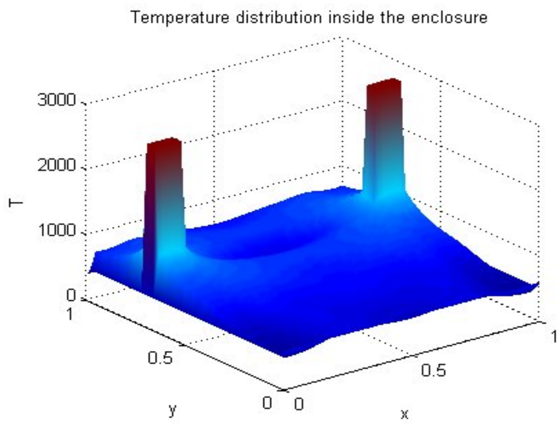
(b)

Figure 2. Comparison of isotherm pattern (a) with that of Ref. [3] (b), $\tau = 1, \omega = 0, \varepsilon = 1, RC = 10$

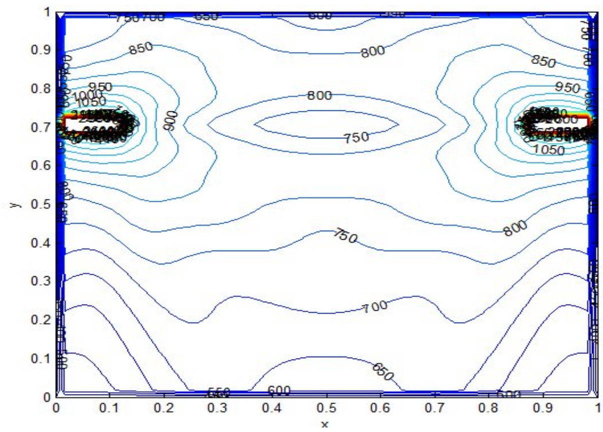
The isotherms inside the enclosure are plotted in Fig. 3 when two heat sources are considered to be attached on the left and right walls.



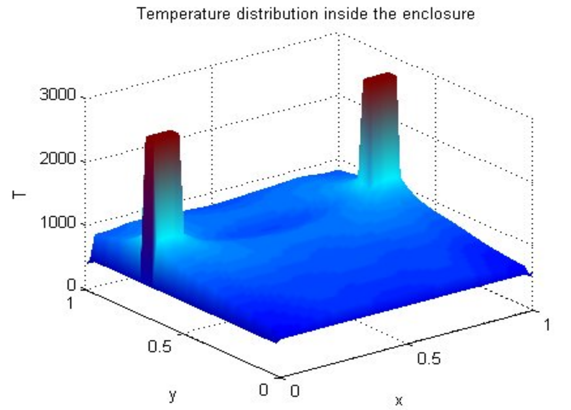
(a-1)



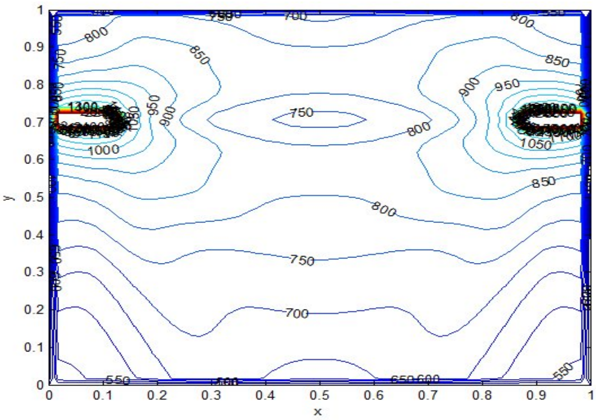
(a-2)



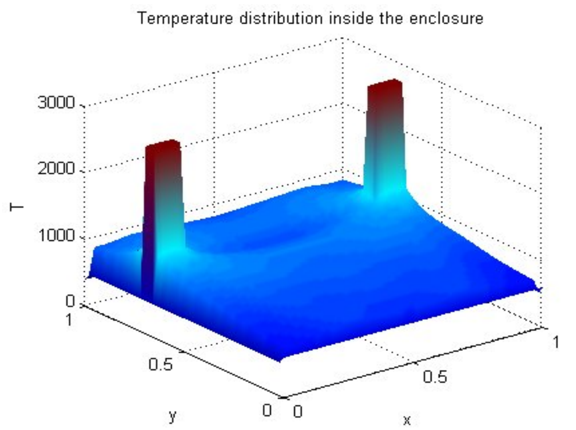
(b-1)



(b-2)



(c-1)



(c-2)

Figure 3. Effect of RC on the isotherm pattern, $\tau = 1, \omega = 0.2, k_a = 7$ and $q_{heat\,source} = 10^6\ (W/m^3)$, (a) $RC = 1$, (b) $RC = 10$, (c) $RC = 250$

The temperature distribution inside the furnace can be observed via these isotherms such that there are two hot regions near the heat source zone, after which the gas temperature decreases near the walls. The effect of RC on temperature distribution is also studied in Fig. 3 by considering three different values of RC equal to 1, 10, and 250. For high values of RC in which the radiation heat transfer is dominant, more or less uniform intense heating of core is observed. At the small values of RC in which most of heat transfer takes place by conduction, isotherms become equally spaced because of low effect of radiation in the mechanism of heat transfer.

The heat flux distributions along the bottom, top, left and right walls are plotted in Figs. 4-6 in which four different values of RC were used in the computations. It should be noted that because of symmetry, the heat flux distributions on the left and right walls are identical. It is seen that there is a non-uniform heat flux distribution on the walls with considerable difference between minimum and maximum values. From these figures, it can be concluded that when the radiation process is dominate which is related to high values of RC, the rate of surface heat fluxes increases, which shows more heat transfer from the two sources into the surrounding walls.

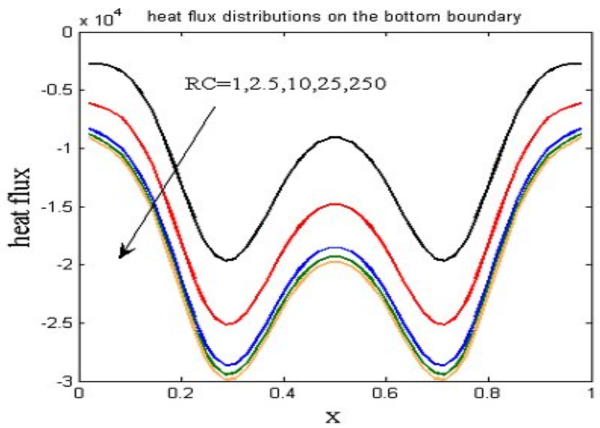


Figure 4. Effect of RC on the heat flux of bottom wall, $\tau = 1, \omega = 0.2, k_a = 7$ and $q_{heat\,source} = 10^6\ (W/m^3)$

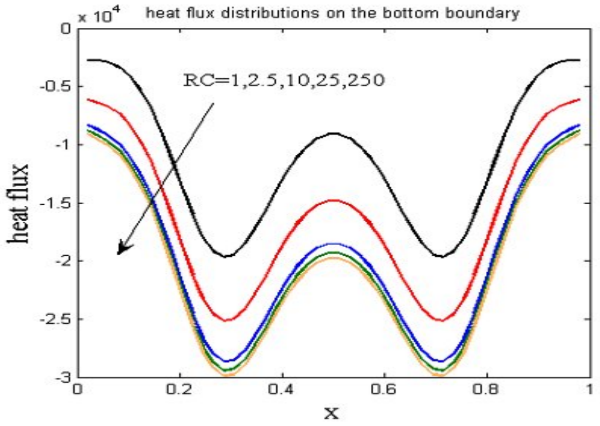


Figure 5. Effect of RC on the heat flux of top wall, $\tau = 1, \omega = 0.2, k_a = 7$ and $q_{heat\,source} = 10^6\,(W/m^3)$

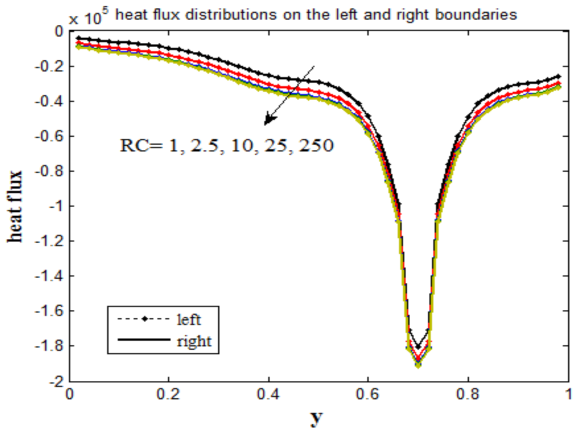


Figure 6. Effect of RC on the heat flux of left and right wall, $\tau = 1, \omega = 0.2, k_a = 7$ and $q_{heat\,source} = 10^6\,(W/m^3)$

4. CONCLUSION

The theoretical modeling aspects of coupled conductive and radiative heat transfer in the presence of absorbing, emitting and scattering gray medium within two-dimensional square furnace including two heat sources have been conducted. The set of governing equations are solved by numerical techniques. On the basis of the results, the following conclusions have been drawn:

- For high values of RC in which the radiation heat transfer is dominant, more or less uniform intense heating of core is observed. At the small values of RC in which most of heat transfer takes place by conduction, isotherms become equally spaced because of low effect of radiation in the mechanism of heat transfer.
- For high values of RC in which radiation is dominant, the values of gas temperature increases.
- When the radiation process is dominate which is related to high values of RC, the rate of surface heat fluxes increases,which shows that there is more heat transfer from the two sources into the surrounding walls.

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