SOME PERSPECTIVES OF MACHINE REPAIR PROBLEMS

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Abstract In this article, we survey machine repair problems (MRP) with an emphasis on historical developments of queuing models of practical importance. The survey proceeds historically, starting with developments in 1985, when the first published review on machine interference models appeared. We attempt to elaborate some basic MRP models of the real life congestion situations. The brief survey of some notable contributions done in the area of machining system has been provided. Many concepts have been incorporated while discussing the machine repair problems under consideration. The purpose of this article is to expose queuing theorists and applied probability analysts to the real time problems that arise in machine repair modeling. This survey may also be useful for machine repair modeling to the practitioners as well as to the researchers.

Keywords Machine Repair, Queue, Finite Population, Spares, Survey, Applications, Stochastic Analysis.

چکیده در این مقاله به بررسی مشکلات تعمیر ماشین (MPR) با تاکید بر پیشرفت تاریخی مدل های صف پرداخته شده است. این بررسی با مرور گذشته و با پیشرفت ها در سال ۱۹۸۵ شروع می شود ، زمانی که اولین مقاله مروری در مورد مدل های تداخل ماشین ها انتشاریافته است. ما سعی کرده ایم که بعضی از مدل های MRP پایه را برای موقعیت های شلوغ زندگی واقعی شرح دهیم. خلاصه ای از مطالعه ها در مورد کمک های قابل توجه انجام شده در سیستم های ماشینی در این مقاله تهیه شده است. مفاهیم زیادی، هنگام بحث در مورد مشکلات تعمیر ماشین تحت بررسی با هم ترکیب می شوند. هدف این مقاله، نشان دادن تئوری های صف و تحلیل احتمالات کاربردی برای مشکلات واقعی زمان است که از مدل سازی تعمیر ماشین ناشی می شود. همچنین، این بررسی برای مدل سازی تعمیر ماشین برای شاغل ها و محققان مفید است.

1. INTRODUCTION

Machine repair problems studied via queuing theory can be thought equivalent to a finite population model. Machines are integral part of all industries and also subject to failure, which result into interference. It is well realized that the manufacturer's aim is to increase the quality and minimize the cost of production in several industries, while working in machining environments. The operation of any machining system may be stopped due to the failure of the machines and in such a situation; the repair facility should be available to repair of the failed machines. These situations occur in almost all areas such as computer networks, communication systems, production systems, transportation systems, flexible manufacturing systems, etc.

The importance of allowing the repair of failed machines in machining systems is obvious while considering systems with redundant machines. If repair of failed machine is possible without affecting the overall system operation, then it is desirable to know that how much are the chances of returning this machine to either operation or an operable state before the complete system failure. Consequently, we need some additional measures of system effectiveness that should be taken into account for smooth running of a machining system.

When one repairman is assigned to two or more failed machines for repair, there is a possibility of interference depending on the ratio of the man time to repair time. If the number of the machines is increased, the probability of the interference with the normal cycle is also increased. The ratio of the man time to machine time or the variation of the number of machines only affects the degree of interference. A machine interference problem is said to be Markovian, if the inter-arrival time or service time are both exponential.

Machine interference problem is the best example of the finite source queuing system where machines fail at irregular interval and are sent for repairing to repair facility having one or more repairmen. When all repairmen are busy and the system is failed, then production stops. Whenever a machine fails, it results in a loss to the organization in terms of production consequently in money. An interruption in the operation of a device can lead not only to the deterioration of the quality of manufacturing production but also increases the cost of production. The schematic diagram of multi-server machine repair model is shown in Figure 1.

With the growing complexity of equipment and components of various machining systems, it is essential to understand the role of redundancy and spare part provisioning. The system designer is always brought face-to-face with the problem for trading of reliability, productability, flexibility etc. In many machine repair systems such as in automobiles, airp[illegible]ters, and air-conditioners, it is desirable to predict how many spares and how many repairmen are required in order to ensure high reliability while minimizing the total cost.

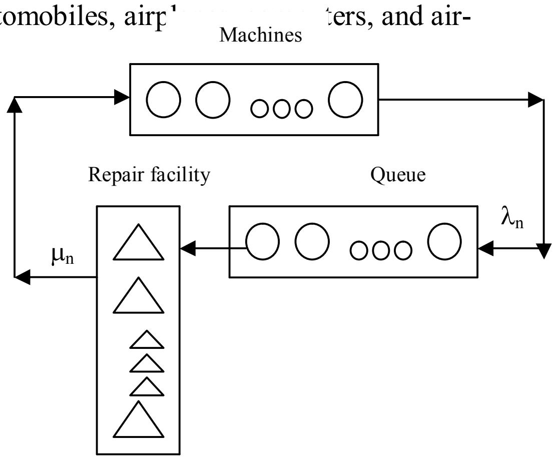

Figure 1. Machine repair problem

In the rest of this survey paper, we will precede more or less historically. In section 2, we briefly explain machine repair model without or with spares and provide some important contributions of MRP and MRP with spare, MRP with state dependent rates, MRP with additional repairmen and MRP with multi-modes of failure. The review of MRP with no passing constraint is presented in section 3. Section 4 provides the noble features and overview of MRP with time-sharing. The MRP with discouragement is outlined in the section 5. In section 6, we provide the review of queue with unreliable server. Section 7 is devoted to the developments of K-out-of-N: G system. Finally, conclusion is drawn in section 8.

2. MACHINE REPAIR MODELS

A typical application of finite source queuing model is that of machine repair, where the calling population is machines. An arrival corresponds to a machine breakdown, and the repair crews are the repairmen. We consider a machine repair problem wherein M operating machines and R repairmen are available and the repair times are identical exponential distributed random variables with mean $1/\mu$. The arrival process is described as follows. If a calling unit is not in the system at time t, the probability it will have entered by time $t+\Delta t$ is $\lambda\Delta t+0(\Delta t)$; that is, the time a calling unit spends outside the system is exponential with mean $1/\lambda$. The repair of failed machines is rendered according to FCFS fashion. After repair, the machine works as good as new one. Under these assumptions, the system can be easily modeled as M/M/R queue with finite source. Let p_n be the steady state probability that there are n failed machines in the system. These state probabilities can be further used to derive important measures of system performance.

The failure and repair rates λ_n and μ_n respectively are given by

$$\lambda_n = \begin{cases} (M-n)\lambda, & 0 \le n < M \\ 0, & n = M \end{cases} \quad (1)$$

and

$$\mu_n = \begin{cases} n\mu, & 0 \le n < R \\ R\mu, & R \le n \le M \end{cases} \quad (2)$$

The steady state probabilities are obtained by solving the Chapman-Kolmogorov equations which govern the model using above transition rates in (1) and (2). The product type solution can be obtained as follows:

$$p_n = \begin{cases} \binom{M}{n} r^n p_0, & 1 \le n < R \\ \binom{M}{n} \frac{n!}{R^{n-R} R!} r^n p_0, & R \le n \le M \end{cases} \quad (3)$$

where,

$$r = \frac{\lambda}{\mu}$$

In many fast growing industries, the operations of the machining system may be interrupted due to failure of machines involved in the system. Therefore, the service facility is to be so adjusted such that the machines are repaired instantly and continue the operation without much delay. The factors affecting the machining systems are running times of a machine before breakdown; machine's waiting time for repair until the repair of the other failed machines is completed, etc. In the case of several repairmen, if the machines are failed, repairmen repair these failed machines and the excess number of machines beyond the number of repairmen wait until repairmen are available. This affects the production and results into interference loss.

The system performance for the machining systems can be predicted using queuing theory. Most of the queuing systems have stochastic elements. These measures are often random variables and are obtained using and probability distributions of queue size, waiting time and busy period. Some measures of performance of interest for M/M/R/M/M model are calculated as follows:

Average number of failed machines in the system is

$$L_s = \sum_{n=0}^{M} nP_n$$

Expected number of failed machines in the queue waiting for repair is

$$L_q = \sum_{n=R}^{M} (n - R)P_n$$

Expected waiting time in the system is

$$W_s = \frac{L_s}{\lambda_{eff}}$$

where $\lambda_{eff} = \lambda(M - L_s)$.

Expected waiting time in the queue is

$$W_q = \frac{L_q}{\lambda}$$

Expected number of machines being repaired is

$$M_s = L_s - L_q = \sum_{n=0}^{R-1} nP_n + R\sum_{n=R}^{M} P_n$$

Average server utilization is

$$U = \frac{M_s}{R} = \frac{1}{R}\left(\sum_{n=0}^{R-1} nP_n\right) + \sum_{n=R}^{M} P_n$$

Related literature Sufficient works have been done on the performance analysis of machine repair problems. Now, we provide a brief survey of the literature in this area. Palm [1] considered the single server machine interference problem with Poisson input and exponential service time distribution. Phipps [2], Naor [3] and Benson [4] studied machine repair problems and suggested various measures of prevention. Morse [5] analyzed some machine repair problems with a cost proportion to the rate and machine probability dependent on the machine uptime. Jaiswal and Thiruvengadem [6] explored the busy period distribution of a machine repair model. Nakagawa and Osaki [7] considered the busy period of a repairman in a system having unreliable units. The numerical solution for $M/E_k/1$ machine interference model was provided by Maritas and Xirokastas [8].

Elsayed [9] discussed an optimum repair policy for the machine interference problem. Jain and Sharma [10] employed the semi numerical iterative method for solving two machine interference problems. Albright and Soni [11] studied the machine repair problem where failures may be irreparable. They derived computationally tractable formulae for the steady state probabilities, the long run average cost per unit time and the expected discounted cost. Stecke and Aranson [12] presented a review on the machine interference problem having different standbys. Wang [13] developed a machine repair model with single repair facility. A machine repair model of finite capacity assuming delayed repairs was investigated by Jain and Singh [14].

Jiang et al. [15] considered an optimal repair/replacement policy for a general machine repair model. Armstrong [16] investigated a machine repair problem with the perspective of designing optimal age-repair policies. Jain et al. [17] discussed a machine repair system with spares, state dependent rates and having the provision of additional repairmen. Kenne and Gharbi [18] presented the optimal flow control for a one-machine, two-product manufacturing system subject to random failures and repairs. Li et al. [19] investigated the case of process delays in assembly for reasons such as machine breakdowns, shortages of materials, etc. Ke and Wang [20] studied vacation policies for machine repair problem in co-operative with two types of spares.

Now, we discuss some machine repair models including spares, state dependent rates, additional removable repairmen, multi-mode failure, etc.

2.1 MRP with Spares The manufacturing/ production system may not operate with full capacity during the period of break down and this may lead to the loss of production. However, the organization or industry may avoid any loss of production with a proper combination of spare part support and repair facility. Spare machines may be either cold standby, or warm standby, or hot standby as defined below. A standby machine is said to be cold standby when its failure rate is zero. It is said to be warm standby when its failure rate is non-zero, but less than the failure rate of an operating machine. In case of hot standby, its failure rate is equal to that of an operating machine.

Machine repair can be generalized to include the use of spares. We assume that system consists of M operating and Y cold spares, so when a machine fails, a spare is immediately substituted for it. If all spares are used and a breakdown occurs, then the system becomes short. When a machine is repaired, it becomes a spare (unless the system is short, in which case the repaired machine goes immediately into service). For this model, the state dependent failure rate and repair rate are given by

$$\lambda_n = \begin{cases} M\lambda, & 0 \le n < Y \\ (M-n+Y)\lambda, & Y \le n < Y+M \\ 0, & n \ge Y+M \end{cases} \tag{4}$$

and

$$\mu_n = \begin{cases} n\mu, & 0 \le n < R \\ R\mu, & R \le n \le M+Y \end{cases} \tag{5}$$

For $R \le Y,$ we obtain steady state queue size distribution as follows:

$$p_n = \begin{cases} \dfrac{M^n}{n!} r^n p_0, & 0 \le n < R \\ \dfrac{M^n}{R^{n-R}\, R!} r^n p_0, & R \le n < Y \\ \dfrac{M^Y}{(M-n+Y)!\, R^{n-R}\, R!} r^n p_0, & Y \le n \le Y+M \end{cases} \tag{6}$$

When R>Y, we have

$$p_n = \begin{cases} \dfrac{M^n}{n!} r^n p_0, & 0 \le n < Y \\ \dfrac{M^Y M!}{(M-n+Y)!\, R!} r^n p_0, & Y+1 \le n < R \\ \dfrac{M^Y M!}{(M-n+Y)!\, R^{n-R}\, R!} r^n p_0, & R \le n \le Y+M \end{cases} \tag{7}$$

The expected number of spares in the system is obtained by

$$E[S] = Y\sum_{n=0}^{Y} P_n + \sum_{n=Y+1}^{Y+M} (M-n+Y)P_n \tag{8}$$

Related Literature Now, we give a brief review of the important research works done in the area of machine repair problems with spares. Taylor and Jackson [21] first studied the Markovian machine repair problem with spares. An M/M/C/M/M machine repair problem with standby units was investigated by Gross and Harris [22]. Economic analysis of M/M/R machine repair problem with warm standbys was made by Sivazlian and Wang [23].

Singh et al. [24] gave the profit analysis of a two unit standby system having two types of independent repair facilities (i) cheaper and (ii) costlier. A profit model was developed by Wang [25] in order to determine the optimal values of the number of servers and spares for M/M/R machine repair problem. Sridharan and Mohanavadivu [26] developed a model for a two units-identical system with one operating unit and the other warm standby unit in a human and machine system. Zhang [27] addressed an optimal geometric process model for a cold standby repairable system.

Teixeirade [28] presented multi-criteria decision models for two maintenance problems in which one is based on repair contract selection and other one on spares provisioning. Sharma et al. [29] worked on machine repair problem with balking and reneging. They gave a cost function to find out the optimal number of spares and repairmen. More recently, Jain and Saxena [30] studied performance analysis of state dependent machining system with mixed standby. Wang et al. [31] discussed a profit analysis of M/M/R machine repair problem with balking, reneging and standby switching failures.

2.2 MRP with State-dependent Rates In machine repair system, sometimes we treat queues with state dependent rates; in such systems the failure and repair rates depend on the state of the system. The server may speed up on seeing a long queue. On the other hand, it may happen if the server is inexperienced. Then, it becomes flustered and the mean service rate actually decreases as the system becomes more congested.

State-dependent queues play important role and have wide applicability in real life congestion situations arising in production/manufacturing systems, computer/ communication systems, etc. There is a great deal of literature concerning the state dependent machine repair model.

The state dependent rates for machine repair problems with spares are given by

$$\lambda_n = \begin{cases} M\lambda_0, & 0 \le n < Y \\ (M-n+Y)\lambda_1, & Y \le n < Y+M \\ 0, & n \ge Y+M \end{cases} \tag{9}$$

and

$$\mu_n = \begin{cases} n\mu_0, & 0 \le n < R \\ R\mu_1, & n \ge R \end{cases} \tag{10}$$

Related Literature The important contributions in the area of state dependent rates are due to Yechaili and Naor [32] who studied a queuing problem with heterogeneous arrival and service rates. The steady state behavior of processing system having Erlang service and Markovian state dependent arrivals was studied by Conolly [33]. Morrison [34] calculated the sojourn time and waiting time for a state dependent queue.

Sztrik [35] studied the G/M/r/FIFO machine interference model with state dependent speeds. Again, Sztrik [36] modified G/M/r/SIRO queue for machine interference model with state dependent speeds. Jain [37] employed diffusion approximation technique to study the (m, M) machine repair problem with spares and state dependent rates. Jain and Baghel [38] studied machine repair problem with spare part support and state-dependent rates. Cheng et al. [39] discovered single machine batch scheduling with resource dependent set up and processing times. Jain and Shekhar [40] provided transient analysis of state-dependent multiprocessor system. Jain and Mishra [41] studied multistage degraded machining system with common cause shock failure and state dependent rates.

2.3 MRP with Additional Removable Repairmen The provision of additional removable repairmen in case of long queue may be helpful in reducing the waiting time as well as balking behavior of the care taker of failed machines in many congestion situations. For economic feasibility of the model, additional repairmen can be removed in case of low traffic.

We consider the state dependent arrival rate for finite population M/M/R model with r additional repairmen as follows:

$$\lambda_n = \begin{cases} (M-n)\lambda, & 0 \le n < M \\ 0, & otherwise \end{cases} \quad (11)$$

The state dependent service rate is given by

$$\mu_n = \begin{cases} \mu\, n, & 1 \le n \le R \\ \mu R, & R < n \le k \\ \mu(R+j), & jk < n \le (j+1)k \\ \mu(R+r), & jr < n \le M \end{cases} \quad (12)$$

Related Literature Machining systems with additional repairmen have been studied by many researchers and have been applied to many practical situations. Makaddis and Zaki [42] studied the M/M/1 queue with additional servers. Shawky [43] examined the single server machine interference model with balking, reneging and an additional server for longer queues. Jain [44] considered an M/M/m queue with discouragement and additional servers. Jain et al. [45] investigated M/M/C/K/N machine repair problem with balking, reneging, spares and additional repairman. Al-Seedy and Al-Ibraheem [46] discussed an inter arrival hyper-exponential machine interference model with balking, reneging, state dependent rates, spares and an additional server for longer queues. Jain and Maheshwari [47] gave a transient analysis of redundant repairable system with additional repairmen. Jain and Singh [48] analyzed a multi-server queuing model with discouragement and additional servers. Sharma et al. [49] investigated loss and delay multi-server queuing model with discouragement and additional repairmen.

2.4 Multi-modes Failure Machine repair problems have focused mainly for single mode of failure. However, some attempts have also been made to investigate the two modes and multi-modes failure models as well.

Related Literature A few researchers have studied various machine repair problems for multi-modes of failure. Some of them considered two-mode failure models. Goyal and Sharma [50] gave the stochastic analysis of two unit standby system with two failure modes. Pham and Pham [51] considered optimal designs of (k, n-k+1)-out-of-n: F-systems subject to two failure modes. Reddy and Rao [52] obtained the optimization of parallel system subject to two modes of failure and repair provision. The cost analysis of the M/M/R machine repair problem with two modes of failure was provided by Wang and Wu [53]. Moustafa [54] considered Markov models for transient analysis of reliability for the system, which is subject to two failure modes. Sharma and Sharma [55] considered M/M/R machine repair problem with spares and three modes of failure. Again, Moustafa [54] extended it for multi-modes. Wang and Lee [56] developed the cold-standby M/M/R machine repair problem where a group of identical and independent operating machines have $K(K \ge 1)$ failure modes, and are maintained by one or more repairmen in the repair facilities. Jain et al. [57] considered machine repair problem with additional repairmen and two modes of failure. M/M/R machine interference model with balking, reneging, spares and two modes of failure was considered by Jain et al. [58]. The developed model generalizes the linear consecutive k-out-of-r-from-n system to the case of multiple failure criteria. Assessment of reversible multi-state k-out-of-n: G/F system was discussed by incorporating load sharing. Jain et al. [59] discussed performance modeling of state dependent system with mixed spares and two modes of failures.

3. QUEUE WITH NO PASSING RESTRICTION

In many critical applications of computer, communication, manufacturing and production systems, the provision of no passing restriction may be proved as an important architectural attribute for machining system. Due to no passing restriction, the machines are allowed to depart from the system in the same sequential order in which they join the system. The worth nothing aspects of no passing restriction in queuing models are:

The jobs are classified into two categories due to restriction of no passing. The type A jobs have zero service time and type B jobs have exponential service time. All the jobs leave the system in the chronological order in which they arrive.
Let p be the probability that a job is of type B

while (1-p) be the probability that a job is type A. Thus, the cumulative distribution function for a tagged job is given by

$$F(x) = (1-p) + p(1-e^{-\mu_n x}) \quad for \;\; x \ge 0, 0 < p \le 1 \qquad (13)$$

Let the expected waiting time of type A jobs and type B jobs be $E(W_A)$ and $E(W_B)$, respectively. The expected waiting time E(W) for a tagged job is given by

$$E(W) = (1-p)E(W_A) + pE(W_B) \qquad (14)$$

Related Literature Many mathematicians have contributed significantly towards queuing systems with no passing restriction. Washburn [60] introduced the concepts of no passing, which also has practical utility in machining systems. Sharma et al. [61] studied a limited multi-server queuing model with no passing restriction. Jain et al. [62] analyzed multi-server queuing model with discouragement and two types of customers, under restriction of no passing. They obtained some numerical results to facilitate the comparison between systems with constant and state dependent arrivals rates. Jain [63] considered no passing queue with finite capacity and finite population. Jain and Ghimire [64] incorporated additional servers for M/M/m/K queue with no passing. Jain and Singh [65] analyzed the effect of implementing additional service positions in case of no passing time-sharing queuing model. Jain and Singh [66] provided various performance indices for Markovian loss and delay queuing model with no passing and additional removable servers. Jain and Agrawal [67] discussed loss and delay time-shared model under no passing constraint for machining system with spares and additional service positions.

4. TIME SHARING MODELS

The time-sharing concept can play significant role to regulate the service provided by machines and is vital importance in some critical applications of computer, communication, automated manufacturing systems, etc.

We address the formulation aspects of time-sharing finite population queue under the restriction of no passing and with the provision of additional service position by stating the underlying assumptions and notations as follows:

The jobs arrive at a single processor in Poisson manner with mean arrival rate1/λ.

The processing time requirements of each job are exponentially distributed with mean service rate 1/μ.

The single processor services the jobs with different rates depending upon the number of jobs present in the system.

The numbers of service positions are the available space for service depending upon the number of jobs present in the system in the following manner:

When there are less than k jobs, only a service positions are available for the jobs.

When there are greater than jk and less than or equal to (j+1)k jobs, then (R+j)(j=1,2,..,r-1) service positions are available for providing service. If the number of jobs drops to jk-1, then j^{th} (j=1, 2, …, r-1) additional service position is removed from the system. Therefore, one additional service position is removed with decrease in queue length by k jobs.

If there are greater than rk jobs in the system, then all the (R+r) service positions are available.

We also use the following notations for mathematical model formulation:

$\phi(n)$ service effect for sharing jobs at a time.

$\Phi(n)$ $\prod_{i=1}^{n} \phi(i), \quad 0 < n \le R$

R(r) number of permanent (additional) repairmen

$$p_n = \begin{cases} \dfrac{M!\,\rho^n}{(M-n)!\,\Phi(n)}\,p_0, & 1 \le n \le R \\ \dfrac{M!\,\rho^R}{(M-n)!\,\Phi(R)\{\phi(R)\}^{n-R}}\,p_0, & R < n \le k \\ \dfrac{M!\,\rho^R}{(M-n)!\,\Phi(R)\{\phi(R)\}^{k-R}\prod_{i=1}^{j-1}\{\phi(R+i)\}^k\{\phi(R+j)\}^{n-jk}}\,p_0, & jk < n \le (j+1)k \\ \dfrac{M!\,\rho^R}{(M-n)!\,\Phi(R)\{\phi(R)\}^{k-R}\prod_{i=1}^{r}\{\phi(R+i)\}^k\{\phi(R+j)\}^{n-rk}}\,p_0, & rk < n \le M \end{cases} \tag{17}$$

$$\begin{aligned} \mu E(W_A) = \Bigg[& \sum_{n=0}^{R} a_n \frac{M!\,\rho^n}{(M-n)!\,\phi(n)} + \frac{M!\,\rho^R}{(M-n)!\,\phi(R)\{\phi(R)\}^{n-R}} \sum_{n=R+1}^{k} \left\{ \frac{n-R+1}{R} + a_{R-1} \right\} \\ & + \frac{M!\,\rho^R}{(M-n)!\,\phi(R)\{\phi(R)\}^{k-R}\prod_{i=1}^{j-1}\{\phi(R+i)\}^k\{\phi(R+j)\}^{n-jk}} \sum_{j=1}^{r-1} \sum_{n=jk+1}^{(j+1)k} \left\{ \frac{n-(R+j)+1}{R+j} + a_{R+j-1} \right\} \\ & + \frac{M!\,\rho^R}{(M-n)!\,\phi(R)\{\phi(R)\}^{k-R}\prod_{i=1}^{r}\{\phi(R+i)\}^k\{\phi(R+j)\}^{n-rk}} \sum_{n=rk+1}^{M} \left\{ \frac{n-(R+r)+1}{R+r} + a_{R+r-1} \right\} \Bigg] p_0 \end{aligned} \tag{18}$$

$$\begin{aligned} \mu E(W_B) = \Bigg[& \sum_{n=0}^{R} a_{n+1} \frac{M!\,\rho^n}{(M-n)!\,\phi(n)} + \frac{M!\,\rho^R}{(M-n)!\,\phi(R)\{\phi(R)\}^{n-R}} \sum_{n=R+1}^{k} \left\{ \frac{n-R+1}{R} + a_{R} \right\} \\ & + \frac{M!\,\rho^R}{(M-n)!\,\phi(R)\{\phi(R)\}^{k-R}\prod_{i=1}^{j-1}\{\phi(R+i)\}^k\{\phi(R+j)\}^{n-jk}} \sum_{j=1}^{r-1} \sum_{n=jk+1}^{(j+1)k} \left\{ \frac{n-(R+j)+1}{R+j} + a_{R+j} \right\} \\ & + \frac{M!\,\rho^R}{(M-n)!\,\phi(R)\{\phi(R)\}^{k-R}\prod_{i=1}^{r}\{\phi(R+i)\}^k\{\phi(R+j)\}^{n-rk}} \sum_{n=rk+1}^{M} \left\{ \frac{n-(R+r)+1}{R+r} + a_{R+r} \right\} \Bigg] p_0 \end{aligned} \tag{19}$$

$\rho = \dfrac{\lambda p}{\mu}$ traffic intensity

To determine the steady state queue size distribution for machine repair model, we consider the state dependent failure and repair rates by

$$\lambda_n = \begin{cases} (M-n)\lambda, & 0 \le n < M \\ 0, & otherwise \end{cases} \tag{15}$$

and

$$\mu_n = \begin{cases} \mu\phi(n), & 1 \le n \le R \\ \mu\phi(R), & R < n \le k \\ \mu\phi(R+j), & jk < n \le (j+1)k \\ \mu\phi(R+r), & rj < n \le M \end{cases} \tag{16}$$

Now the steady state queue size distribution p_n is given by (cf. Jain et al. [68])

Related Literature The earlier published analysis of queuing models of a time-sharing system can be found in Kleinrock [69] and numerous papers referred therein. A mathematical idealization of round robin scheduling called processor sharing in which quantum size shrinks to zero was introduced by Kleinrock [70]. The response time for M/M/1 time-sharing queues with limited number of service positions was studied by Avi-Izhak and Halfin [71]. Avi-Izhak [72] suggested approximation for the moments of response time in the time-sharing queues. Jain and Premlata [73] considered a time-sharing queue with a limited number of service positions and limited number of waiting space. Sharma et al. [29] derived numerical solution of processor-sharing queuing model. Many researchers have paid considerable attention to study the machine repair problem with time-sharing. Scherr [74] developed a finite source model of a time-sharing system. He used this model to analyze the Compatible Time Sharing System (CTSS) that was developed at MIT by Project MAC. Jain and Premlata [75]) examined accumulated work process in a time-sharing queue with finite population. Jain et al. [76] incorporated additional service positions to analyze loss and delay queuing model for time-shared system with no passing restriction. Singh et al. [77] discussed no passing M/M/ϕ(.) time-sharing queuing system. Jain et al. [78] applied optimal control for the finite capacity Markovian queuing network for the time-sharing.

5. MRP WITH DISCOURAGEMENT

Customers are said to be impatient if they tend to join the queue only when a short wait is expected and tend to remain in line if the wait are sufficiently small. The impatience results due to an excessive wait is just as important as the manager of the enterprise. The enterprise must take action to reduce the congestion to the levels that customers can tolerate.

Discouragement generally takes three forms. The first is *balking* which is due to the reluctance of a customer to join a queue upon arrival. The second one is *reneging* and is due to reluctance to remain in line after joining and waiting for some time. The third one is ***jockeying*** between lines when each of a number of parallel lines has its own queue.

Now, we describe the state dependent M/M/R/M/M machining system with balking, reneging and spares as follows:

There are M operating and Y cold spare units in the system. The life times of the units are exponentially distributed with mean rate λ. Also the repair times are identical exponentially distribution random variables. There is a provision of Y cold spares in the system, so that when a unit fails, a spare is immediately substituted for it. When the repairing of a failed unit is completed, it joins standby group and is as well as new one. Let us assume that the failed units may balk (β be the joining probability) and renege exponentially (γ being the reneging rate).

The failure and repair rates are given by

$$\lambda_n = \begin{cases} M\lambda, & 0 \le n < R \\ M\lambda\beta, & R \le n \le Y \\ (M+Y-n)\beta\lambda, & Y < n < M+Y \end{cases} \quad (20)$$

$$\mu_n = \begin{cases} n\mu, & 0 \le n < R \\ R\mu + (n-R)\gamma, & R \le n \le M+Y \end{cases} \quad (21)$$

To obtain steady state queue size distribution, the product type solution can be employed.

Relevant Literature Some investigators have developed machine repair models with discouragements. Gupta [79] studied interrelationship between queuing models with balking, reneging and machine repair problem with warm spares. Al Seedy [80] treated the truncated Poissonian M/M/2/M/M+Y machine interference queue with balking concept, heterogeneous servers, spares and an additional server for longer queues. Ke and Wang [81] studied cost analysis of M/M/R machine repair problem with balking, reneging and server breakdown. Shawky [82] investigated an interarrival hyper exponential machine interference model with balking and reneging. Ke and Wang [83] developed a repairable system with warm standbys, balking and reneging. They obtained the reliability characteristics viz. the system reliability, mean time to failure, etc. They obtained queue size distribution in explicit form. The expected cost is also provided to determine the allocation of repairmen and spares. Jain et al. [84] investigated the queue size distribution of G/G/R machining system with cold standbys via a diffusion approximation technique. The balking and reneging behavior of the failed machines are incorporated.

6. UNRELIABLE SERVER MODEL

In many queuing systems the server is subject to breakdown. If the server is not repaired, the interruption of service continues. It is essential to see how the level of performance of the system can be maintained by providing proper maintenance facility.

We assume that the customers arrive from a Poisson stream with rate λ. Also, the service requests are exponentially distributed with parameter μ, and hence have a mean service time of $1/\mu$. The single server is subjected to breakdown with rate γ. The repair rate of server is η, which is exponentially distributed with mean length $1/\eta$. Both breakdown and repair rates which are independent to the number of jobs presented in the system.

The state of the system can be represented by a pair of integer, (i, j), with i=0,1 representing the number of operating server and breakdown server, respectively and j representing the number of jobs present in the system. Assuming that steady state conditions will be reached, we denote the probability of state (i, j) by $p_{i,j}$.

The steady state equations are constructed with the help of transition rate diagram (see Figure 2) as follows:

$$(\lambda+\eta)\,p_{0,0} = \gamma\; p_{1,0} \tag{22}$$

$$(\lambda+\eta)\,p_{0,1} = \gamma\; p_{1,1} + \lambda p_{0,0} \tag{23}$$

$$(\lambda+\eta)\,p_{0,j} = \gamma\; p_{1,j} + \lambda\; p_{0,j-1}, \quad j = 2,3,... \tag{24}$$

$$(\lambda+\gamma)\,p_{1,0} = \mu\; p_{1,1} + \eta\; p_{0,0} \tag{25}$$

$$(\lambda+\gamma+\mu)\,p_{1,1} = \lambda\;\; p_{1,0} + \mu\; p_{1,2} + \eta\; p_{0,1} \tag{26}$$

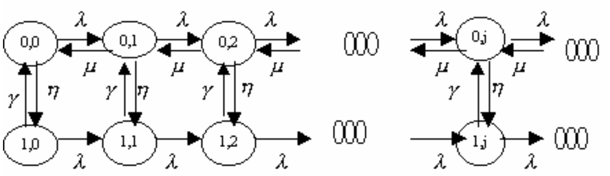

Figure 2. Transition rate diagram

$$(\lambda+\gamma+\mu)\,p_{1,j} = \lambda\;\; p_{1,j-1} + \mu\; p_{1,j+1} + \eta\; p_{0,j}, \quad j = 2,3... \tag{27}$$

We define two generating functions $P_0(z)$ and $P_1(z)$ as

$$P_i(z) = \sum_{j=0}^{\infty} p_{i,j} z^j \quad for\ i = 0,1 \quad . \tag{28}$$

Multiplying the equations (20)-(25) by appropriate powers of z, summing and using normalization condition, we get

$$P(z) = P_0(z) + P_1(z) = \frac{[(\mu-\lambda)\eta-\lambda\gamma]\,[\gamma+\lambda(1-z)+\eta]}{(\gamma+\eta)\,\{(\lambda(1-z)+\eta)(\mu-\lambda z-\gamma\lambda z)\}} \tag{29}$$

The mean number of the customers in the system, independent of the state of the server, can now be easily obtained by

$$E(N) = \lim_{z\to 1}\left[\frac{dP(z)}{dz}\right] = \frac{\lambda\,[(\gamma+\eta)^2+\mu\gamma]}{(\gamma+\eta)[\eta(\mu-\lambda)-\lambda\gamma]} \tag{30}$$

Related Literature In the direction of unreliable server queue, many researchers have studied the queuing models with interruption. Numerous papers have been published on queuing models with server breakdown and vacation. In past, Graver [85] , Lee [86], Sztrik and Gal [87], Hsieh and Andersland [88], Tang [89] and many others, have studied the Markovian queuing models with server breakdown and vacation. Shogan [90] analyzed a single server queue with arrival rate dependent on server breakdown. An optimal splitting of the input stream in queuing model in which the server is subjected to breakdown was studied by Mitrani and Wright [91]. Wartenhorst [92] analyzed a queuing model for n-parallel servers with breakdowns and repairs. Wang et al. [93] considered a model with single removable and non-reliable server in both infinite and finite $M/H_2/1$ queuing systems. Perry and Posner [94] considered a single machine system, subjected to breakdown that produces items to inventory, continuously and uniformly. Grey et al. [95] described a vacation queuing model with server breakdown. Masuyama and Takine [96] considered a FIFO single server queue with service interruption and multiple batches Markovian arrival streams. Ke and Pern [97] analyzed optimal management policy for heterogeneous arrival queuing systems with server breakdowns and vacations. Jain et al. [98] studied bi-level control of degraded machining system with warm standbys, setup and vacation. Almási et al. [99] considered homogeneous finite-source retrial queues with server subjected to breakdowns and repairs. Gharbi and Ioualalen [100] studied GSPN analysis of retrial systems with server's breakdowns and repairs. Ke and Lin [101] analyzed the $M^{[x]}/G/1$ queuing system with server vacations. Wang et al. [31] discussed maximum entropy analysis of the $M^{[x]}/M/1$ queuing system with multiple vacations and server breakdowns. Charlot et al. [102] considered a production control problem for a manufacturing system subjected to random failures and repairs.

7. K-OUT-OF-N: G SYSTEM

In a K-out-of-N system, (N-K) machines are redundant machines, which are used for the purpose of improving the system reliability. Sometimes such systems are known as parallel-redundant system; the K machines are known as basic machines whose survival is for the successful operation of the system. The 1-out-of-4 machining system is shown in Figure 3.

The K-out-of-N system can be modeled by continuous-time Morkov process. Consider a system consisting of N identical and independent components which may fail in M-modes.

Each of them is either up (good), or failed by mode-m with constant failure rate λ_m, m=1,2,..,M. There is a multiple repair facility having R repairmen. The time to repair j-failed component due to m^{th} failure mode is exponentially distributed with mean equal to $1/j\mu_m$. The repair is perfect i.e. after the repair; the component is as good as new one. The system is up if at least K components are up.

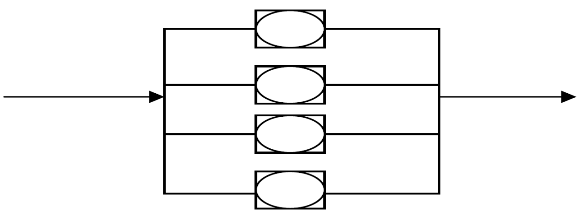

Figure 3. 1-out-of-4 Machining system

Let (je_m) be the state of the system representing the number of the failed components due to failure mode-m (j=0,1,2,…, N-K+1), where e_m is unit row of dimension M which has unity in the m^{th} position and zero every where else. Let $P_t(je_m)$ be the probability of being in this state at time t when the system states at time t=0 in the state (0).

The set of Kolmogorov's equations constructed are as follows.

(i) For j=0

$$\frac{dP_t(0)}{dt} = -[N\lambda]P_t(0) + \sum_{m=1}^{M} \mu_m P_t(e_m) \tag{31}$$

where $\lambda = \sum_{m=1}^{M} \lambda_m$, $\alpha = \sum_{m=1}^{M} \alpha_m$.

(ii) For state je_m $(1 \le j \le K-1)$.

$$\frac{dP_t(je_m)}{dt} = -[(N-j)\lambda_m + j\mu_m]P_t(je_m) + [N-(j-1)]\lambda_m P_t((j-1)e_m) + (j+1)\mu_m P_t((j+1)e_m) \tag{32}$$

(iii) **For j=(N-K+1)**

$$\frac{dP_t((N-K+1)e_m)}{dt} = K\lambda'_m \, P_t((N-K)e_m) \tag{33}$$

where the initial conditions are:

$$P_0(0) = 1 \quad and \quad P_0(je_m) = 0 \qquad for\ j > 0.$$

If μ_m=0 then, it is a case without repair and the probability can be obtained as

(i) For state 0

$$P_t(0) = e^{-(N\lambda)t} \tag{34}$$

(ii) For state je_m $(1 \le j \le K-1)$.

$$P_t(je_m) = M_j \left[\frac{e^{-Nt\lambda}}{\prod_{n=1}^{j} [(N-n)\lambda_m - N\lambda]} \right] + \left[\sum_{n=1}^{j} \frac{e^{-(N-n)\lambda_m t}}{(N\lambda - (N-n)\lambda_m) \prod_{\substack{p=1 \\ p \ne n}}^{j} [(N-p)\lambda_m - (N-n)\lambda_m]} \right] \tag{35}$$

The reliability R(t) and mean time to failure (MTTF) of the system with repair can be calculated using

$$R_t\,(with\,repair) = P_t(0) + \sum_{j=1}^{N-K} \sum_{m=1}^{M} P_t\,(je_m) \qquad (36)$$

and

$$MTTF = \int_0^{\infty} R_t\,(with\,repair)\,dt \qquad (37)$$

Related Literature Numerous works are reported in literature on the reliability analysis of such systems. Boland and Proschan [103] provided an interesting survey on the reliability of k-out-of-n systems. Barlow and Heidtmann [104] gave a procedure to compute the reliability of k-out-of-n system. Jain and Gopal [105] analyzed reliability of k-*l*-out of-n systems. Pham and Upadhyaya [106] considered the efficiency of computing the reliability of k-out-of-n system. Sarje and Prasad [107] provided an efficient non-recursive algorithm for computing the reliability of k-out-of-n systems. Bai et al. [108] studied redundancy optimization for k-out of-n system with common cause failures. Das and Wortman [109] discussed the performance of N machine centers of k-out-of-n: G type maintained by a single repairman. Coit and Lim [110] obtained system reliability optimization for k- out of-n sub systems. Li et al. [111] investigated k-out-of-n system.

Chakravarthy et al. [112] developed a k-out-of-n reliability system with an unreliability server and phase type repairs and services. A finite capacity interference model for common mode failure 1-out-of-2: G system was examined by Lewis [113]. Arulmozhi [114] developed a closed form equation for system reliability of an M-out of-N warm standby system with R repairmen. Frostig and Levikson [115] considered R-out-of-N systems with general distributed repair time and exponentially distributed lifetime and derived formulae for the expected cycle time and the availability of the system. Chen [116] developed a method for analyzing component reliability as well as system reliability of k-out-of-n systems with independent and identically distributed components. Ushakumari and Krishnamoorthy [117] developed k-out-of-n system with repair. Amari et al. [118] considered optimal design of k-out-of-n: G subsystems subjected to imperfect fault-coverage. Da Costa Bueno [119] considered minimal standby redundancy allocation in a *k*-out-of-*n*: F-system of dependent components. Barron et al. [120] analyzed an R-out-of-N repairable system. The system failed whenever the number of good components decreased from R to R-1. Zhang et al. [121] obtained availability and reliability of k-out-of–(M+N): G warm standbys systems. Reliability analysis of K-out-of-N: G machining systems with spares and imperfect coverage was studied by Jain et al. [77]. Da Costa Bueno and Do Carmo [122] studied active redundancy allocation for a k-out-of-n: F-system of dependent components. De Smidt-Destombes et al. [123] developed a k-out-of-N system under block replacement sharing limited spares and repair capacity.

8. CONCLUSION

We have provided the brief survey of some notable contributions done in the area of machining system. With growing need for better performance of machining systems, queue theoretic approach has under gone significant transformation over the past few decades. The machine repair problems are likely to be more complex in future and it is natural to pay attention towards new technologies and congestion problems arising out of machine schedules. To justify this, we have facilitated an overview of important contributions in the area of machine repair models in different frameworks.

The performance indices established for the various models provide an insight on the performance prediction issues of these systems. We have stressed on the need of standby units in various models for an efficient handling of the problem of breakdown of operating machines. In some cases, provision of removable additional repairmen has been recommended to control the congestion in the system. It would also be helpful in preventing the failed machines from being discouraged and leaving the system. Various aspects of the server including vacation and breakdown have treated to account for the situations of real time machining systems wherein servers are subject to breakdown. Also, some notable works devoted to the reliability analysis of machining systems have been discussed, which is needed of the time to achieve desired reliability and efficiency. In K-out of–N: G system, our interest mostly lies in surveying the availability/reliability issues of concerned machining systems.

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