

MATHEMATICAL MODELING AND PERFORMANCE OPTIMIZATION FOR THE DIGESTING SYSTEM OF A PAPER PLANT

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Abstract This paper deals with the mathematical modeling and performance optimization for the Digesting system of a Paper Plant using Genetic Algorithm. The Digesting system of a Paper Plant has four main subsystems, arranged in series and parallel. Considering exponential distribution for the probable failures and repairs, the mathematical formulation of the problem is done using probabilistic approach and differential equations are developed based on Markov birth-death process. Then, these equations are solved using normalizing conditions to determine the steady state availability of the Digesting system. The performance of each subsystem of the Digesting system in a Paper Plant has also been optimized using Genetic Algorithm. Therefore, the findings of the present paper will be highly useful to the plant management for the timely execution of proper maintenance decisions and hence to enhance the system performance.

Keywords: Performance Optimization, Digesting System, Genetic Algorithm

چکیده این مقاله با مدل سازی ریاضی و بهینه سازی عملکردی با استفاده از الگوریتم ژنتیک برای سیستم هضم یک کارخانه کاغذ کاربرد دارد. سیستم هضم یک کارخانه کاغذ دارای چهار زیرسیستم اصلی است که به صورت سری و موازی مرتب شده اند. توزیع توانی در نظر گرفته شده برای نقایص و تعمیرات احتمالی، فرمولاسیون ریاضی مسئله با استفاده از معادلات دیفرانسیلی و روش احتمال پذیری بر پایه فرایند تولد- مرگ مارکوف (Markov) توسعه یافته است. سپس، این معادلات با استفاده از شرایط نرمال سازی برای تعیین موجودیت حالت یکنواخت سیستم هضم، حل شده است. همچنین، عملکرد هر زیرسیستم از سیستم هضم در کارخانه کاغذ با استفاده از الگوریتم ژنتیک بهینه شده است. بنابراین، یافته های مقاله حاضر برای مدیریت کارخانه برای اجرا تصمیمهای در مورد نگهداری مناسب و در نتیجه افزایش عملکرد سیستم بسیار مفید خواهد بود.

1. INTRODUCTION

The paper industry comprises of large complex engineering systems arranged in

series, parallel or a combination of both the configurations. Some of these systems are chipping, cooking, washing, bleaching, screening, stock preparation and paper

making, etc. These systems are normally arranged in hybrid configuration. The important process of a paper industry, upon which the quality of paper depends, is the Digesting process. In the process of paper formation, the chips from storage are fed in to a digester. A chemical compound, Sodium Hydroxide (NaOH) is added to digester and the steam is forced from the bottom. The wooden chips are cooked for about 8 to 10 hrs

so that lignin and other fibers present in the wooden chips get dissolved. Then, these cooked chips are flown to blow tank in which black liquid is added for dilution. These cooked chips are passed through various systems like washing, bleaching, screening, stock preparation and paper making, etc. to convert into the paper. The schematic flow diagram of the Digesting system is shown in Figure 1.

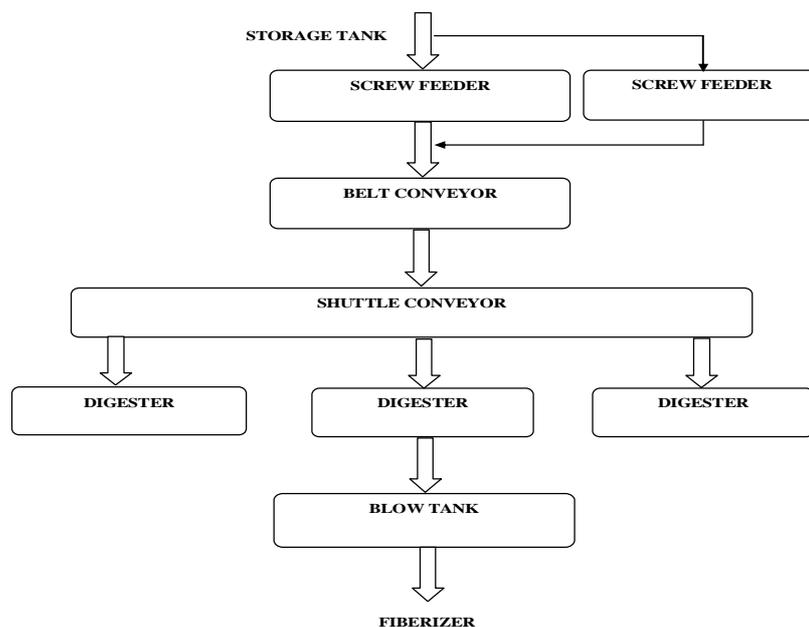


Figure 1. Schematic flow diagram of digesting system

1.1 Literature Review The available literature reflects that several approaches have been used to analyze the steady state behavior of various systems. Dhillon et al. [1] have frequently used the Markovian approach for the availability analysis, using exponential distribution for failure and repair times. Kumar, D. et al. [2, 3, 4] dealt with reliability, availability and operational behavior analysis for different systems in paper plant. Kumar et al. [2, 4] dealt with maintenance planning for the systems in fertilizer and thermal plants. Srinath [5] has proposed the mathematical models for the prediction of reliability and availability. Kalyanmoy Deb [6] has explained the optimization techniques and how they can be used in the engineering problems. Shooman [7] suggested different methods for the reliability computations of systems with

dependent failures. Sunand et al. [8] dealt with maintenance management for Ammonia Synthesis System in fertilizer plant. Singh and Jain [9] dealt with transient analysis of cold standby system with n units. They developed Chapman Kolmogorov equations for repair facility with two repairmen and solved them by using matrix technique. Availability and reliability factors have been obtained with probability of the system. Goldberg [10] has given the detail of Optimization technique of Genetic Algorithm. Tsai et al. [11] dealt with the optimizing preventive maintenance for mechanical components using genetic algorithms. Castro and Cavalca [12] presented an availability optimization problem of an engineering system assembled in series configuration which has the redundancy of units and teams of maintenance as

optimization parameters. Genetic Algorithm was used to reach the objective of availability, considering installation and maintenance costs. Chales and Kondo [13] tackled a multiobjective combinatorial optimization problem. They used Genetic Algorithm to optimize the availability and cost of a series and parallel repairable system. Fricks and Trivedi [14] stressed upon importance analysis with Markov Chains. Tewari et al. [15] dealt with the determination of availability for the systems with elements exhibiting independent failures and repairs or the operation with standby elements for sugar industry. Madhu Jain et al. [16] discussed the reliability analysis of redundant repairable system with degraded failure. Kumral [17] dealt with reliability-based optimization of a mine production system using Genetic Algorithm where as Tewari et al. [18] dealt with mathematical modeling and behavioral analysis for a refining system of a sugar industry using Genetic Algorithm. Marseguerra et al. [19] has explained the basics of Genetic Algorithm (GA) optimization for RAMS applications i.e. use of Genetic Algorithm within the area of reliability, availability, maintainability and safety (RAMS) optimization. Sunand et al. [20] discussed simulated availability of CO₂ cooling system in a fertilizer plant. Barabady [21] presented a case study describing the reliability and availability analysis of crushing plant in Jajarm Bauxite mine of Iran. Rajiv et al. [22, 23] have developed performance evaluation system for Screening Unit of Paper Plant. He also dealt with availability of bleaching system of paper plant. Sachdeva et al. [24] dealt with the reliability analysis of pulping system in paper industry Ying-Shen Juang et al. [25] proposed a Genetic Algorithm based optimization model to optimize the availability for a series parallel system. The objective was to determine the most economical policy of component's mean time between failure (MTBF) and mean time to repair (MTTR).. S. Gupta et al. [26] dealt with the reliability and availability analysis of coal handling unit of a steam thermal power plant using the concept of performance modeling and analysis. Michael et al. [27] dealt with availability analysis of transmission system (live case of Goa Electricity Department) using Markov model. In this, the laplace transition

matrix involved in stochastic modeling has been solved by using algorithm developed in MATLAB. Nakagawa et al. [28] analyzed the reliability optimization problem using the concept of the reversed failure rate. Sanjeev et al. [29] dealt with simulation model for evaluating the performance of Urea Decomposition System in a Fertilizer Plant. Wang et al. [30] performed the reliability optimization of a series-parallel system with fuzzy approach. Garg et al. [31] discussed about the availability and maintenance scheduling of a repairable block-board manufacturing system.

In this paper, the mathematical (availability) model has been developed to evaluate the performance of Digesting system of a paper plant on the basis of certain assumptions. After that, the performance optimization using Genetic Algorithm Technique (G.A.T.) is done, which gives the optimum system availability levels for different combinations of failure and repair rates of the subsystems of Digesting system for improving the performance of the paper plant. So, the findings of the present paper will be highly useful to the plant management in futuristic maintenance planning and control to enhance the system performance.

2. SYSTEM DESCRIPTION

The Digesting system of a paper plant comprises of four main subsystems with following description:

- I. *Subsystem B₁*: It consists of two screw feeder unit out of which one is standby. The function of screw feeder is to extract the wooden chips from storage silos and transfer it to the belt conveyor. When both screw feeders fails, it causes the complete failure of the system.
- II. *Subsystem B₂*: It consists of one belt conveyor unit to carry the chips . When belt conveyor fails, it causes the complete failure of the system.
- III. *Subsystem B₃*: It consists of one shuttle conveyor unit to feed wooden chips from belt conveyor to digester. When the shuttle

conveyor fails, it causes the complete failure of the system.

- IV. *Subsystem B₄*: It consists of three digester units in parallel to cook the wooden chips. If one digester fails, the system is subjected to reduce capacity. When all the three digesters fail at a time, it causes the complete failure of the system.

3. ASSUMPTIONS

The transition diagram (figure-2) of the Digesting system shows the three states, the system can acquire i.e. full working, reduced capacity and failed state. Based on the transition diagram, a performance-evaluating model has been developed. The following assumptions are addressed in developing the probabilistic models for the Digesting system of the paper plant concerned:

1. Failure/repair rates are constant over time and statistically independent.
2. A repaired subsystem is as well as new, performance wise for a specified duration.
3. Sufficient repair facilities are provided, i.e. No waiting time to start the repairs.
4. Standby units (if any) are of the same nature and capacity as the active units.
5. System failure /repair follow exponential distribution.
6. Service includes repair and /or replacement.
7. System may work at a reduced capacity.
8. There are no simultaneous failures among systems.

4. MATHEMATICAL MODELING

The mathematical modeling is carried out using simple probabilistic considerations and differential equations are developed on the basis of Markov birth-death process. These equations are further solved for determining the steady state availability of the Digesting system. Various probability considerations give the following differential equations associated with the Digesting system:

This system consists of twenty three states as:

States 0 to 1 show Full capacity working. States 2 to 5 show Reduced capacity working. States 6 to 22 show that unit is in failed state due to complete failure of one or the other subsystem of the unit.

The difference differential equations associated with the transition diagram (Figure 2) of Digesting system are:

$$P'_0(t) + \sum_{i=9}^{12} I_i P_0(t) = \mu_1 P_1(t) + \sum_{i=10}^{11} m_i P_{i-4}(t) + \mu_{12} P_2(t) \quad (1)$$

$$P'_1(t) + \sum_{i=9}^{12} I_i P_1(t) + \mu_9 P_1(t) = \sum_{i=9}^{11} m_i P_{i-1}(t) + \mu_{12} P_3(t) + \lambda_9 P_0(t) \quad (2)$$

$$P'_2(t) + \sum_{i=9}^{12} I_i P_2(t) + \mu_{12} P_2(t) = \sum_{i=10}^{11} m_i P_{i+1}(t) + \lambda_{12} P_0(t) + \mu_9 P_3(t) + \mu_{12} P_4(t) \quad (3)$$

$$P'_3(t) + \sum_{i=9}^{12} I_i P_3(t) + \mu_9 P_3(t) + \mu_{12} P_3(t) = \sum_{i=9}^{11} m_i P_{i+4}(t) + \lambda_9 P_2(t) + \lambda_{12} P_1(t) + \mu_{12} P_5(t) \quad (4)$$

$$P'_4(t) + \sum_{i=9}^{12} I_i P_4(t) + \mu_{12} P_4(t) = \sum_{i=10}^{12} m_i P_{i+6}(t) + \lambda_{12} P_2(t) + \mu_9 P_5(t) \quad (5)$$

$$P'_5(t) + \sum_{i=9}^{12} I_i P_5(t) + \mu_9 P_5(t) + \mu_{12} P_5(t) = \sum_{i=9}^{12} m_i P_{i+10}(t) + \lambda_9 P_4(t) + \lambda_{12} P_3(t) \quad (6)$$

$$P'_i(t) + \mu_9 P_i(t) = \lambda_9 P_j(t) \quad \text{where } j=1,3,5 \\ i=8,13,19 \quad (7)$$

$$P'_i(t) + \mu_{10} P_i(t) = \lambda_{10} P_j(t) \quad \text{where } \\ j=0,1,2,3,4,5 \quad i=6,9,11,14,16,20 \quad (8)$$

$$P'_i(t) + \mu_{11} P_i(t) = \lambda_{11} P_j(t) \quad \text{where } \\ j=0,1,2,3,4,5 \quad i=7,10,12,15,17,21 \quad (9)$$

$$P'_i(t) + \mu_{12} P_i(t) = \lambda_{12} P_j(t) \quad \text{where } \\ j=4,5 \quad i=18,22 \quad (10)$$

With initial conditions at time $t = 0$
 $P_i(t) = 1$ for $i = 0$
 $= 0$ for $i \neq 0$

**Solution of Equations:
 Steady State Behaviour**

The steady state behavior of the Digesting system can be analyzed by setting $d/dt = 0$ and $t \rightarrow \infty$ in equations (1) to (10) and solving

these equations recursively, we get:
 $P_1 = T_{13}P_0$ $P_2 = T_{14}P_0$ $P_3 = T_{12}P_0$

TOTAL STATES = 23
 REDUCED CAPACITY STATES = 04

FULL WORKING STATES = 02
 FAILED STATES = 17

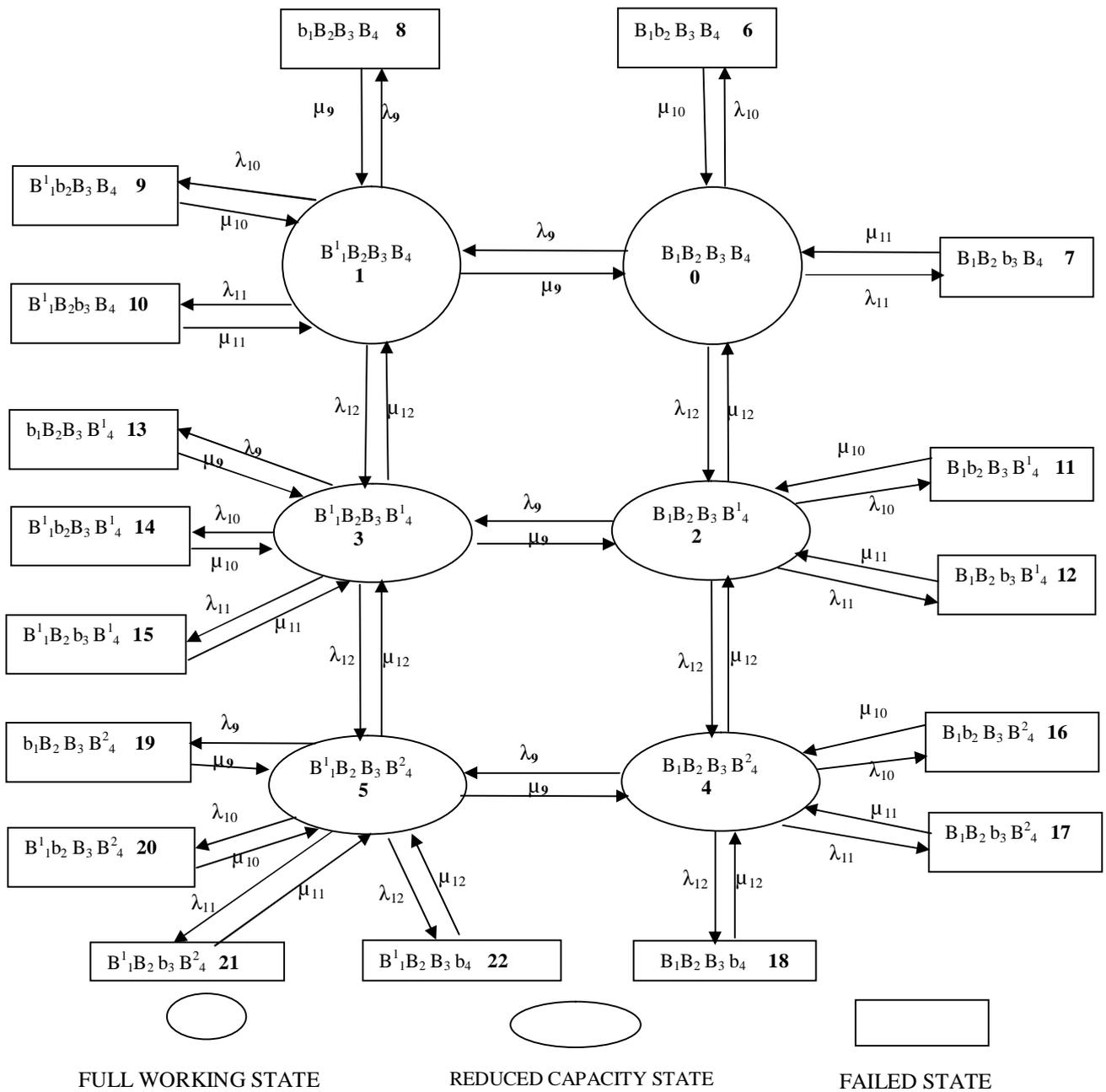


Figure 2: Transition Diagram of Digesting System

$$\begin{aligned}
P_4 &= T_{15}P_0 & P_5 &= T_{16}P_0 & P_6 &= B_2P_0 \\
P_7 &= B_3P_0 & P_8 &= B_1T_{13}P_0 & P_9 &= B_2T_{13}P_0 \\
P_{16} &= B_2T_{15}P_0 & P_{17} &= B_3T_{15}P_0 & P_{18} &= B_4T_{15}P_0 \\
P_{19} &= B_1T_{16}P_0 & P_{20} &= B_2T_{16}P_0 & P_{21} &= B_3T_{16}P_0 \\
P_{22} &= B_4T_{16}P_0
\end{aligned}$$

$$\begin{aligned}
P_{10} &= B_3T_{13}P_0 & P_{11} &= B_2T_{14}P_0 & P_{12} &= B_3T_{14}P_0 \\
P_{13} &= B_1T_{12}P_0 & P_{14} &= B_2T_{12}P_0 & P_{15} &= B_3T_{12}P_0 \\
& & & & & + B_1(T_{13}+T_{12}+T_{16})+B_4(T_{15}+T_{16})
\end{aligned} \tag{12}$$

Here, system performance was evaluated in terms of availability.

where, $B_i = \lambda_i/\mu_i$ $i=9,10,11,12$
 $T = \lambda_9 + \lambda_{12}$, $T_1 = \lambda_{12} + \mu_9$, $T_2 = \lambda_9 + \lambda_{12} + \mu_{12}$
 $T_3 = \lambda_{12} + \mu_9 + \mu_{12}$, $T_4 = \lambda_9 + \mu_{12}$, $T_5 = \mu_9 + \mu_{12}$
 $T_7 = [(T/\mu_{12}) - \{(\lambda_9 * \mu_9)/(T_1 * \mu_{12})\}]$, $T_8 = \mu_9/T_1$
 $T_9 = [(T_2 * T_4)/(\mu_9 * \mu_{12}) - \{(\lambda_9 * T_2)/(T_5 * \mu_{12})\} - (\lambda_{12}/\mu_9)]$
 $T_{10} = [(\lambda_{12} * T_4)/(\mu_{12} * \mu_9) + \{(\lambda_9 * \lambda_{12}/T_5 * \mu_{12})\}]$
 $T_{11} = [(T_4/\mu_{12}) + \{(\lambda_9 * \mu_9/T_5 * \mu_{12}) + (\lambda_{12}/T_5)\}]$,
 $T_{12} = [\{(T_7 * T_9) - T_{10}\}/\{T_{11} + (T_8 * T_9)\}]$
 $T_{13} = [(\lambda_9 + (T_{12} * \mu_{12}))/T_1]$,
 $T_{14} = [T - (\mu_9 * T_{13})/\mu_{12}]$
 $T_{15} = [(T_2 * T_{14}) - \lambda_{12} - (\mu_9 * T_{12})/\mu_{12}]$,
 $T_{16} = [(\lambda_9 * T_{15}) + (\lambda_{12} * T_{12})/T_5]$

Use of Normalizing condition i.e. sum of all the state probabilities is equal to one

$[\sum_{i=0}^{22} P_i = 1]$ gives:

$$\begin{aligned}
& T_{13}P_0 + T_{14}P_0 + T_{12}P_0 + T_{15}P_0 + T_{16}P_0 + B_2P_0 + B_3P_0 + \\
& B_1T_{13}P_0 + B_2T_{13}P_0 + B_3T_{13}P_0 + B_2T_{14}P_0 + \\
& B_3T_{14}P_0 + B_1T_{12}P_0 + B_2T_{12}P_0 + B_3T_{12}P_0 + B_2T_{15}P_0 + B_3T_{15}P_0 + \\
& B_4T_{15}P_0 + B_1T_{16}P_0 + B_2T_{16}P_0 + B_3T_{16}P_0 + B_4T_{16}P_0 = 1 \\
& P_0 = 1/[T_{13} + T_{14} + T_{12} + T_{15} + T_{16} + B_2 + B_3 + B_1T_{13} + B_2T_{13} + \\
& B_3T_{13} + B_2T_{14} + B_3T_{14} + B_1T_{12} + B_2T_{12} + B_3T_{12} + B_2T_{15} + B_3T_{15} + \\
& B_4T_{15} + B_1T_{16} + B_2T_{16} + B_3T_{16} + B_4T_{16}] \tag{11}
\end{aligned}$$

Now, the steady state availability (Av.) of the Digesting system is given by summation of all the full working and reduced capacity states probabilities.

Hence, $Av. = \sum_{i=0}^5 P_i$
 $Av. = P_0 + P_1 + P_2 + P_3 + P_4 + P_5$
 $Availability(Av.) = 1 + T_{13} + T_{14} + T_{12} + T_{15} + T_{16} / [1 + T_{13} + T_{14} + T_{12} + T_{15} + T_{16}(1 + B_2 + B_3)]$

5. GENETIC ALGORITHM

Genetic Algorithms are computerized search and optimization algorithms based on the mechanics of natural genetics and natural selection. Genetic Algorithms have become important because they are found to be potential search and optimization techniques for complex engineering optimization problems. The action of Genetic Algorithm shown in figure 3 for parameter optimization in the present problem can be stated as follows:

1. Initialize the parameters of the Genetic Algorithm.
2. Randomly generate the initial population and prepare the coded strings.
3. Compute the fitness of each individual in the old population.
4. Form the mating pool from the old population.
5. Select two parents from the mating pool randomly.
6. Perform the crossover of the parents to produce two off springs.
7. Mutate if required.
8. Place the child strings to new population.
9. Compute the fitness of each individual in new population.
10. Create best-fit population from the previous and new population.
11. Repeat the steps 4 to 10 until the best individuals in new population represent the optimum value of the performance function (System Availability)

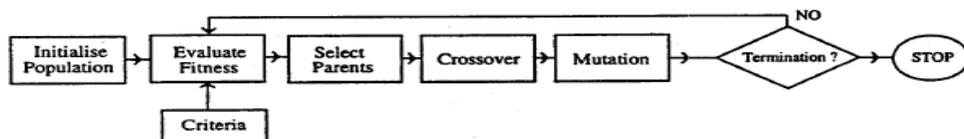


Figure 3: Flow chart for typical genetic algorithm process

6. PERFORMANCE OPTIMIZATION USING GENETIC ALGORITHM

The performance optimization of the Digesting system is highly influenced by the failure and repair parameters of each subsystem. These parameters ensure high performance of the Digesting system. Genetic Algorithm is hereby proposed to coordinate the failure and repair parameters of each subsystem for stable system performance i.e. high availability. Here, number of parameters is four (four failure parameters and four repair parameters). To use Genetic Algorithm for solving the given problem, the chromosomes are to be coded in real structures. Unlike, unsigned fixed point integer coding parameters are mapped to a specified interval $[X_{\min}, X_{\max}]$, where X_{\min} and X_{\max} are the minimum and maximum values of system parameters, respectively. The maximum value of the availability function corresponds to optimum values of system parameters. These parameters are optimized according to the performance index i.e. desired availability level. To test the proposed method, failure and repair rates are determined simultaneously for optimal value of unit availability. Effect of number of generations, population and crossover probability size on the availability of the Digesting system is shown in Table 1, 2 and 3. To specify the computed simulation more precisely, trial sets are also chosen for Genetic Algorithm and system parameters. The performance [availability] of the Digesting system is evaluated using the designed values of the unit parameters.

Parameters	Minimum	Maximum
λ_9	0.002	0.008
μ_9	0.2	0.5
λ_{10}	0.002	0.008
μ_{10}	0.2	0.5
λ_{11}	0.002	0.008
μ_{11}	0.2	0.5
λ_{12}	0.001	0.007
μ_{12}	0.05	0.125

Failure and Repair Rate Parameter Constraints are $\lambda_9, \mu_9, \lambda_{10}, \mu_{10}, \lambda_{11}, \mu_{11}, \lambda_{12}, \mu_{12}$.

Here, real-coded structures are used. The simulation is done to maximum number of generations, which is varying from 20 to 100. The effect of number of generations on availability of the Digesting system is shown in Figure 4.

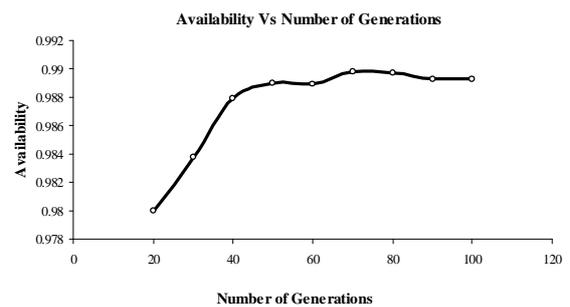


Figure 4. Effect of number of generations on fitness (digesting unit availability)

The optimum value of system's performance is 98.98%, for which the best possible combination of failure and repair rates is $\lambda_9 = 0.008, \mu_9 = 0.2210, \lambda_{10} = 0.0077, \mu_{10} = 0.2019, \lambda_{11} = 0.008, \mu_{11} = 0.2010, \lambda_{12} = 0.0069, \mu_{12} = 0.0510$ at generation size 70 as given in Table 1.

Now, the simulation is done to maximum number of population size, which is varying from 20 to 100. The effect of population size on availability of the Digesting system is shown in Figure 5.

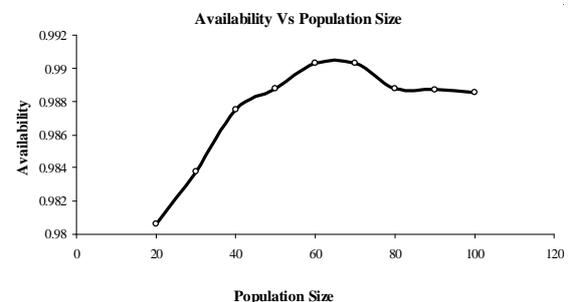


Figure 5. Effect of population size on fitness (digesting unit availability)

Table 1: Effect of Number of Generations on Availability of the Digesting System Using Genetic Algorithm.
(Mutation Probability = 0.015, Population Size = 80, Crossover Probability = 0.90)

Number of Generations	Availability	λ_9	μ_9	λ_{10}	μ_{10}	λ_{11}	μ_{11}	λ_{12}	μ_{12}
20	0.98	0.0074	0.2937	0.0078	0.2013	0.0079	0.20	0.0066	0.0526
30	0.9838	0.0079	0.2033	0.0079	0.20	0.008	0.20	0.007	0.05
40	0.9879	0.008	0.2213	0.008	0.20	0.0079	0.20	0.007	0.05
50	0.989	0.008	0.2690	0.008	0.20	0.008	0.20	0.007	0.05
60	0.9889	0.008	0.2177	0.008	0.20	0.008	0.20	0.007	0.0576
70	0.9898	0.008	0.2210	0.0077	0.2019	0.008	0.2010	0.0069	0.0510
80	0.9897	0.0079	0.2100	0.0078	0.2016	0.0078	0.2013	0.0069	0.0516
90	0.9893	0.008	0.2410	0.0079	0.2015	0.0078	0.2017	0.0069	0.05
100	0.9893	0.008	0.2412	0.0079	0.2016	0.008	0.2014	0.0069	0.0510

Table 2: Effect of Population Size on Availability of the Digesting System Using Genetic Algorithm.
(Mutation Probability=0.015, Number of Generations=80, Crossover Probability=0.90)

Population Size	Availability	λ_9	μ_9	λ_{10}	μ_{10}	λ_{11}	μ_{11}	λ_{12}	μ_{12}
20	0.9806	0.006	0.2128	0.008	0.2135	0.0075	0.2001	0.0065	0.0541
30	0.9838	0.008	0.20	0.008	0.20	0.008	0.20	0.007	0.05
40	0.9875	0.008	0.2011	0.0079	0.2100	0.008	0.2012	0.0068	0.0511
50	0.9888	0.0076	0.2112	0.008	0.20	0.0079	0.2119	0.007	0.0782
60	0.9903	0.0078	0.2098	0.008	0.20	0.0079	0.2100	0.007	0.0691
70	0.9901	0.008	0.2011	0.0078	0.2090	0.008	0.2040	0.0069	0.0710
80	0.9888	0.008	0.20	0.008	0.20	0.008	0.20	0.007	0.0510
90	0.9887	0.008	0.2015	0.008	0.20	0.008	0.20	0.007	0.0518
100	0.9885	0.008	0.20	0.008	0.20	0.008	0.20	0.007	0.05

The optimum value of system's performance is 99.03%, for which the best possible combination of failure and repair rates is $\lambda_9 = 0.0078$, $\mu_9 = 0.2098$, $\lambda_{10} = 0.008$, $\mu_{10} = 0.20$, $\lambda_{11} = 0.0079$, $\mu_{11} = 0.2100$, $\lambda_{12} = 0.007$, $\mu_{12} = 0.0691$ at population size 60 as given in Table 2.

Again, the simulation is done for maximum number of crossover probability, which is varying from 0.10 to 0.90. The effect of crossover probability on availability of the Digesting system is shown in Figure 6.

Table 3: Effect of crossover probability on availability of the digesting system using genetic algorithm (Mutation Probability=0.015, Number of Generations=80, Population Size=80)

Crossover Probability	Availability	λ_9	μ_9	λ_{10}	μ_{10}	λ_{11}	μ_{11}	λ_{12}	μ_{12}
0.10	0.9794	0.008	0.2067	0.008	0.20	0.008	0.20	0.007	0.05
0.20	0.985	0.008	0.3104	0.008	0.20	0.008	0.20	0.007	0.05
0.30	0.9869	0.0079	0.2621	0.008	0.2100	0.0079	0.2013	0.006	0.0684
0.40	0.9883	0.008	0.2177	0.0078	0.2016	0.0079	0.2211	0.0069	0.0611
0.50	0.9874	0.008	0.2067	0.008	0.2013	0.008	0.2028	0.007	0.051
0.60	0.9884	0.008	0.20	0.008	0.20	0.008	0.20	0.007	0.05
0.70	0.9893	0.0079	0.2610	0.008	0.2015	0.0078	0.2330	0.0069	0.0580
0.80	0.9892	0.0079	0.3003	0.0079	0.2019	0.008	0.2014	0.0069	0.0590
0.90	0.9883	0.0079	0.2611	0.0079	0.2113	0.008	0.2011	0.0069	0.0580

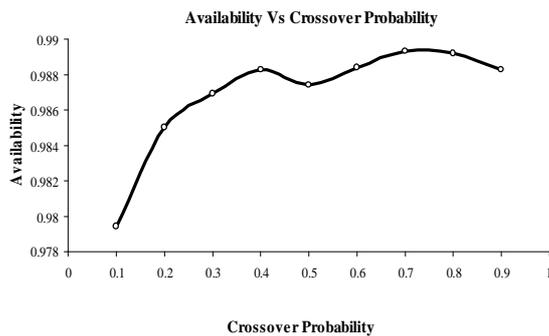


Figure 6. Effect of Crossover Probability on Fitness (Digesting Unit Availability)

The optimum value of system's performance is 98.93%, for which the best possible combination of failure and repair rates is $\lambda_9 = 0.0079$, $\mu_9 = 0.2610$, $\lambda_{10} = 0.008$, $\mu_{10} = 0.2015$, $\lambda_{11} = 0.0078$, $\mu_{11} = 0.2330$, $\lambda_{12} = 0.0069$, $\mu_{12} = 0.0580$ at crossover probability 0.70 as given in Table 3.

The above values reveal that to achieve the optimum unit availability level, the corresponding repair and failure rates of the subsystems should be maintained. The failure rates can be reduced using good design, reliable machines and providing standbys etc.

The repair rates can be improved by employing more workers for repair and utilizing better repair facilities. The maintenance workers can be trained beforehand so that quick repairs are possible.

7. CONCLUSIONS

The mathematical modeling and performance optimization of the Digesting system of a paper plant has been carried out in this paper. Genetic Algorithm is hereby proposed to select the various feasible values of the system failure and repair parameters. Then, Genetic Algorithm is successfully applied to coordinate simultaneously these parameters for an optimum level of system performance. Besides, the effect of Genetic Algorithm parameters such as number of generations, population size and crossover probability on the system performance i.e. availability has also been analyzed. Then, the findings of this paper are discussed with the concerned paper plant management. Such results are found highly beneficial for the purpose of performance optimization of the Digesting system in the paper plant concerned.

Nomenclature

The symbols and notations associated with the transition diagram of the Digesting system are given below:

○ Indicates the system in operating condition.

□ Indicates the system in breakdown condition

◌ Indicates the system in reduced capacity state.

B_1, B_2, B_3, B_4 : Represent good working states of respective screw feeder, belt conveyor, shuttle conveyor, digester.

b_1, b_2, b_3, b_4 : Represent failed states of respective screw feeder, belt conveyor, shuttle conveyor, digester.

$\lambda_9, \lambda_{10}, \lambda_{11}, \lambda_{12}$: Respective mean constant failure rates of B_1, B_2, B_3, B_4 .

$\mu_9, \mu_{10}, \mu_{11}, \mu_{12}$: Respective mean constant repair rates of b_1, b_2, b_3, b_4 .

$P_0(t)$: Probability that the system is working at full capacity at time t .

$P_i(t)$: Probability that the system is in the i^{th} state at time t .

P' : First-order derivative of the probabilities.

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