SOLVING A NEW MULTI-PERIOD MATHEMATICAL MODEL OF THE RAIL-CAR FLEET SIZE AND CAR UTILIZATION BY SIMULATED ANNEALING

R. Tavakkoli-Moghaddam*

Department of Industrial Engineering and Department of Engineering Science
College of Engineering, University of Tehran
P.O. Box 11155-4563, Tehran, Iran
tavakoli@ut.ac.ir

H.R. Sayarshad

Department of Industrial Engineering, Mazandaran University of Science and Technology
P.O. Box 734, Bobol, Iran
hamsayar@yahoo.com

T.Y. ElMekkawy

Department of Mechanical and Manufacturing Engineering, University of Manitoba
Winnipeg, MB, R3T 2N2, Canada
tmekkawy@cc.umanitoba.ca

*Corresponding Author

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Abstract There is a significant interaction between sizing a fleet of rail cars and its utilization. This paper presents a new multi-period mathematical model and a solution procedure to optimize the rail-car fleet size and freight car allocation, wherein car demands, and travel times, are assumed to be deterministic, and unmet demands are backordered. This problem is considered NP-complete. In other words, the traditional exact optimization approaches cannot solve a real-life size problem of this kind in a reasonable time. To tackle this problem, an efficient meta-heuristic algorithm based on simulated annealing (SA) is proposed. This algorithm works efficiently on a neighborhood search within solution space and probable acceptance of inferior solutions to escape from being trapped in local optima. A number of numerical examples are solved to check for efficiency and validity of the proposed SA algorithm. We conclude that the proposed model and algorithm are useful to identify good strategies for the sizing of rail car fleets and allocation of related cars.

Keywords Multi-period model, Fleet sizing, Freight car allocation, Railroad transportation, Simulated Annealing

چکیده رابطه مهمی بین اندازه ناوگان و بهره وری از ناوگان وجود دارد. در این مقاله، یک مدل جدید چند پریودی و روش حلی ارایه می‌شود که تعداد ناوگان مورد نیاز و همچنین تخصیص واگن‌های باری را بهینه کند در حالی که تقاضاها و زمان سفر بطور قطعی فرض می‌شود و همچنین تقاضاهای تامین نشده در انتهای پریود زمانی نخواهیم داشت. ولی از آنجایی که حتی مدل یشنهادی در اندازه بالا و متوسط دارای پیچیدگی بوده، یک الگوریتم شبیه سازی تبرید (SA) برای حل آن توسعه داده می‌شود. الگوریتم SA یکی از کاراترین روش های حل است که اساس کار آن روی جستجو همسایگی در فضاهای حل می‌باشد همچنین از نقاط قوت این الگوریتم این است که به راحتی می‌تواند از نقاط بهینه موضعی رهایی پیدا کرده و در جهت بهبود تابع هدف حرکت نماید. برای تایید عملکرد و کارایی الگوریتم پیشنهادی SA ارایه شده، مسایل گوناگونی با اندازه های مختلف حل شده و مقدار تابع هدف و زمان محاسباتی مقایسه می گردد. در انتها نتیجه گیری می شود که این مدل و الگوریتم حل پیشنهادی استراتژی خوبی برای تعیین اندازه ناوگان و همچنین تخصیص واگن است.

1. INTRODUCTION

Transportation systems frequently contain fleets of vehicles that circulate a network to carry people or goods. The capacity of a transportation system is directly proportional to the number of available vehicles. Determining the optimal number of vehicles that satisfy a certain demand for a particular system, requires a tradeoff between the ownership costs of the vehicles, and the potential costs or penalties associated with unmet demands. Serving demands results in the relocation of rail-cars. The consequent movement of rail-cars between various locations is often imbalanced, and this implies the need for optimal allocation of empty rail-cars over the network. Thus, the fleet of cars, which is available for service at any given time (and their locations), depends on the car redistribution strategy. The interaction between fleet sizing decisions and car distribution or utilization decisions is the main focus of this paper.

There is an increasing interest of investment in rail freight cars. The management of these systems is a very complex issue. Therefore, it has received the attention of both practitioners and researchers. Studies in this area can be categorized in two main directions: 1) determining an optimal fleet size. 2) allocating the available car capacity to various destinations, in order to calculate the required empty flows. The main goal of this paper is considering the simultaneous optimization of fleet sizing decisions and the car utilization. By having direct impact on the level of investment in capital resources, the potential benefits from improved utilization of cars is much greater than the reduced operating costs.

Transportation vehicles are expensive capital, and fleet sizing is an important issue for both researchers and service providers. Fleet sizing is related to overall service design [1], and there has been recent work related to trucking [2-4], and airline express package service [5] that emphasizes these connections. Fleet sizing is also important in material handling systems used for manufacturing operations [6,7]. Even more generally, sizing the fleet's vehicle is a specific example of sizing a system for reusable resources. This task has been treated as non-dynamic for a long time, and formulated as a linear programming problem with known supply and demand, and the objective of maximizing revenue. These models are most often solved by using the standard simplex algorithm [8,9].

Dejax, et al [10] surveyed models of fleet management and distribution of empty vehicles. Network flow models for empty vehicle distribution are presented on the allocation of empty vehicle, not on the fleet size decisions [11-18]. Frantzeskakis, et al [19] considered the problem of fleet sizing and empty equipment redistribution from the standpoint of inventory theory and developed decentralized stock control policies for empty equipment; however, their focus is on utilization of a given vehicle fleet, not on fleet sizing decisions. Taxonomy of rail car fleet sizes by distinguishing between deterministic and stochastic models and dividing them into the sub problems of fully and partially loaded rail cars in transportation systems [20-27]. Therefore, it can be noticed that these studies followed constraints and more general objectives.

Literature related to vehicle fleet sizing has not specifically addressed the fleet sizing of rail-cars. Sherali, et al [28,29] proposed a time-space network representation of practical fleet sizing models for the automobile and railroad industries, concerned with the problem of shipping automobiles via railroad auto racks; however, it has very simple network structure and also has limitations stemming from its simplifying assumptions. Bojovic, et al [30] addressed the problem of determining an optimal number of rail-cars to satisfy demand by minimizing the total cost. This indirectly reflects fleet sizing concerns; but, their primary focus is on the allocation decisions. The proposed optimization model provides rail network information, such as yard capacity, unmet demands, and number of loaded and empty rail-car at any given time and location. Moreover, the optimum use of rail-cars for demands response in the length of the time periods is one of the main advantages of the proposed model.

The remaining of this paper is organized as follows: Section 2 presents an exact mathematical formulation of the given problem. In Section 3, a simulated annealing (SA) algorithm is proposed to solve the developed mathematical model and then a numerical example is presented. Section 4 describes the experiments of testing the convergence behavior of the solution procedure. Concluding remarks and future research directions are given in Section 5.

2. MATHEMATICAL MODEL

It is also assumed that the planning horizon (T) has been divided into discrete "decision periods" and using t to denote one such period. A set of rail network location is denoted by N which is divided into two subsets, N_1 and N_2, representing the number of origins and destinations points, respectively.

2.1. Variables

$X_{ij}(t)$: Number of loaded cars dispatched from $i \in N_1$ to $j \in N_2$ in period $t \in T$.

$Y_{ji}(t)$: Number of empty cars dispatched from $i \in N_1$ to $j \in N_2$ in period $t \in T$.

$V_i(0)$: Number of cars initially allocated to origin $i \in N_1$.

$VV_j(0)$: Number of cars initially allocated to destination $j \in N_2$.

$V_i(t)$: Number of cars present at origin $i \in N_1$ at the end of period $t \in T$.

$VV_j(t)$: Number of cars present at destination $j \in N_2$ at the end of period $t \in T$.

$U_{ij}(t)$: Unmet demand from $i \in N_1$ to $j \in N_2$ in period $t \in T$.

2.2. Input Data

r_{ij}: Revenues per loaded car sent from $i \in N_1$ to $j \in N_2$.

l_{ij}: Cost of moving a loaded car from $i \in N_1$ to $j \in N_2$.

e_{ji}: Cost of moving a empty car from $j \in N_2$ to $i \in N_1$.

q: Cost per car per period to own or lease a car.

h_i: Cost of holding a car for one period at origin $i \in N_1$.

w_j: Cost of holding a car for one period at destination $j \in N_2$.

P_{ij}: Penalty cost per period for one unit of unmet demand from $i \in N_1$ to $j \in N_2$.

SC_{it}: Yard capacity at origin yard $i \in N_1$ at the end of period $t \in T$.

SC_{jt}: Yard capacity at destination yard $j \in N_2$ at the end of period $t \in T$.

$\alpha_{ij}(\tau,t)$: Proportion of loaded cars dispatched from $i \in N_1$ to $j \in N_2$ in period $t \in T$ which arrive in period $t \in T$, such that:

$$\sum_{\tau < t} \alpha_{ij}(\tau,t) = 1 \qquad \forall i,j,t \tag{1}$$

$\beta_{ji}(\tau,t)$: Proportion of empty cars dispatched from $j \in N_2$ to $i \in N_1$ in period $t \in T$ which arrive in period $t \in T$, such that:

$$\sum_{\tau < t} \beta_{ji}(\tau,t) = 1 \qquad \forall i,j,t \tag{2}$$

$d_{ij}(t)$: Demand for transportation service between $i \in N_1$ and $j \in N_2$ in period $t \in T$.

The model is formulated as follows:

Max φ:

$$\begin{aligned} &\sum_i\sum_j\sum_t r_{ij} X_{ij}(t) - \sum_i\sum_j\sum_t \left[l_{ij} X_{ij}(t) + e_{ji} Y_{ji}(t) \right] - \\ &- \sum_i\sum_j\sum_\tau \left\{ \begin{matrix} \left[X_{ij}(\tau) \sum_{t>\tau} (t-\tau) q \alpha_{ij}(\tau,t) \right] + \\ \left[Y_{ji}(\tau) \sum_{t>\tau} (t-\tau) q \beta_{ji}(\tau,t) \right] \end{matrix} \right\} \\ &- \sum_i\sum_t h_i V_i(t) - \sum_j\sum_t w_j VV_j(t) - \sum_i\sum_j\sum_t p_{ij} U_{ij}(t) \end{aligned} \tag{3}$$

s.t.

$$\begin{aligned} &U_{ij}(t) = U_{ij}(t-1) + \\ &d_{ij}(t) - \sum_{\tau < t} X_{ij}(\tau) \alpha_{ij}(\tau,t) \qquad \forall i,j,t \end{aligned} \tag{4}$$

$$\begin{aligned} &V_i(t) = V_i(t-1) + \\ &\sum_j \sum_{\tau < t} \beta_{ji}(\tau,t) Y_{ji}(\tau) - \sum_j X_{ij}(t-1) \qquad \forall i,t \end{aligned} \tag{5}$$

$$\begin{aligned} &VV_j(t) = VV_j(t-1) + \\ &\sum_i \sum_{\tau < t} \alpha_{ij}(\tau,t) X_{ij}(\tau) - \sum_i Y_{ji}(t-1) \qquad \forall j,t \end{aligned} \tag{6}$$

$$\sum_j X_{ij}(t) \le V_i(t) \qquad \forall i,t \quad (7)$$

$$\sum_i Y_{ji}(t) \le VV_j(t) \qquad \forall j,t \quad (8)$$

$$U_{ij}(T) = 0 \qquad \forall i,j \quad (9)$$

$$\sum_j \sum_{\tau < t} \beta_{ji}(\tau,t) Y_{ji}(\tau) + \sum_j X_{ij}(t) \le SC_{it} \qquad \forall i,t \quad (10)$$

$$\sum_i \sum_{\tau < t} \alpha_{ij}(\tau,t) X_{ij}(\tau) + \sum_i Y_{ji}(t) \le SC_{jt} \qquad \forall j,t \quad (11)$$

$$X_{ij}(t),\ \ Y_{ji}(t),\ \ U_{ij}(t),\ \ V_i(t),\ \ VV_j(t) \ge 0, \quad Integer\ \forall i,j,t \qquad (12)$$

The objective function (3) includes terms for revenues, direct transportation cost, ownership cost for cars per route, holding costs for idle cars, and penalty costs for unmet demand. Constraint (4) ensures that all demand is accounted for; unmet demand in period *t* must equal to unmet demand from the previous period plus new demand minus the loaded movements. Constraints (5) and (6) are conservation of flow constraints for cars at each location in each time period, which include the effects of deterministic travel times for car movements through α and β terms, representing the certain arrival times of cars at their destinations. Constraints (7) and (8) are balancing constraints for cars at each location in each period. Constraint (9) ensures that unmet demands become zero at the end of the planning horizon.

Constraint (10) estimated capacity of yard at a station with respect to the summation of the number of loaded and empty railcars also outbound of the railcars at original nodes of the network. Constraint (11) computes the summation of the number of the inbound loaded railcars and number of the outbound empty railcars reflecting the needed capacity at the destination rail yards. Constraint (12) ensures that $X_{ij}(t)$, $Y_{ji}(t)$, $U_{ij}(t)$, $V_i(t)$, and $VV_j(t)$ are always nonnegative and integer.

3. PROPOSED SA ALGORITHM

Simulated annealing (SA) was first introduced as an intriguing technique for optimizing functions of many variables (Kirkpatrick, et al [31]). Simulated annealing is a heuristic strategy that provides a means for optimization of NP complete problems: those for which an exponentially increasing number of steps are required to generate the/an exact answer. Although such a heuristic (logical) approach can't guarantee to produce the exact optimum, an acceptable optimum can be found in a reasonable time, while keeping the computational expense is dependent on low powers of the dimension of the problem. Simulated annealing is based on an analogy to the cooling the heated metals.

In any heated metal sample the probability of some cluster of atoms as a position, r_i, exhibiting a specific energy state, $E(r_i)$, at some temperature T, is defined by the Boltzmann probability factor:

$$P(E(r_i)) = exp[-E(r_i)/k_B T] \qquad (13)$$

Where, k_B is Boltzmann's constant. As a metal is slowly cooled, atoms will fluctuate between relatively higher and lower energy levels and allowed to equilibrate at each temperature T.

The material will approach a ground state, a highly ordered form in which, there is very little probability for the existence of a high energy state throughout the material. Figure 1 provides a flowchart representation of the annealing algorithm. In standard iterative improvement methods, a series of trial point are generated until an improvement in the objective function is noted, in which case the trial point is accepted. However, this process only allows for downhill movements to be made over the domain. In order to generate the annealing behavior, a secondary criterion is added to the process. If the *k*-th trial point generates a large value of the objective function then the probability of accepting this trial point is determined using the Boltzmann probability distribution:

$$P\left[accept\ X^k, Y^k, U^k, V^k, VV^k\right] = exp\left[-\left(\varphi^k - \varphi^0\right)/CT\right] = exp[-\Delta\varphi/CT] \qquad (14)$$

Where, $\varphi^k = \varphi\left(X^k, Y^k, U^k, V^k, VV^k\right)$ and φ^0 corresponds to the initial starting point. This

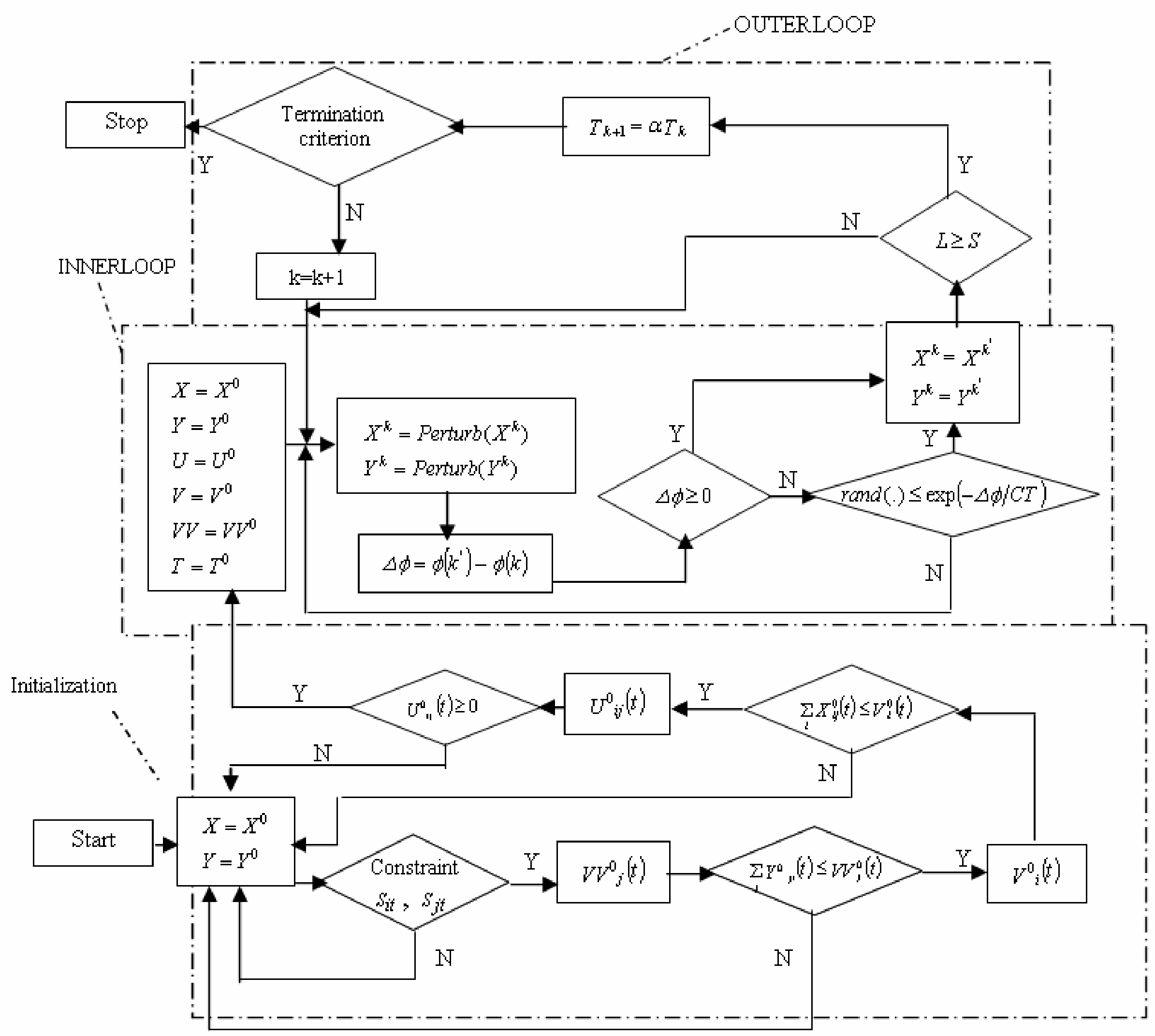

Figure 1. A flowchart representation of the annealing process.

probability is compared with a randomly generated number over the range [0,1].

If $P\left[accept\, X^k, Y^k, U^k, V^k, VV^k\right] \geq random\ [0..1]$,

then the trial point is accepted. This dependence on random numbers makes simulated annealing a stochastic method.

3.1. Initial Temperature In physical analogy, the initial temperature should be large enough to heat up the solid until all particles are randomly arranged in the liquid phase. This means that in the beginning, the temperature of the annealing process must be high enough to make sure that the system can be shifted to all possible states. By this property, the algorithm can find a solution that does not strongly depend upon the initial configuration. Since the probability to accept the worse solutions is exp(-$\Delta\phi$/CT), the initial temperature T_0 can be determined by means of the objective function transitions which would be accepted in the beginning of the annealing process with a probability P_0.

Pilot runs are performed, and the mean benefitincreasing $\overline{\Delta}$ of the objective function increasing is then computed. In the calculation, T_0 is calculated as follows:

$$T_0 \approx \overline{\Delta\varphi} \Big/ ln\left(P_0^{-1} \right) \qquad (15)$$

3.2. Number of Iterations

Various implementations use various methods of random number generation (e.g., the Lehmer generator [32]). Repeating this iterative improvement many times at each value of the control parameter *T*, the methodical thermal rearrangement of atoms within a metal at temperature *T* is simulated [31]. In addition, the pseudo code of the developed SA is illustrated in Figure 2.

The annealing process transfers from one configuration to one of its neighbors with certain probability; this is equivalent to a Markov chain. Therefore, we should determine the number of iterations at each temperature. In our problem, L, the length of the *k*-th Markov chain, L, is a value that depends on the size of the problem. Alternatively it can be argued that a min number of transitions should be accepted at each temperature.

3.3. Rules for Decreasing the Temperature

For a certain value of temperature, the temperature is reduced when the numbers of transitions reach the upper bound of the Markov chain length. The control parameter, i.e. the reduction ratio of temperature, usually is chosen for small temperature changes. The Markov chain more easily leads to an equilibrium state if the temperature change is small. Hence, we use the decrement rule as follows:

$$T_{k+1} = \alpha T_k \qquad k = 0,1,2,3,... \qquad (16)$$

The control parameter α, called cooling rate, is small; however, it is close to 1. It is normally between 0.85 and 0.99.

3.4. Stopping Condition

The annealing process is terminated when the system is frozen, i.e. the value of the objective function of the solution does not improve after a certain number of consecutive Markov chain. Termination criterion is determined by:

$$V(T) \Big/ T\left(\overline{C}(T) - \overline{C}\left(T_0\right)\right) \le \varepsilon \qquad (17)$$

Step 1: Select the initial temperature T_0, cooling rate α, termination criterion π and Markov chain L.
Set $\left(X^0, Y^0\right) = (0,0)$, $k = 0$, $S = 0$.

Step 2: Compute $V^0{}_i(t)$, $U^0{}_{ij}(t)$, $VV^0{}_j(t)$, φ^0.
Set $k = k+1$, $\varphi^* = \varphi^0$.

Step 3: Determine the neighborhood $\left(X^k, Y^k\right)$ using perturbation.

Step 4: If $\sum_j \sum_{\tau<t} \beta_{ji}(\tau,t) Y_{ji}(\tau) + \sum_j X_{ij}(t) \le SC_{it}$, $\sum_i \sum_{\tau<t} \alpha_{ij}(\tau,t) X_{ij}(\tau) + \sum_i Y_{ji}(t) \le SC_{jt}$
compute $VV^k{}_j(t)$ and check feasibility of the solution through Step 5.
Else if Go to Step 3.

Step 5: If $\sum_j Y^k{}_{ji}(t) \le VV^k{}_j(t)$ then compute $V^k{}_i(t)$.
Else if Go to step 3.
If $\sum_i X^k{}_{ij}(t) \le V^k{}_i(t)$ then compute $U^k{}_{ij}(t)$
Else if Go to Step 3.
If $U^k{}_{ij}(t) \ge 0$ then determine $\varphi^k = \varphi\left(X^k, Y^k, U^k, V^k, VV^k\right)$
Else if Go to Step 3.

Step 6: If $\Delta\varphi = \varphi^k - \varphi^* \ge 0$ or $rand(.) \le exp(-\Delta\varphi/CT)$ where $rand(.)$ is a random number in the range of 0 and 1 then Set $k = k + 1$, $S = S + 1$ and Go to Step 7.
Else if then Go to step 3.

Step 7: $\varphi^* = \varphi^k$.

Step 8: If $L \ge S$ then Reduce temperature by cooling rate α.
Else if Set $S = 0$ and Go to Step 3.

Step 9: If termination condition π is reached then Go to Step 10.
Else if Go to Step 3.

Step 10: Report φ^* and $\left(X^*, Y^*, U^*, V^*, VV^*\right)$.

Figure 2. Pseudo code of the proposed SA.

$V(T)$: Variance of the accepted objective function value in temperature T.

$\overline{C}(T)$: Mean of the accepted objective function value in temperature T.

$\overline{C}(T_0)$: Mean of the accepted objective function value in initial temperature.

ε: Positive small number.

A run is ended if after a specified number of temperature decrements are made without any improvement in objective function, or if number of neighbors tested exceeds an iteration limit.

4. NUMERICAL EXAMPLES

The proposed new mathematical model has been tested on the example of a hypothetical network with four origins, four destinations on a 6-day planning horizon using the simulated annealing (SA) algorithm with starting temperature of 1000 (see Equation 15), final temperature 0.05, cooling rate 0.99 (i.e., α = 0.99), and number of iterations per temperature 20 (i.e., L = 20).

Table 1 presents transportation demands for all days over the planning horizon and all origin-destination combinations. Table 2 illustrates values of unit holding costs of cars at all stations. Table 3 shows the input data on revenue per loaded car sent from i to j, cost of moving an empty car from i to j, penalty cost per period for one unit of unmet demand from i to j. The unit ownership cost for a car traveling between stations per unit time is 5 (q = 5). The shunting yard capacity at all origins and destinations, at the end of period is 150 (S_{it} = 150, S_{jt} = 150). A computer program has been developed using Visual Basic 6 and the obtained results have been summarized and are shown in Figure 3. The number of cars present at origin i at initial period ($V_1(1)$ = 266, $V_2(1)$ = 301, $V_3(1)$ = 314, $V_4(1)$ = 272) have been determined after ten iterations. It has been concluded that a fleet consisting of 1153 (i.e., 266+301+314+272) cars is required for a proper functioning of the described system.

4.1. Experimentation

The experiments are designed to test the convergence behavior of our solution procedure. As it is shown in Table 4, test problems are solved to check for the efficiency and validity of SA algorithm in comparison with the exact algorithm. We solved nine small-sized instances by lingo software using branch-and-bound (B and B) method according to Table 4. Table 5 presents the computational results obtained on nine large-sized test problems with application dimensions. Consequently, the SA solution is compared with the upper bound (UB) solution. It is worthy noting that the average difference between the upper bound and the SA solution is nearly 11 %, which is very satisfactory.

To analyze the sensitivity of the algorithm to the number of time periods, we fixed number of the network locations while increasing the number of time periods. The results are reported in Table 6 and Figure 4. As it can is seen, the optimal total fleet size found in the range of 25 and 35 time periods (e.g. monthly planning). In order to analyze the sensitivity of the algorithm to the number of network locations, number of time periods is fixed while increasing the number of locations. Table 7 and Figure 5 show the associated results.

4.2. Effect of the SA Parameters

Figure 6 shows the objective function value of the solutions found at different stages of the SA algorithm for a moderate-sized example network of five origins, five destinations, and five time periods. At the initial stages, since the temperature is very high, the proposed SA algorithm accepts nearly all solutions. It acts as random search first, and the objective function value of the accepted solutions changes in wide range as seen in Figure 6. As the temperature decreases, the probability of accepting the worse solutions also decreases. Because of that, at later stages of the run, the search becomes greedy and only better solutions are accepted.

The parameter used in the proposed SA algorithm is the alpha (α) value, used to decrease the temperature of the system as in Equation 16. The higher its value, the slower the system cools down. The range of α is experimented between 0.5 and 1. By increasing the alpha value, larger portion of the solution space can be searched, but run time gets longer. Figure 7, states that value 0.99 performs better compared to the other values in the ranges from 0.5 to 0.99.

TABLE 1. Demand Scenarios for the Illustrative.

Origin	Destination	Periods					
		1	2	3	4	5	6
1	1	17	12	21	27	27	29
1	2	10	18	27	12	14	10
1	3	10	13	14	10	15	16
1	4	21	17	17	17	28	19
2	1	18	16	29	26	29	15
2	2	29	11	24	26	29	19
2	3	16	25	17	25	11	13
2	4	11	17	19	20	17	29
3	1	10	14	10	28	12	15
3	2	25	23	26	24	11	12
3	3	25	22	13	18	21	24
3	4	21	13	29	23	20	25
4	1	26	15	13	27	12	11
4	2	25	24	29	28	28	20
4	3	20	23	19	12	10	25
4	4	23	24	21	13	27	17

TABLE 2. Holding Cost for the Illustrative.

Origin	*hi* ($)	Destination	*wj* ($)
1	5	1	5
2	5	2	5
3	5	3	5
4	5	4	5

TABLE 3. Parameter Values for the Illustrative.

Origin	Destination	*lij* ($)	*rij* ($)	*eij* ($)	*pij* ($)
1	1	3	110	4	2
1	2	1	107	5	1
2	1	3	117	5	2
2	2	1	106	6	3

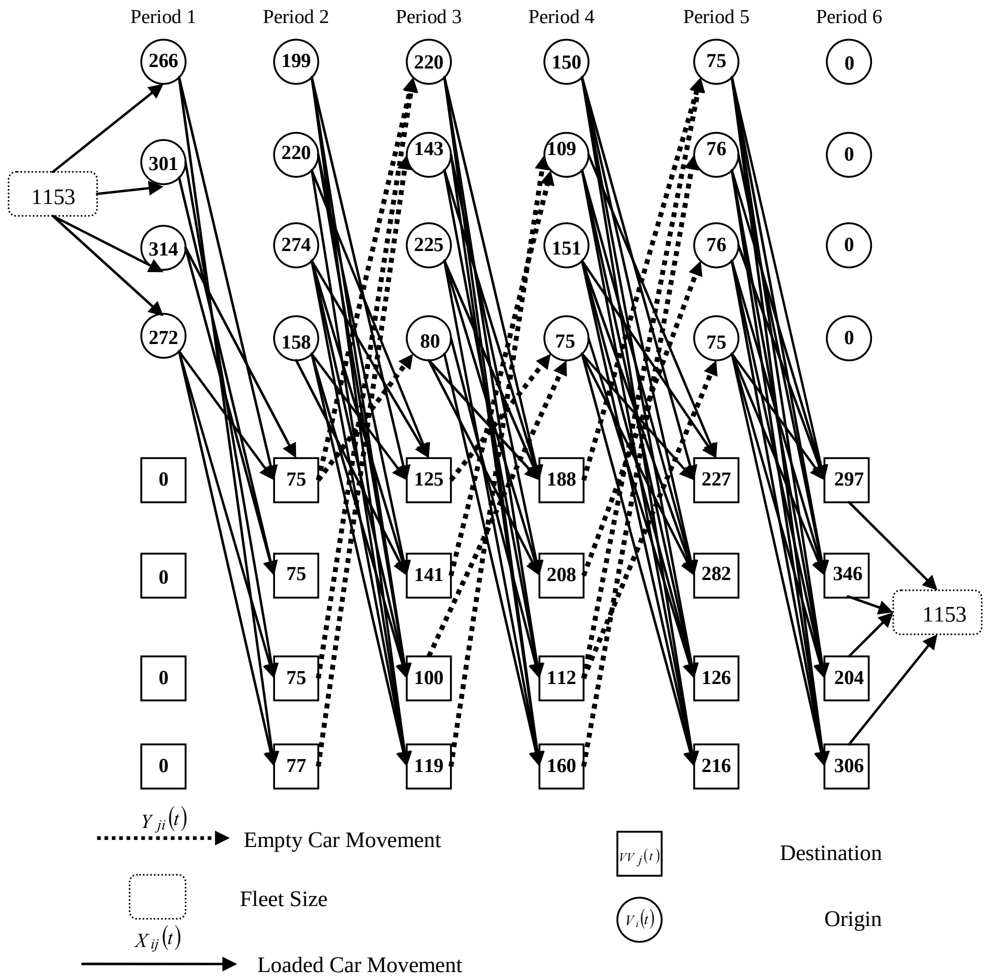

Figure 3. Control actions (loaded and empty cars).

TABLE 4. Comparison Results of the Simulated Annealing with the Exact Algorithm.

Pro.	$\|N_1\|$	$\|N_2\|$	T_h	FS[b]		OFV[a]		CPU Time		Gap
				SA	Optimal	SA	Optimal	SA	Optimal	
1	2	2	3	315	315	16 820	16 831	1s	2s	0.0006
2	2	2	4	178	166	20 430	20 450	3s	3s	0.0009
3	2	2	5	196	192	25 613	25 633	3s	3s	0.0007
4	2	2	6	190	182	32 053	32 151	3s	3s	0.003
5	3	3	3	526	526	38 781	38 960	2s	2s	0.004
6	3	3	4	580	580	49 821	50 122	3s	3s	0.006
7	5	5	3	1760	1760	109 056	110 821	2s	2s	0.015
8	4	4	3	1125	1125	69 651	70 128	3s	3s	0.006
9	4	4	4	935	893	99 163	101 920	5s	5s	0.02
Objective Function Value[a], Fleet Size[b]										Mean Gap = 0.006

TABLE 5. Computational Results for Real-Life Dimension Size Problems.

Pro.	$\lvert N_1 \rvert$	$\lvert N_2 \rvert$	T_h	E_T[d]	FS[b]	D_T[c]	FS/D_T	OFV[a]		CPU Time (Sec.)	Gap
								SA	Upper Bound		
1	20	20	7	18 411	36 012	54 423	0.66	3 743 121	4 177 590	27	0.104
2	7	7	30	24 321	4531	28 852	0.16	1 703 963	1 893 292	20	0.1
3	30	30	5	20 884	66 923	87 807	0.76	6 732 798	7 564 941	38	0.11
4	40	40	6	56 766	130 461	187 227	0.70	14 791 230	16 751109	43	0.117
5	25	25	10	71 505	50 422	121 927	0.41	8 993 439	10 127746	74	0.112
6	50	50	5	58 563	185 144	243 707	0.76	19 371 162	21 987698	74	0.119
7	15	15	15	50 159	16 034	66 193	0.24	4 655 921	5 225 500	35	0.109
8	5	5	90	40 640	3462	44 102	0.07	1 942 631	2 165 697	43	0.103
9	15	15	20	67 507	14 818	82 325	0.18	5 437 689	6 109 762	42	0.11
Objective Function Value[a], Fleet Size[b], Sum of the Demands[c] Total Empty Cars[d]											Mean Gap = 0.11

TABLE 6. Computational Results for the Sensitivity Analysis of the Time Periods.

Pro.	$\lvert N_1 \rvert$	$\lvert N_2 \rvert$	T	FS[a]	D_T[b]	E_T[c]	FS/D_T	E_T/D_T
1	5	5	10	750	1000	250	0.75	0.25
2	5	5	15	1000	2770	1770	0.36	0.64
3	5	5	20	1126	4000	2874	0.28	0.72
4	5	5	25	720	3229	2509	0.22	0.78
5	5	5	30	1300	6000	4700	0.21	0.79
6	5	5	35	1730	7000	5270	0.24	0.76
7	5	5	40	2120	7900	5780	0.26	0.74
8	5	5	45	2435	8500	6065	0.28	0.72
9	5	5	50	3165	10 000	6835	0.31	0.68

[a]Fleet Size, [b]Sum of the Demands, [c]Total Empty Cars.

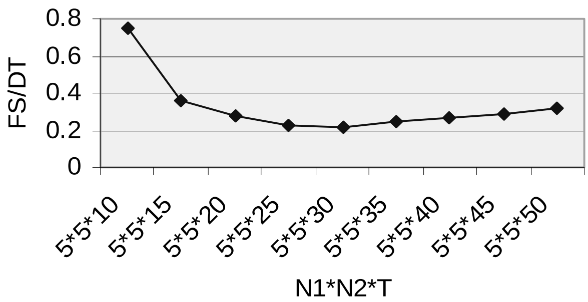

Figure 4. Sensitivity analysis of *FS/DT* with respect to the change in the number of time periods.

TABLE 7. Computational Results for the Sensitivity Analysis of the Number of the Network Locations.

Pro.	$\|N_1\|$	$\|N_2\|$	T	FS[a]	D_T[b]	E_T[c]	FS/D_T	E_T/D_T
1	2	2	4	200	271	71	0.74	0.26
2	3	3	4	598	692	94	0.86	0.14
3	4	4	4	960	1279	319	0.75	0.25
4	5	5	4	1320	1928	608	0.68	0.32
5	6	6	4	1858	2866	1008	0.64	0.36
6	7	7	4	2293	3915	1622	0.58	0.42
7	8	8	4	2940	5098	2158	0.57	0.43

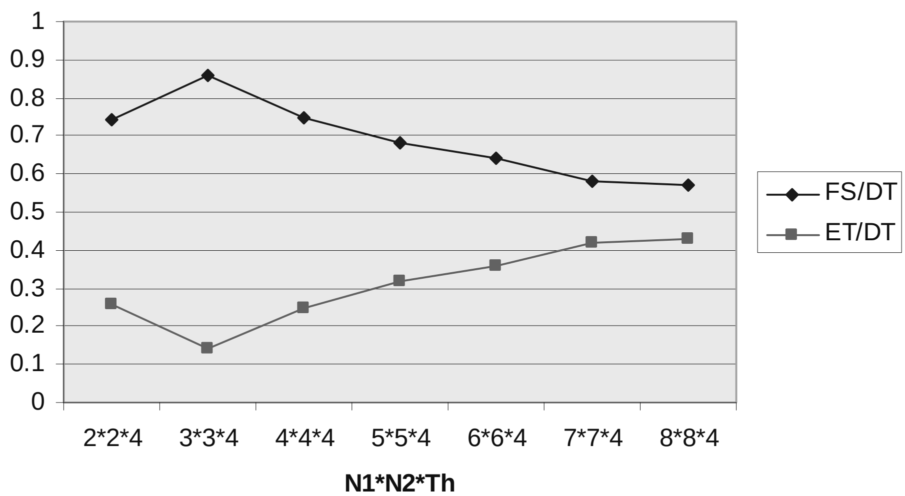

Figure 5. Sensitivity analysis of FS/D_T , E_T/D_T with respect to the change in number of network locations.

Number of accepted solutions to decrease the temperature (L: Accepted to Decrease) is the third parameter used. If this value is small, then the SA algorithm converges faster. Values assigned to this parameter had a range from 10 to 150 in our experiments. As Figure 8 denotes, although the value of L does not have much effect on the quality of results, the value 10 gives slightly better results than the others.

5. CONCLUSION

We presented a new formulation and a solution procedure to optimize the fleet size and freight car allocation wherein car demands and travel times were assumed to be deterministic and unmet demands were backordered. We assumed that unmet demands become zero at the end of the planning horizon, i.e., the car demands would be totally responded through the horizon. We believe that our model is able to support all following features:

- The model provides rail network information such as yard capacity, unmet demands, and number of loaded and empty rail-cars at any given time and location.
- The optimal use of empty rail-cars for responded demand, during the time periods of summarizing this model and decreased car purchasing costs.

Numerical examples in small sizes show that the exact solution for the given problem is capable of reporting solutions in a fair amount of CPU time; however, it was unable to solve the problem in medium and large-sized instances. To tackle this problem, a simulated annealing (SA) algorithm is proposed to solve the presented model. The algorithm worked efficiently on a neighborhood search within the solution space, acceptance probability, and inferior solutions to escape from trap (i.e., local optimal solution). Numerical examples were solved to check the efficiency and validity of the proposed SA algorithm. We concluded that the model was useful in identifying good strategies to size rail car fleets and allocation of the freight cars. A primary direction for further research is the extension of the current model to a

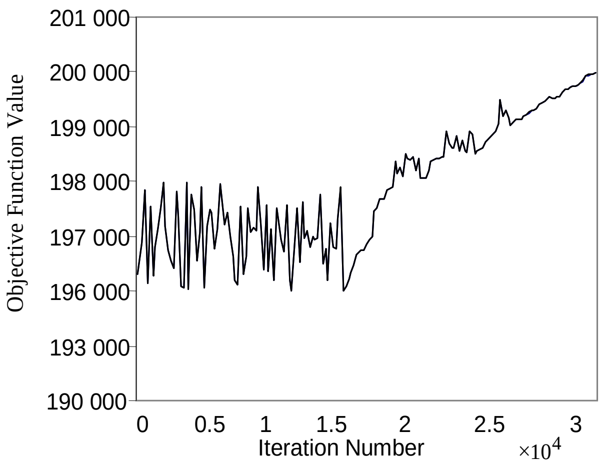

Figure 6. Convergence indicator in improving the solutions.

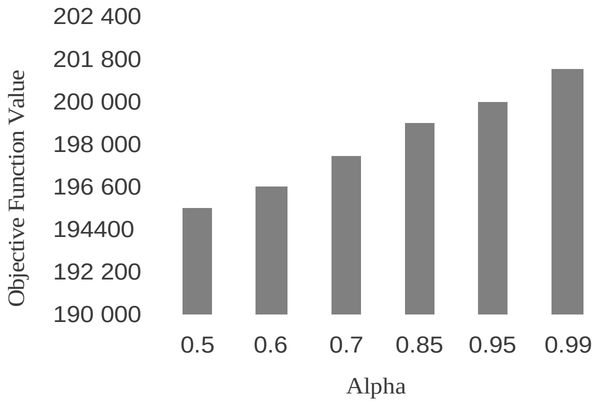

Figure 7. Effect of parameter α on the OFV.

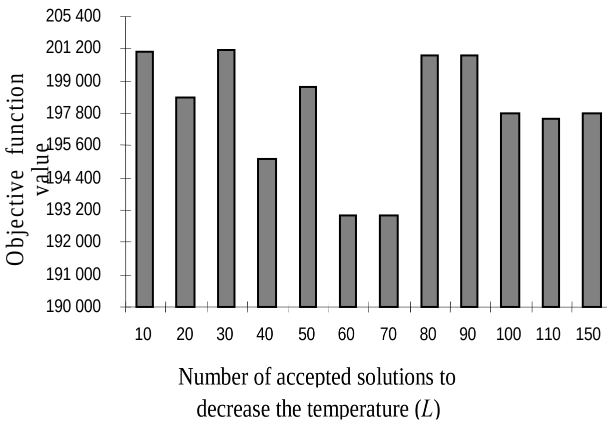

Figure 8. Effect of parameter L on the OFV.

stochastic formulation wherein car demands and travel times are assumed to be stochastic. The second direction for further research is to create a multi-objective optimization model for rail-car fleet sizing.

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