

A STOCHASTIC MODEL FOR INDIRECT CONDITION MONITORING USING PROPORTIONAL COVARIATE MODEL

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Abstract This paper introduces a model to make decision on the maintenance of a mechanical component subject to condition monitoring. A stochastic model is used to determine what maintenance action should be taken at a monitoring check and the follow up inspection times. The condition of component has a stochastic relation with measurements. A new state space model is developed and used, to predict the hazard rate and condition monitoring measurements, to indirectly assess the hazard rate of the system. The Proportional Covariate Model (PCM) which was proposed by Yong Sun (2004) was also used to develop the model. The known Kalman Filter was employed to derive the probability of the conditional hazard rate, which is predicted and updated for condition monitoring. The maintenance is being performed based on the estimated hazard rate so that the desired level of reliability is achieved, in a cost effective approach. This approach is validated by using the experimental data obtained from gearboxes which ran and failed on the Mechanical Diagnostic Test Bed (MDTB) at the Penn State University Applied Research Laboratory.

Keywords Maintenance, Reliability, Kalman Filter, Condition Monitoring, Hazard Rate

چکیده این مقاله یک مدل تصمیم‌گیری را برای قطعه‌ای که دارای پایش شرطی است ارائه می‌کند. براساس مدل ارائه شده می‌توان نوع اقدام نگهداری و تعمیرات لازم و زمان بازرسی بعدی را معین نمود. پایش‌ها به وضعیت قطعه و در یک شرایط احتمالی قرار دارند. یک مدل حالت-فضا ساخته شده و به منظور پیش‌بینی مقدار نرخ خطر و اندازه‌گیری‌های لازم برای تعیین شرایط به کار برده شده است. مدل طراحی شده بر پایه مدل یونگ (۲۰۰۴) می‌باشد. از فیلتر کالمن برای استخراج احتمال شرطی نرخ خطر با توجه به پایش انجام شده، استفاده گردیده است. نگهداری و تعمیرات ناشی از مدل‌های این مقاله با هدف ایجاد پایایی مطلوب و هزینه‌های قابل قبول تعیین می‌گردند. مدل با اطلاعات واقعی برخی قطعات مکانیکی که از مراجع رسمی و معتبر بین‌المللی (مرکز تحقیقات دانشگاه پن) اخذ گردید، اعتبارسنجی شده است.

1. INTRODUCTION

Production systems have critical components and their failure may lead to total shut down of a whole production line, which will result in a substantial loss. The failure of mechanical components may occur because of gradual deterioration, that comes from fatigue, like crack propagation, erosion and tool blunting. In addition, these kinds of failures in mechanical systems decrease safety and therefore cause more irreparable damages. The importance of preventive maintenance, whose aim is to repair

or replace the components before complete failure, is notable.

If deterioration level of components correlated strongly with a control parameter that shows system's state, it is better to make a decision about preventive maintenance operations, based on system's condition which is called, Condition Based Maintenance (CBM).

In general CBM has two categories: Direct CBM and Indirect CBM. In CBM the decision about maintenance time and operation is made, based on the measurements of control parameters at

inspection points. In direct CBM, the control parameter is the original state of the component and shows its depreciation, so that, it can be measured directly, like the thicknesses of a brake pad. Sometimes it is not possible to measure the amount of depreciation during the operation. In this case some variables which are stochastically correlated with the amount of depreciation are measured. For example, the cutting forces, vibration or temperatures are measured to estimate the state of a machining tool in operation. In most of CBM models, a decision is made based on the thresholds which have been defined for control parameters. Maintenance should be carried out before functional failure, when these parameters exceed a defined limit. In many instances no clear set of limits or rules have been developed, to indicate whether or not a failure process is underway and how much time is available before the component is no longer able to perform one or all of its functions. In Wang and Christer's model [1] the follow up inspection time is determined with the obtained data about the conditions, regarding critical values for the system's state. Based on their models, some rules and policies for inspection, repair or replacement were proposed and because of the system's diversity, they checked the mechanical parts of the proposed models. In this paper, the proposed model and the applied rules were tested on gearboxes as an important mechanical system in car industries. All previous models in the literature usually concentrated on periodic inspection. Chelbi, et al [2] presented a mathematical model for optimizing expected total cost in order to determine optimal inspection time interval. Barbara, et al [3] have used a dynamic method to make a decision about maintenance operation of a system with two series unit and fixed inspection intervals. The aim was minimizing the expected total cost.

Grall [4] developed a model for determining inspection points, based on some critical depreciation level. Another method is discussed by Chen, et al [5], in which the optimum amount of critical threshold for depreciation is determined for different values of the inspection rate, by using Semi Markov Process. When indirect information is involved such as vibration monitoring, or covariates such as the oil temperature of an engine, the ordinary approach is to model the hazard rate. One of the most widely used methods for the study of the

effect of covariates is the Proportional Hazard Model (PHM). The basis of the model is the simple assumption that the hazard rate is affected in a multiplicative way by a risk factor. In all, condition monitoring models, in which PHM is used, the covariates are assumed to follow a Markov process. In most of these models the goal is taking a critical threshold for the hazard rate in order to minimize expected cost and the inspections are taken periodically [6-9]. Wang, et al [10] developed a general approach in modeling indirect CBM to determine periodic inspection times for maintenance, based upon the condition monitoring and preventive maintenance information obtained by stochastic filtering theory. Wang, et al [11] provided a CBM model for a factory in which the decision was made based on the residual life time, estimated at periodic inspection times. They considered the maintenance history and the expert judgment as indirect information. The hazard rate was modeled as a continuous stochastic process.

In this paper an indirect condition monitoring decision model is proposed for mechanical components, which can be applied in different conditions, even when the data on the history of failure is limited.

Proportional Covariate Model (PCM) which was proposed by Sun, et al [12] is used to define a relationship between the covariates and the hazard rate. A state space model described for predicting the hazard rate. Kalman filter is used to estimate and update the hazard rate according to condition monitoring information obtained, at inspection times; this approach is discussed in Section 4.2. A decision model is proposed based on the estimated hazard rate to determine what maintenance action to take and when the next measurement shall be taken, in order to achieve a desired level of reliability for a cost effective way. This model can be used for mechanical systems whose hazard rates depend on deterioration.

To illustrate the model and modeling process in a non-maintenance case, a numerical example based on the experimental data was taken from a gearbox ran to failure on the Mechanical Diagnostic Test Bed (MDTB) at Penn State University Applied Research Laboratory (ARL) is presented.

Some approaches were introduced to avoid unnecessary maintenance based on condition monitoring by Jardine, et al [13].

They used mathematical models to optimize the related targets. Also, Zhou, et al [14] considered a predictive model for condition monitoring on mechanical system especially on monitored mechanical system. As a new approach for robust detection on gearbox, Zhan et al [15] developed a mathematical and statistical model that used previous data and autoregressive concepts.

2. FAILURE PREDICTION APPROACH FOR MECHANICAL SYSTEMS

Accurate estimation and prediction of the hazard rates of mechanical systems are critical for predictive maintenance activities. The failure prediction of mechanical systems can be conducted in two ways: fault diagnosis from condition monitoring signals and statistical analysis of the data on its history of failures.

Fault diagnosis techniques mainly focuses on feature extraction and defects detection, using different signal processing techniques and pattern recognition. In these methods, different patterns should be recognized due to different failure modes, based on the operator's job training, hence they can not be used in critical applications.

The failure of a mechanical component with a specific failure mode is usually defined as inability to perform its predefined function. Unlike routine failures in electrical components, the failure of a mechanical component usually occurs more gradually rather than being a sudden occurrence. This feature enables the quantification of the hazard rate of mechanical components using deterioration indicators such as the increment of crack's depth or the degree of misalignment. The probability of failure of such systems is dependent on two items: the initiation of the failure, and its propagation.

In this paper a prognostic approach that takes into account both of these events for estimating the hazard rate is proposed.

3. PROPORTIONAL COVARIATE MODEL (PCM)

Condition monitoring data are commonly termed

as covariates in reliability theory and can be classified into two categories [12]:

- Environmental covariates $Z_e(t)$. The changes of these covariates will cause the characteristics of the hazard to change.
- Response covariates $Z_r(t)$. The changes of these covariates are caused by changes of the system's hazards.

The majority of condition monitoring data can be classified as response covariates. These are symptoms that reflect the deterioration of a system.

The proportion hazard model (PHM), which was introduced by Cox [16] and was developed to predict system's hazard with a combination of historical failure data and on-line condition monitoring data. The hazard rate at time t ; $h(t)$ of an item is modeled as a product of the baseline hazard function $h_0(t)$ and a covariate function $\psi(Z_e(t), \gamma)$ as follows:

$$H(t) = h_0(t) \psi(Z_e(t), \gamma) \quad (1)$$

PHM needs sufficient failing data to estimate the baseline function and to weight parameters for each covariate. The effectiveness of PHM is significantly reduced, where the data on the history of its failure is insufficient. In PHM, it is assumed that covariates are explanatory variables and hazard is the response variable. However in practice, response covariates are often monitored to determine the state of the system and hence their responses vary and so the covariates are explanatory variables. PHM is not a perfectly suitable and satisfactory model for this scenario as Moore, et al [17] have demonstrated.

In order to predict the hazard rate of a mechanical system when the history of failure is not available, the PCM was proposed by Sun, et al [12].

In PCM, a function of covariates $\psi(Z_r(t))$ is expressed as follows:

$$\Psi(Z_r(t)) = C(t).h(t) \quad (2)$$

Where $Z_r(t)$ is the covariate function and $C(t)$ is the baseline covariate function and both of them are usually time dependent.

The PCM represented by Equation 2 indicates

that the covariate of a system may change, based on the changes of hazard rate.

$C(t)$ is typically estimated from historical failure data and even in the case of sparse or even zero historical data, it can be determined according to anecdotal experience of the operators of a plant and/or using supplementary information such as data from accelerated life tests.

PCM is used to update the hazard function of a system. The changes of this hazard function are independent of the covariate and hence the updated hazard function can be used to predict the failure time.

In this paper a stochastic model based on the well known Kalman filter is described, so that the relationship between the covariates and the hazard rate is determined by PCM in a state space model.

4. KALMAN FILTER AND THE STATE SPACE MODEL

As mentioned in the previous sections, the focus will be on the hazard rate, rather than the distribution function, to formulate the behavior of the mechanical system.

In this paper the hazard rate is considered to be partially stochastic. There is some random variable, $\eta(t)$, which contributes to the hazard rate at time t . Wang, et al [11] developed a CBM model based on the same assumption that the hazard rate is stochastic and can be described by a Gamma process.

The fault propagation of mechanical systems is sensitive to varying environmental and operational conditions. Besides a common understanding of such systems integrity, that is increased deterioration and the likelihood of failure have positive correlation [18,19]. Therefore it is appropriate to consider stochastic models in hazard prediction.

Given the monitoring information available to date, the key concern is how to predict the hazard rate, as a probabilistic estimation. The Kalman filter is employed to provide an optimal solution to the problems of prediction and updating.

The Kalman filter is a set of mathematical equations that provides an efficient computational (recursive) means to estimate the state of a process,

in a way that minimizes the mean of the squared error [20].

This filter is carried out in two steps. The first is to form the optimal prediction of the state variable, $h(t)$, The second is to incorporate the new observation into the estimator of the state variable using updating equations.

Based on the estimated and predicted hazard rate, an online model is developed to achieve the optimal CBM point.

4.1. The State Space Model It is supposed that the increments of $h(t)$ are described by stochastic model. In this section we formulate the correlation between the condition monitoring measurements, covariates. The hazard rate will be formulated to establish a state space model.

The structure of such a space model is:

$$Z(t_i) = C(t_i).h(t_i) + \xi(t_i) \quad i = 1,2,\dots,n \quad (3)$$

$$h(t_i) = A(t_i).h(t_{i-1}) + \eta(t_i) \quad i = 1,2,\dots,n \quad (4)$$

$Z(t_i)$: The Covariate, Condition at Time t_i
 $A(t_i)$ and $C(t_i)$: The Scalar Coefficients at Time t_i , Which Relates the $h(t_i)$ to $Z(t_i)$
 $h(t_i)$: The Hazard Rate at Time(t_i)
 t_i : The Inspection Point.

At discrete inspection point, a new measurement obtained and the relation between this covariate and the hazard rate is described via Equation 3, which is called measurement equation.

$\xi(t_i)$ is the disturbance at time t_i which is assumed to follow $N(0,R(t))$, where $R(t)$ is the covariance of the disturbances and in our case, is assumed as time varying.

Equation 4 is a transition equation which describes the stochastic behavior of $h(t)$ between two measurement points t_i and t_{i-1} .

$A(t_i)$ relates the unobservable $h(t_i)$ to the previous one, $h(t_{i-1})$.

$\eta(t_i)$ is the stochastic part of the equation which was introduced before and is assumed to follow $N(0,Q(t))$. It should be noted that all disturbances are uncorrelated.

As mentioned before because of the common property of deterioration increments of mechanical

components a very suitable function for their hazard rate is Weibull, which has the form:

$$h(t) = \alpha^\beta \cdot \beta (t) \beta^{-1} \quad (5)$$

With $\beta > 0$ and $\alpha > 0$ called the shape and scale parameter, respectively. In industrial applications, time usually represents working age. The shape parameter is related to the nature of failure, while the scale parameter is related to the size and operational conditions of the component.

Therefore it is reasonable to assume that similar components should have approximately the same shape parameter as a general characteristic.

There are also some references to achieve this parameter for different components, like the manufacturers of the components.

Based on this reality, $A(t)$ can be specified in transition equation in the form:

$$A(t_i) = (t_i/t_{i-1})^{\beta-1} \quad (6)$$

Where t_i and t_{i-1} are two simultaneous measurement times.

The relationship between the covariate and the hazard rate in Equation 3 is determined based on the proportional Covariate Model (PCM), that is:

$$Z(t) = C(t) \cdot h(t)$$

Where $C(t)$ is the baseline covariate function.

4.2. Prediction and Updating $\hat{h}(t_i)$ is defined to be a priori hazard estimate at time t_i given knowledge of the stochastic behavior prior to measurement point t_i and $\hat{h}(t_i)$ to be a posteriori hazard estimate at time t_i based on previous measurements.

So the priori and posteriori estimate errors are

$$\begin{aligned} \bar{e}(t_i) &= h(t_i) - \hat{h}(t_i) \\ e(t_i) &= h(t_i) - \hat{h}(t_i) \end{aligned} \quad (7)$$

Then priori and posteriori estimate error co variances are:

$$\begin{aligned} \bar{P}(t_i) &= E[\bar{e}(t_i)^2] \\ P(t_i) &= E[e(t_i)^2] \end{aligned} \quad (8)$$

The estimator of Kalman filter provides an optimal solution to the problems of prediction and updating.

At time t_i , the state of knowledge about $h(t_i)$ is embodied in the following probability statement for $h(t_i)$.

$$(h(t_i) | Z(t_i)) \sim N(\hat{h}(t_i), P(t_i)) \quad (9)$$

Where

$$Z(t_i) = (Z(t_i), Z(t_{i-1}), \dots, Z(t_1))$$

$\hat{h}(t_i)$ is the posteriori estimate of $h(t_i)$ and $p(t_i)$ is the covariance which is defined by Equation 8. A recursive procedure commences at time $t = 0$ by choosing $\hat{h}(t_0)$.

p_0 is the best estimates of the mean and covariance of the hazard rate at time t_0 .

$$\begin{aligned} \hat{h}(t_i) &= A(t_i) \cdot \hat{h}(t_{i-1}) \\ \bar{P}(t_i) &= A(t_i) \cdot p(t_{i-1}) + Q(t_i) \end{aligned} \quad (10)$$

$\hat{h}(t_i)$ is estimated in two stages:

- Prior to observing $Z(t_i)$:

$$h(t_i) | Z(t_{i-1}) \quad (11)$$

- After observing $Z(t_i)$:

$$h(t_i) | Z(t_i) \quad (12)$$

These are affected by means of prediction and updating stages.

Prediction, At time t_{i-1} , our estimate for $h(t_i)$ is governed by the transition Equation 4, that is:

$$(h(t_i) | Z(t_{i-1})) = (A(t_i) \cdot h(t_{i-1}) + \eta(t_i) \eta | Z(t_{i-1})) \quad (13)$$

Because of the well known result for normal probability distributions and due to the fact that the covariances in Equation 3 and 4 are normally distributed, it can be found that [1]:

$$(h(t_i) | Z(t_{i-1})) \sim N(\hat{h}(t_i), \bar{P}(t_i))$$

Updating, The second stage, namely updating, is to

re-evaluate $h(t_i) | Z(t_{i-1})$ given $Z(t_i)$. The updating equation is essentially

$$h(t_i) | Z(t_i) = ((h(t_i) | Z(t_{i-1}), Z(t_i))) \quad (14)$$

The error between the actual observed condition at time t_i and its value predicted at time t_{i-1} is:

$$e(t_i) = Z(t_i) - \hat{Z}^-(t_i) \quad (15)$$

Where from Equation 3,

$$\hat{Z}^-(t_i) = C(t_i) \cdot \hat{h}^-(t_i) \quad (16)$$

The above quantity is the estimated of $Z(t_i)$. Using results in multivariate statistics and standard properties of normal distribution, it could be resulted that:

$$h(t_i) | Z(t_i) = ((h(t_i) | e(t_i), Z(t_{i-1}))) \quad (17)$$

$$h(t_i) | e(t_i), Z(t_{i-1}) \sim$$

$$N(\hat{h}(t_i) + K(t_i)(Z(t_i) - C(t_i) \cdot \hat{h}^-(t_i)), P(t_i)) \quad (18)$$

Where

$$p(t_i) = p^-(t_i)(1 - k(t_i) \cdot C(t_i))$$

$K(t_i)$: the gain or blending factor that minimizes the posteriori error covariance (8).

The difference $(Z(t_i) - C(t_i) \cdot \hat{h}^-(t_i))$ in (18) is called the measurement innovation, or residual. The residual reflects the discrepancy between the predicted measurement, $C(t_i) \cdot \hat{h}^-(t_i)$, and the actual measurement $Z(t_i)$.

4.3. Estimation of the System Parameters

To apply the state space model, values of parameters are required.

The classical theory of maximum likelihood of observed events is used for estimation. In this approach all events are independent and identically distributed. However, in our case the observations are not independent and a conditional probability density function is used to formulate the joint

density functions as:

$$L = \prod_{i=1}^n p(Z(t_i) | Z(t_{i-1})) \quad (19)$$

Where $P(Z(t_i) | Z(t_{i-1}))$ denotes the pdf of $Z(t_i)$ conditional on the information set at time t_i .

$$1 \leq i \leq n.$$

Since at time t , the estimate $Z^-(t_i)$ is known, it is resulted as before

$$(Z(t_i) | Z(t_{i-1})) = (e(t_i) | Z(t_{i-1})) \quad (20)$$

Further, it can be shown that

$$(e(t_i) | Z(t_{i-1})) \sim N(0, F(t_i)) \quad (21)$$

Where:

$$F(t_i) = C^2(t_i) \cdot P^-(t_i) + R(t_i) \quad (22)$$

It follows that the log likelihood function for the observed covariates based upon the multivariate normal distribution is given by:

$$\log - L = -\frac{n}{2} \log(2\pi) - \frac{1}{2} \sum_{i=1}^n \log |F(t_i)| - \frac{1}{2} \sum_{i=1}^n e^2(t_i) \cdot F^{-1}(t_i) \quad (23)$$

Maximizing in terms of unknown parameters, Equation 23 will give the estimated values of these parameters. It is noted that the likelihood, Equation 23, is very unstable and produce a large variance in parameter estimates. Christer, et al [7] have resolved this problem with introducing just one actual measure of the state variable.

However, in our case the state variable is the hazard rate. In this case the problem has been observed to be resolved by enforcing two limitations and maximizing Equation 23 subject to these limitations.

Mechanical components have approximately the same shape parameter as a general characteristic, so bounds can be determined for this parameter.

Also, the reliability of the component should be at neighborhood of failure time, so with due attention to the relationship between the hazard

rate and the reliability, the area under the obtained $h(t)$ should reflect reliability.

5. THE DECISION RULE

In this section, the decision rule is developed. The proposed rule has a preventive approach for maintenance planning. It is assumed maintenance has to provide the right Reliability of production equipment (or system) and give an economic value to the maintenance result. In most production processes there are critical components which their failure may result in substantial costs. In these cases, the down time cost is much greater than the costs of maintenance, then the primary objective will achieve high reliability.

A decision model is developed to aid in choosing, the best maintenance option and a follow up time to inspect, at each inspection time so that a predefined level of reliability is achieved in a cost effective way. This is a dynamic condition based maintenance model. It is assumed that failure is detected instantaneously upon a failure and the component is renewed and the process of condition monitoring is restarted.

At each monitoring check, a decision is made based on the latest condition monitoring obtained, where permitted actions are:

- To leave the unit as it is ($a = 0$)
- To carry out preventive maintenance on the unit with a specific level of efficiency ε .
 $a = \varepsilon, 0 < \varepsilon < 1$
- To replace the unit with an identical new item immediately, $a = r$ (a is a maintenance action index)

The next best inspection point is determined by the action taken.

Assuming that the unit monitored, is still operative at the i 'th monitoring check time t_i , based on the estimated hazard rate $\hat{h}(t_i)$, an estimate of cumulative hazard rate from t_i to $t_i + \Delta t$ is given by:

$$\hat{H}(t_i + \Delta t) = \int_0^{\Delta t} \left(\frac{t+x}{t}\right)^{\beta-1} \times \hat{h}(t_i) dx = \frac{((t_i + \Delta t)^\beta - t_i^\beta)}{\beta \times t^{\beta-1}} \times \hat{h}(t_i) \quad (24)$$

and an expected reliability at time $t_i + \Delta t$ is approximately given by

$$R(t_i + \Delta t) \approx \exp(-\hat{H}(t_i + \Delta t)) \quad (25)$$

Since the primary objective is achieving high reliability, the next inspection time is defined in such way, that the expected reliability is greater than or equal to a desired level.

Assuming that α denotes the desired level of reliability, then we have:

$$\Delta t^* = \inf(\hat{R}(t_i + \Delta t) \geq \alpha) \quad (26)$$

Where Δt^* is an optimal period from the current time, at the end of which, the next inspection should be done. Because of the dynamic updating nature of condition monitoring only the expected cost per unit time within a period from current time t_i to

$t_i + \Delta t$ is considered. Let new definitions for cost:

- C_i : The average cost per monitoring
- C_f : The average cost of a failure replacement
- C_R : The average cost of a preventive replacement
- $C_{p(\varepsilon)}$: The average cost of a preventive maintenance with efficiency of ε .

Denote that C_R and $C_{p(\varepsilon)} \ll C_f$

The expected cost per unit time, given no preventive maintenance at time t_i and the next inspection at $t_i + \Delta t$, is determined by:

$$ECT_0(\Delta t) = \frac{c_i \cdot R(t_i + \Delta t) + c_f \cdot (1 - R(t_i + \Delta t)) + c_p \cdot p_{rep} \cdot R(t_i + \Delta t)}{\int_0^{\Delta t} \frac{\partial(1 - R(t_i + x))}{\partial X} dx + R(t_i + \Delta t)\Delta t} \quad (27)$$

Where

$$P_{rep} = P(h(t_i + \Delta t) > h^*(t_i + \Delta t))$$

$$h^*(t_i + \Delta t) = \frac{-\ln R^* \cdot \beta \cdot (t_i + \Delta t)^{\beta-1}}{(t_i + \Delta t + t_p)^\beta - (t_i + \Delta t)^\beta} \quad (28)$$

If a preventive maintenance is undertaken at t_i

(Immediately after monitoring check), the estimated hazard rate which should be used in Equation 24 is:

$$\hat{h}^+(t_i) = \hat{h}(t_i)(1 - \varepsilon) \quad (29)$$

ε is the influence of preventive maintenance and if $\varepsilon = 0$ maintenance is ineffective and if $\varepsilon = 1$ the maintenance is perfect and the hazard rate will be reduced to zero.

In this case the expected cost is determined by:

$$ECT_{\varepsilon}(\Delta t) = \frac{c_{p_{\varepsilon}} + C_i \cdot R(t_i + \Delta t) + c_f \cdot (1 - R(t_i + \Delta t)) + c_p \cdot p_{rep} \cdot R(t_i + \Delta t)}{\int_0^{\Delta t} \frac{\partial(1 - R(t_i + x))}{\partial x} dx + R(t_i + \Delta t) \Delta t} \quad (30)$$

The optimal next inspection time is computed by maximizing the efficiency value $\{E_0(\Delta t), E_{\varepsilon}(\Delta t)\}$ describing in the following equations, subject to Equation 26.

$$E_0(\Delta t) = \frac{R(t_i + \Delta t)}{ECT_0(\Delta t)}$$

and

$$E_{\varepsilon}(\Delta t) = \frac{R(t_i + \Delta t)}{ECT_{\varepsilon}(\Delta t)} \quad (31)$$

The component will be replaced if Δt^* to be smaller than or equal to the time needed for preventive replacement preparation.

$$\Delta t^* \leq t_p \quad (32)$$

In other words, preventive replacement is taken when Equation 26 is cancelled by Equation 32.

6. NUMERICAL RESULTS

In this section a numerical result is presented to

predict the hazard rate based on a proposed model. The data used here is from three gearboxes run to failure on the Mechanical Diagnostic Test Bed (MDTB) at Penn State University Applied Research Laboratory (ARL). This source was selected after some negotiation with two car producing research centers in Iran. ARL is one of the reference laboratories for mechanical systems data in the world.

The gearboxes were run at 540 in lb torque for the first 96 hours of each test and overloaded during the tests to accelerate the onset of failure. The gearboxes consist of a gear and a pinion with gear ratio 1:1.533.

Figure 1 shows the kurtosis of the residual signal measured at operation hours for test run 7. A residual signal is obtained from the signal average by filtering out gear meshing harmonics. It represents random transmission errors for healthy gears. For faulty gears, the transmission errors will include a sudden change, which becomes non-Gaussian. Kurtosis is a good measure of non-Gaussian (e.g. spikiness) in a signal. Vibration acceleration readings were taken at 8-h intervals during the 96 run in period and at 30 minutes intervals during the high load operational phase.

The failure mode examined here is "Gear tooth fracture" [21,22].

Previous researches [1,18,19] have revealed that the kurtosis of the residual signal has a good relation with crack of the test gear.

Four options for the format of the parameters have been tested:

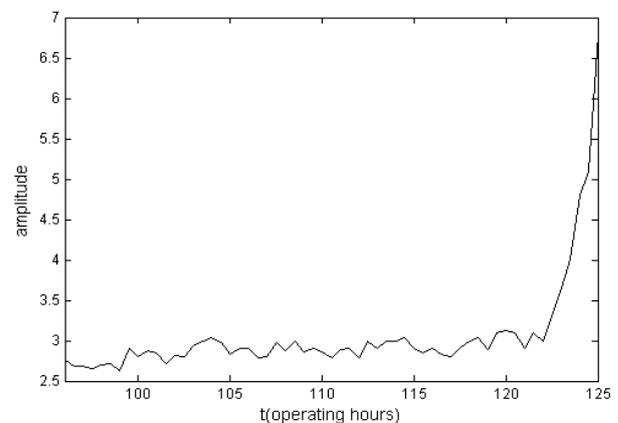


Figure 1. The kurtosis of the residual signal for test run 7.

- $Q(t)$ and $R(t)$ are time-invariant
- $Q(t)$ takes a time-invariant, and $R(t)$ has the form of $Q(t) = ct^d$
- $Q(t) = ct^d$ and $R(t) = rt^h$
- $Q(t)$ is time-invariant and $R(t) = rt^h$

$A(t)$ is undertaken the form of $(t_i/t_{i-1})^{\beta-1}$ as mentioned before. It has been shown that $C(t)$ is taken the form of $C(t) = ct^d$ for gearboxes [1].

The largest log-likelihood value using Equation 23 is model option (2) (see Table 1).

The parameter values obtained for test run 7 based upon option (2) is shown in Table 2.

The potential number of time-variant model options is unlimited. However, the model option (2) has produced a good fit to the data. The form of $Q(t)$, ct^d are reasonable, since hazard rate increase rapidly near and after crack initiation, the magnitude of $Q(t)$ is increased in order to account for both uncertainty about hazard's increments and uncertainty in the model. Figure 2 shows the estimated hazard rate using the estimated parameter values from Table 2 based on the data obtained over test run 7.

In order to show the applied estimated hazard rate in the proposed decision model, the following

data were used:

C_i	15 Unit Cost;
C_R	100 Unit Cost;
C_f	1000 Unit Cost;
t_p	0.5 Hour
R^*	0.95

In this case, since there is no preventive maintenance data available, $ECT_e(\Delta t)$ is excluded and therefore the decision variable at each decision epoch is whether or not to replace the gearbox; and if not, when the next measurement should be taken. If the current time is set at 122.5 which is the time near the crack initiation point [21] using Equation 26, it is resulted that $\Delta t^* \leq 1.14$.

Since it is smaller than $t_p = (0.5)$, The next step is calculating $E_0(\Delta t)$ to identify the best next inspection point.

Figure 3 shows the efficiency value graphically, which has a maximum at $t = 123.6$. It is noted that in this case, since the measurements has been taken only at 30 minutes intervals, Δt^* is restricted to be approximated to 123.5. Similarly at time $t = 123.5$ it is $\Delta t^* \leq 1.06$ and subsequently the efficiency value will be as in Figure 4, which is maximized at

TABLE 1. Log-Likelihood Values of Parameter Options.

Option	(1)	(2)	(3)	(4)
Log-likelihood	98.25	185.72	148.45	100.43

TABLE 2. Fitted Results for Option (1).

Parameter	A	B	r	β	C	D
Value	$8.16(10^8)$	-3.51	0.5	4.65	$4(10^{-7})$	2.55
Standard error	0.4	0.7	0.004	0.04	$1(10^{-6})$	0.03

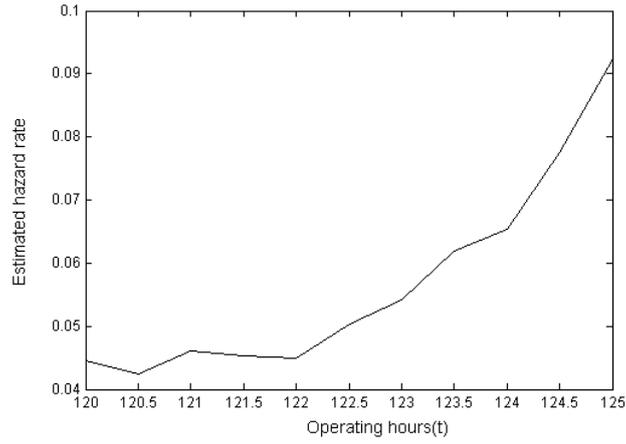


Figure 2. Estimated hazard rate for test run 7.

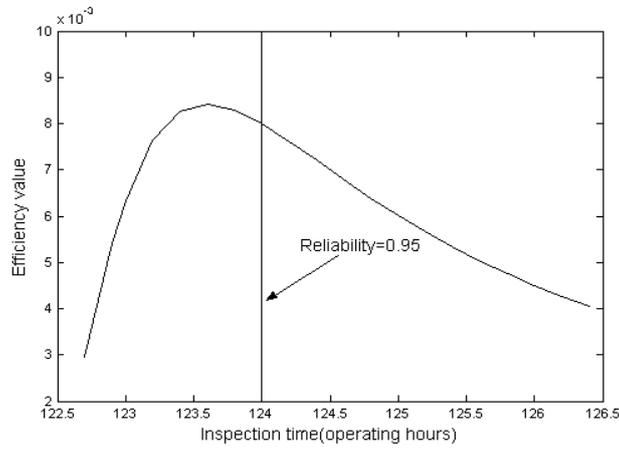


Figure 3. The efficiency value calculated at $t = 122.5$.

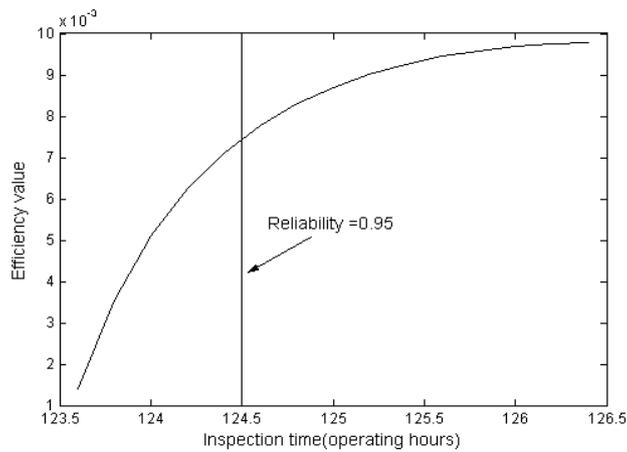


Figure 4. The efficiency value calculated at $t = 123.5$.

time $t = 124.5$.

At this time point Δt^* is smaller than $t_p(\Delta t^* \leq 0.4)$, so the optimal policy is preventively replacing the gearbox. As expected and also as time increases, the optimal inspection time decreases to satisfy the desired level of reliability. Furthermore, it is also observed that preventive replacement is made before failure, without an unnecessary removal of the un-failed gearbox.

7. CONCLUSION

A preventive maintenance policy based on indirect condition monitoring information was proposed. A stochastic model using Kalman filter was presented to estimate and predict the hazard rate based on monitoring up dated data. The optimal policy at each inspection point advises what maintenance action to take and also determine the optimal next inspection time. It was shown that the proposed policy is a good one when cost of loosing production is much greater than the maintenance costs and the primary objective is achieving high reliability. The proposed decision model can also be applied to other maintenance models; in which hazard rate is estimated from the indirect condition monitoring. Some simplifying restrictions in this paper can be lifted in future researches to yield a better filter. For example, a nonlinear state space model could be developed, which of course, increases computational problems.

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