CONTINUOUS DISCRETE VARIABLE OPTIMIZATION OF STRUCTURES USING APPROXIMATION METHODS

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Abstract Optimum design of structures is achieved while the design variables are continuous and discrete. To reduce the computational work involved in the optimization process, all the functions that are expensive to evaluate, are approximated. To approximate these functions, a semi quadratic function is employed. Only the diagonal terms of the Hessian matrix are used and these elements are estimated from the first derivatives that are available from the previous iterations. The second order approximation is obtained for both direct and reciprocal approximations. In addition, a hybrid form of the approximation is introduced. With the help of this approximation, the continuous optimization is obtained. The results are used as the starting point for the discrete optimization. A new penalty function is introduced for discrete optimum design and the discrete variables are obtained in conjunction with the same function approximation. Examples are given and the numerical results are discussed.

Key Words Continuous Optimization, Discrete Optimization, Approximation Concepts, Penalty Functions

چکیده در این مقاله طرح بهینهٔ سازه‌ها با استفاده از متغیرهای پیوسته و گسسته مورد نظر است. برای کاهش محاسبات در پروسهٔ بهینه‌سازی تمامی توابعی که محاسبهٔ آنها وقت‌گیر است، تقریب زده می‌شوند. برای تقریب این توابع، از یک تابع شبه درجه دو استفاده می‌شود. تنها جملات قطری ماتریس هسیان استفاده شده و با استفاده از مشتق اول که از تکرار قبل بدست آمده ساخته می‌شود. یک تابع ترکیبی تقریب نیز ساخته می‌شود. با استفاده از این تقریب‌ها طرح بهینهٔ پیوسته بدست آمده و نتایج بدست آمده بعنوان نقطهٔ شروع بهینه‌سازی گسسته در نظر گرفته می‌شود. تابع پنالتی جدیدی برای طرح بهینهٔ گسسته ایجاد شده و مقادیر بهینهٔ گسسته همانند قبل با استفاده از روش تقریبی محاسبه می‌شود. نتایج عددی چند مثال بیان می‌شود.

1. INTRODUCTION

The idea of approximation concepts is now well established in numerical optimization techniques. The optimum design procedure requires the evaluation of the objective function and the constraints at a number of design points. The evaluation of some of the functions such as member forces, displacements, frequencies are time consuming. These functions are approximated and an approximate design problem is solved with move limits. The results are employed as a starting point for the next design iteration. The process is continued until the optimum design process converges. The use of function approximation was first introduced by implementing a first order Taylor series expansion for design constraints [1]. To increase the quality of approximation, the cross sectional properties were employed as the intermediate variables [2]. These variables for frame structures are taken as cross sectional areas and moments of inertia. The idea of force approximation was later introduced in order to enhance further the quality of stress approximation [3]. In this case, first the element forces are approximated with respect to cross sectional properties and then the stresses are recovered from the approximate forces. The numerical results indicate that the number of design iterations is decreased when member forces are taken as the intermediate responses. This is basically due to the fact that the variation of forces is not very sensitive to the cross sectional properties.

Some midrange approximations were proposed by using two-point approximation [4]. The approximate functions were considered linear or quadratic by considering some non-linearity indices. The indices were selected from the data available in the previous iterations. A three-point function approximation was introduced by using the information of three consecutive design points [5]. In all the multi-point approximation, the unknown coefficients should be evaluated by solving some algebraic equations and in some cases numerical errors may encounter.

A quadratic function approximation was outlined in which all the elements of the Hessian matrix were estimated from the existing data [6]. In fact, an approximate Hessian matrix was developed based on the first derivatives. This approach is effective for design optimization as it reduces the design cycles. However, for all the constraints under consideration, the Hessian matrices must be created and stored. For further discussion of the approximation concepts Ref. [7] can be consulted.

As far as the discrete variable optimization is concerned, less research work has been carried out. Practical and efficient methods of optimum design of structures is based on choosing the design variables from a set of available values, while the computational work involved in the design process is reduced as much as possible. There are several methods for optimum design of structures with discrete variables such as branch and bound, duality theory, penalty functions, genetic algorithms, simulated annealing, etc. Each of the methods has some limitations and difficulties [8]. Among these methods, penalty functions are easy to implement and if they are combined with approximation concepts, efficient methods for both continuous and discrete optimization can be achieved. There are various techniques for employing the penalty functions for continuous optimization. However, a few penalty functions exist for the solution of problems with discrete variables. In the literature, a sine-function [9] and a quadratic function [10] have been introduced in this regard. These methods have been combined with approximation of structural responses to enhance the efficiency of the methods [11-13].

In the present work, a semi quadratic function is developed in which only the diagonal elements of the Hessian matrix are estimated. The diagonal elements are evaluated by matching the first derivatives of the function with those of the previous iteration. Explicit relations are obtained to find the necessary unknowns, thus numerical procedures are not necessary to evaluate the second derivatives. The function is expressed in terms of the direct variables as well as the reciprocal variables. The necessary criteria are established to create a hybrid form of the direct and reciprocal approximations. In addition, a new penalty function is proposed for discrete optimization. The process of discrete variable optimum design is also combined with the function approximation in order to reduce the number of iterations.

2. FUNCTION APPROXIMATION

Given a function G(X), the quadratic approximation with the diagonal elements of the Hessian matrix is expressed as

$$G_Q(X) = G_Q(X_1) + \sum_{i=1}^{n} G_{,i}(X_1)(x_i - x_{1i}) + \frac{1}{2}\sum_{i=1}^{n} G_{,ii}(X_1)(x_i - x_{1i})^2 \quad (1)$$

where $X=[x_1, x_2, \ldots, x_i, \ldots, x_n]$ is the vector of design variables with n variables and X_1 is the current design point. x_{1i} is the i^{th} element of X_1. The notations $G_{,i}$ and $G_{,ii}$ represent the first and second derivatives, respectively. The subscript Q reflects the quadratic approximation. The quadratic reciprocal approximation with the use of intermediate variables

$$y_i = \frac{1}{x_i} \quad (2)$$

is arranged as follows by substituting Equation 2 into Equation 1

$$G_{QR}(X) = G_Q(X_1) + \sum_{i=1}^{n} G_{,i}(X_1)(x_i - x_{1i})\frac{x_{1i}}{x_i}(2 - \frac{x_{1i}}{x_1}) + \frac{1}{2}\sum_{i=1}^{n} G_{,ii}(X_1)(x_i - x_{1i})^2 \ (\frac{x_{1i}}{x_i})^2 \quad (3)$$

First subtracting Equation 1 from Equation 3 establishes the conservative approximation, which is a hybrid form of Equations 1 and 3

$$G_Q(X) - G_{QR}(X) = \sum_{i=1}^{n} G_{,i}(X_1)[(x_i - x_{1i}) - (x_i - x_{1i})(2 - \frac{x_{1i}}{x_1})\frac{x_{1i}}{x_i}] + \frac{1}{2}\sum_{i=1}^{n} G_{,ii}(X_1)[(x_i - x_{1i})^2 - (\frac{x_{1i}}{x_i})^2 (x_i - x_{1i})^2] \quad (4)$$

After some mathematical manipulation Equation 4 can be expressed as

$$G_Q - G_{QR} = \sum_{i=1}^{n} (\frac{x_i - x_{1i}}{x_i})^2 [G_{,i}(X_1)(x_i - x_{1i}) + \frac{1}{2} G_{,ii}(X_1)(x_i^2 - x_{1i}^2)] \quad (5)$$

Equation 5 can be shown as

$$G_Q - G_{QR} = \sum (\frac{x_i - x_{1i}}{x_i})^2 . \ \alpha_i \quad (6)$$

where

$$\alpha_i = G_{,i}(X_1)(x_i - x_{1i}) + \frac{1}{2} G_{,ii}(X_1)(x_i^2 - x_{1i}^2) \quad (7)$$

It can be seen that the sign of (G_Q-G_{QR}) depends only on α_i. Suppose that G(X) represents a constraint of the form G(X)≤0, then

If $\alpha_i \leq 0$ implies $G_Q \leq G_{QR}$

Thus G_{QR} is more conservative (less negative) than G_Q. In this case G_{QR} is more effective. On the other hand, if $\alpha_i > 0$, then the use of G_Q will be more conservative. Thus the criteria for the conservative (hybrid) approximation can be stated as

If $\alpha_i \leq 0$ use G_{QR}

Otherwise use G_Q.

3. ESTIMATION OF SECOND ORDER DERIVATIVES

The evaluation of the exact second order derivatives is time consuming. The approximate values of $G_{,ii}$ are found from the condition that the first derivatives of G(X) match those of G_Q or G_{QR} at the previous design point X_0. Therefore depending on the sign of α_i the first derivatives of Equation 1 or Equation 3 is matched with that of the previous point as follows:

(a) If $\alpha_i > 0$, by using Equation 1,

$$G_{,i}(X_0) = G(X_1) + G_{,ii}(X_1)(x_{01} - x_{1i}) \quad (8)$$

yields

$$G_{,ii}(X_1) = \frac{G_{,i}(X_0) - G_{,i}(X_1)}{x_{0i} - x_{1i}} \quad (9)$$

(b) if $\alpha_i \leq 0$, by using Equation 3 after the necessary manipulation

$$G_{,ii}(X_1) = \frac{x_{0i}^3 G_{,i}(X_0) - x_{1i}^2(3x_{0i} - 2x_{1i})G_{,i}(X_1)}{x_{1i}^3(x_{0i} - x_{1i})} \quad (10)$$

It can be seen from Equations 9 and 10 that for the evaluation of the second derivatives, only the first derivatives of the function under consideration are required at two design points. It is to be mentioned that in the first iteration a linear function approximation must be used.

4. CONTINUOUS OPTIMIZATION

Suppose a constrained optimization problem with m inequality constraints is expressed as

Minimize F(X), subject to $g_j(X) \leq 0$, j=1, m (7)

where F(X) is the objective function and $g_j(X)$, j=1,m represent the constraints. Penalty function methods are employed to solve the optimization problem. This can be achieved by solving the following unconstrained problems

$$\text{Minimize}\,\phi(X,r) = F(X) + rP(X) \quad r = r_1, r_2, ... \tag{12}$$

where ϕ is an auxiliary function, r is the penalty multiplier and P(X) is the imposed penalty function which is the function of the constraints. Any appropriate penalty function can be used for continuous optimization. In this work a quadratic extended interior penalty function has been employed [14]. By changing the value of r the minimum of ϕ approaches the minimum of F. The main steps for the solution of this problem in conjunction with approximation concepts can be summarized as follows:

(a) Perform a finite element analysis of the structure and evaluate all the structural responses such as element forces, nodal displacements, etc.
(b) Find the first derivatives of the necessary responses with respect to the intermediate variables and establish the approximate relations for the responses under consideration.
(c) Formulate the approximate optimization problem and evaluate the initial value of multiplier r. By gradually reducing the value of r, solve the approximate problems until the continuous problem converges.
(d) If the overall design optimization has not converged then update the analysis model and go to step (a).

5. DISCRETE OPTIMIZATION

The results of the continuous optimization are used as the starting point for the discrete variable optimization. To perform the discrete optimization, now the following problem is considered:

$$\begin{aligned} &\text{Minimize}\,\phi(X,r,s) = F(X) + rP(X) + sQ(X) \\ &\qquad r: \text{ constant} \\ &\qquad s = s_1, s_2, ... \end{aligned} \tag{13}$$

in which Q(X) is the imposed penalty function for discrete variables for which the following new function is presented:

$$Q(X) = \sum_{i=1}^{n} (e^{\beta.q(x)} - 1) \tag{14}$$

where

$$q(x) = \frac{(x_i - d_i^l)(d_i^u - x_i)}{(d_i^u - d_i^l)^{\gamma}} \tag{15}$$

and d_i^l and d_i^u are lower and upper discrete values for x_i. The value of γ can be taken as either of 0, 1 and 2, depending on the space of discrete values. A suitable value of γ can be considered as 2. Equation 14 can be scaled such that the value of Q(X) at midpoint becomes unity, i.e.

$$Q(X) = 1 \text{ at } x_i = \frac{1}{2}(d_i^l + d_i^u)$$

which yields $\beta = 2.7726$ for $\gamma = 2$. The numerical results indicate that this form of scaling produces a smooth function and thus makes the discrete variable optimization easier. In addition, the proposed function is parameter independent.

In this step the multiplier r is kept constant and by changing s, the discrete variable optimization is achieved for one design cycle. The numerical results show that the performance of the function is better than the existing functions. The nature of the function is such that it changes smoothly at the discrete points. The main steps in the discrete optimization are as follows:

(a) Repeat similar Steps a and b in the process of continuous optimization to establish the approximate discrete design problems.
(b) Freeze the multiplier r and find the initial value of the multiplier s. Solve the approximate design problems by gradually increasing the multiplier s, until the discrete problems converge. Check if the solution is feasible and discrete, if it is not feasible, increase r and reinitialize s and go to Step a.
(c) Check the overall convergence. If converged terminate, otherwise update the discrete problem and go to Step a.

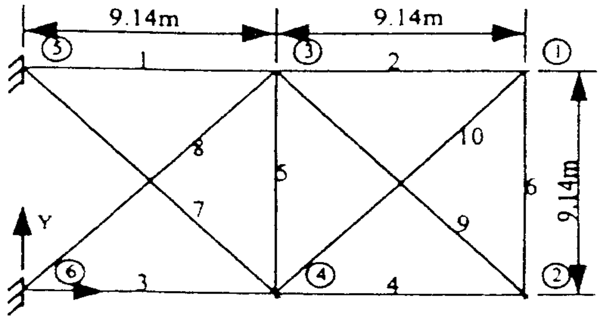

Figure 1. Ten-bar truss.

6. INITIAL VALUES OF MULTIPLIERS r AND s

The multiplier r is only used for the continuous optimization. The necessary condition for $\phi(X, r)$ represented by Equation 12 to be minimized is that the first partial derivatives must vanish. Therefore a suitable choice for the initial value of r would be given by the r that minimizes the magnitude of the squire of the gradient of ϕ (X, r) at the starting point X_0, that is

$$\left\|\varphi_{,}(X_0, r)\right\|^2 = \min_{r}\left\|F_{,}(X_0) + r\,P_{,}(X_0)\right\|^2 \tag{16}$$

where $\left\|\varphi_{,}(X_0, r)\right\|^2$ denotes the Euclidean norm of $\phi_{,}(X_0,r)$ and $\phi_{,}(X_0,r)$ is the gradient of $\phi(X_0,r)$. Then the value of r can be obtained from Equation 16 as

$$r = \frac{-F_{,}(X_0)^T P_{,}(X_0)}{\left\|P_{,}(X_0)\right\|^2} \tag{17}$$

Equation 17 can be used providing it yields r>0. Because of the initial value of X_0, it may happen that r < 0. In such a case either X_0 may be changed or the initial value of r can be chosen in such a way that at X_0, the two terms F(X) and r P(X) do not differ greatly in value. Hence, a reasonable value of r can be obtained when

$$F(X_0) = r\,P(X_0) \rightarrow r = F(X_0)/P(X_0) \tag{18}$$

The same approach can be argued to estimate the initial value of s [11]. Thus

$$s = \frac{-F_{,}(X_0)^T Q_{,}(X_0)}{\left\|Q_{,}(X_0)\right\|^2} \tag{19}$$

provided s>0, otherwise $s=F(X_0)/Q(X_0)$.

7. NUMERICAL RESULTS

A computer program has been developed based on the preceding discussion. In this study the results of four examples are presented. To compare the results, the examples are solved by the following methods:

1. Linear Approximation (LA): In this method only the first two terms of the Taylor series are chosen. For stress constraints, the direct approximation and for displacement constraints the reciprocal approximations are employed.
2. Quadratic Approximation (QA): In this method the first three terms of the Taylor series expansion with diagonal Hessian matrix are used. Again for stress constraints the direct quadratic approximation (Equation 1) and for displacements the reciprocal quadratic approximation (Equation 3) are considered.
3. Hybrid Linear Approximation (HLA): The hybrid approximation presented in Ref. [15] is employed. In this method, based on the sign of the first derivatives, the direct or reciprocal approximation is used.
4. Hybrid Quadratic Approximation (HQA): The method outlined in the present study.

In all cases, the member forces are first approximated and then the approximate stress constraints are established [16]. The initial move limit is considered as 90% and is gradually reduced by 10% in each iteration.

7.1. Problem 1. Ten-Bar Truss

The standard test problem shown in Figure 1 is solved with stress and displacement constraints. The material properties are given as Young's modulus, $E = 6.9\times10^{10}$ N/m^2, weight density, $\rho = 2.77\times10^3$ Kg/m^3 and allowable stresses, $\sigma_a = \pm\ 1.72\times10^8$

TABLE 1. Iteration Histories of 10-bar Truss for Continuous Variables (Weight: Kg).

Iteration No.	LA	QA	HLA	HQA
0	1904.1	1904.1	1904.1	1904.1
1	2016.9	2016.9	1537.2	2016.9
2	2202.5	1033.0	1735.2	2022.1
3	2160.4	1328.5	1729.3	2119.6
4	2160.4	2071.8	1595.7	2119.6
5	2294.2	2243.6	1831.8	3352.9
6	2305.5	2243.0	1911.8	2298.2
7	2310.6	2317.5	2137.6	2298.2
8	2318.6	2294.4	2203.8	
9	2315.6	2294.8	2286.2	
10	2315.6		2286.3	

TABLE 2. Iteration Histories of 10-bar Truss for Discrete Variables (Weight: Kg).

Iteration No.	LA	QA	HLA	HQA
1	2315.8	2318.9	2293.6	2334.5
2	2326.6	2322.8	2303.8	2335.1
3	2322.2	2332.7	2310.8	2335.1
4	2319.3	2342.4	2321.8	
5	2317.2	2344.3	2328.6	
6	2317.2	2344.3	2328.6	

TABLE 3. Optimum Design for 10-bar Truss (Continuous): cm^2.

Member	LA	QA	HLA	HQA
1	181.89	180.59	180.37	189.02
2	15.08	2.85	3.47	9.10
3	161.20	154.39	153.18	158.38
4	82.84	86.55	95.79	87.32
5	9.30	2.84	2.47	4.90
6	0.645	1.04	1.43	0.766
7	73.23	67.22	54.28	59.33
8	116.60	132.54	136.91	129.69
9	116.73	133.31	133.98	126.25
10	21.28	4.99	4.43	8.74

N/m^2 for all members. One load case is considered as $P_{2y}=P_{4y}=-4.45\times10^5$ N. In addition to the stress constraints, displacement limits of $\pm5.08\times10^{-2}$ m are imposed on the vertical direction of each joint. The cross-sectional areas of members are considered as design variables. The initial value and minimum size limits are 6.45×10^{-3} m^2 and 6.45×10^{-5} m^2, respectively. The set of available discrete values for the cross-sectional areas is

$$A_i \in \{0.645,\ 1,\ 5,\ 10,\ 15,\ 20,\ 25,\ ...\} \quad (\text{cm}^2)$$

Iteration histories of the above mentioned four

TABLE 4. Optimum Design for 10-bar Truss (Discrete): cm^2.

Member	LA	QA	HLA	HQA
1	180	180	180	190
2	15	1	5	10
3	165	155	155	160
4	85	85	100	90
5	10	5	5	5
6	0.645	5	1	0.645
7	75	70	55	60
8	115	140	140	130
9	115	135	135	130
10	20	5	5	10

TABLE 5. Comparison of the Results for 10-bar Truss.

	No. conti. anal.	No. discr. anal.	Cont./discr. Weig.	Execution time (sec.)	Max. constraint
LA	9	5	2315/2317	4.0	0.066
QA	8	5	2294/2344	6.0	0.003
HLA	9	5	2286/2328	6.0	0.026
HQA	6	2	2298/2335	5.0	0.003

approximation methods are presented in Tables 1 and 2 for continuous and discrete optimization, respectively. In these tables the weight of the structure is given in Kg. Iteration number 0 indicates the initial design point. The optimal continuous and discrete design variables are given in Tables 3 and 4, respectively. Comparison of the methods in terms of the number of analyses, weight, execution time (wall-clock time) and maximum constraints for continuous optimization is presented in Table 5. From the numerical results, it can be seen that the number of required iterations in the proposed method is less than other methods. The execution time in all the methods is near, however, this is a small structure and the time can not be considered as a major factor.

7.2. Problem 2. 25-Bar Space Truss The structure is shown in Figure 2 and the material properties are given as Young's modulus, $E = 6.9\times10^{10}$ N/m^2, weight density, $\rho = 2.77\times10^3$ Kg/m^3 and allowable stresses, $\sigma_t = +\ 2.76\times10^8$ N/m^2 for all tensile members [17]. Compression stress limits and linking variables are given in Table 6. The structure is subjected to two load cases as shown in Table 7. The displacement limit

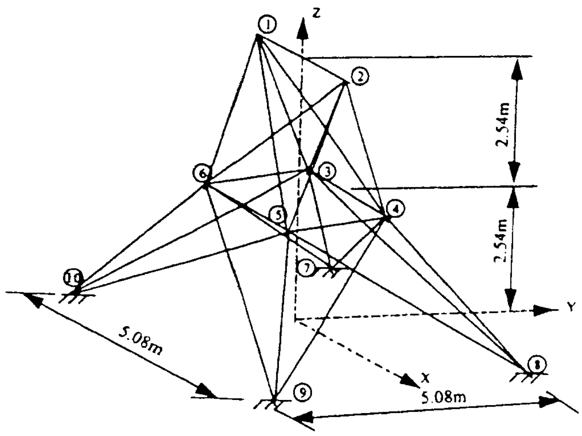

Figure 2. 25-bar space truss.

TABLE 6. Variable Linking and Allowable Stresses for 25-bar Truss.

Variable	Compression stress limit, N/m^2	Linking members
1	-2.42×10^{8}	1-2
2	-7.99×10^{7}	1-4; 2-3; 1-5; 2-6
3	-1.19×10^{8}	2-4; 2-5; 1-6; 1-3
4	-2.42×10^{8}	4-5; 3-6
5	-2.42×10^{8}	3-4; 5-6
6	-4.66×10^{7}	3-10; 6-7; 5-8; 4-9
7	-4.80×10^{7}	4-7; 3-8; 5-10; 6-9
8	-7.64×10^{7}	6-10; 3-7; 4-8; 5-9

TABLE 7. Load Condition for 25-bar Truss (N).

Load case	Joint	X dir.	Y dir.	Z dir.
1	1	4450	44500	-22250
	2	0	44500	-22250
	3	2225	0	0
	6	2225	0	0
2	5	0	89000	-22250
	6	0	-89000	-22250

TABLE 8. Iteration Histories of 25-bar Truss for Continuous Variables.

Iteration No.	LA	QA	HLA	HQA
0	1500	1500	1500	1500
1	951	951	951	951
2	309	459	278	493
3	270	318	267	270
4	261	235	254	252
5	258	269	255	252
6	255	252	255	
7	253	256		
8	252	256		
9	252			

is $\pm8.89\times10^{-3}$ m. the initial value and minimum size limits are 6.45×10^{-3} m^2 and 6.45×10^{-5} m^2, respectively. The set of available discrete values for the cross-sectional areas is

$$A_i \in \{0.645,\ 1,\ 2,\ 3,\ 4,\ 5,\ ...\} \quad (cm^2)$$

The results of this problem are given in Tables 8-12. In both continuous and discrete optimization, the number of required analyses in the present method is less than other methods.

7.3. Problem 3. 132-Bar Grid Dome The 132 bar grid dome shown in Figure 3 chosen from Reference 16 is designed to support four independent load conditions and subjected to stress

TABLE 9. Iteration Histories of 25-bar Truss for Discrete Variables.

Iteration No.	LA	QA	HLA	HQA
1	252	259	256	255
2	254	259	259	255
3	254	262	259	
4		262		
3	19.39	20.29	17.42	19.03
4	9.13	13.40	6.55	8.13
5	6.61	5.51	4.16	5.14
6	12.83	12.25	13.19	12.65
7	4.00	4.25	4.69	4.84
8	21.10	21.85	20.95	20.75

TABLE 10. Optimum Design for 25-bar Truss (Continuous): cm^2.

Variable	LA	QA	HLA	HQA
1	0.645	2.04	0.712	12.20
2	3.37	2.34	5.86	2.22

TABLE 11. Optimum Design for 25-bar Truss (Discrete): cm^2.

Variable	LA	QA	HLA	HQA
1	0.645	2	0.645	12
2	4	3	7	2
3	19	20	17	19
4	9	14	7	8
5	7	6	4	5
6	13	12	13	13
7	4	5	5	5
8	21	22	21	21

and displacements constraints. The allowable member stresses are taken as

$$-1723.75 \prec \sigma_{ij} \prec 1723.75\,\text{kg/cm}^2, i = 1,36 \,\&\, j = 1,4$$

where the subscripts i and j represent the member number and load condition, respectively. Minimum area constraints of 0.6452 cm^2 are imposed on all members and displacement constraints of ±0.254 cm are prescribed at all joints in each coordinate direction. The structure is supported at all exterior joints. Young's modules is taken as, 6.895×10^5 Kg/cm^2 and the material density as, 0.002768 Kg/cm^3 for all members. Loads of 444.8 Kg are applied in the negative Z-direction for four independent load conditions given in Table 13. An initial area of 6.452 cm^2 is prescribed for all members.

The structure is assumed to be symmetric about a vertical plane through joints 1, 40 and 52 and about a vertical plane through joints 1, 46 and 58. Thus the problem has 36 independent design variables, which are the areas of the members, A_i. All the design variables are allowed to take the following discrete values:

$$A_i \in \{1,\ 2,\ 3,\ 4,\ 5, ...\} \qquad (\text{cm}^2)$$

TABLE 12. Comparison of the Results for 25-bar Truss.

	No. conti. anal.	No. discr. anal.	Cont./discr. weig.	Execution time (sec.)	Max. constraint
LA	8	2	252/254	4.0	0.007
QA	7	3	256/262	7.0	0.001
HLA	5	2	255/259	3.0	0.030
HQA	4	1	252/255	6.0	0.038

TABLE 13. Load Condition for Dome.

Load cond.	Loaded joints
1	1
2	1,2,3,4,7,8,9,10,11,12,13,19,20,21,22,23,24,25,26,27,28,37
3	All joints are loaded
4	1,4,5,6,7,13,14,15,16,17,18,19,28,29,30,31,32,33,34,35,36,37

TABLE 14. Iteration Histories of 132-bar Dome for Continuous Variables.

Iteration No.	LA	QA	HLA	HQA
0	197.7	197.7	197.7	197.7
1	203.3	200.3	203.2	200.3
2	104.5	181.6	57.56	152.5
3	86.23	87.35	75.38	96.35
4	83.33	85.01	80.41	80.13
5	82.56	86.68	84.10	79.76
6	83.47	84.62	83.64	79.79
7	79.97	84.68	81.53	79.76
8	78.93	84.55	81.53	79.77
9	78.93	84.55	81.53	79.77

The continuous optimization is achieved with five analyses of the structure. One extra analysis is required to complete the discrete solution. The results are presented in Tables 14-18. For comparison, the problem is also solved by LA, QA and HLA methods. The performance of the present approach in both continuous and discrete optimization is better than other techniques. However, the execution time in LA and HLA is less, indicating that in this problem, the time required by the analysis is not significant.

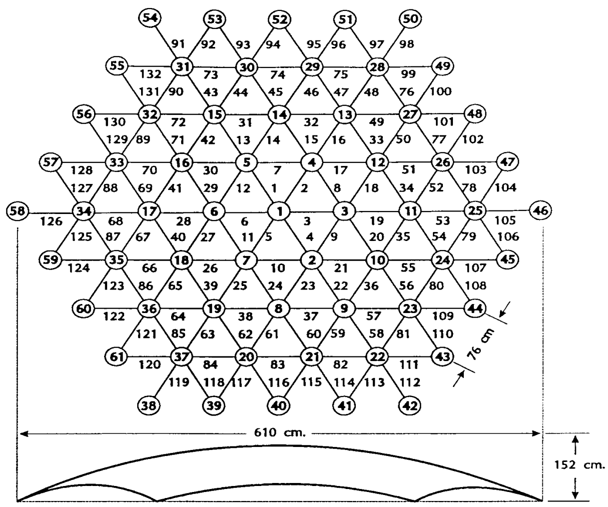

Figure 3. 132-bar grid dome.

7.4. Problem 4. Double-Layer Grid

A double-layer grid of the type shown in Figure 4 with a span of 21 m and height of 1.5 m is chosen from Ref. [3]. The structure is simply supported at every other boundary joint of the bottom layer. The

TABLE 15. Iteration Histories of 132-bar Dome for Discrete Variables.

Iteration No.	LA	QA	HLA	HQA
1	81.92	85.85	82.57	81.70
2	81.69	86.63	83.08	81.04
3	82.37	87.23	83.87	81.04
4	82.98	88.03	84.70	
5	82.98	88.03	84.70	

TABLE 16. Optimum Design for 132-bar Dome (Continuous): cm^2.

Var.	Member	LA	QA	HLA	HQA
1	3	6.44	6.70	5.97	5.58
2	4	6.42	5.59	6.53	6.96
3	9	5.66	5.57	5.40	5.37
4	10	5.61	6.90	5.25	5.61
5	19	2.43	4.95	3.62	2.36
6	20	3.06	2.86	3.19	3.47
7	21	2.83	3.55	3.02	2.96
8	22	3.41	3.21	3.25	3.43
9	23	3.14	3.00	3.31	3.33
10	35	2.56	2.85	2.59	2.68
11	36	2.82	3.26	2.82	2.90
12	37	2.55	2.74	2.70	2.58
13	53	2.08	3.78	3.20	2.44
14	54	2.30	1.68	1.85	1.97
15	55	2.56	2.75	2.83	2.92
16	56	2.80	2.19	2.67	2.65
17	57	2.29	2.30	2.38	2.08
18	58	3.24	3.36	3.29	3.20
19	59	2.51	1.78	2.21	2.47
20	60	3.01	3.31	3.36	3.38
21	79	1.26	1.27	1.26	0.98
22	80	1.08	1.20	1.15	1.62
23	81	1.62	1.74	1.56	1.60
24	82	1.11	1.08	1.10	0.91
25	83	0.698	5.14	3.36	1.00
26	105	1.92	2.07	2.17	2.45
27	106	1.35	1.02	0.862	0.99
28	107	2.41	2.67	2.52	2.71
29	108	2.67	2.63	2.99	2.41
30	109	1.90	2.40	2.28	1.98
31	110	2.93	2.74	2.46	2.72
32	111	1.51	1.37	1.51	1.70
33	112	0.835	1.39	0.658	0.669
34	113	2.16	2.40	2.38	2.12
35	114	2.29	2.12	2.16	2.01
36	115	2.28	3.41	2.95	2.65

loading is assumed a uniformly distributed load on the top layer of intensity of 155.5 Kg/m^2 and it is

TABLE 17. Optimum Design for 132-bar Dome (Discrete): cm^2.

Var.	Member	LA	QA	HLA	HQA
1	3	6	7	6	6
2	4	6	6	7	7
3	9	6	6	5	5
4	10	6	7	5	6
5	19	2	5	4	2
6	20	3	3	3	3
7	21	3	4	3	3
8	22	3	3	3	3
9	23	3	3	3	3
10	35	3	3	3	3
11	36	3	3	3	3
12	37	3	3	3	3
13	53	2	4	3	2
14	54	2	2	2	2
15	55	3	3	3	3
16	56	3	2	3	3
17	57	2	3	2	2
18	58	3	3	3	3
19	59	3	2	2	2
20	60	3	3	3	3
21	79	1	1	2	1
22	80	1	2	2	2
23	81	2	2	2	2
24	82	2	1	2	1
25	83	1	5	3	1
26	105	2	2	2	2
27	106	2	1	1	1
28	107	3	3	3	3
29	108	3	3	3	2
30	109	2	2	2	2
31	110	3	3	2	3
32	111	2	2	2	2
33	112	1	2	2	1
34	113	2	3	2	2
35	114	3	2	2	2
36	115	2	3	3	4

TABLE 18. Comparison of the results for 132-bar dome.

	No. con.	No. dis.	Con./discr. weig.	Execution time (sec.)	Max. constraint
LA	8	4	78.9/82.9	45.0	0.000001
QA	6	4	84.6/88.0	80.0	0.00033
HLA	7	4	81.5/84.7	40.0	0.00014
HQA	5	2	79.7/81.0	70.0	0.00046

transmitted to the joints acting as concentrated vertical loads only. The structure is assumed pin

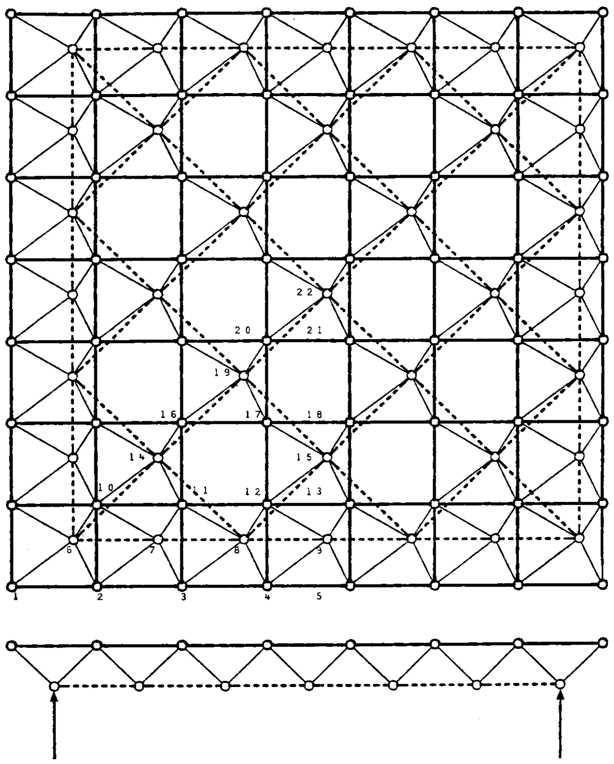

Figure 4. Double layer grid

TABLE 19. Iteration Histories of Double Layer Grid for Continuous Variables.

Iteration	LA	QA	HLA	HQA
0	3781.0	3781.0	3781.0	3781.0
1	3650.6	3650.6	3405.9	3650.6
2	1377.3	3015.4	1177.0	2964.5
3	1210.2	1218.4	1136.5	1360.2
4	1144.8	1201.6	1134.2	1092.6
5	1142.8	1147.1	1132.6	1056.0
6	1138.8	1136.4	1124.7	1056.1
7	1128.8	1136.4	1106.9	1056.1
8	1128.8		1106.9	

jointed with Young's modules, 2.1×10^6 Kg/cm^2 and material density, 0.008 Kg/cm^3 for all members. Member areas are linked to maintain symmetry about the four lines of symmetry in the plane of the grid. Thus the problem has 47 design variables. The initial areas are considered 20 cm^2 with a lower limit of 0.1 cm^2. The available

TABLE 20. Iteration Histories of Double Layer Grid for Discrete Variables.

Iteration	LA	QA	HLA	HQA
1	1131.9	1144.6	1127.1	1060.5
2	1132.4	1147.6	1122.0	1060.5
3	1134.2	1150.7	1117.5	
4	1134.7	1154.6	1117.5	
5	1139.3	1154.6		
6	1141.5			
7	1141.8			

discrete values are

$$A_i \in \{0.1,\ 0.3,\ 0.5,\ 1,\ 2,\ 3,\ 4,\ 5, ...\} \quad (\mathrm{cm}^2)$$

Stress, Euler buckling and displacement constraints are considered in this problem. All the elements are subjected to the following stress constraints:

$$-1000 \prec \sigma_i \prec 1400\,\mathrm{kg/cm}^2, \quad i = 1,47$$

where i is the element number. Tubular members are considered with a diameter to thickness ratio of 10. Thus Euler buckling is considered as

$$\sigma_i \geq \sigma_{bi} = -10.1 E A_i / 8 L_i^2, \qquad i = 1,47$$

where A_i and L_i are the cross-sectional area and length of the ith element, respectively.

In addition, displacement constraints are imposed on the vertical component of the three central joints along the diagonal of the grid (joints 19, 20 and 22) as

$$-1.5\mathrm{cm} \leq \delta_i \leq 1.5\mathrm{cm}, \quad i = 1,2,3$$

The results are presented in Tables 19-23. The efficiency of HQA method in terms of the number of iterations is better than other approaches. In all the problems under investigation, it was noticed that this method is very stable and the changes in the parameters such as multipliers r and s do not influence the convergence process. In some methods like LA, sometimes difficulties arise for problems to converge and changing

TABLE 21. Optimum Design for Double Layer Grid (Continuous): cm^2.

Variable	Member	LA	QA	HLA	HQA
1	1-2	0.42	0.42	0.27	0.27
2	2-3	0.59	1.97	0.20	1.34
3	3-4	1.65	8.31	2.31	4.06
4	4-5	8.61	5.45	9.74	4.49
5	10-11	4.54	3.09	5.87	3.95
6	11-12	3.05	5.31	0.92	4.59
7	12-13	9.69	5.12	10.32	5.39
8	16-17	12.95	16.01	16.28	13.56
9	17-18	10.14	5.55	13.10	7.27
10	20-21	23.76	19.73	20.18	24.93
11	2-10	0.51	0.50	0.50	0.50
12	3-11	0.50	0.50	0.50	0.54
13	11-16	4.87	5.69	5.19	5.97
14	4-12	0.52	0.5	0.50	0.56
15	12-17	10.01	9.09	7.16	8.76
16	17-20	16.00	15.85	12.67	15.23
17	6-7	1.93	5.51	3.57	5.42
18	7-8	0.97	0.40	0.14	2.01
19	8-9	7.03	5.84	6.23	5.62
20	14-8	9.31	9.02	9.63	12.21
21	19-15	11.96	13.63	11.02	11.98
22	8-15	0.86	1.78	1.03	0.93
23	6-14	2.36	6.95	9.50	5.71
24	14-19	7.65	2.24	13.62	8.06
25	19-22	23.29	16.11	26.97	22.06
26	6-2	3.28	4.50	3.14	3.24
27	10-7	2.31	2.44	2.69	2.79
28	7-3	0.62	4.80	1.01	1.74
29	14-11	3.29	4.69	2.81	3.22
30	11-8	6.64	12.72	7.64	7.40
31	8-4	3.20	3.14	3.23	3.13
32	12-9	0.101	1.64	0.14	0.22
33	19-17	2.63	3.58	1.18	2.16
34	17-15	8.00	7.25	5.61	6.66
35	2-7	1.49	4.99	1.74	2.57
36	7-11	0.97	2.20	1.48	1.86
37	3-8	4.72	6.70	3.57	3.90
38	8-12	10.16	9.49	7.70	9.31
39	4-9	0.12	1.64	0.14	0.21
40	12-15	8.12	7.29	5.61	6.58
41	1-6	12.34	5.41	11.72	2.22
42	6-10	12.30	5.39	12.81	5.62
43	10-14	12.84	5.19	12.82	4.84
44	14-16	14.64	8.88	15.27	10.21
45	16-19	13.24	6.97	13.54	6.04
46	19-20	15.54	10.14	14.82	11.80
47	20-22	12.33	5.41	11.71	0.82

TABLE 22. Optimum Design for double layer grid (discrete): cm^2.

Variable	Member	LA	QA	HLA	HQA
1	1-2	0.5	0.5	0.3	0.3
2	2-3	1	2	0.3	1
3	3-4	2	8	2	4
4	4-5	9	5	10	5
5	10-11	6	3	6	4
6	11-12	3	5	1	5
7	12-13	10	5	10	5
8	16-17	13	16	16	14
9	17-18	10	6	13	7
10	20-21	24	20	20	25
11	2-10	0.5	0.5	0.5	0.5
12	3-11	0.5	0.5	0.5	0.5
13	11-16	5	6	5	6
14	4-12	0.5	0.5	0.5	0.5
15	12-17	10	9	7	9
16	17-20	16	16	13	15
17	6-7	2	6	4	5
18	7-8	1	0.5	0.1	2
19	8-9	7	6	6	6
20	14-8	9	9	10	12
21	19-15	12	14	11	12
22	8-15	1	2	1	1
23	6-14	2	7	1	6
24	14-19	8	2	14	8
25	19-22	23	16	27	22
26	6-2	3	5	3	3
27	10-7	2	3	3	3
28	7-3	1	5	1	2
29	14-11	3	5	3	3
30	11-8	7	13	8	7
31	8-4	3	3	3	3
32	12-9	0.1	2	0.1	0.3
33	19-17	3	5	1	2
34	17-15	8	7	6	7
35	2-7	2	5	2	3
36	7-11	1	2	1	2
37	3-8	5	7	4	4
38	8-12	10	10	8	9
39	4-9	0.1	2	0.1	0.3
40	12-15	8	7	6	7
41	1-6	12	5	12	2
42	6-10	12	5	13	6
43	10-14	13	5	13	5
44	14-16	15	9	15	10
45	16-19	13	7	14	6
46	19-20	16	10	15	12
47	20-22	12	5	12	1

move limits, initial design point and other optimization parameters is necessary to get a proper convergence.

For the size of problems considered in this study, the computational time of QHA is slightly larger than other methods. For these problems this

TABLE 23. Comparison of the Results for Double Layer Grid.

	No. con.	No. Dis.	Conti./Discr. weight	Execution time (sec.)	Max. constra.
LA	7	6	1128.8/1141.5	30.0	0.061
QA	6	4	1136.4/1154.6	105.0	0.087
HLA	7	3	1106.9/1117.5	30.0	0.086
HQA	5	2	1056.0/1060.5	100.0	0.047

is reasonable as the time taken by the optimization process is greater than the time required by the analysis. The overall computational time to achieve an optimal solution depends on the number of design variables, number of constraints and the size of the problem in terms of the degrees of freedom. Usually, all the constraints are not considered in each design iteration and some of the critical or near critical constraints are retained. The number of retained constraints is two to three times the number of design variables. Thus only the number of variables and the number of degrees of freedom has a great influence on the computational time. Practical design problems have 10 to 50 variables and several thousand degrees of freedom. In such cases, the cost of analysis dominates the overall cost. Therefore, in large-scale problems, reducing the number of iterations has an important role on the overall computational cost of optimization.

8. CONCLUSIONS

An efficient second order hybrid approximation is presented for the functions that are required in the process of continuous and discrete optimization. The exact evaluation of these functions is computationally expensive, thus the introduction of the approximate functions creates a robust optimization process. First the continuous optimization is obtained by a penalty function, then the discrete variables are obtained by presenting a new penalty function with the use of the same approximation concepts. The main features of the proposed technique are that the second order derivatives are established by the available first order derivatives. In addition, only the diagonal elements of the Hessian matrix are estimated. The numerical results indicate that with this form of simplification, a high quality approximation is established. Also the hybrid second order approximation is stable to converge and parameters such as penalty function multipliers, initial point and move limits do not change the convergence process. In this method for small size problems, the execution time is slightly higher, however, for large structures in terms of the number of degrees of freedom, the overall computational cost would decrease.

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