RELIABILITY ANALYSIS OF REDUNDANT REPAIRABLE SYSTEM WITH DEGRADED FAILURE

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(Received: October 25, 2002 - Accepted in Revised Form: June 10, 2004)

Abstract This investigation deals with the transient analysis of the machine repair system consisting of M-operating units operating under the care of single repairman. To improve the system reliability/availability, Y warm standby and S cold standby units are provided to replace the failed units. In case when all spares are being used, the failure of units occurs in degraded fashion. In such situation there is a facility of one additional repairman to speed up the repair. The lifetime and repair time of units are exponentially distributed. We use matrix method to solve the governing Chapman-Kolmogorov equations. Expressions for the system reliability, availability, mean time to system failure, etc. are established in terms of transient probability. Computational scheme is discussed to facilitate the numerical results. Sensitivity analysis is also carried out to depict the effect of various parameters on the system reliability.

Key Words Machine Repair, Warm Standby, Cold Standby, Degraded, Additional Repairman, Mean Time to Failure (MTTF), Reliability, Availability

چکیده این مقاله به تحلیل گذرای یک سیستم تعمیر ماشین شامل M واحد عملیاتی تحت نظارت یک تعمیر کار می‌پردازد. تعداد Y ماشین رزرو فعال و S ماشین رزرو غیر فعال برای بهبود قابلیت دسترسی و پایایی سیستم فراهم شده است. در حالتی که تمامی قطعات یدکی مصرف شده‌اند، خرابی ماشینها در حالت نزولی اتفاق می‌افتد. در چنین وضعیتی امکان استفاده از یک تعمیر کار اضافی برای تسریع عملیات تعمیر وجود دارد. طول عمر و مدت زمان تعمیر ماشین‌ها دارای توزیع نمایی فرض شده‌اند. در این مقاله از روش ماتریسی برای حل معادلات کالموگورف - چپمن (GK) استفاده شده است. معادلات برای قابلیت دسترسی، پایایی، میانگین مدت زمان تا خرابی سیستم و غیره با استفاده از احتمالات گذرا تعریف شده‌اند. روش محاسباتی و نتایج عددی ارایه شده‌اند. همچنین تحلیل حساسیت برای نشان دادن تاثیر پارامترهای مختلف بر قابلیت اطمینان سیستم صورت گرفته است.

1. INTRODUCTION

The performance of any machining system is highly influenced by component failures. The machine failure may be balanced either by providing spare part support or by facilitating better repair facility so as the production may not suffer. Several researchers have done work in the area of transient analysis of redundant repairable system. Kumagai [1] provided reliability analysis of an n-spare system with a single repair facility. Kalpakam and Hameed [2] obtained the availability and reliability of an n-unit warm standby redundant system. Sivazlian and Wang [3] studied economic analysis of the M/M/R machine repair problem with warm standbys. Shao and Lamberson [4] discussed reliability of series system with spares. Goel and Srivastava [5] investigated a transient behavior of an n-unit cold standby system with a single repair facility. They developed the Chapman-Kolmogorov equations and solved by using matrix approach. Jain [6] employed the diffusion approximation technique to study the (m, M) machine repair problem with spares and state dependent rates. Jain and Dhyani [7] developed transient analysis of M/M/C machine repair problem with spares. Jain and Singh [8] considered reliability of repairable multi-component redundant system. Jain and Singh [9] studied optimal N-policy for single server markovian queue with breakdown, repair and state dependent arrival rate.

One can improve the system reliability either by providing sufficient spare parts or by facilitating the additional repairman to speed up the repair. Many mathematicians have contributed significantly towards machine repair problem with spares and additional repairmen. Chandre and Shanti Kumar [10] discussed a machine interference problem with several types of machines attended by a single repairman. Jain [11] analyzed reliability analysis for M/M/C repairable system with spares and additional repairmen. Jain [12] studied M/M/R machine repair problem with spares and additional repairman. Jain et al. [8] discussed M/M/C/K/N machine repair problem with balking, reneging, spares and additional repairman. Jain et al. [13] analyzed machine repair problem with spares, reneging, additional repairman and two modes of failure was by. Jain and Maheshwari [14] gave a transient analysis of redundant repairable system with additional repairmen.

When all spares are being used, the failure of units may occur in degraded fashion. Significant works have been done on the machine repair problem with degraded failure rate. Goyal and Tantawi [15] discussed evaluation of performability for degradable computer systems. Meyer [16] provided the performance measures for degradable computer systems. Najjar and Gaudiot [17] gave scalability analysis in gracefully degradable large systems. Pham et al. [18] considered availability and mean lifetime prediction of multistage degraded system with partial repairs. Jain et al. [19,20] studied reliability analysis of gracefully degradable multiprocessor system. However much work has not been done for the improvement in availability of such a system. The issue of improvement in reliability/availability of a repairable degraded multi-component system can be tackled to some extents by employing cold and warm spares and additional repairman.

In this paper we extend the work of Goel and Srivastava [5] on machining system with cold standbys by incorporating the degraded failure and additional repairman. We have also considered mixed standbys by including warm spares along with cold standbys, which was not taken into consideration by Goel and Srivastva [5]. The main contents of this paper are organized as follows: In Section 2, the structure of the problem and related notations to describe the model are given. The governing differential difference equations are constructed for state probabilities using suitable transition rates. By taking the Laplace transform of state transition equations, the queue size distribution is given in Section 3. The rearrangement of the set of steady state equations in matrix form and the analysis are done in Section 4. In Section 5, some performance measures are established. For illustration purpose, the numerical results are provided in Section 6. Finally conclusion is given in Section 7.

2. MODEL DESCRIPTION

We consider a machining system consisting of N = M + Y + S mixed units where M are the operating units, Y are the warm standby units and S are the cold standby units. To develop the mathematical model for the system, the following assumptions are made:

- The lifetime and the repair time distributions of the units are exponentially distributed.
- The repair facility consists of one permanent and one additional removable repairman. Whereas the additional repairman turns on when all standby units are exhausted and is renewed as soon as standby unit repaired.
- There is a provision of cold standby units and warm standby units to replace the failed units.
- Once standby (cold or warm) unit replaces the failed operating unit, its characteristics are the same as that of operating unit.
- The system will fail when there are less than m units in the system. The failure rates of operating and warm standby units are λ and α $(0 \leq \alpha \leq \lambda)$,
- respectively, while cold standby units have the failure rate zero.
- When all standbys are used then

the failure rate of operating units is assumed to be state dependent and is denoted by: $\lambda'_{i-\overline{Y+S}}$ $(0 \le i < M+Y+S-m)$.

- When all standbys (warm and cold) are being used and a unit fails, the system operates in degraded manner being in short mode. To cope up with this situation, additional repairman is called to reduce the backlog of failed units.
- The permanent repairman repairs the failed units with rate μ while the additional repairman repairs the failed units with rate μ_1.
- The repairmen can repair only one failed unit at a time and performs service in the FIFO discipline.
- After the completion of repair, the failed units join the operating units if system is short otherwise become part of the standby group to which it belongs.
- The switchover times from standby to operating state and from repair to standby state are negligible.

Let X(t) be the number of operable units at time t. Then the state dependent failure and repair rates are given by:

$$\lambda_{N-i} = \begin{cases} M\lambda + (Y-i)\alpha, & 0 \le i \le Y-1 \\ M\lambda, & Y \le i < Y+S \\ (M+Y+S-i)\lambda'_{i-\overline{Y+S}}, & Y+S \le i < L \\ 0, & otherwise \end{cases} \tag{1}$$

and

$$\mu_{N-i} = \begin{cases} \mu, & 1 \le i \le Y+S \\ \mu + \mu_1, & Y+S < i \le L \\ 0, & otherwise \end{cases} \tag{2}$$

where: L = M+Y+S-m

Let $p_j(t)$ be the probability that the system is in state j ($m-1 \le j \le N$) at time t, it is to be noted that state (m-1) is an absorbing state. Again assume that $q_j(t)$ be the probability that the system is in state j ($m \le j \le N$) in case when it will not ever reach the failed state. The state transition flow diagram is depicted in Figure 1. The state 'N' indicates that all the (M+Y+S) units are in working state. The state (N-i), (1≤ i ≤ Y-1) represents that (N-i) units are in working state due to failure of i units which belong to either operating group or warm standby group. In these states, if failure of operating units occurs then this failure unit is replaced by warm standby so that there are M operating units along with (Y-i) warm standby units and S cold standby units in the system. The state (N-i), (Y≤ i < Y+S) denotes that all warm standby units are exhausted either due to failure or in replacing the failed operating units, so that there are only M operating units and (Y+S-i) cold standby units in the system. In this state whenever an operating unit fails, it is replaced by cold standby unit. The state (N-i), (Y+S ≤ i ≤ M+Y+S-m) represents the situation when all cold and warm standby units are being used and system is operating in degraded mode with i-(Y+S) operating units. The system fails when m^{th} operating unit fails and the state changes from m to $(m-1)^{th}$ as shown in the Figure 1.

The Chapman-Kolmogorov equations governing the model are as follows:

$$\frac{d}{dt} p_{m-1}(t) = \lambda_m p_m(t) \tag{3}$$

$$\frac{d}{dt} p_m(t) = -(\lambda_m + \mu_{m+1}) p_m(t)$$

$$+\mu_m P_{m-1}(t)+\lambda_{m+1} p_{m+1}(t) \tag{4}$$

$$\frac{d}{dt} p_{N-i}(t) = -(\lambda_{N-i} + \mu_{N-\overline{i-1}}) p_{N-i}(t) +$$

$$+\mu_{N-i} p_{N-\overline{i+1}}(t) + \lambda_{N-\overline{i-1}} p_{N-\overline{i-1}}(t),$$

$$Y + S \le i < L \tag{5}$$

$$\frac{d}{dt} p_M(t) = -(\lambda_M + \mu_{M+1}) p_M(t)$$

$$+\mu_M p_{M-1}(t) + \lambda_{M+1} p_{M+1}(t) \tag{6}$$

$$\frac{d}{dt} p_{N-i}(t) = -(\lambda_{N-i} + \mu_{N-\overline{i-1}}) p_{N-i}(t)$$

$$+\mu_{N-i} p_{N-\overline{i+1}}(t) + \lambda_{N-\overline{i-1}} p_{N-\overline{i-1}}(t),$$

$$Y \le i < Y + S \tag{7}$$

$$\frac{d}{dt} p_{N-Y}(t) = -(\lambda_{N-Y} + \mu_{N-\overline{Y-1}}) p_{N-Y}(t) +$$

$$\mu_{N-Y} p_{N-\overline{Y+1}}(t) + \lambda_{N-\overline{Y-1}} p_{N-\overline{Y-1}}(t) \tag{8}$$

$$\frac{d}{dt} p_{N-i}(t) = -(\lambda_{N-i} + \mu_{N-\overline{i-1}}) p_{N-i}(t)$$

$$+\mu_{N-i} p_{N-\overline{i+1}}(t) + \lambda_{N-\overline{i-1}} p_{N-\overline{i-1}}(t),$$

$$0 \le \mathrm{i} \le \mathrm{Y} - 1 \tag{9}$$

$$\frac{d}{dt} p_{N-1}(t) = -(\lambda_{N-1} + \mu_N) p_{N-1}(t) +$$

$$\mu_{N-1} p_{N-2}(t) + \lambda_N p_N(t) \tag{10}$$

$$\frac{d}{dt} p_N(t) = -\lambda_N p_N(t) + \mu_N p_{N-1}(t) \tag{11}$$

and

$$\frac{d}{dt} q_m(t) = -(\lambda_m + \mu_{m+1}) q_m(t)$$

$$+\lambda_{m+1} q_{m+1}(t) \tag{12}$$

$$\frac{d}{dt} q_{N-i}(t) = -(\lambda_{N-i} + \mu_{N-\overline{i-1}}) q_{N-i}(t) +$$

$$\mu_{N-i} q_{N-\overline{i+1}}(t) + \lambda_{N-\overline{i-1}} q_{N-\overline{i-1}}(t),$$

$$Y + S \le i < L \tag{13}$$

$$\frac{d}{dt} q_M(t) = -(\lambda_M + \mu_{M+1}) q_M(t) +$$

$$\mu_M q_{M-1}(t) + \lambda_{M+1} q_{M+1}(t) \tag{14}$$

$$\frac{d}{dt} q_{N-i}(t) = -(\lambda_{N-i} + \mu_{N-\overline{i-1}}) q_{N-i}(t) +$$

$$\mu_{N-i} q_{N-\overline{i+1}}(t) + \lambda_{N-\overline{i-1}} q_{N-\overline{i-1}}(t),$$

$$\mathrm{Y} \le \mathrm{i} < \mathrm{Y} + \mathrm{S} \tag{15}$$

$$\frac{d}{dt} q_{N-Y}(t) = -(\lambda_{N-Y} + \mu_{N-\overline{Y-1}}) q_{N-Y}(t)$$

$$+\mu_{N-Y} q_{N-\overline{Y+1}}(t) + \lambda_{N-\overline{Y-1}} q_{N-\overline{Y-1}}(t) \tag{16}$$

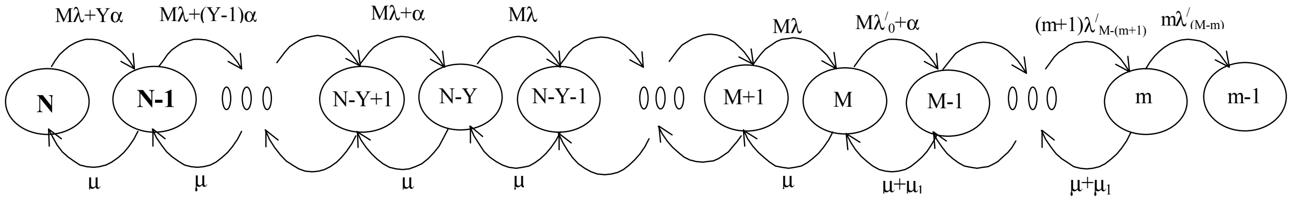

Figure 1. The state transition diagram.

$$\frac{d}{dt}q_{N-i}(t) = -(\lambda_{N-i} + \mu_{N-\overline{i-1}})q_{N-i}(t)$$

$$+\mu_{N-i}q_{N-\overline{i+1}}(t) + \lambda_{N-\overline{i-1}}q_{N-\overline{i-1}}(t),$$

$$0 \le i \le Y-1 \tag{17}$$

$$\frac{d}{dt}q_{N-1}(t) = -(\lambda_{N-1} + \mu_N)q_{N-1}(t)$$

$$+\mu_{N-1}q_{N-2}(t) + \lambda_N q_N(t) \tag{18}$$

$$\frac{d}{dt}q_N(t) = -\lambda_N q_N(t) + \mu_N q_{N-1}(t) \tag{19}$$

3. THE QUEUE SIZE DISTRIBUTION

The stationary distribution π_j (j = m-1, m, …,N) of the stochastic process { X (t), t $\geq$ 0} can be obtained by setting the derivatives equal to zero in Equations 3 to 11. Letting $\pi_j = \lim_{t\to\infty} p_j(t)$ and substituting the values of λ_{N-i} and μ_{N-i} $(0 \le i \le L)$, we obtain

$$m\lambda'_{M-m}\pi_m = 0 \tag{20}$$

$$-(m\lambda'_{M-m} + \mu + \mu_1)\pi_m + (\mu+\mu_1)\pi_{m-1} +$$

$$(m+1)\lambda'_{M-\overline{m+1}}\pi_{m+1} = 0 \tag{21}$$

$$-[(N-i)\lambda'_{i-\overline{Y+S}} + \mu + \mu_1]\pi_{N-i} +$$

$$(\mu+\mu_1)\pi_{N-\overline{i+1}} + (N-\overline{i-1})\lambda'_{\overline{i-1}-\overline{Y+S}}\pi_{N-\overline{i-1}} = 0 \tag{22}$$

$$-(M\lambda'_0 + \mu)\pi_M + (\mu+\mu_1)\pi_{M-1}$$

$$+M\lambda\pi_{M+1} = 0 \tag{23}$$

$$-(M\lambda + \mu)\pi_{N-i} + \mu\pi_{N-\overline{i+1}}$$

$$+M\lambda\pi_{N-\overline{i-1}} = 0 \tag{24}$$

$$-(M\lambda + \mu)\pi_{N-Y} + \mu\pi_{N-\overline{Y+1}} +$$

$$(M\lambda + \alpha)\pi_{N-\overline{Y-1}} = 0 \tag{25}$$

$$-[M\lambda+(Y-i)\alpha+\mu]\pi_{N-i}+\mu\pi_{N-\overline{i+1}}$$
$$+[M\lambda+(Y-\overline{i-1})\alpha]\pi_{N-\overline{i-1}}=0 \tag{26}$$

$$-[M\lambda+(Y-1)\alpha+\mu]\pi_{N-1}+\mu\pi_{N-2}$$

$$+[M\lambda+Y\alpha]\pi_{N}=0 \tag{27}$$

$$-(M\lambda+Y\alpha)\pi_{N}+\mu\pi_{N-1}=0 \tag{28}$$

Equations 20 to 28 can be solved by using product type solution given as

$$\pi_{j}=\pi_{0}\prod_{i=1}^{j}\frac{\mu_{i}}{\lambda_{i}} \tag{29}$$

where:

$$\pi_{0}=\left[1+\sum_{j=m-1}^{N}\left\{\prod_{i=1}^{j}\left(\frac{\mu_{i}}{\lambda_{i}}\right)\right\}\right]^{-1}$$

4. THE TRANSIENT ANALYSIS

Taking Laplace transform of Equations 3 to 11 and denoting the Laplace transform of function f(t) by f(s), the governing equations reduce to

$$A(s)\,P=I_{N-m+2} \tag{30}$$

where A(s) is a (N-m+2) $\times$ (N-m+2) real symmetric tridigonal matrix, denoted by:

A(s)=

$$\begin{bmatrix}
s & -\lambda_m & 0 & .. & 0 & 0 & 0 & .. & 0 & 0 & 0 & .. & 0 & 0 & 0\\
-\mu_m & \begin{pmatrix} s+\lambda_m+\\ \mu_{m+1}\end{pmatrix} & -\lambda_{m+1} & .. & 0 & 0 & 0 & .. & 0 & 0 & 0 & .. & 0 & 0 & 0\\
.. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & ..\\
0 & 0 & 0 & .. & -\mu_M & \begin{pmatrix} s+\lambda_M+\\ \mu_{M+1}\end{pmatrix} & -\lambda_{M+1} & .. & 0 & 0 & 0 & .. & 0 & 0 & 0\\
.. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & ..\\
0 & 0 & 0 & .. & 0 & 0 & 0 & .. & -\mu_{N-Y} & \begin{pmatrix} s+\lambda_{N-Y}+\\ \mu_{N-\overline{Y-1}}\end{pmatrix} & -\lambda_{N-\overline{Y-1}} & .. & 0 & 0 & 0\\
.. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & ..\\
0 & 0 & 0 & .. & 0 & 0 & 0 & .. & 0 & 0 & 0 & .. & -\mu_{N-1} & \begin{pmatrix} s+\lambda_{N-1}+\\ \mu_{N}\end{pmatrix} & -\lambda_N\\
0 & 0 & 0 & .. & 0 & 0 & 0 & .. & 0 & 0 & 0 & .. & 0 & -\mu_N & (s+\lambda_N)
\end{bmatrix}$$

$P=[p_{m-1}(s),p_m(s),\ldots,p_{N-i}(s),\ldots,p_M(s),\ldots,$

$$p_{N-i}(s),\ldots,p_{N-Y}(s),\ldots,p_{N-i}(s),\ldots,p_{N-1}(s),p_N(s)]^T \tag{31}$$

and

$$\mathbf{I}_{N-m+2}=(0,0,0,\ldots,0,1)^T \tag{32}$$

Similarly the Laplace transform of Equations 12 to 19 gives

$$B(s)\,Q=I_{N-m+1} \tag{33}$$

where B(s) is the (N-m+1)$\times$(N-m+1) tridigonal matrix obtained by suppressing the first row and first column of A,

$\mathbf{Q} = [q_m(s), \ldots, q_{N-i}(s), \ldots, q_M(s), \ldots, q_{N-i}(s), \ldots, q_{N-Y}(s), \ldots, q_{N-i}(s), \ldots, q_{N-1}(s), q_N(t)\,]^T$

(34)

and I_{N-m+1} is a unit column vector with unity in the last place. Using Cramer's rule, Equations 30 and 33 yield

$$p_j(s) = \frac{|A_j(s)|}{|A(s)|}, \qquad m-1 \le j \le N \tag{35}$$

and

$$q_j(s) = \frac{|B_j(s)|}{|B(s)|}, \qquad m \le j \le N \tag{36}$$

where $\mathbf{A}_j(s)$ and $\mathbf{B}_j(s)$ are obtained from the matrices $\mathbf{A}(s)$ and $\mathbf{B}(s)$ by replacing the j^{th} column by the unit vector in the RHS of Equations 34 and 37.

Now we proceed to show that the polynomials $|\mathbf{A}(s)|$ and $|\mathbf{B}(s)|$ have real and distinct zeros and hence the expressions for $p_j(s)$ and $q_j(s)$ can be broken into partial fractions. After applying some elementary row and column transformations on $|A(s)|$ and $|B(s)|$, we have:

$$|A(s)| = s|\alpha(s)| \qquad \text{and} \qquad |B(s)| = |\beta(s)|$$

where

$\alpha(s) =$

$$\begin{bmatrix}
(s+\lambda_m) & \sqrt{\lambda_m\mu_{m+1}} & 0 & .. & 0 & 0 & 0 & .. & 0 & 0 & 0 \\
\sqrt{\lambda_m\mu_{m+1}} & \begin{pmatrix} s+\lambda_m \\ +\mu_{m+1} \end{pmatrix} & \sqrt{\lambda_{m+1}\mu_{m+2}} & .. & 0 & 0 & 0 & .. & 0 & 0 & 0 \\
.. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\
0 & 0 & 0 & .. & \sqrt{\lambda_{N-Y}\mu_{N-\overline{Y-1}}} & \begin{pmatrix} s+\lambda_{N-\overline{Y-1}} \\ +\mu_{N-\overline{Y-1}} \end{pmatrix} & \sqrt{\lambda_{N-\overline{Y-1}}\mu_{N-\overline{Y-2}}} & .. & 0 & 0 & 0 \\
.. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\
0 & 0 & 0 & .. & 0 & 0 & 0 & .. & \sqrt{\lambda_{N-2}\mu_{N-1}} & \begin{pmatrix} s+\lambda_{N-1} \\ +\mu_{N-1} \end{pmatrix} & \sqrt{\lambda_{N-2}\mu_N} \\
0 & 0 & 0 & .. & 0 & 0 & 0 & .. & 0 & \sqrt{\lambda_{N-1}\mu_N} & \begin{pmatrix} s+\lambda_N \\ +\mu_N \end{pmatrix}
\end{bmatrix}$$

$\beta(s) \;\; =$

$$\begin{bmatrix}
(s+\lambda_m) & \sqrt{\lambda_m\mu_{m+1}} & 0 & .. & 0 & 0 & 0 & .. & 0 & 0 & 0 \\
\sqrt{\lambda_m\mu_{m+1}} & \begin{pmatrix} s+\lambda_m \\ +\mu_{m+1} \end{pmatrix} & \sqrt{\lambda_{m+1}\mu_{m+2}} & .. & 0 & 0 & 0 & .. & 0 & 0 & 0 \\
.. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\
0 & 0 & 0 & .. & \sqrt{\lambda_{N-Y}\mu_{N-\overline{Y-1}}} & \begin{pmatrix} s+\lambda_{N-\overline{Y-1}} \\ +\mu_{N-\overline{Y-1}} \end{pmatrix} & \sqrt{\lambda_{N-\overline{Y-1}}\mu_{N-\overline{Y-2}}} & .. & 0 & 0 & 0 \\
.. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\
0 & 0 & 0 & .. & 0 & 0 & 0 & .. & \sqrt{\lambda_{N-2}\mu_{N-1}} & \begin{pmatrix} s+\lambda_{N-1} \\ +\mu_{N-1} \end{pmatrix} & \sqrt{\lambda_{N-2}\mu_N} \\
0 & 0 & 0 & .. & 0 & 0 & 0 & .. & 0 & \sqrt{\lambda_{N-1}\mu_N} & \begin{pmatrix} s+\lambda_N \\ +\mu_N \end{pmatrix}
\end{bmatrix}$$

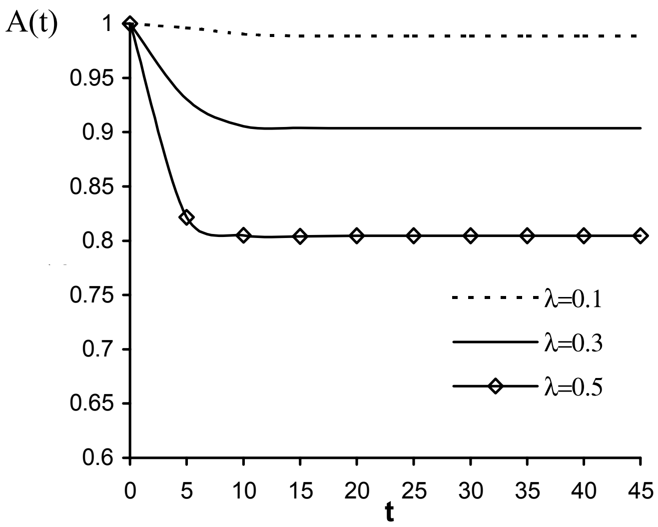

Figure 2. Effect of λ on availability.

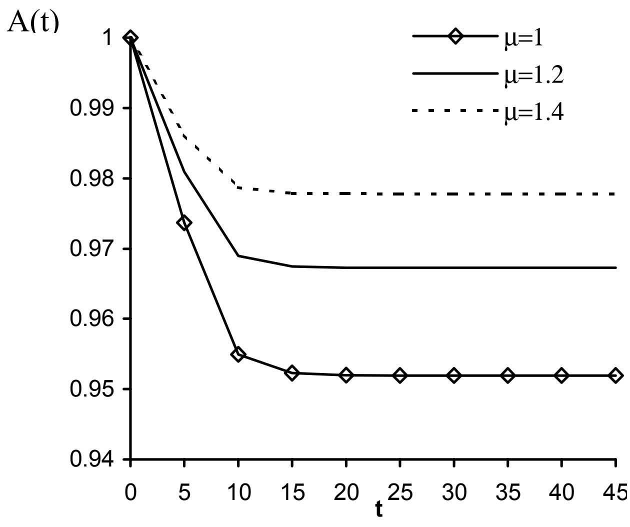

Figure 3. Effect of μ on availability.

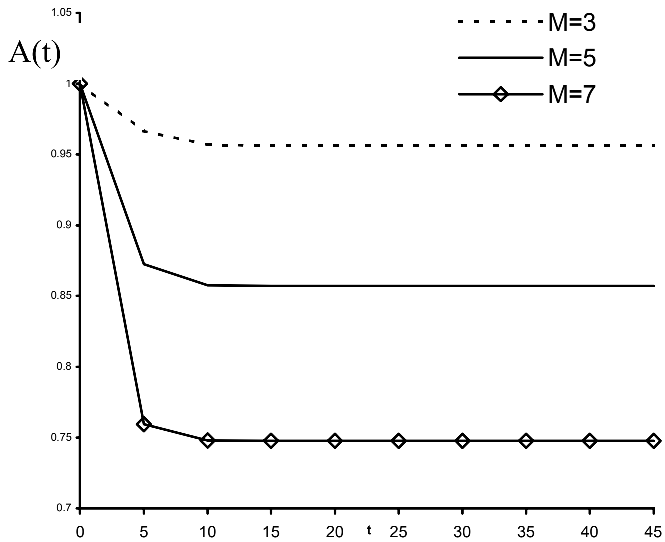

Figure 4. Effect of M on availability.

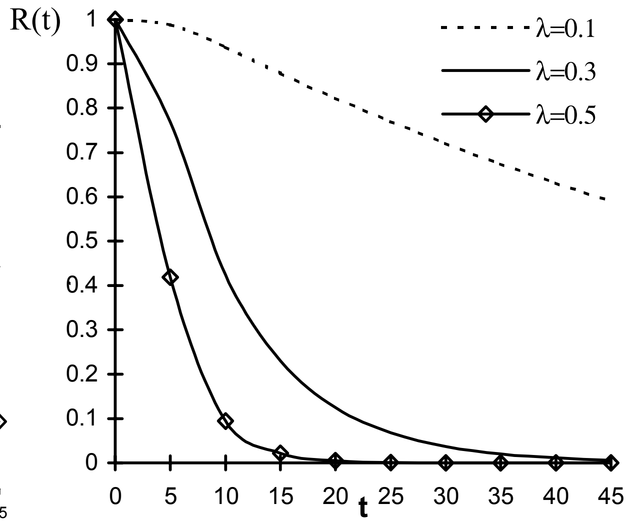

Figure 5. Effect of λ on reliability.

From Equation 29, we note that the eigen values of $\alpha(0)$ and $\beta(0)$ are real, distinct and positive which is denoted by α_k and β_k. Then we have

$$|A(s)| = s\prod_{k=m}^{N}(s-\alpha_k)$$

and

$$|B(s)| = \prod_{k=m}^{N}(s-\beta_k)$$

Hence

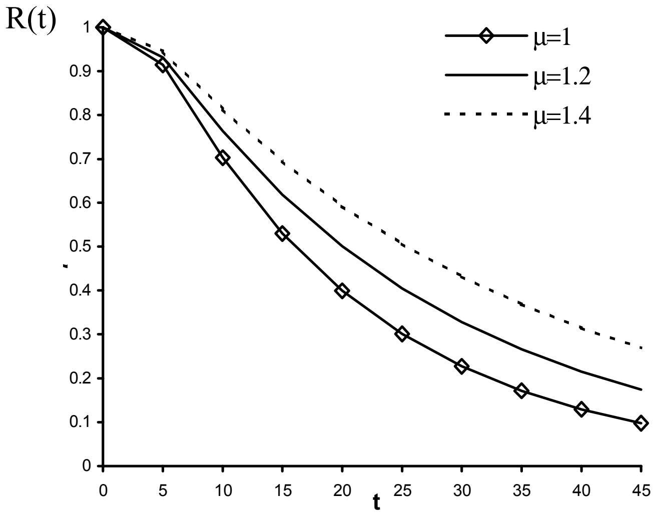

Figure 6. Effect of μ on reliability.

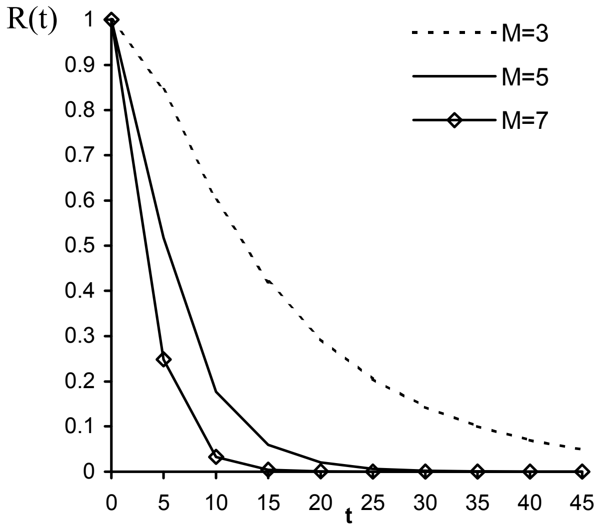

Figure 7. Effect of M on reliability.

$$p_j(s) = \frac{|A_j(s)|}{s\prod_{k=m}^{N}(s - \alpha_k)}, \qquad m-1 \le j \le N \tag{37}$$

$$q_j(s) = \frac{|B_j(s)|}{\prod_{k=m}^{N}(s - \beta_k)}, \qquad m \le j \le N \tag{38}$$

Taking the partial fractions of Equations 37 to 38, we get

$$p_j(s) = \frac{a_{0j}}{s} + \sum_{k=m}^{N} \frac{a_{kj}}{(s-\alpha_k)}, \qquad m-1 \le j \le N \tag{39}$$

and

$$q_j(s) = \sum_{k=m}^{N} \frac{b_{kj}}{(s-\beta_k)}, \qquad m \le j \le N \tag{40}$$

where

$$a_{0j} = \frac{|A_j(0)|}{\prod_{k=m}^{N}\alpha_k} \tag{41}$$

$$a_{kj} = \frac{|A_j(-\alpha_k)|}{\alpha_k \prod_{i=m\neq k}^{N}(\alpha_k - \alpha_i)} \tag{42}$$

and

$$b_{kj} = \frac{|B_j(-\beta_k)|}{\prod_{i=m\neq k}^{N}(\beta_k - \beta_i)} \tag{43}$$

5. SOME PERFORMANCE MEASURES

Now we obtain some operating characteristics of the system as follows:

The system availability is given by:

$$A(t) = \sum_{j=m}^{N} p_j(t) \tag{44}$$

The reliability of the system is given by:

$$R(t) = \sum_{j=m}^{N} q_j(t) \tag{45}$$

The mean time to system failure is obtained as:

$$MTTF = \int_0^{\infty} R(u)du = \sum_{j=m}^{N} \int_0^{\infty} q_j(u)du \tag{46}$$

Next, the probability that the repairman is busy is

$$B(t) = 1 - p_n(t) \tag{47}$$

Taking the inverse Laplace transform of (39) and (40), we find

$$p_j(t) = a_{0j} + \sum_{k=m}^{N} a_{kj} e^{-\alpha_k t}$$

$$= \frac{|A_j(0)|}{\prod_{k=m}^{N} \alpha_k} - \sum_{k=m}^{N} \frac{|A_j(-\alpha_k)|}{\alpha_k \prod_{i=m \neq k}^{N} (\alpha_k - \alpha_i)} e^{-\alpha_k t} \tag{48}$$

and

$$q_j(t) = \sum_{k=m}^{N} b_{kj} e^{-\beta_k t} = \sum_{k=m}^{N} \frac{|B_j(-\beta_k)|}{\prod_{i=m \neq k}^{N} (\beta_k - \beta_i)} e^{-\beta_k t} \tag{49}$$

6. NUMERICAL RESULTS

In this section, numerical results for reliability and availability are calculated using MATLAB software and are summarized in Tables 1 and 2. The graphical presentation has also been shown in Figures 2-7.

In Table 1, we display reliability and availability for different values of failure rate (α) of warm standbys. We notice that the reliability and availability decrease as α increases but for larger values of time t remain constant for the increasing values of α. In Table 2, we depict the results for reliability and availability for the various repair rates of additional repairman. We observe that there is an increment in reliability and availability with the repair rate μ_1 of additional repairman but both availability and reliability decrease as time t increases.

In Figure 2, we illustrate the effect of failure rate λ of the operating units on the availability by varying t. We see from the graph that availability decreases sharply when λ increases for lower value of t, but as time t increases the availability remains constant. Figure 3 depicts the effect of repair rate μ of permanent repairman on availability. We note that the availability increases when μ increases for small value of t but as time t increases, the availability becomes almost constant. Figure 4 displays the effect of the number of operable units (M) on the availability. It is easily observed from the graph that the availability decreases for smaller time t when the number of operable units increases but for larger time t it remains almost constant.

In Figure 5, the effect of failure rate λ of operating units on reliability is shown. It is noted that the reliability decreases with the increase in λ. The effect of repair rate μ of permanent repairman on the reliability by varying t is displayed in Figure 6. We observe that the reliability increases with μ but decreases as t increases. Figure 7 illustrates the effect of number of the operable units (M) on the reliability. The decreasing trend in reliability with the increase in M is found.

TABLE 1. Reliability and Availability for different values of α.

t	a = 0.01		a = 0.02		a = 0.03	
	A(t)	R(t)	A(t)	R(t)	A(t)	R(t)
0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
5	0.973820	0.914973	0.973779	0.914797	0.973738	0.914622
10	0.954971	0.703409	0.954962	0.703206	0.954954	0.703006
15	0.952301	0.530769	0.952300	0.530613	0.952299	0.530459
20	0.951964	0.400148	0.951964	0.400031	0.951964	0.399915
25	0.951921	0.301661	0.951921	0.301573	0.951921	0.301485
30	0.951916	0.227414	0.951916	0.227347	0.951916	0.227281
35	0.951915	0.171441	0.951915	0.171391	0.951915	0.171341
40	0.951915	0.129245	0.951915	0.129207	0.951915	0.129169
45	0.951915	0.097434	0.951915	0.097405	0.951915	0.097377

TABLE 2. Reliability and Availability for different values of m_1.

t	m_1 = 1.2		m_1 = 1.4		m_1 = 1.6	
	A(t)	R(t)	A(t)	R(t)	A(t)	R(t)
0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
5	0.966753	0.901446	0.973659	0.914280	0.979080	0.925392
10	0.939781	0.657966	0.954937	0.702615	0.966125	0.742016
15	0.935229	0.468348	0.952297	0.530159	0.964621	0.587050
20	0.934537	0.332911	0.951963	0.399688	0.964465	0.464201
25	0.934433	0.236622	0.951921	0.301314	0.964449	0.367052
30	0.934417	0.168182	0.951916	0.227152	0.964448	0.290234
35	0.934415	0.119538	0.951915	0.171244	0.964447	0.229493
40	0.934414	0.084963	0.951915	0.129096	0.964447	0.181465
45	0.934414	0.060389	0.951915	0.097322	0.964447	0.143487

7. CONCLUSION

In this paper, we have considered repairable system having warm as well as cold standby units. A permanent repairman together with additional repairman is facilitated so that the reliability of the system can be improved up to a desired grade in particular when there is constraint of limited spare support. The numerical results provided demonstrate the computational tractability of the analytical results as well as give insight how the system reliability/availability can be modified by the appropriate choice of standbys and repair facility. From the tables and graphs, we conclude that

- The reliability and availability decrease as the failure rate (λ) of operable units and the failure rate (α) of warm standby units increase.

- With the increase in repair rate of permanent and additional repairmen, the reliability and availability both increase.
- The reliability and availability decrease as the number of operable units increase. Furthermore as time goes on, the reliability tends to zero asymptotically however availability becomes almost constant.

8. ACKNOWLEDGEMENTS

The authors are thankful to University Grants Commission, New Delhi, for providing financial support vide Project No. F 8-5/98 (SR-I). The authors are indebted to learned referees and the Editor-in-Chief for their constructive comments and suggestions, which have improved the paper considerably.

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