

# STAGewise MODELING OF LIQUID-LIQUID EXTRACTION COLUMN (RDC)

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**Abstract** Stagewise forward mixing model considering coalescence and redispersion of drops was used to predict the performance of Rotating Disc Liquid-Liquid Extraction Contactors. Experimental data previously obtained in two RDC columns of 7.62 cm diameter, 73.6cm height and 21.9cm diameter, 150cm height were used to evaluate the model predictions. Drop-side mass transfer coefficients were predicted applying Handlos-Baron drop model and Olney's model was used to predict drop velocities. According to the results obtained the followings could be concluded: (1) If the height of coalescence and redispersion i.e.:  $h = h_p \phi_p / \phi$  could be estimated, the stagewise forward mixing with coalescence and redispersion model will predict the column height and efficiency with the acceptable accuracy, (2) The stagewise modeling predictions are highly dependent on the number of stages used when the number of stages is less than 10 and (3) Application of continuous phase mass transfer and axial dispersion coefficients ( $k_c$  and  $E_c$ ) obtained from the solute concentration profile along the column height will predict the column performance more accurately than the Calderbank and Moo-Young (for  $k_c$ ) and Kumar-Hartland (for  $E_c$ ) correlations.

**Key Words** Stagewise Modeling, Liquid-Liquid Extraction, Coalescence

**چکیده** با استفاده از مدل مرحله‌ای برای بیان انتقال جرم داخل برج استخراج مایع-مایع RDC، توجه به اختلاط پیش رو در فاز پراکنده و در نظر گرفتن ائتلاف و توزیع مجدد قطرات، عملکرد برج پیش بینی گردید. برای پیش بینی عملکرد دستگاه از داده های تجربی که قبلاً در دو تماس دهنده استخراج مایع-مایع RDC با قطرهای ۷/۶۲ cm و ۲۱/۹ cm و ارتفاعهای به ترتیب ۷۳/۶ cm و ۱۵۰ cm بدست آمده بودند، استفاده گردیده و برای ارزیابی پیش بینی مدل مورد استفاده قرار گرفتند. ضرایب انتقال جرم داخل قطرات با استفاده از مدل ارائه شده توسط Handles and Baron پیش بینی شدند و مدل Olney برای محاسبات مربوط به سرعت و هیدرودینامیک فازها، بکار گرفته شد. بر اساس نتایج بدست آمده، موارد زیر را می توان نتیجه گیری نمود: (۱) اگر بتوان ارتفاع ائتلاف و توزیع مجدد قطرات را که بر اساس رابطه  $h = h_p \phi_p / \phi$  محاسبه می شود، پیش بینی کرد، مدل مرحله‌ای اختلاط پیش رو با در نظر گرفتن ائتلاف و توزیع مجدد قطرات می تواند ارتفاع و راندمان ستون را با دقت قابل قبولی پیش بینی نماید. (۲) پیش بینی های مدل مرحله‌ای هنگامی که تعداد مراحل کمتر از ۱۰ در نظر گرفته شود تا حد زیادی وابسته به تعداد مراحل است. (۳) با استفاده از مقادیر ضرایب انتقال جرم و پراکندگی محوری فاز پیوسته ( $k_c$  and  $E_c$ ) که از روش توزیع غلظت ماده حل شونده در فاز پیوسته در طول ستون بدست آمده اند، عملکرد ستون بسیار بهتر از زمانی که این ضرایب با استفاده از رابطه Calderbank and Moo-Young برای  $k_c$  و Kumar-Hartland برای  $E_c$  بدست آمده اند، پیش بینی خواهد شد.

## 1. INTRODUCTION

Early models of extraction columns were formulated by analogy with gas-liquid contacting in absorption columns [1]. A common feature of these models was the assumption that both phases are in plug-flow. Later the influence of axial mixing in one, or both, phases was included [2].

Axial mixing in countercurrent flow liquid-liquid extraction columns refer to the combination

of influences on phase flow which lead to a departure from pure countercurrent plug flow and uniform residence times within each phase, and consequently to a reduction in mass transfer driving force and performance. Factors like the column internals, drop size and velocity distribution, the extent and type of agitation, concentration gradients within each phase, channeling, etc, will play roles and affect the axial mixing in the column. Hartland and

Mecklenburgh [3] took these effects into account and developed models based on continuous and stage-wise contacting over a considerable period. Misek and Marek [4] included the influence of variable drop velocity and coalescence in a stage model.

They assumed that, different drop size fractions move from one compartment to the next one with different velocities. They also assumed that the solute concentration within a given drop size fraction is uniform (unrealistic assumption). Azimzadeh [5] developed a stage-wise model, which included the influence of drop size distribution and solved the resulting equations numerically.

Qian and Wang [6] developed a stage-wise model including coalescence and redispersion of drops to predict the pulsed sieved plate column performance. The same approach with some differences was used in this work to model and predict the performance of a rotating disc contactor (RDC).

Drop coalescence and break-up may be included in models in at least two ways. A coalescence height may be assumed, so that at repeated intervals of column height all drops will coalesce and redisperse, maintaining the distribution at these intervals of height. Or perhaps more accurately, a coalescence frequency can be assumed. The first approach was used in Qian and Wang [6] work in concern with the application of stage model, which is easy for modeling and computation.

Bahmanyar and Slater [7] studied the break-up of drops in rotating disc contactors for conditions of no mass transfer. Their observations on the break up of single drops in RDC showed that there was a critical rotor speed below which drops of a given size do not break. They correlated data as a function of Weber and Reynolds numbers. The probability of break-up of a drop at the rotor edge was expressed in terms of a Weber number with a lower limit for critical conditions. The mean number of daughter drops produced on break-up was also correlated as a simple function of drop diameter based on the critical diameter.

Later on, Bahmanyar et al. [8] studied the effect of mass transfer and its direction on the drop break-up in liquid-liquid system in RDC columns. They found out that critical rotor speeds for a

given drop size undergoing mass transfer can be used to find an effective interfacial tension. Using this interfacial tension value, the break-up fractions were correlated within experimental uncertainties in the same manner as for no mass transfer. Drop break-up fractions depend on column size and relevant empirical correlations of the data were presented.

Tsouris et al. [9] considered drop size distribution and hold-up profiles in a multistage extraction column on a population-balance-equation model. In their model they considered the drop breakage, coalescence and existence phenomena of the drop phase caused by drop-drop and drop-continuous phase interactions. They employed drop breakage and coalescence rates in order to develop a drop exit frequency based on a stochastic modeling approach. The model was tested by drop size distribution and dispersed phase volume fraction data obtained for a multistage column contactor of pilot-plant scale. The authors found out that their model could be used for the control of extraction columns and could be extended to include mass transfer calculations for the prediction of extraction efficiency.

Ghalehchian and Slater [10] studied the possible approach to improving disc contactor design accounting for drop breakage and mass transfer in the presence contamination.

They postulated a model of a liquid-liquid rotating disc contactor which takes into account drop breakage but not coalescence, individual drop motion and contact time in a stage, mass transfer coefficients which are affected by contamination, and axial mixing in the continuous phase. Their model is intended to guide industrial design work under conditions of interfacial contamination, which are expected in industrial practice. Academic work has concerned itself primarily with very clean systems resulting in mass transfer correlations, which are not appropriate for industrial design work.

The work addresses the question whether mass transfer data obtained experimentally for single drops can be used with confidence in design of columns. They showed that average drop size and hold ups can be simulated well without using adjustable parameters for each experiment for hold up not exceeding 20%. The mass transfer performance can be modeled using a

contamination factor, which varies with drop diameter and residence time. However the effect of contamination is apparently reduced in the RDC perhaps by virtue of reduced contamination gradient on the drop surface brought about by non-uniform drop motions. Their work indicates that mass transfer data for single drops in vertical motion may not be valid for column design (although giving conservative results) unless drops can be shown to be very clean in both cases Kostoglou and Karabelas [11] studied the contribution towards predicting the evaluation of droplet size distribution in flowing dilute liquid-liquid dispersions.

Their observations showed that in common flow fields, such as stirred vessels and pipelines, the steady state of the dispersed phase size distribution (including the maximum stable size  $X_m$ ) may be unattainable over a time period of practical interest. Therefore computation of the temporal evaluation of the dispersion (through the breakage population balance equation) is indispensable. They showed that the necessary breakage rate and breakage kernel could be determined from experimental data by solving the so-called *inverse* problem. In their paper they represented that to tackle the latter, an improvement of the method originally developed by Sathyagal et al. [12] is necessary. The method was based on the concept of self-similarity of size distributions, which was shown to hold even if the evolving maximum particle size is relatively close to an existing maximum stable size  $X_m$ . Their proposed improved inversion procedure relies on the observation that the form of the breakage kernel can be inferred from the form of the self-similar distribution representing the experimental data. They also showed that the new method was very stable with respect to noise in the experimental data.

## 2. THEORY

In extraction columns with sieve plates, the free void area is the smallest when drops pass through the holes on perforated plates, where it is the most probable that drops coalesce. It was observed from experiments of Grag and Pratt [13] and Qian [14]

that coalescence and breakage of drops mainly occur when they pass through plate holes, but not between plates. Obviously, the increase in dispersed phase hold-up  $\phi$ , or the decrease of the free void area  $\phi_p$  of sieve plates, will increase the frequency  $f_c$  of collision and coalescence between drops. When  $\phi = \phi_p$ , it is reasonable to assume  $f_c$  equal to one. Assuming linear relationship,  $f_c$  can be expressed as:

$$f_c = \frac{\phi}{\phi_p} \quad (1)$$

and the drops coalescence-redispersion height may, therefore, be estimated as:

$$h = \frac{h_p}{f_c} = h_p \frac{\phi_p}{\phi} \quad (2)$$

In which  $h_p$  is the plate spacing. This equation is quantitatively consistent with the experimental observation and practical design rules that the decrease of plate spacing or the fraction of void area will promote coalescence and reduce forward mixing.

Qian and Wang [6] used Equation 2 in a stage-wise model considering drop forward-mixing and coalescence-redispersion effect. They used different extraction systems and operating conditions and observed the resultant extraction column designs were reasonably satisfactory.

Some models (like work of Korchinsky and Azimzadeh) of extraction process accounted for the influence of forward-mixing, but not for coalescence and redispersion [15]. This is an idealized assumption, because except for spray columns, drop breakage and coalescence actually occur in most types of commercial extraction columns. For example, in RDCs, drops coalesce under the fixed rings and break on the rotating discs.

Similar to Qian and Wang's [6] approach, in this work a coalescence-redispersion height in RDC columns was defined and used as:

$$h = h_c \frac{\phi_p}{\phi} \quad (3)$$

where  $h_c$  and  $\phi_p$  are the column compartment height and the free void area, defined as:

$$\phi_p = \left( \frac{D_s}{D_c} \right)^2 \quad (4)$$

In Equation 4  $D_s$  and  $D_c$  are the stator and column diameters, respectively.

**2.1. Hydrodynamic Equations** The basic equations below are used to calculate individual drop velocities through the column in terms of some predicted drop-size dependent slip velocity, which has been corrected by a constriction factor  $C_R$ .

$$V_{d,i} = C_R V'_{s,i} - \frac{V_c}{(1-\phi)} = V_{s,i} - \frac{V_c}{(1-\phi)} \quad (5)$$

Summing up the flow of all drops gives:

$$\frac{Q_D}{S} = V_D = \sum_{i=1}^n f_i V_{d,i} \phi \quad (6)$$

where  $f_i$  represent the static volume fraction of drops of size  $d_i$ .

Equations 5 and 6 may be combined and rearranged to give:

$$C_R = \frac{\frac{V_D}{\phi} + \frac{V_c}{(1-\phi)}}{\sum_{i=1}^n f_i V'_{s,i}} \quad (7)$$

experimental measurements of superficial velocities, the dispersed phase hold-up and the drop size distributions, permit the calculation of  $C_R$  values for any assumed relationship for  $V'_{s,i}$ . One would expect that, if the drop-size dependency of the slip velocities were correctly predicted, the values of  $C_R$  would vary only with internal column dimensions and agitator speed.

The Olney [16] slip velocity prediction method was used to determine drop-size dependent slip

velocities as follows:

$$V'_{s,i} = V_{t,i} (1-\phi) \quad (8)$$

in which,  $V_{t,i}$  is the drop terminal velocity. Correlation proposed by Klee and Treyball [17] were used to estimate the drops terminal velocities.

The kinetic volume fraction,  $g_i$ , for each fraction could be calculated through the following relationship:

$$g_i V_D = \phi V_{d,i} f_i \quad (9)$$

**2.2. Mass Transfer Equations** The basic assumptions for mass transfer model are:

1. The height of the column is  $H$ , which is divided into  $N$  stages, giving the stage spacing as  $h=H/N$ . The height is defined as the coalescence-redispersion height, which is dependent on the operating condition and column geometry, but is not necessarily the same as the actual compartment height.

2. Drops are assumed to be spherical, with a volume fraction distribution  $f(d)$ . There is no interaction between drops, within stages. However, when drops reach the end of a stage they are assumed to coalesce, producing a uniform solute concentration in the dispersed phase. Then they are redispersed. When they enter the next stage, maintaining the same distribution of drop sizes. This implies that the solute concentration in drops is identical in a given size fraction.

3. The continuous phase is assumed to be completely mixed within each stage. To account for axial mixing of continuous phase, back flow between stages is assumed.

The liquid-liquid extraction column mass transfer model is illustrated in Figure 1.

The rising head  $z$  of a drop with diameter  $d_i$  in a differential time  $dt$  is as follows:

$$dz = V_{d,i} dt \quad (10)$$

Differential mass balance of solute transferred through the interface of drop with time  $dt$  is,

$$dR_{i,j} = K_{OD,i} \pi d_i^2 [y_j^* - y_{i,j}] dt \quad (11)$$

where  $y^*$  denotes the solute equilibrium

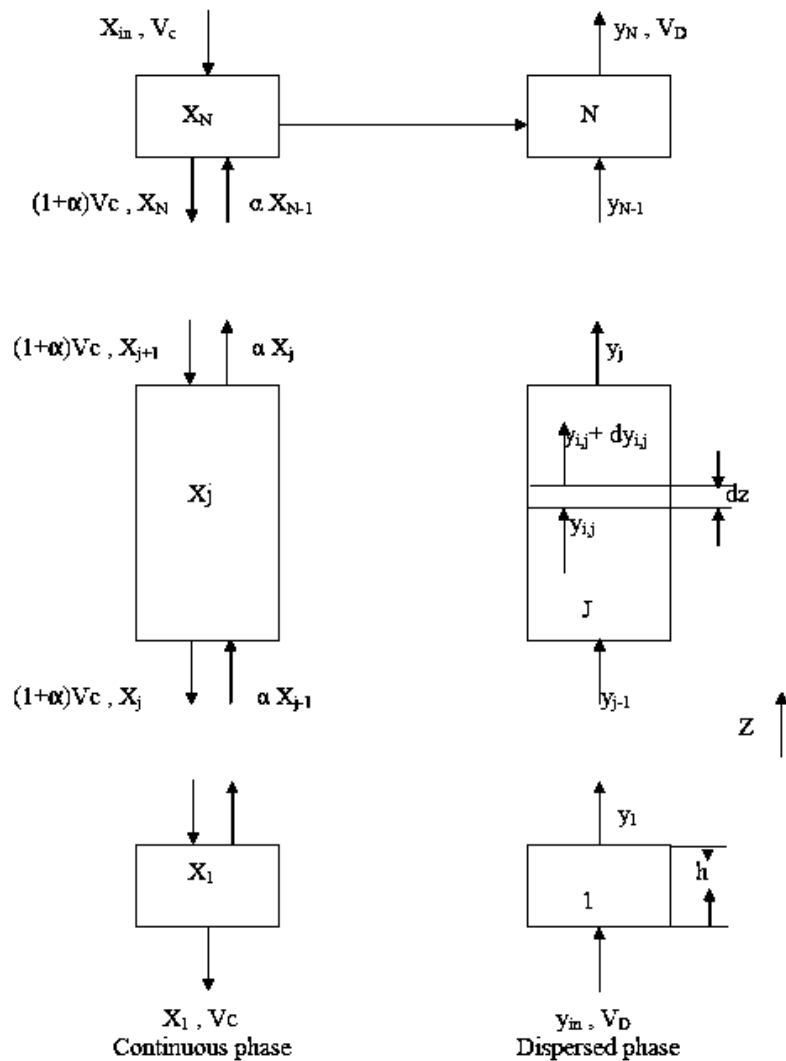


Figure 1. Forward mixing with coalescence and redispersion

concentration in drop in corresponding to the concentration of continuous phase, i.e.  $y^* = f_c(x)$ . Differential mass balance in drop gives the following equation:

$$V_{d,i} \left( \frac{\pi d_i^3}{6} \right) dy_{i,j} = K_{OD,i} \pi d_i^2 [y_j^* - y_{i,j}] dz \quad (12)$$

$y_j^*$  is constant in a stage since  $x_j$  does.

Integration of Equation 12 between  $z = (j-1)h$

and  $z = jh$  with boundary condition:

$$y_{i,j} = y_{j-1} \text{ at } z = (j-1)h$$

$$\frac{[y_{i,j}|_{z=jh} - y_j^*]}{[y_{i,j-1} - y_j^*]} = \exp[-6 K_{OD,i} h / d_i V_{d,i}] \quad (13)$$

coalescence at the end of the stage equalizes the solute concentration in all drops:

$$y_j = \sum_{i=1}^n y_{i,j} \Big|_{z=Jh} g_i \quad (14)$$

inserting Equation 14 into Equation 13 gives:

$$\frac{(y_j - y_j^*)}{y_{j-1} - y_j} = \sum_{i=1}^n g_i \exp[-6K_{OD,i}h/d_iV_{d,i}] \quad (15)$$

the overall material balance of the dispersed phase and the continuous phase in J.th stage is:

$$(1 + \alpha)V_c(x_{j+1} - x_j) = \alpha V_c(x_j - x_{j-1}) = V_D(y_j - y_{j-1}) \quad (16)$$

by introducing a new variable defined by:

$$X_j = x_j + \alpha(x_j - x_{j-1}) \quad (17)$$

which represents the concentration in the continuous phase without backflow. Equation 16 becomes:

$$V_c(X_{j+1} - X_j) = V_D(y_j - y_{j-1}) \quad (18)$$

the boundary conditions required to solve these equations are given as:

$$y_{in} = y_o \text{ and } X_1 = x_i = x_{out} \quad (19)$$

The model may be simplified further. If we define an effective mass transfer coefficient  $K_{OD}^e a$ , differential solute balance on the element shown in Figure 1 will give the following differential equation;

$$V_D dy_{i,j} = K_{OD}^e a (y_j^* - y_{i,j}) dz \quad (20)$$

Integrating Equation 20 within the J.th stage gives:

$$\frac{(y_j - y_j^*)}{y_{j-1} - y_j} = \exp[-hK_{OD}^e a / V_D] \quad (21)$$

Where the effective mass transfer coefficient is:

$$K_{OD}^e a = -\left(\frac{V_D}{h}\right) \ln\left(\sum_{i=1}^n g_i \exp\left[\frac{-6K_{OD,i}h}{d_iV_{d,i}}\right]\right) \quad (22)$$

The physical meaning of Equation 20 is that the dispersed phase passes through the column in plug flow with a specific interfacial area  $a$ . Application of the effective mass transfer coefficient simplifies the model's equations and it can be said that all the non-ideal effects included in the dispersed phase are lumped into this coefficient.

### 3. EXPERIMENTAL WORK

The data to be used were obtained in two rotating disc contactors of 7.62 cm diameter, 73.6 cm height and 21.9 cm diameter, 150 cm height. Two different systems, low (n-Butanol Succinic-Acid Water) and high (Toluene-Acetone-Water) interfacial tension (1.4 & 32 dyne/cm, respectively) were used to obtain the experimental data (45 runs all together).

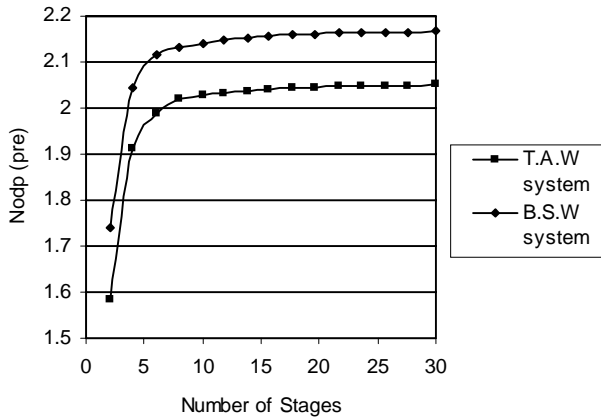
The dimensions of columns and operating conditions of the experiments are given in [18]. The experimental data were including the measured dispersed phase hold-up, drop size distribution, and concentration profile of continuous phase along the column height.

The data mentioned above were used to predict the continuous and axial dispersion coefficients [18].

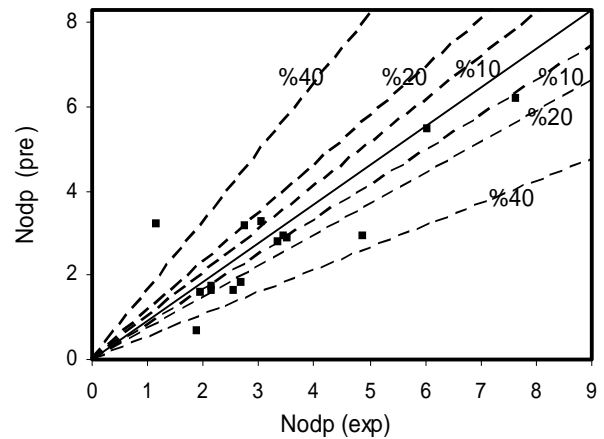
### 4. RESULTS AND DISCUSSIONS

The equations of the model and experimental data were used to predict the performance of the columns. In order to evaluate the accuracy of the column prediction, instead of comparison of efficiencies, the number of experimental and predicted transfer units based on dispersed phase and plug flow number [ $N_{odp}(\text{exp})$  and  $N_{odp}(\text{pre})$ ] were compared.

In addition to the described model, a stagewise model; neglecting coalescence and redispersion was used to predict the column performance. In the latter model,  $h$  (the coalescence-redispersion



**Figure 2.** Effect of the number of the stages on prediction of  $N_{odp}$ .



**Figure 3.** Comparison between  $N_{odp}$  (exp) and  $N_{odp}$  (pre),  $k_c$  and  $E_c$  from conc. profile, B.S.W. system.

height) was substituted by:

$$h = \frac{Z}{N} \quad (23)$$

where  $Z$  stands for column height and  $N$  for the number of the stages.

The drop-size overall mass transfer coefficient values,  $K_{OD}(d)$ , were calculated from the model proposed by Handlos and Baron [19]. The continuous phase mass transfer coefficient,  $k_c$ , in addition to those values evaluated by Bastani [18] were calculated using the correlation given by Calderbank and Moo-Young [20]. The continuous phase backflow ratio  $\alpha$  was calculated from the following relation:

$$\frac{E_c (1 - \phi)}{V_c h_c} = 0.5 + \alpha \quad (24)$$

where  $E_c$  is the continuous phase axial dispersion coefficient given by Kumar and Hartland [21].

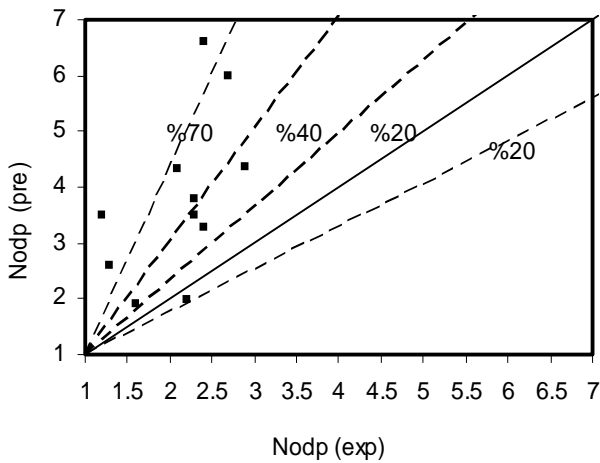
The effect of the number of stages on the predicted plug flow number of transfer units (or extraction efficiency) is shown in Figure 2. As the figure shows model's prediction is highly dependent on the number of stages when they are

low (less than 8), and relatively independent when the number of stages are high (more than 15). At low stage numbers, differences between  $N_{odp}$  (exp) and  $N_{odp}$  (pre) were very high (more than 300% in some cases), therefore they are not brought here. In contrast the predicted and experimental values of  $N_{odp}$  were much closer at high stage numbers.

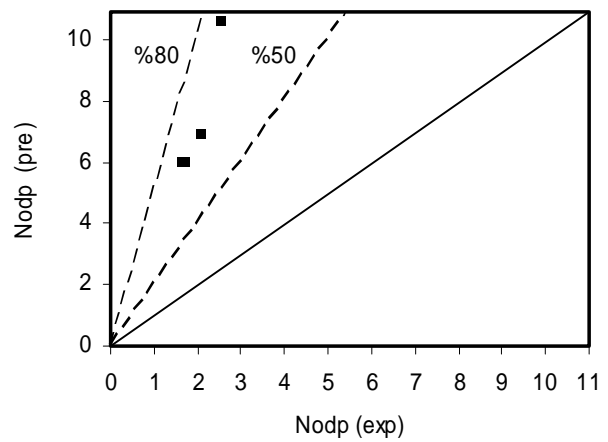
Also it was observed that there was a good agreement between the predicted number of transfer units when the number of stages were very high (more than 50) and those predicted when axial dispersion model was used. This confirms the close prediction of these two models when in the stage model the number of stages is high enough.

In spite of the more accuracy prediction of the axial dispersion model, in this work stage model was selected because the author believes that model should be constructed from first principles of the process as close as possible and be applied in design and simulation as simply as possible. The hydrodynamic results are previously discussed in [12] in detail, therefore they are not brought in this work.

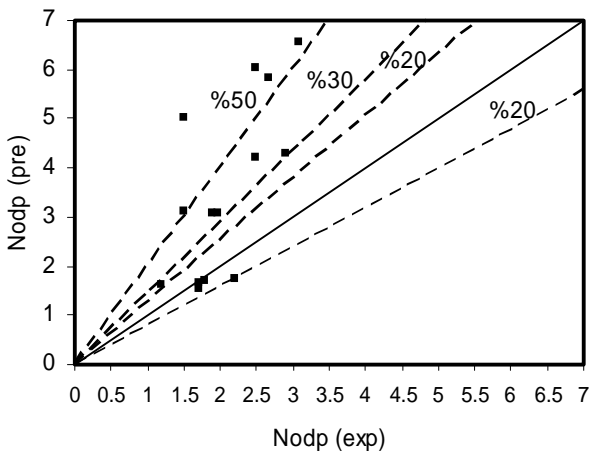
The predicted and experimental plug flow number of transfer units for small and large columns using both systems are compared in Figures 3 to 6. In all of this figures the continuous phase axial dispersion and mass transfer coefficients were those evaluated by Bastani [18]



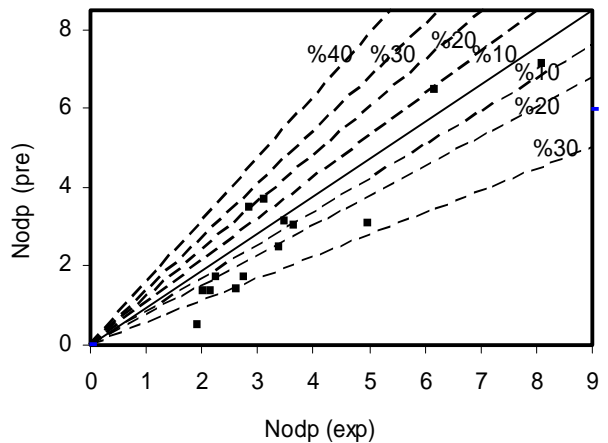
**Figure 4.** Comparison between  $N_{odp} (exp)$  and  $N_{odp} (pre)$ ,  $k_c$  and  $E_c$  from conc. profile, T.A.W. system.



**Figure 6.** Comparison between  $N_{odp} (exp)$  and  $N_{odp} (pre)$ ,  $k_c$  and  $E_c$  from conc. profile, T.A.W. system.



**Figure 5.** Comparison between  $N_{odp} (exp)$  and  $N_{odp} (pre)$ ,  $k_c$  and  $E_c$  from conc. profile, B.S.W. system.



**Figure 7.** Comparison between  $N_{odp} (exp)$  and  $N_{odp} (pre)$ ,  $k_c$  and  $E_c$  from correlation, B.S.W. system.

from the continuous phase solute concentration profiles measurements The predicted plug flow number of transfer units,  $N_{odp} (pre)$  values using Kumar and Hartland [21] correlation for  $E_c$ , and Calderbank and Moo-Young [20] correlation for  $k_c$  calculations are also compared with  $N_{odp} (exp)$  values in Figures 7 to 10.

As Figures 3 and 4 show, differences between

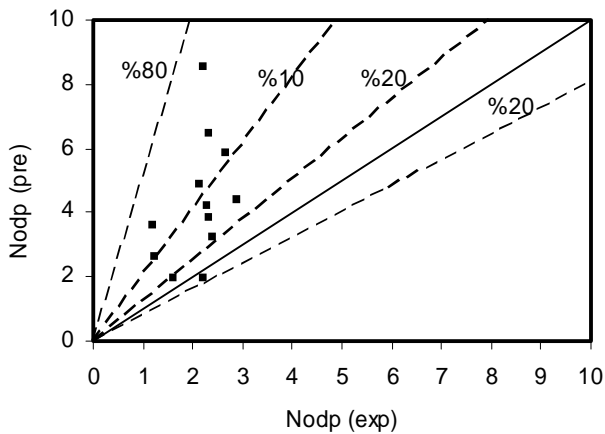
$N_{odp} (pre)$  and  $N_{odp} (exp)$  in large column using B.S.W. system are about  $\pm 20\%$ , while they are about 45% when T.A.W. system is used.

The same trend of differences (35% and 70%) can be seen in Figures 5 and 6 for small column.

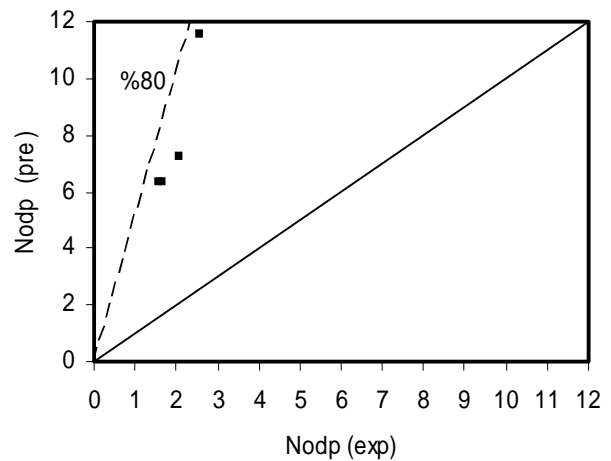
According to Figures 7 and 8 differences between  $N_{odp} (pre)$  and  $N_{odp} (exp)$  in large column using both systems are about  $\pm 30\%$  and 50%.

Figures 9 and 10 show 45% and 70%

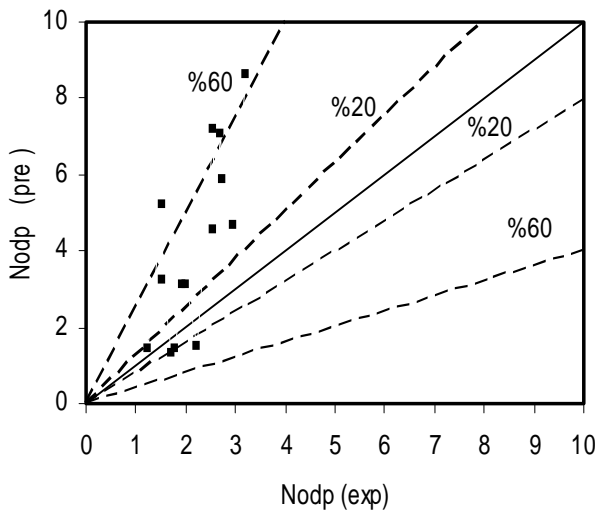




**Figure 8.** Comparison between  $N_{odp} (exp)$  and  $N_{odp} (pre)$ ,  $k_c$  and  $E_c$  from correlation, T.A.W. system.



**Figure 10.** Comparison between  $N_{odp} (exp)$  and  $N_{odp} (pre)$ ,  $k_c$  and  $E_c$  from correlation, T.A.W. system.



**Figure 9.** Comparison between  $N_{odp} (exp)$  and  $N_{odp} (pre)$ ,  $k_c$  and  $E_c$  from correlation, B.S.W. system.

differences in small column using B.S.W. and T.A.W. systems, respectively.

The results show that the model prediction are more accurate for low interfacial tension system, it is perhaps due to narrower drop size distributions and/or higher rate of coalescence and redispersion in these systems. An important parameter, which affects the prediction of the

model, is the continuous backflow ratio  $\alpha$ . The calculated values of  $\alpha$  were between 0.05 and 0.2. The parameter values depend on calculated  $E_c$  and its accuracy. Calculated  $E_c$  values using Kumar & Hartland [21] correlation are less accurate than those predicted using concentration profile approach. Kumar and Hartland [21] correlation for  $E_c$  prediction is based on experimental data obtained by tracer injection method, which is in the absence of solute mass transfer operations.

## 5. CONCLUSION

Conclusions may be summarized as follows:

1. Stagewise forward mixing model considering coalescence and redispersion of drops, can be used to evaluate the liquid-liquid extraction column performance.
2. The model prediction for low interfacial tension system is more accurate, probably due to narrower drop size distribution and/or higher rate of coalescence and redispersion.
3. The stagewise modeling predictions (as stated

by Azimzadeh) are highly dependent on the number of stages used (the results of this section due to high scatterness of results are not stated here).

4. Application of continuous phase mass transfer and axial dispersion coefficients ( $k_c$  and  $E_c$ ) obtained from the solute concentration profile along the column height, will predict the column performance more accurate than those correlations given by Calderhank and Moo-Young (for  $k_c$ ) and Kumar and Hartland (for  $E_c$ ).

## 6. ACKNOWLEDGEMENT

The financial support for this project was provided in parts by the Research Administration of the Sharif University of Technology and is gratefully acknowledged.

## 7. NOMENCLATURE

$a$	specific interfacial area, $m^2/m^3$
$C_R$	constriction factor of the hydrodynamic equation.
$d$	drop diameter, m
$d_i$	drop diameter, $i$ th fraction, m
$D_s$	stator ring diameter, m
$D_c$	column diameter, m
$E$	extraction efficiency
$E_c$	axial dispersion coefficient, $m^2/s$
$f(d)$	drop volume fraction distribution, $m^{-1}$
$f_i$	static volume fraction of drops with diameter $d_i$
$f_c$	collision and coalescence frequency of drops, $t^{-1}$
$g_i$	dynamic volume fraction of drops with diameter $d_i$
$g_d$	drop flow rate fraction distribution, $m^{-1}$

$H$	effective height of extraction column, m
$h$	drops coalescence-redispersion height, m
$h_c$	compartment height, m
$K_{OD}$	overall mass transfer coefficient, dispersed phase basis, m/s
$K_{OD,i}^e$	effective overall mass transfer coefficient, m/s
$k_c$	continuous phase mass transfer coefficient, m/s
$N$	number of stages in model
$n$	number of drop fractions
$Q_D$	dispersed phase volumetric flow rate, $m^3/s$
$Q_C$	continuous phase volumetric flow rate, $m^3/s$
$R$	mass transfer rate of solute, kmol/s
$Re$	drop Reynolds number
$S$	column cross-sectional area, $m^2$
$t$	time, seconds
$V$	superficial velocity, m/s
$V_d$	drop velocity, m/s
$V'_s$	Drop slip velocity (before correction), m/s
$V_{s,i}$	drop slip velocity, m/s
$V_C$	continuous phase superficial velocity, m/s
$V_D$	dispersed phase superficial velocity, m/s
$V_{t,i}$	drop terminal velocity, m/s
$X$	Solute mole fraction in continuous phase, plug flow, kmol/kmol
$x_j$	Solute mole fraction in continuous phase, $j$ th stage, kmol/kmol
$x$	Solute mole fraction in continuous phase, kmol/kmol
$y_{i,j}$	solute mole fraction in dispersed phase, $i$ th fraction in $j$ th stage, kmol/kmol
$y_j^*$	equilibrium mole fraction with $x$ , $y_j^* = f_e$ , kmol/kmol
$Z$	vertical position along the column height, m

## Greek Letters

$\phi_p$	fractional free area of plates
$\phi$	dispersed phase hold-up
$\alpha$	continuous phase backflow ratio

## Subscripts

C,D	refer to continuous or dispersed phase
d	refers to drop
in, out	refer to inlet or outlet stream
j	refer to jth stage
odp	overall dispersed phase plug flow
pre	predicted
exp	experimental

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