INTEGRATOR BACKSTEPPING CONTROL OF A 5 DOF ROBOT MANIPULATOR WITH CASCADED DYNAMICS

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Abstract In this paper, dynamic equations of motion of a 5 DoF robot manipulator including mechanical arms with revolute joints and their electrical actuators are considered. The application of integrator backstepping technique for trajectory tracking in presence of parameters of uncertainty and disturbance is studied. The advantage of this control technique is that it imposes the desired properties of stability by fixing the candidate Lyapunov functions initially, then by calculating the other functions in a recursive way. Simulation results are presented in order to evaluate the tracking performance and the global stability of the closed loop system. The validity and usefulness of the proposed technique for robot motion control when the system dynamics including both mechanical arms and electrical actuators become more complex is obtained from the results.

Key Words Integrator Backstepping Control, Actuators Dynamics, 5 DoF Robot Manipulator, Robot Control

چکیده در این مقاله، معادلات دینامیکی یک ربات فضایی شامل بازوهای مکانیکی با مفصل‌های دورانی به همراه محرک‌های الکتریکی بررسی شده و در حالتی که پارامترهای اینرسی ربات معلوم فرض شده‌اند، اعمال روش کنترلی گام بعقب انتگرالی بر این ربات مورد مطالعه قرار گرفته و دستور کنترل مناسب ارائه می‌گردد. مزیت این روش کنترلی در این است که خواص مطلوب پایداری را با تعیین تابع لیاپانف کاندیدا شده اعمال کرده و سپس دیگر توابع لازم را با یک روش برگشتی محاسبه می‌نماید . نتایج مدلسازی شامل ارزیابی عملکرد و کارآیی روش گام بعقب در پیروی مسیر و پایداری مجانبی سراسری سیستم کنترلی مدار بسته بوده و نشان می‌دهد که روش پیشنهادی برای کنترل حرکت ربات با توجه به پیچیده‌تر شدن دینامیک سیستم که شامل قسمتهای مکانیکی و الکتریکی است، بسیار مناسب می‌باشد.

1. INTRODUCTION

Recently robot's actuator dynamics has been explicitly included in control schemes. This dynamics becomes extremely important during fast robot motion and highly varying loads, [1-10]. However, the inclusion of actuators in dynamic equations complicates both the controller structure and its stability analysis [2]. We can avoid increasing the order of dynamic equations by considering the arm's dynamics and motor's dynamics as two cascade loops [3].

Lyapunov theory has for a long time been an important tool in linear as well as nonlinear control theory. However, its use within nonlinear control programming has been hampered by difficulties to find a Lyapunov function for a given system. If one can be found, the system is known to be stable, but the task of finding such a function has often been left to the imagination and experience of the designer [11]. Also the more complicated the dynamics of the nonlinear system is, the more important and sophisticated this task will appear to be.

Over the past ten years, focus in the areas of control theory and control engineering has shifted from linear to nonlinear systems providing control algorithms for systems that are both more general and more realistic. Nonlinear control has, therefore, shown strong presence in academic curricula, in industry and in conferences during recent years [12]. The control community has, therefore, been receiving with open arms the invention of constructive tools for nonlinear control design based on Lyapunov theory like backstepping and forwarding [11].

In this paper, using the integrator backstepping method develops the problem of nonlinear position control of a 5 DoF robot manipulator. It consists of elaborating a control method [13] that guarantees the asymptotic stability and the tracking of desired position and velocity trajectories. A major advantage of this method is its flexibility to build the control law by avoiding the cancellation of useful non-linearities [8]. Simulation results presented in this paper show that the system has global stability.

2. ROBOT DYNAMIC EQUATIONS

The popular second order model of robot manipulators, which disregards the actuator dynamics, leads to unsatisfactory performance of nonlinear feedback controllers [14,15]. Especially, in fast tracking problems, with high-speed microprocessors, the small electric time constant of the servomotors cannot be neglected any more and a more precise model is needed [14]. Based on this fact, a third order model of robotic system is usually used. To avoid the need to measure the accelerations, in this paper, the arm's dynamics and motor's dynamics are considered as two cascade loops [3].

Consider an n-links manipulator. Let $q \in R^n$ denote the vector of generalized displacements of joint space coordinates. Generally, the second order dynamic model of the mechanical system can be written as [16,17]:

$$\tau = \left(M(q) + J_m\right).\ddot{q} + C(q,\dot{q}).\dot{q} + G(q) + F(\dot{q}) + T_L \qquad (1)$$

where robot parameters are:

$M(q)$: n×n positive definite manipulator inertia matrix,

J_m: n×n diagonal matrix of motors and gear inertias,

$C(q,\dot{q})$: n×n diagonal matrix due to Coriolis and centrifugal forces,

$G(q)$: n×1 vector of joint torques and forces due to gravity,

$F(\dot{q})$: n×1 vector containing the static and dynamic friction terms,

T_L: n×1 vector representing an additive bounded joint torque disturbance,

τ: n×1 vector of joint torques,

Also with DC motors as system actuators, actuator dynamics is [1,3,8]

$$L.\dot{I} + R.I + K_e.\dot{\theta} + T_E = v \qquad (2)$$

where,

L: positive definite constant diagonal n×n matrix used to represent the electrical inductance,

R: n×1 vector used to represent the electrical resistance,

I: n×1 vector of armature current in each joint actuator,

K_e: n×n positive definite constant diagonal matrix of back electromotive coefficients,

θ: n×1 vector of angular positions of the rotors,

T_E: n×1 vector representing an additive bounded voltage disturbance,

v: n×1 input armature voltage vector.

The angular position of the rotors and generalized displacement in articulation coordinates are related by

$$\theta = A.q \qquad (3)$$

where A is an n×n positive definite constant diagonal matrix of gear ratios.

The relationship between the joint's torques vector and the armature current vector is described by:

$$\tau = AK_T.I \qquad (4)$$

where K_T is n×n positive definite constant diagonal matrix of actuator torque coefficients.

Substituting Equation 4 in Equation 1, one obtains the following equation:

$$A.K_T.I = \left(M(q) + J_m\right)\ddot{q} + C(q,\dot{q}).\dot{q} + G(q) + F(\dot{q}) + T_L \qquad (5)$$

Also, if we combine Equation 3 with Equation

2, dynamic equation of actuators can be written as:

$$L.\dot{I} + R.I + K_e.A.\dot{q} + T_E = v \qquad (6)$$

According to Reference 6, we can have a cascaded form of the dynamic equations by considering Equations 5 and 6 together:

$$\begin{cases} (M(q) + J_m).\ddot{q} + C(q,\dot{q}).\dot{q} + G(q) + F(\dot{q}) + T_L = A.K_T.I \\ L.\dot{I} + R.I + K_e.A.\dot{q} + T_E = v \end{cases} \qquad (7)$$

3. STATE SPACE REPRESENTATION

Let us put the system's dynamics in a state space form. With the state vectors chosen as

$$\xi = \left[\xi_1^T, \xi_2^T\right]^T \quad , \quad \xi_1 = q \quad , \quad \xi_2 = \dot{q} \qquad (8)$$

and

$$\xi_3 = [I] \qquad (9)$$

then, from Equations 7, 8 and 9, we have:

$$\xi_1 = q \;\Rightarrow\; \dot{\xi}_1 = \dot{q} = \xi_2 \qquad (10)$$

$$\xi_2 = \dot{q} \;\Rightarrow$$
$$\dot{\xi}_2 = \ddot{q} = \left(M(\xi_1) + J_m\right)^{-1} \times$$
$$\left(-C(\xi_1, \xi_2).\xi_2 - G(\xi_1) - F(\xi_2) - T_L + A.K_T.\xi_3\right) \qquad (11)$$

$$\xi_3 = I \;\Rightarrow\; \dot{\xi}_3 = L^{-1}.\left(-R.\xi_3 - K_e.A.\xi_2 - T_E + v\right) \qquad (12)$$

$$\dot{\xi} = \begin{bmatrix} \xi_2 \\ -(M(\xi_1) + J_m)^{-1}\left(C(\xi_1, \xi_2).\xi_2 + G(\xi_1) + F(\xi_2) + T_L\right) \end{bmatrix} + \begin{bmatrix} 0 \\ (M(\xi_1) + J_m)^{-1} A.K_T \end{bmatrix} \xi_3$$
$$\dot{\xi}_3 = \left[-L^{-1}\left(R.\xi_3 + K_e.A.\xi_2 + T_E\right)\right] + \left[L^{-1}\right] v \qquad (13)$$

This dynamics can be written in the following general form:

$$\begin{aligned} \dot{\xi} &= f(\xi) + g(\xi).\xi_3 \\ \dot{\xi}_3 &= f_1(\xi, \xi_3) + g_1(\xi, \xi_3).v \end{aligned} \qquad (14)$$

where, $\xi \in R^{2n}$, $\xi_3 \in R^n$ and $v \in R^n$.

4. INTEGRATOR BACKSTEPPING TECHNIQUE

Backstepping is a recursive design methodology for constructing both feedback control laws and associated Lyapunov functions in a systematic manner, whose significance for nonlinear control can be compared to root locus or Nyquist's method for linear systems. Its root is in the theory of feedback linearization of the 1980's, [12,18,19]. The key idea in backstepping is to let certain states act as "virtual controls" of others, [11,19]. The same idea can be found in cascade control design and singular perturbation theory [20].

In this method, by recursive manner we introduce feedback control laws and their Lyapunov functions, storage functions and their stabilizing functions, in a systematic way [11].

Nonlinear backstepping designs are strongly related to feedback linearization. However, while feedback linearization method cancels all nonlinearities in the system, it will be shown that when applying the backstepping design methodology, the designer obtains flexibility to exploit "good" nonlinearities while "bad" or destabilizing nonlinearities are dominated e.g. by adding nonlinear dampings. Hence, additional robustness is obtained. This is important in industrial control systems since cancellation of all nonlinearities require precise models which are difficult to obtain in practice [11,18].

The reference [1] applied this procedure in robot motion control. Also the references [6,8] used this method for the case of uncertainty in combined dynamics. But these studies were for planar or low DoF robot manipulators. We consider this technique to control the whole dynamics of a 5-Dof robot manipulator. The proposed technique in this paper does not need measuring joint accelerations and some difficult calculations are avoided although system dynamics is still complex.

5. THE BACKSTEPPING CONTROL LAW FOR ROBOT TRAJECTORY TRACKING

According to Reference 21, the control law will be conceived by applying the backstepping technique. To ensure the asymptotic stability of the system, the backstepping technique consists of considering a given output, fixing a storage function and calculating the candidate and stabilizing functions valid for each system step. In the present case, we have only one step. The objective is to find a control law *v* to stabilize the system of Equation 7. System of Equation 13 satisfies all requirements of the class of strict-feedback systems [21].

Considering the output,

$$y = \beta.(\xi_3 - \xi_{3d}) - \alpha_0(\xi) \tag{15}$$

the candidate Lyapunov function,

$$V(\xi,\xi_3) = W(\xi) + \tfrac{1}{2} y^T.y \tag{16}$$

and the storage function,

$$W(\xi) = \frac{1}{2}(\xi_1 - \xi_{1d})^T.(\xi_1 - \xi_{1d}) \tag{17}$$

such that the stabilizing function is:

$$\alpha_0(\xi) = \beta.(\xi_3 - \xi_{3d}) - (\xi_2 - \xi_{2d}) \tag{18}$$

with selecting $\beta = I_{n\times n}$ the identity matrix, and using Equations 8 and 9, the time derivatives of $W(\xi)$, $V(\xi,\xi_3)$ and *y* are:

$$\dot{W}(\xi) = \frac{1}{2}(\dot{\xi}_1 - \dot{\xi}_{1d})^T.(\xi_1 - \xi_{1d}) + \frac{1}{2}(\xi_1 - \xi_{1d})^T.(\dot{\xi}_1 - \dot{\xi}_{1d}) = (\xi_1 - \xi_{1d})^T.(\xi_2 - \xi_{2d}) \tag{19}$$

$$\dot{V}(\xi,\xi_3) = \dot{W}(\xi) + \tfrac{1}{2}\dot{y}^T.y + \tfrac{1}{2}y^T.\dot{y} = (\xi_1 - \xi_{1d})^T.(\xi_2 - \xi_{2d}) + y^T.\dot{y} \tag{20}$$

$$\dot{y} = (\dot{\xi}_3 - \xi_{3d}) - \frac{\partial \alpha_0(\xi)}{\partial \xi}.\dot{\xi} \tag{21}$$

From Equation 18:

$$\frac{\partial \alpha_0(\xi)}{\partial \xi} = \frac{\partial}{\partial \xi}\left((\xi_3 - \xi_{3d}) - (\xi_2 - \xi_{2d})\right) = \left\{[0]_{n\times n}, [-1]_{n\times n}\right\} \tag{22}$$

So, we can rewrite Equation 21 as:

$$\dot{y} = (\dot{\xi}_3 - \xi_{3d}) - \frac{\partial \alpha_0(\xi)}{\partial \xi}.\dot{\xi} = (\dot{\xi}_3 - \xi_{3d}) - \{[0]_{n\times n}, [-1]_{n\times n}\}.\dot{\xi} = (\dot{\xi}_3 - \xi_{3d}) + \dot{\xi}_2 \tag{23}$$

Substituting Equation 23 into Equation 20, one obtains

$$\dot{V}(\xi,\xi_3) = (\xi_1 - \xi_{1d})^T.(\xi_2 - \xi_{2d}) + y^T.\left((\dot{\xi}_3 - \xi_{3d}) + \dot{\xi}_2\right) \tag{24}$$

Now, by replacing $\dot{\xi}$ and $\dot{\xi}_3$ in Equation 13, we will have

$$\dot{V}(\xi,\xi_3) = (\xi_1 - \xi_{1d})^T.(\xi_2 - \xi_{2d}) + y^T.\Big(-L^{-1}\big(R.\xi_3 + K_e.A.\xi_2 + T_E\big) + L^{-1}.v - \xi_{3d} + \big(M(\xi_1) + J_m\big)^{-1}.\big(A.K_T.\xi_3 - \big(C(\xi_1,\xi_2).\xi_2 + G(\xi_1) + F(\xi_2) + T_L\big)\big)\Big) \tag{25}$$

By using Lyapunov stability condition, one obtains

$$\dot{V}(\xi,\xi_3) \le -y^T.y \tag{26}$$

Then we have

$$-y^T.y \ge (\xi_1 - \xi_{1d})^T.(\xi_2 - \xi_{2d}) + y^T.\Big(-L^{-1}\big(R.\xi_3 + K_e.A.\xi_2 + T_E\big) + L^{-1}.v - \xi_{3d} + \big(M(\xi_1) + J_m\big)^{-1}.\big(A.K_T.\xi_3 - \big(C(\xi_1,\xi_2).\xi_2 + G(\xi_1) + F(\xi_2) + T_L\big)\big)\Big) \tag{27}$$

By rewriting the above equation, one gets

$$-y^T.y-(\xi_1-\xi_{1d})^T.(\xi_2-\xi_{2d})\geq$$
$$y^T.\left(-L^{-1}\left(R.\xi_3+K_e.A.\xi_2+T_E\right)+L^{-1}.v-\xi_{3d}+\left(M(\xi_1)+J_m\right)^{-1}.\left(A.K_T.\xi_3-\left(C(\xi_1,\xi_2).\xi_2+G(\xi_1)+F(\xi_2)+T_L\right)\right)\right) \quad (28)$$

In the other hand, combining Equation 15 and Equation 18 gives,

$$y=(\xi_2-\xi_{2d}) \quad (29)$$

Therefore, we obtain from Equation 28:

$$-y^T.(y+(\xi_1-\xi_{1d}))\geq$$
$$y^T.\left(-L^{-1}\left(R.\xi_3+K_e.A.\xi_2+T_E\right)+L^{-1}.v-\xi_{3d}+\left(M(\xi_1)+J_m\right)^{-1}.\left(A.K_T.\xi_3--\left(C(\xi_1,\xi_2).\xi_2+G(\xi_1)+F(\xi_2)+T_L\right)\right)\right) \quad (30)$$

One sufficient condition for Inequality 30 to be satisfied is:

$$-(y+(\xi_1-\xi_{1d}))\geq$$
$$\left(-L^{-1}\left(R.\xi_3+K_e.A.\xi_2+T_E\right)+L^{-1}.v-\xi_{3d}+\left(M(\xi_1)+J_m\right)^{-1}.\left(A.K_T.\xi_3--\left(C(\xi_1,\xi_2).\xi_2+G(\xi_1)+F(\xi_2)+T_L\right)\right)\right) \quad (31)$$

The final form of the above equation is,

$$-((\xi_2-\xi_{2d})+(\xi_1-\xi_{1d}))\geq$$
$$\left(-L^{-1}\left(R.\xi_3+K_e.A.\xi_2+T_E\right)+L^{-1}.v-\xi_{3d}++\left(M(\xi_1)+J_m\right)^{-1}.\left(A.K_T.\xi_3-\left(C(\xi_1,\xi_2).\xi_2+G(\xi_1)+F(\xi_2)+T_L\right)\right)\right) \quad (32)$$

Noticing that

$$g_1(\xi,\xi_3)=L\neq 0 \quad (33)$$

where g_1 is nonsingular for all values of ξ_1, ξ_2 and ξ_3, we can present a particular control law of the model of Equation 13,

$$v=L.\left(-((\xi_2-\xi_{2d})+(\xi_1-\xi_{1d}))+L^{-1}\left(R.\xi_3+K_e.A.\xi_2+T_E\right)+\xi_{3d}+\left(M(\xi_1)+J_m\right)^{-1}.\left(-A.K_T.\xi_3+\left(C(\xi_1,\xi_2).\xi_2+G(\xi_1)+F(\xi_2)+T_L\right)\right)\right) \quad (34)$$

The global stability of the closed loop system is ensured by the recursive nature of the backstepping technique. If we consider that all the parameters are known, in addition to the desired positions, velocities and accelerations trajectories, the expression of the transformed desired current I_d from Equation 7 can be obtained as follow,

$$\begin{cases}\left(M(q)+J_m\right).\ddot{q}+C(q,\dot{q}).\dot{q}+G(q)+F(\dot{q})+T_L= \\ A.K_T.I_d+A.K_T.\tilde{I} \\ L.\dot{I}+R.I+K_e.A.\dot{q}+T_E=v\end{cases} \quad (35)$$

where $\tilde{I}$, is the current error and I_d is the desired current.

Then dynamics of the robotic arm can be seen as a subsystem, which has a disturbance $A.K_T.\tilde{I}$ and is controlled by $A.K_T.I_d$. Thus, by transforming the coordinates into state variable vectors, we have

$$\xi_{3d}=K_T^{-1}.A^{-1}\left(\left(M(\xi_1)+J_m\right).\dot{\xi}_2^*+C(\xi_1,\xi_2).\xi_2^*+G(\xi_1)+F(\xi_2^*)+T_L-K_d.\varepsilon\right) \quad (36)$$

with ξ_2^* and ε defined as follow

$$\xi_2^*=\xi_{2d}-\Lambda.(\xi_1-\xi_{1d}) \quad (37)$$

$$\varepsilon=\xi_2-\xi_{2d}+\Lambda.(\xi_1-\xi_{1d})=\xi_2-\xi_2^* \quad (38)$$

where ε is the auxiliary error and Λ is an n×n positive definite matrix. Vector of desired state variables and their errors can be presented as,

TABLE 1. Kinematic Specifications of The Robot.

Joint	θ_i [rad]	d_i [mm]	a_i [mm]	α_i [rad]
1	θ_1	125	0	$-\frac{\pi}{2}$
2	θ_2	0	200	0
3	θ_3	0	200	0
4	θ_4	0	0	$-\frac{\pi}{2}$
5	θ_5	148	0	0

TABLE 2. Desired Trajectory Data.

Link	1	2	3	4	5
$\theta_{m,i}$	$-\frac{2\pi}{3}$	$\frac{\pi}{2}$	$-\frac{\pi}{3}$	$\frac{\pi}{6}$	π

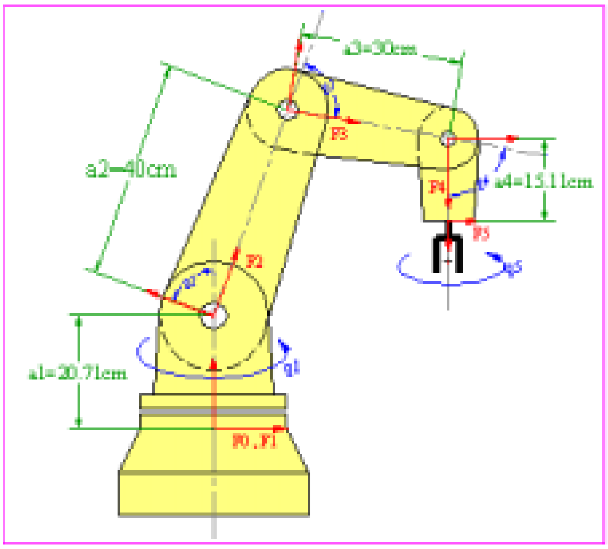

Figure 1. Schematic of the 5 DoF robot.

$$\tilde{\xi}_1 = \xi_1 - \xi_{1d}, \tilde{\xi}_2 = \xi_2 - \xi_{2d}, \tilde{\xi}_3 = \xi_3 - \xi_{3d} \quad (39)$$

6. THE ROBOTIC SYSTEM

The considered robotic system is a 5Dof three-dimensional robot manipulator with revolute joints. Its kinematic specifications are given in Table 1 [16,17]. Schematic of the robot is presented in Figure 1.

Masses and inertias, link's center of mass vectors in corresponding frames and motors specifications are considered [22].

The desired trajectory is defined in joint space in which the arms start from zero and go to $\theta_{m,i}$, then pauses for 20 sec at the position and finally return to the initial point. In the first part of the trajectory, all joints have motions together and in the final part, they have separate motions. Table 2 shows the desired trajectory data in joint space.

7. SIMULATION RESULTS

The simulation results are for two cases. In first, we have simulated the robot for tracking a trajectory when there is no uncertainty in the parameters and no disturbance exists. In this case, gains of the controller are chosen as $\Lambda = 10.I_{5\times5}$, $K_d = 10.I_{5\times5}$. Figs. 2-4 show the controlled voltage commands, positions and velocities of links for 100 sec.

For the second case, we have simulated the system for 5~10% uncertainty in the parameters and the existence of voltage disturbances as $V_i = 0.1\times\mathrm{Sin}(\pi.t/4)$ for i=1… 5. The controller gains are selected as $\Lambda = 100.I_{5\times5}$, $K_d = 80.I_{5\times5}$. Results for this case are shown in Figures 5-7.

8. CONCLUSION

The simulation results show the ability and merit of the backstepping technique. The results of both cases were fairly good. All control commands are admissible and trajectory was tracked with good accuracy.

The first case of simulation shows high tracking accuracy of backstepping when just some parts of robot dynamics are used. The second case shows the power of the proposed control technique for eliminating disturbances and the robot's acceptable and accurate tracking of the desired trajectory.

The results show the integrator backstepping control method is very suitable since it does not use all system nonlinearities. It only uses good

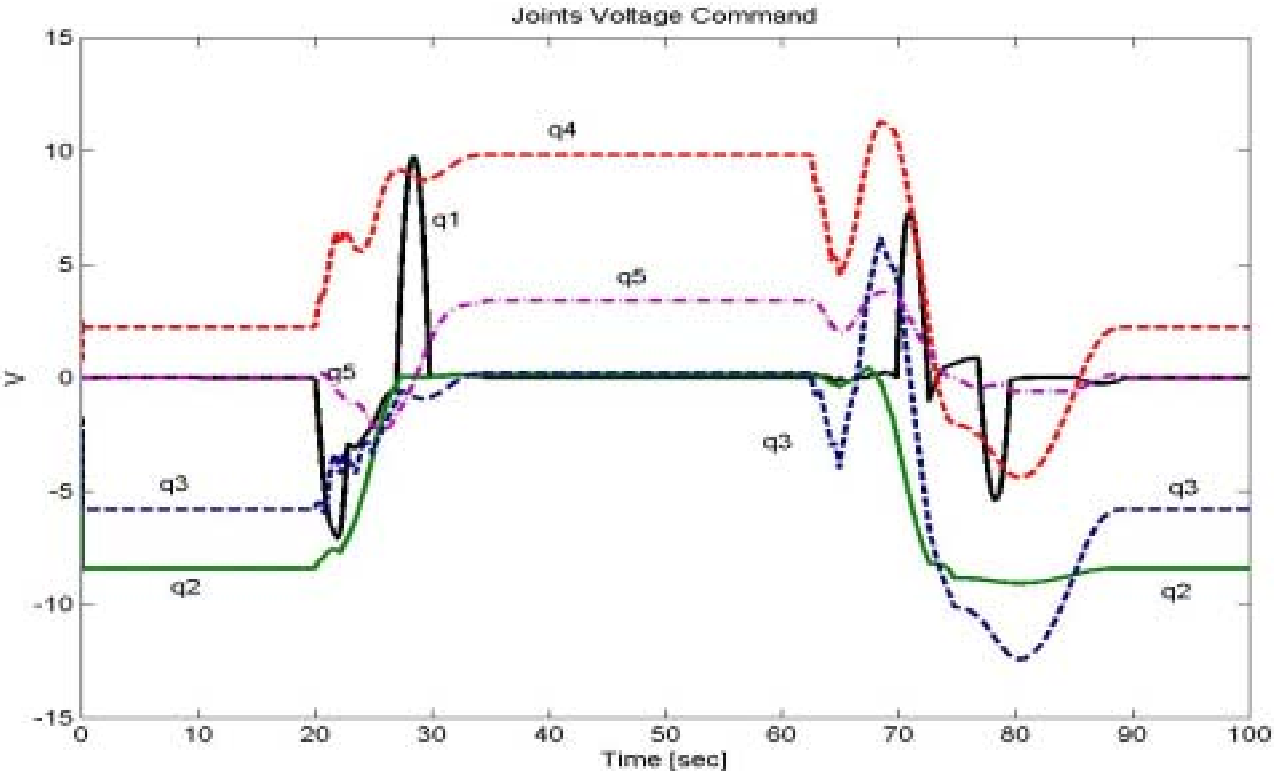

Figure 2. Control commands for the first case.

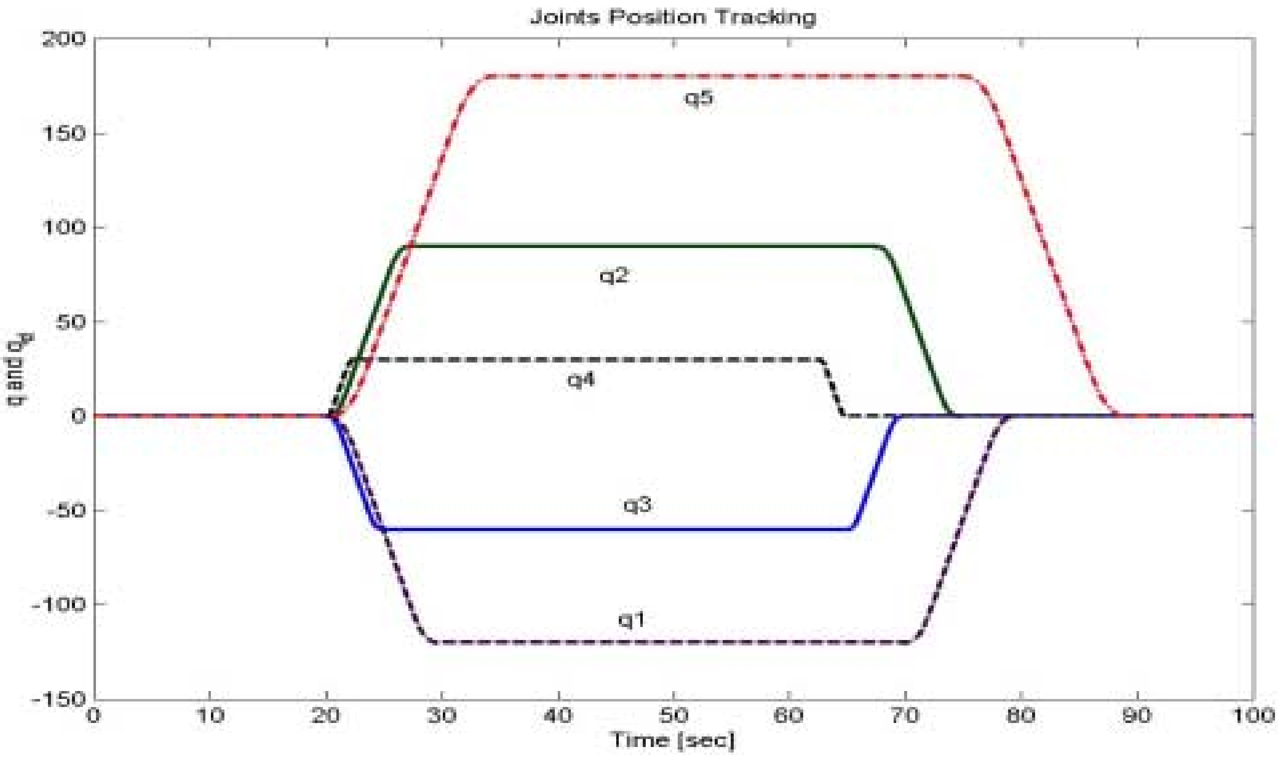

Figure 3. Positions in trajectory tracking for the first case.

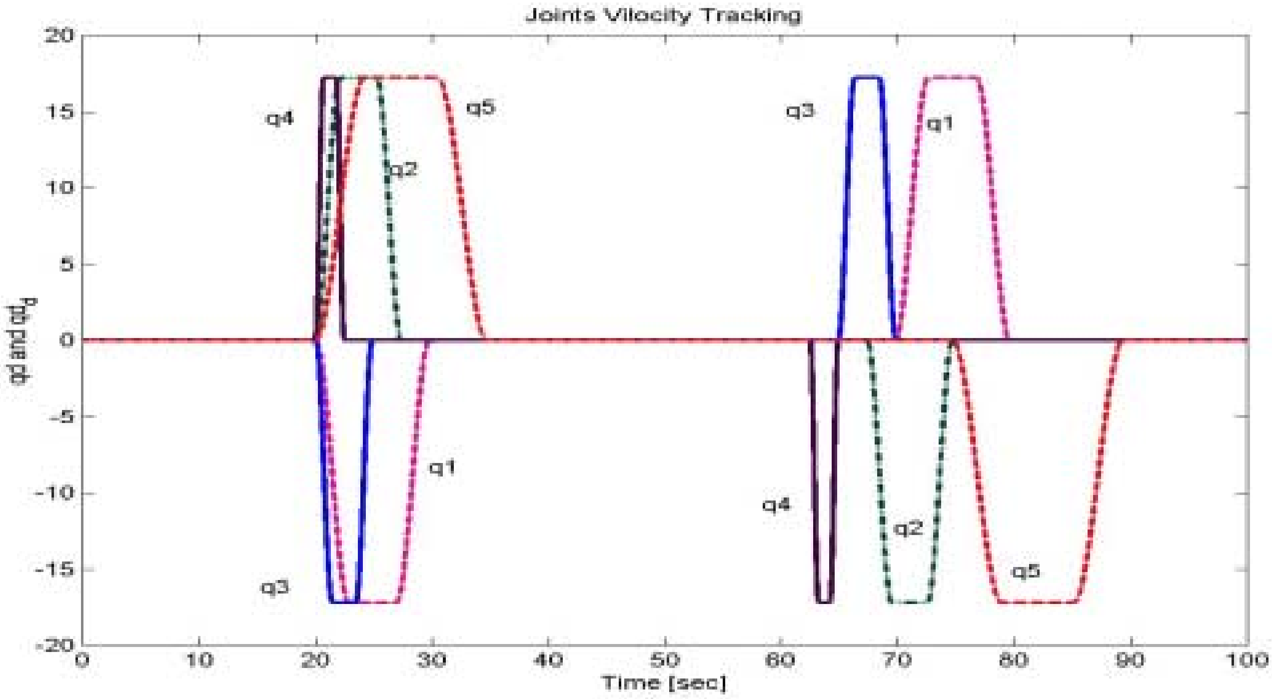

Figure 4. Velocities in trajectory tracking for the first case.

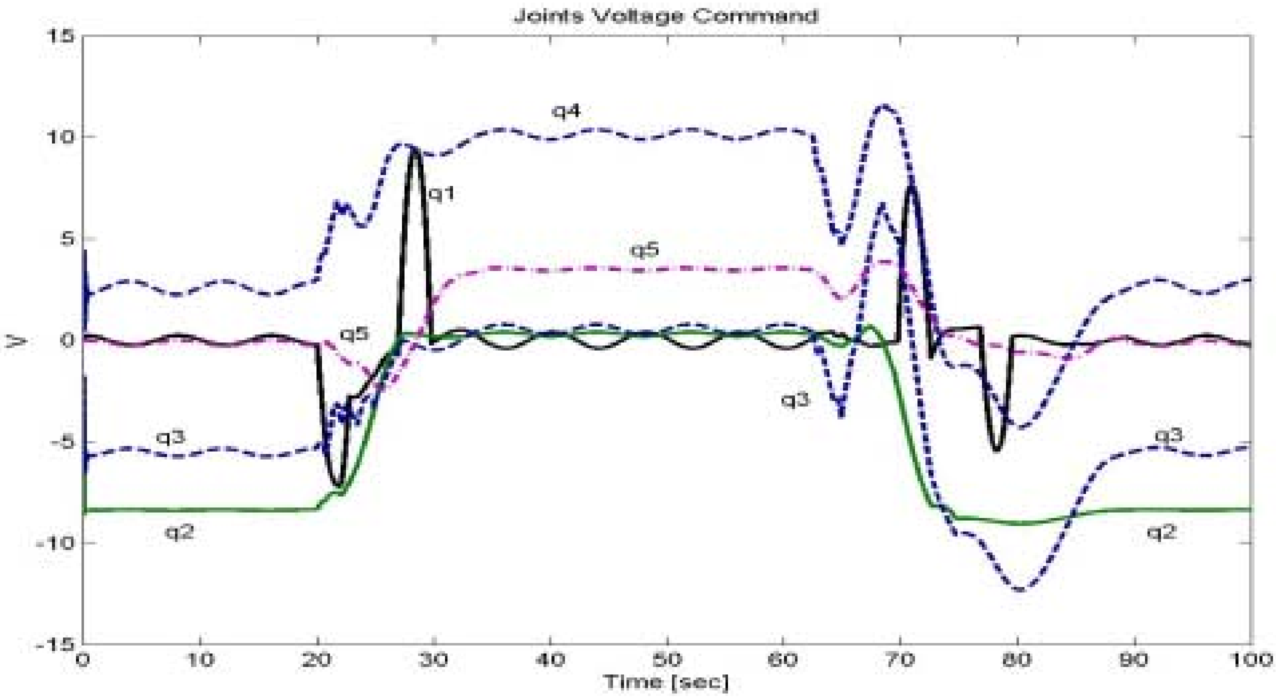

Figure 5. Control commands for the second case.

nonlinearities and stabilizing properties of the system dynamics.

Also, it can be seen that the proposed technique is robust against parameter uncertainty and

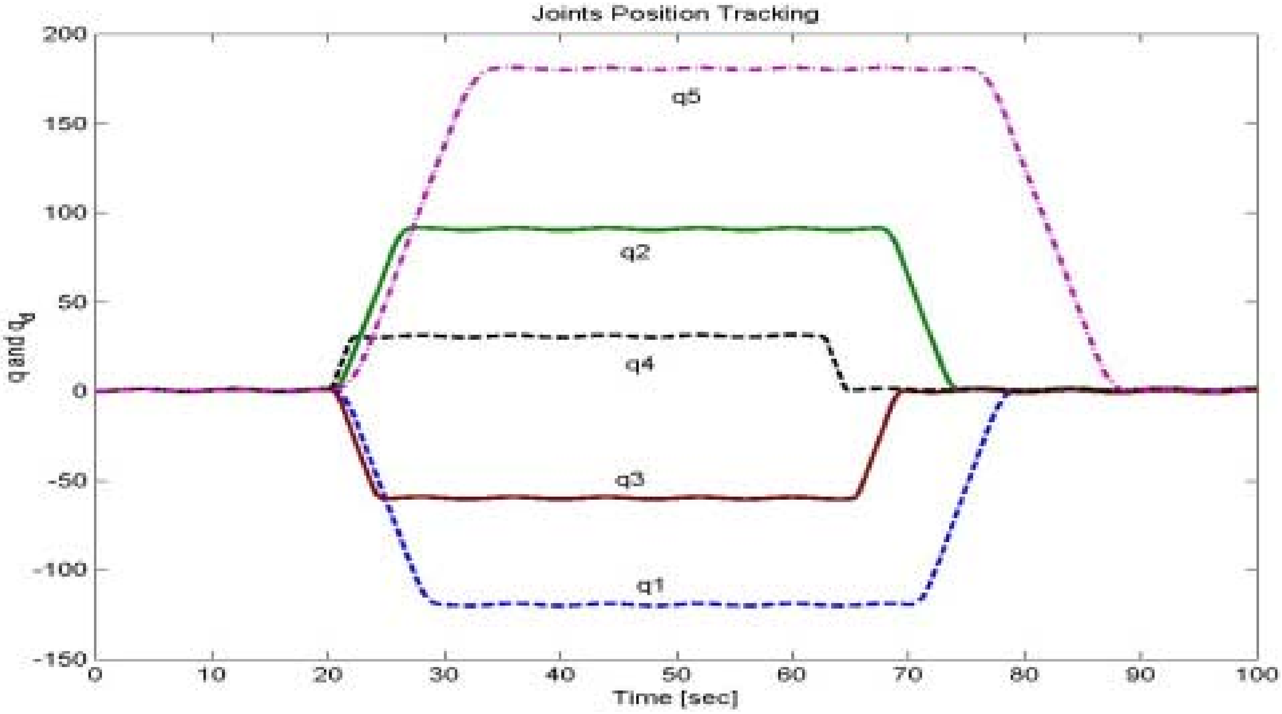

Figure 6. Positions in trajectory tracking for the second case.

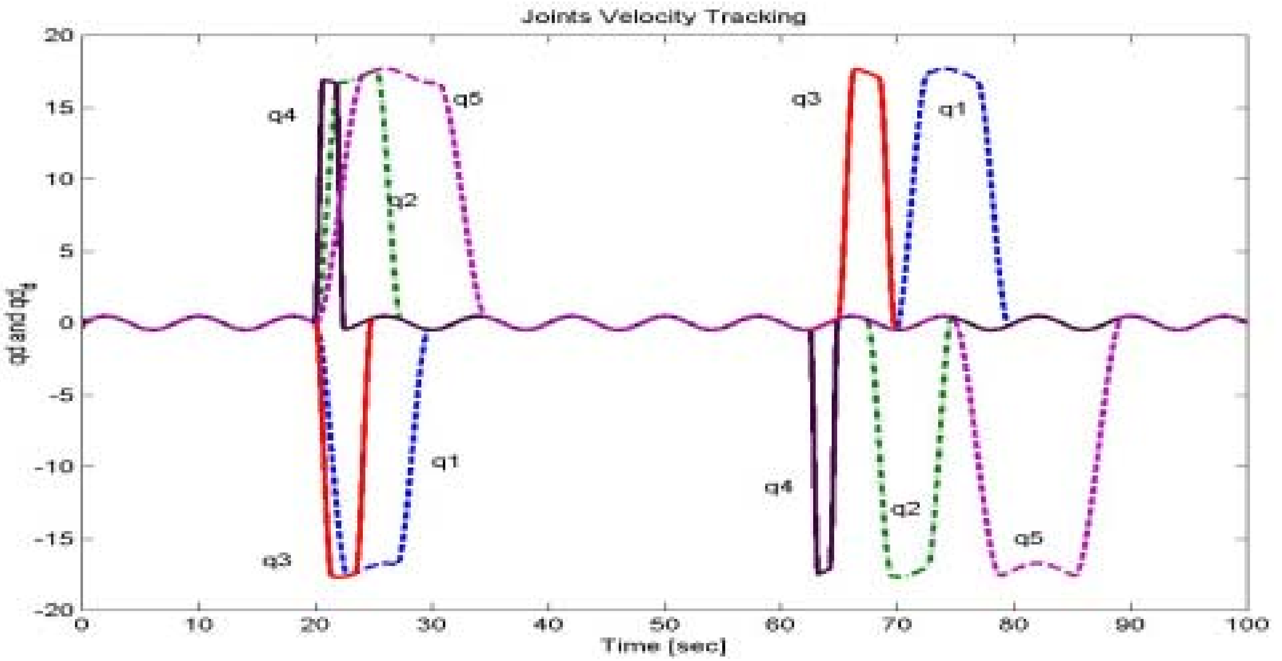

Figure 7. Velocities in trajectory tracking for the second case.

disturbance. Therefore, if we have not complete knowledge of the system, we can propose using this method to improve the system performance and to decrease the possibility of controlled system instability. This is the subject of our continuing research. Also, we are testing this method on an experimental setup and the results will be reported in the near future.

9. NOMENCLATURE

$\alpha_0(\xi)$: stabilizing function,

$\beta = I_{n\times n}$ the identity matrix,

ε: auxilliary error,

Λ: an n×n positive definite matrix,

τ: n×1 vector of joint torques,

θ: n×1 vector of angular positions of the rotors,

v: n×1 input armature voltage vector,

$\xi = [\xi_1^T, \xi_2^T]^T \quad \xi_3 = [I]$ state vectors,

A: an n×n positive definite constant diagonal matrix of gear ratios,

$C(q,\dot{q})$: n×n diagonal matrix due to Coriolis and centrifugal forces,

$F(\dot{q})$: n×1 vector containing the static and dynamic friction terms,

$G(q)$: n×1 vector of joint torques and forces, due to gravity,

I: n×1 vector of armature current in each joint actuator,

$\tilde{I}$: current error,

I_d: desired current,

J_m: n×n diagonal matrix of motors and gear inertias,

K_e: n×n positive definite constant diagonal matrix of back electromotive coefficients,

K_T: n×n positive definite constant diagonal matrix of actuator torque coefficients,

L: positive definite constant diagonal n×n matrix used to represent the electrical inductance,

$M(q)$: n×n positive definite manipulator inertia matrix,

$q \in R^n$:vector of generalized displacements of joint space coordinates,

$\dot{q} \in R^n$:vector of generalized velosities of joint space coordinates,

$\ddot{q} \in R^n$: vector of generalized accelerations of joint space coordinates,

R: n×1 vector used to represent the electrical resistance,

T_L: n×1 vector representing an additive bounded joint torque disturbance,

t: time

T_E: n×1 vector representing an additive bounded voltage disturbance,

$V(\xi,\xi_3)$: candidate Lyapunov function,

V_i: voltage disturbance,

$W(\xi)$: storage function,

y: output vector,

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