RESEARCH NOTE

BIT ERROR PERFORMANCE FOR ASYNCHRONOUS DS/CDMA SYSTEMS OVER MULTIPATH RAYLEIGH FADING CHANNELS

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Abstract In recent years, there has been considerable interest in the use of CDMA in mobile communications. Bit error rate is one of the most important parameters in the evaluation of CDMA systems. In this paper, we develop a technique to find an accurate approximation to the probability of bit error for asynchronous direct–sequence code division multiple–access (DS/CDMA) systems by modeling the multiple access interference (MAI) as an improved Gaussian process. The channel is modeled as a multipath flat Rayleigh fading channel. Numerical results are obtained by using the improved Gaussian approximation (IGA) and compared to the standard Gaussian approximation (SGA).

Key Words DS/CDMA, Bit Error Rate, Multipath Fading, Rayleigh Distribution

چکیده در دهه اخیر سیستمهای طیف گسترده DS/CDMA برای استفاده در مخابرات سلولی مورد توجه فراوان قرار گرفته اند. یکی از مهمترین پارامترها در ارزیابی سیستم DS/CDMA مقدار احتمال خطاست. در این مقاله با مدل نمودن تداخل دسترسی چندگانه به عنوان فرآیند گوسی بهبود یافته، روشی جدید برای محاسبه احتمال خطای بیت در سیستمهای طیف گسترده DS/CDMA پیشنهاد می گردد. همچنین کانال به صورت چند مسیری با محو شدگی از نوع رایلی در نظر گرفته شده است. نتایج حاصله نشان می دهد که احتمال خطا با تقریب گوسی بهبود یافته دارای دقت بیشتری نسبت به احتمال خطا با تقریب گوسی استاندارد است.

1. INTRODUCTION

In recent years, there has been considerable interest in the use of CDMA as an alternative to traditional frequency–division multiple–access (FDMA) and time–division multiple–access (TDMA) techniques in mobile communications [1]. In CDMA, multiple users transmit, often asynchronously, over a common communication channel, typically using the direct–sequence (DS) spread spectrum technique. Multiple access interference (MAI) is the main factor that limits the throughput in a CDMA system by reducing the number of active users operating at a specified system bit error rate (BER).

In practice, the MAI terms in a DS/CDMA system are independent only when conditioned on a set of specific operating circumstances for every user [2]. Thus, the SGA is not always accurate enough. This observation has led to the proposal of an alternative procedure based on the improved Gaussian approximation (IGA). The point of departure in an improved Gaussian approximation is the application of the central limit theorem and the law of total probability to a conditional probability of error.

An IGA with a good accuracy is presented in [3,4] for AWGN channels. In [3,4], evaluation of the expression used to describe the IGA technique, requires significant computational complexity. This is overcome by an accurate Taylor series expansion based on an approximation presented in [5]. Also, an improved Gaussian approximation for single–path Rayleigh fading channel is presented in [6].

In this paper, we extend the works in [3] and [6] to analyze the performance of asynchronous DS/CDMA systems using Gold sequences [7] over the multipath flat Rayleigh fading channel, by modeling the MAI as an improved Gaussian process.

2. SYSTEM MODEL

Consider K users transmitting asynchronously over a multipath-fading environment to a single receiver station. Assume that each transmitter uses a binary phase shift keying (BPSK) DS/CDMA modulation. The kth transmitter generates data bits at a rate of $1/T_b$ bits per second. Its data signal, $b_k(t)$, and the spreading signal, $a_k(t)$, are defined as [8]

$$b_k(t) = \sum_{n=-\infty}^{n=\infty} b_n^{(k)} P_{T_b}(t - nT_b) \quad (1)$$

$$a_k(t) = \sum_{n=-\infty}^{n=\infty} a_n^{(k)} P_{T_c}(t - nT_c) \quad (2)$$

here $P_T(t)$ is unit rectangular pulse. The data symbols, $b_n^{(k)}$, are considered to be independent and identically distributed (iid) sequences with $\Pr[b_n^{(k)} = -1] = \Pr[b_n^{(k)} = +1] = 0.5$. T_c is the chip period and we assume $T_b = N\, T_c$, with N being the period of spreading sequence.

First, data are multiplied by the spreading signal to produce a base band signal. Then, the carrier modulation is used to get

$$s_k(t) = \sqrt{2P}\, b_k(t) a_k(t) \cos(\omega_c t + \theta_k) \quad (3)$$

In (3), θ_k, ω_c, and P represent the phase of the kth carrier, the common center frequency and the signal power respectively.

The low pass equivalent impulse response of the passband-fading channel for the link between the *k*th user transmitter and the receiver is

$$h_k(t) = \sum_{l=1}^{L} \beta_{l,k}\, \delta(t - \tau_{l,k}) \exp(j\phi_{l,k}) \quad (4)$$

where $\beta_{l,k}$, $\phi_{l,k}$, and $\tau_{l,k}$ are, respectively, the lth path gain, phase and time delay of the kth user, and $\delta(.)$ is the Dirac delta function.

For any input signal $s_k(t)$, modeled as (3), the output signal, $y_k(t)$, produced by the composite multipath fading channel, consists of a sum of delayed, phase shifted, attenuated replicas of the input signal,

$$y_k(t) = \sum_{l=1}^{L} \sqrt{2P}\, \beta_{l,k} b_k(t - \tau_{l,k})\, a_k(t - \tau_{l,k}) \cos(\omega_c t + \varphi_{l,k}) \quad (5)$$

where $\varphi_{l,k} = \phi_{l,k} + \theta_k - \omega_c \tau_{l,k}$. In (5), $\tau_{l,k}$ has a uniform distribution over the interval $[0, T_b)$, $\varphi_{l,k}$ is uniform in $[0, 2\pi)$, and $\beta_{l,k}$ is a Rayleigh random variable.

The received signal is the sum of signal components arriving over all paths from the users along with an additive thermal noise. In fact

$$r(t) = \sum_{k=1}^{K} \sum_{l=1}^{L} \sqrt{2P}\, \beta_{l,k} b_k(t - \tau_{l,k}) a_{k,l}(t - \tau_{l,k}) \cos(\omega_c t + \varphi_{l,k}) + n(t), \quad (6)$$

where $n(t)$ is a white zero–mean Gaussian process with a two–sided noise power spectral density $N_0/2$.

It is assumed that the signal of user 1 is to be detected. Furthermore, without loss of generality [8], we assume that the receiver is locked to the component of user 1 on $\tau_{1,1} = 0$ and $\varphi_{1,1} = 0$. The output of a coherent receiver is

$$Z = \int_0^{T_b} r(t) a_1(t) \cos(\omega_c t)\, dt. \quad (7)$$

By substituting (6) into (7), we obtain

$$Z = \sqrt{P/2}\, \beta_{1,1} b_0^{(1)} T_b + \sqrt{P/2} \sum_{\substack{k=1 \\ l,k \neq 1,1}}^{K} \sum_{l=1}^{L} \beta_{l,k} W_{l,k} \cos(\varphi_{l,k}) + \eta \quad (8)$$

where we have

$$W_{l,k} = [b_{-1}^{(k)} R_{1k}(\tau_{l,k}) + b_0^{(k)} \hat{R}_{1k}(\tau_{l,k})] \quad (9)$$

$b_{-1}^{(k)}$ and $b_0^{(k)}$ represent a pair of consecutive data bits of the kth signal. $R_{1k}(\tau_{l,k})$ and $\hat{R}_{1k}(\tau_{l,k})$ are the continuous time partial cross–correlation of the kth and 1st spectral spreading waveforms as defined in [7].

Since data bits are assumed to be equiprobable, the bit error probability is

$$P_b = \Pr(Z < 0 \mid b_0^{(1)} = +1 \qquad (10)$$

3. PROBABILITY OF BIT ERROR WITH IGA

An improved Gaussian approximation was presented in [3,4], and it was shown that this approximation provides an accurate result for the probability of bit error in asynchronous DS/CDMA, using a random sequence in an AWGN channel. In this section, we extend the work in [3] and evaluate the probability of bit error for asynchronous DS/CDMA in multipath Rayleigh fading channel, using the improved Gaussian approximation. Following the approach in [3], let $T_c = 1$ ($T_b = N$), $b_1^0 = 1$, and $P = 2$. Then we have

$$Z = \beta_{1,1} N + \sum_{\substack{k=1 \\ l,k \neq 1,1}}^{K} \sum_{l=1}^{L} \beta_{l,k} W_{l,k} \cos(\varphi_{l,k}) + \eta \qquad (11)$$

As in [3], let Ψ denote the variance of the MAI conditioned on the path gains, delays and phases of all interfering signals and of the desired sequence structure, through a quantity $B = (N-1-C)/2$ where C is the discrete aperiodic autocorrelation of the signature sequence of receiver 1,

$$C = \sum_{j=0}^{N-2} a_j^{(1)} a_{j+1}^{(1)} \qquad (12)$$

and

$$E[B] = \frac{N-1}{2} \qquad (13)$$

$$E[B^2] = \frac{N(N-1)}{4} \qquad (14)$$

Therefore, the conditioned variance, Ψ, is given by

$$\Psi = \mathrm{Var}[MAI \mid \tau_{l,k}, \varphi_{l,k}, \beta_{l,k}, B]$$

$$= \sum_{\substack{k=1 \\ l,k \neq 1,1}}^{K} \sum_{l=1}^{L} E[(\beta_{l,k} W_{l,k} \cos \varphi_{l,k})^2 \mid \tau_{l,k}, \varphi_{l,k}, \beta_{l,k}, B]$$

$$= \sum_{\substack{k=1 \\ l,k \neq 1,1}}^{K} \sum_{l=1}^{L} X_{l,k}, \qquad (15)$$

$$X_{l,k} = E[(\beta_{l,k} \cos\varphi_{l,k})^2 \mid \beta_{l,k}, \varphi_{l,k}]\, E[(W_{l,k})^2 \mid \tau_{l,k}, B]$$

$$= (\beta_{l,k}\cos\varphi_{l,k})^2\, E[(W_{l,k})^2 \mid \tau_{l,k}, B]. \qquad (16)$$

The second term of $X_{l,k}$ is equal to [3]

$$E[(W_{l,k})^2 \mid \tau_{l,k}, B] = 2(2B + 1)(S_{l,k}^2 - S_{l,k}) + N \qquad (17)$$

where $S_{l,k}$ is $\tau_{l,k}$ normalized on T_b. Thus, $X_{l,k}$ becomes

$$X_{l,k} = (\beta_{l,k}\cos\varphi_{l,k})^2 [2(2B + 1)(S_{l,k}^2 - S_{l,k}) + N]. \qquad (18)$$

In what follows, we first assume that MAI is the major source of error. Then, the effect of the thermal noise is also taken into account. The accurate probability of bit error conditioned on $\beta_{1,1}$ can be found by evaluating

$$P_{b|\beta_{1,1}}(\beta_{1,1}) = \int_0^\infty P_{b|\beta_{1,1},\psi}(\beta_{1,1},\Psi) p(\Psi) d\Psi \qquad (19)$$

where $p(\Psi)$ is the pdf of Ψ and $P_{b|\beta,\psi}(\beta,\Psi)$ is the probability of bit error conditioned on Ψ for a fixed β and is given by

$$P_{b|\beta,\psi}(\beta,\Psi) = Q\left(\frac{N\beta}{\sqrt{\Psi + P_{noise}}}\right). \qquad (20)$$

To obtain the probability of bit error, we must evaluate the integral

$$P_b = \int_0^\infty P_{b|\beta_{1,1}}(\beta_{1,1}) p(\beta_{1,1}) d\beta_{1,1} \qquad (21)$$

The computation of (19) requires evaluating the pdf of Ψ, which is very cumbersome, especially when multipath fading is considered. Holtzman [5] has used a simple approximation for the computation of probability of bit error for asynchronous DS/CDMA in an AWGN channel that does not require the pdf of Ψ directly. By using a similar approach, we have computed the probability of bit error over the multipath-fading channel. The objective here is to compute (19) without carrying out the integration. This can be done by expanding $P_{b|\beta,\psi}(\beta, \Psi)$ using the Taylor series.

To simplify the notation, we denote $P_{b|\beta,\psi}(\beta, \Psi)$ as $P(\Psi)$. Using the Taylor series, $P(\Psi)$ can be written as

$$P(\Psi) = P(\mu_\psi) + (\Psi - \mu_\psi)P^{(1)}(\mu_\psi) + \ldots + \frac{(\Psi - \mu_\psi)^n}{n!}P^{(n)}(\mu_\psi) + \qquad (22)$$

where μ_ψ is the mean of Ψ and $P^{(n)}(\mu_\psi)$ is the *n*th derivative of $P(\Psi)$ evaluated at $\Psi = \mu_\psi$. Taking the expectation of (22) with respect to Ψ, one obtains

$$E[P(\Psi)] = P(\mu_\psi) + \frac{\sigma_\psi^2}{2!}P^{(2)}(\mu_\psi) + \ldots + \frac{\mu_{\psi,n}}{n!}P^{(n)}(\mu_\psi) + \ldots \qquad (23)$$

where $\sigma_\psi{}^2$ is the variance of Ψ and $\mu_{\psi,n}$ is the nth moment given by $E[(\Psi - \mu_\psi)^n]$. The second derivative of $P(\Psi)$ − in which Ψ is a random variable − can be approximated by the Stirling's formula:

$$P^{(2)}(\Psi) = \frac{P(\Psi + h) - 2P(\Psi) + P(\Psi - h)}{h^2} \qquad (24)$$

where h is the step−size. A appropriate value for h is $h = \sqrt{3}\,\sigma_\psi$ [5]. We assume that only the first two terms in (23) are important. Substituting (24) into (23), gives

$$E[P(\Psi)] = \frac{2}{3}P(\mu_\psi) + \frac{1}{6}P(\mu_\psi + \sqrt{3}\sigma_\psi) + \frac{1}{6}P(\mu_\psi - \sqrt{3}\sigma_\psi) \qquad (25)$$

Now, we find the mean and the variance of Ψ,

$$\mu_\psi = E\{\sum_{\substack{k=1 \\ l,k \neq 1,1}}^{K}\sum_{l=1}^{L}(\beta_{l,k}\cos\varphi_{l,k})^2[2(2B+1)(S_{l,k}^2 - S_{l,k}) + N]\}$$
$$= \sum_{k=1}^{K}\sum_{l=1}^{L}E[\beta_{l,k}^2]E[\cos^2\varphi_{l,k}]E[2(2B+1)(S_{l,k}^2 - S_{l,k}) + N] \qquad (26)$$

Since $\beta_{l,k}$ is a Rayleigh random variable, using (13), we have

$$\mu_\psi = \frac{KL - 1}{3}(2\sigma^2) \qquad (27)$$

The variance of Ψ can be written as

$$\sigma_\psi^2 = E(\Psi^2) - E^2(\Psi)$$
$$= E\{\sum_{\substack{k=1 \\ l,k \neq 1,1}}^{K}\sum_{l=1}^{L}(\beta_{l,k}\cos\varphi_{l,k})^4[2(2B+1)(S_{l,k}^2 - S_{l,k}) + N]^2\} - (\mu_\psi)^2 \qquad (28)$$

Using (14), one obtains

$$\sigma_\psi^2 = (KL-1)\frac{7N^2 + 2N - 1}{40}(24\sigma^2) - (KL-1)\frac{N^2}{9}(4\sigma^2) + (K^2L^2 - 2KL - K + 2)\frac{N-1}{36}(4\sigma^2) \qquad (29)$$

If we substitute (19), (27) and (29) into (25), and incorporate the effect of thermal noise, the probability of bit error conditioned on $\beta_{1,1}$ becomes

$$P_{b|\beta_{1,1}}(\beta_{1,1}) = \frac{2}{3}P_1(\beta_{1,1}) + \frac{1}{6}P_2(\beta_{1,1}) + \frac{1}{6}P_3(\beta_{1,1}) \qquad (30)$$

where $P_i(\beta_{1,1})$ is given by

$$P_i(\beta_{1,1}) = Q\left(\frac{\beta_{1,1}}{D_i}\right) \qquad (31)$$

and D_i's are

$$D_1 = \left[\frac{N_0}{E_b} + \frac{KL-1}{N}(2\sigma^2)\right]^{0.5} \qquad (32)$$

$$D_2 = \left[\frac{N_0}{E_b} + \frac{KL-1}{N}(2\sigma^2) + \frac{\sqrt{3}}{N}\sigma_\psi\right]^{0.5} \qquad (33)$$

$$D_3 = \left[\frac{N_0}{E_b} + \frac{KL-1}{N}(2\sigma^2) - \frac{\sqrt{3}}{N}\sigma_\psi\right]^{0.5} \qquad (34)$$

Now, substituting (30) into (21), we can calculate the probability of bit error

$$P_b = \frac{2}{3}\int_0^\infty P_1(\beta_{1,1})p(\beta_{1,1})d\beta_{1,1} + \frac{1}{6}\int_0^\infty P_2(\beta_{1,1})p(\beta_{1,1})d\beta_{1,1} + \frac{1}{6}\int_0^\infty P_3(\beta_{1,1})p(\beta_{1,1})d\beta_{1,1}. \qquad (35)$$

4. NUMERICAL RESULTS AND CONCLUSIONS

Figure 1 depicts the probability of bit error for an asynchronous DS/CDMA system over a flat Rayleigh fading channel for $E_b/N_0 = \infty$, N = 63, and parameter L. Note that as L increases, the discrepancy between the SGA and the IGA decreases. This result complies with the central limit theorem.

Figure 2 shows the probability of bit error for an asynchronous DS/CDMA system over a flat Rayleigh fading channel for $E_b/N_0 = \infty$, L = 2, and parameter N. We observe that the accuracy of the SGA does not depend on N. The difference between the bit error rate curve for the SGA and the IGA remains almost the same when *N* changes. Figure 3 illustrates the probability of bit error for an asynchronous DS/CDMA system over a flat Rayleigh fading channel for L = 2, N = 63, and parameter E_b/N_0. From this figure, we note that when E_b/N_0 increases, the probability of bit error decrease. Also, the difference between the SGA and IGA remains the same when E_b/N_0 changes.

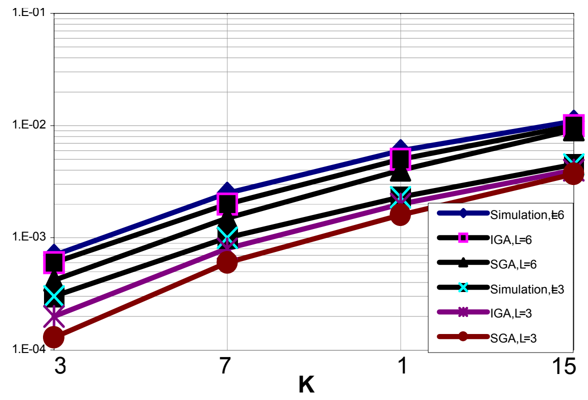

Figure 1. Probability of bit error versus No. of users for IGA, SGA, and the simulation results based on the parameter L (N = 63, $E_b/N_0 = \infty$).

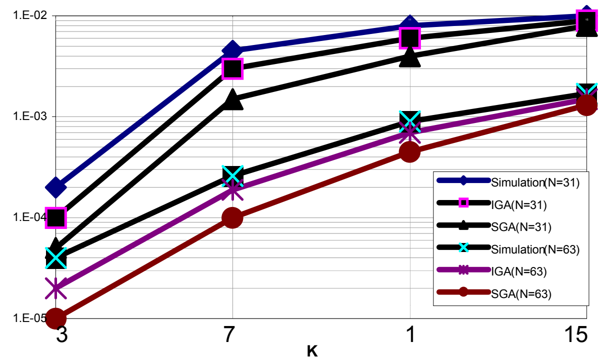

Figure 2. Probability of bit error versus No. of users for IGA, SGA, and the simulation results based on the parameter N (L = 2, $E_b/N_0 = \infty$).

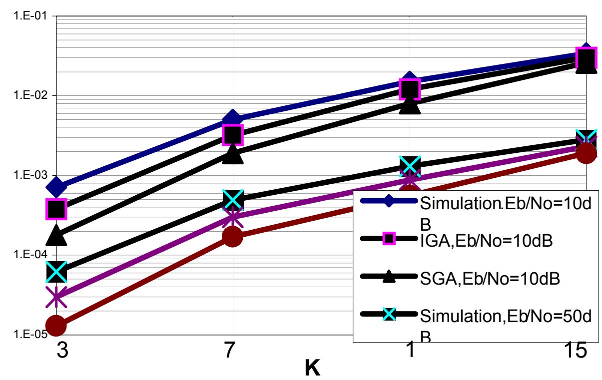

Figure 3. Probability of bit error versus No. of users for IGA, SGA, and the simulation results based on the parameter E_b/N_0 (L = 2, N = 63).

In this paper, we analyzed the performance of asynchronous DS/CDMA system over multipath Rayleigh fading channels. The probability of bit error was computed using two methods. In the first method we assumed that the MAI is Gaussian, while in the second one, we modeled the MAI as an improved Gaussian. The main conclusion is that the computation of the probability of bit error, based on the Gaussian approximation for the MAI, is not accurate, especially for a small number of simultaneous users, which corresponds to a low probability of bit error. However, it was shown that, when the channel exhibits deep fading (large L), the standard Gaussian approximation and the improved Gaussian approximation give similar results.

5. REFRENCES

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