

# CONSTITUTIVE MODEL FOR MULTI-LAMINATE INDUCED ANISOTROPIC DOUBLE HARDENING ELASTIC-PLASTICITY OF SAND

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**Abstract** A constitutive multi-laminate based elastic-plastic model developed to be capable of accounting induced anisotropic behavior of granular material such as sand. The fabric feature or grain orientation characteristic effects through medium are considered in a rational way under any complex stress path, including cyclic loading. The salient feature of the developed model is a non-associative on plane plasticity with biaxial hardening as a function of plastic shear strain components. Generalized form of multi-laminate framework employed to sum up the non-symmetric plastic compliance matrices of sampling planes to build up the main compliance matrix. Two normal translation rules of yield boundary are specified upon the components of plastic shear strain on every sampling plane. The constitutive model is capable of describing expansion of two yield functions upon two predefined coordinate axes. The hardening parameters affect the plastic strain tensor upon the contribution of twenty-six different sliding orientations through any point in medium. This contribution makes a powerful representation of strain hardening due to fabric effects in behavior of material. The comparison of experimental test results with model results represents that this model is more capable in cyclic behavior of porous media such as sand.

**Key Words** Multi-Laminate, Elastic-Plasticity, Double Hardening, Constitutive Model, Induced Anisotropy

**چکیده** روابط اساسی یک مدل ارتجاعی - خمیری چند صفحه ای با قابلیت احتساب ناهمسانی تحمیلی در رفتار مواد دانه ای مانند ماسه گسترش یافته است. آثار ناشی از ویژگیهای خواب دانه ای یا چگونگی بافت دانه ها در ماده تحت اعمال بار با مسیرهای تنش پیچیده متناوب به روشی متناسب دیده شده است. از ویژگیهای مدل گسترش یافته رفتار خمیری صفحه ای با سخت شوندگی تابع کرنش برشی خمیری دوگانه روی دو محور لغزش با قانون خمیری ناهمراه می باشد. چهارچوب عمومی تئوری چند صفحه ای برای جمع انتگرالی ماتریس نامتقارن رفتار خمیری هر صفحه برای بدست آوردن ماتریس کلی رفتار خمیری بکارگرفته شده است. دو انتقال متعامد برای حرکت محدوده مرز سیلان در تغییر شکل های خمیری روی هر صفحه در نظر گرفته شده است. در روابط اساسی این مدل قابلیت تعریف پیشروی توابع سیلان در دو جهت متعامد بر روی صفحه می باشد. تاثیر فراسنجهای سخت شوندگی بر روی تانسور کرنش خمیری بر اساس تجمع آثار بیست و شش امتداد لغزش در هر نقطه از ماده است. این تجمع ارائه گر توان قدرتمند سخت شوندگی کرنشی به خاطر آثار ناشی از خواب دانه ای در رفتار ماده است. مقایسه نتایج مدل با آزمایش ها بیانگر قابلیت بالای مدل در پیش بینی رفتار مواد متخلخل مانند ماسه تحت اعمال بارهای متناوب است.

## 1. INTRODUCTION

Modeling the behavior of soil subjected to any arbitrary stress path, specially, cyclic loading progressed over the world to develop the capability for realistic static/dynamic analysis of geo-mechanical systems. Among proposed models, those with the multi-laminate base are capable of predicting induced

anisotropy, the effect of a rotation of principal stress and strain axes in plasticity, semi-micro mechanical history of plasticity propagation in material and finally the orientation of failure mechanism.

For a granular mass such as sand that supports the overall applied loads through contact friction, the overall mechanical response ideally may be described

on the basis of micro-mechanical behavior of grains interconnections. Naturally, this requires the description of overall stress, characterization of fabric, representation of kinematics, development of local rate constitutive relations and evaluation of the overall differential constitutive relations in terms of the local quantities.

The task of representing the overall stress tensor in terms of micro level stresses and the condition, number and magnitude of contact forces has long been the aim of numerous researchers [1-3].

A multi-laminate model capable of predicting the behavior of granular material with isotropic hardening on sampling planes developed by the authors of [4] and [5]. In this model the hardening rule as function of scalar value of plastic shear strain could not see the full strain history in plasticity.

The concept of proposed model, however, seems to be more realistic and natural, physically meaningful and simple. According to this formulation, which is based on a simple numerical integration, an appropriate connection between averaged micro and macro-mechanical behavior of material has been presented. The inclusion of the rotation of principal stress and strain axes, induced anisotropy and the possibility of supervising pre-failure behavior and even controlling any variation through the medium are the significant of the model.

Multi-laminate framework by defining the small continuum structural units as an assemblage of particles and voids that fill infinite spaces between the sampling planes, has appropriately justified the contribution of interconnection forces in overall macro-mechanics. Plastic deformations are assumed to occur due to sliding, separation/closing of the boundaries and elastic deformations are the overall responses of structural unit bodies. Therefore, the overall deformation of any small part of the medium is composed of total elastic response and an appropriate summation of sliding, separation/closing phenomenon under the current effective normal and shear stresses on sampling planes.

According to these assumptions overall sliding, separation/closing of inter-granular points of grains included in one structural unit are summed up and contributed as the result of sliding, separation/closing surrounding boundary planes. This simply implies yielding/failure or even ill conditioning the and bifurcation response to be possible over

any of randomly oriented sampling planes. Consequently, plasticity control such as yielding should be checked at each of the planes and those of the planes, which are sliding, will contribute to plastic deformation. Therefore, the granular material mass has an infinite number of yield functions usually one for each of the planes in the physical space.

## 2. MODEL CONSTITUTIVE EQUATIONS

The classical decomposition of strain increments under the concept of elastic-plasticity in elastic and plastic parts are schematically written as follows:

$$d\varepsilon = d\varepsilon^e + d\varepsilon^p \quad (1)$$

The increment of elastic strain ( $d\varepsilon^e$ ) is related to the increments of effective stress ( $d\sigma'$ ) by:

$$d\varepsilon^e = C^e \cdot d\sigma' \quad (2)$$

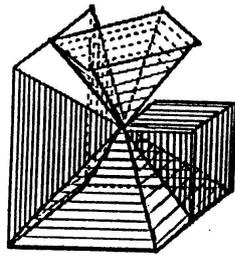
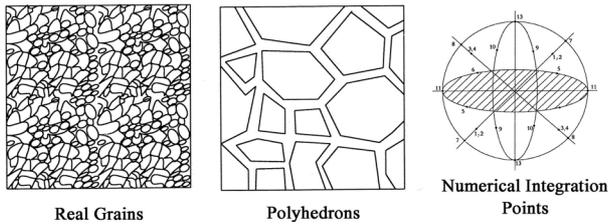
$C^e$  is elastic compliance matrix, usually assumed as linear. Conceptually, it is possible to compute  $C^p$  by using multi-laminate framework. However, if the single structural units are assumed to be elastically isotropic, using a common elasticity tensor, then trivially, the overall elastic response of the collective system will be isotropic, having the same elasticity tensor. Clearly, in this case, computing  $C^e$  by using multi-laminate framework is not fruitful. When single structural unit constituents are anisotropic, then, whether or not the overall elastic response will be isotropic depends on the distribution of the single structural units. For a random distribution, the overall response will be isotropic, whereas this response will be anisotropic if the distribution of particle orientations is biased by prior plastic deformation.

For the soil mass, the overall stress-strain increments relation, to obtain plastic strain increments ( $d\varepsilon^p$ ), is expressed as:

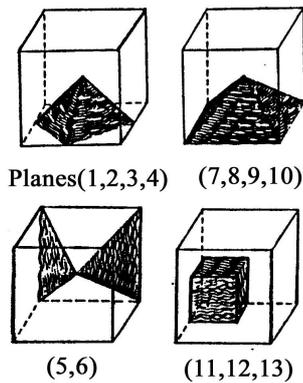
$$d\varepsilon^p = C^p \cdot d\sigma' \quad (3)$$

$C^p$  is plastic compliance matrix.

Clearly, it is expected that all the effects of plastic behavior be included in  $C^p$ . To find out  $C^p$ , the constitutive equations for a typical slip plane



13 Planes



**Figure 1.** The orientation of 13 planes, real and simulated substructure blocks.

must be considered in calculations. Consequently, the appropriate summation of all provided compliance matrices corresponding to defined slip planes yields overall  $C^p$ .

$$C^p = 4\Pi \sum_{i=1}^n W_i \cdot L^T \hat{C}_i^p L \quad (4)$$

$L^T$  is a proper transformation matrix to transform  $\hat{C}_i^p$  from  $i$ th plane coordinate to global coordinate.

### 3. CONSTITUTIVE EQUATIONS FOR A SAMPLING PLANE

A sampling plane is defined as a boundary surface, which is an interconnecting surface between two structural units of polyhedral blocks. These structural units are parts of an inhomogeneous continuum, for simplicity defined as a full homogeneous and isotropic material. Therefore, all inhomogeneities behavior supposed to appear in inelastic behavior of corresponding slip planes. Figure 1 shows these defined planes (say 13).

As already defined, the vector of plastic strain is calculated from the study of the glide motion over an individual sampling plane departed into two perpendicular axes. This hypothesis, of course, may not cope with the reality, which must represent all changes of behavior due to plastic slide on a certain plane, but includes the plastic strain hardening history upon the two perpendicular components. Therefore, the effects of sliding orientation are included in hardening parameters.

To start explaining the plastic constitutive law for a sampling plane, the main features of plasticity law (i.e. yield criterion, plastic potential function, flow rule and hardening rule) must also be considered.

In this constitutive formulation, two yield criteria are defined by two ratios of the shear stress components ( $\tau_{xi}, \tau_{yi}$ ) to the normal effective stress ( $\sigma_{ni}$ ) on  $i$ th sampling plane. A simple form of yield function i.e. a straight line on  $\tau$  versus  $\sigma$  space is adopted. As the ratios  $\tau_{xi}/\sigma_n$  and  $\tau_{yi}/\sigma_n$  increase, the yield boundaries represented by the straight lines rotate anti-clock-wise due to hardening and approaches Mohr-Coulomb's failure surface and finally failure on corresponding plane takes place.

The equation of yield functions for two perpendicular orientations ( $x_i, y_i$ ) on  $i$ th plane are formulated as follows:

$$F_{xi}(\tau_{xi}, \sigma'_{ni}, \eta_{xi}) = \tau_{xi} - \eta_{xi} \sigma'_{ni} \quad (5)$$

$$F_{yi}(\tau_{yi}, \sigma'_{ni}, \eta_{yi}) = \tau_{yi} - \eta_{yi} \sigma'_{ni} \quad (6)$$

$\eta_{xi} = \tan(\alpha_i)$  and  $\eta_{yi} = \tan(\beta_i)$  are hardening parameters for two plasticity rules of orientations  $x_i$  and  $y_i$ . However, they are assumed as a hyperbolic function of plastic shear strain components on the  $i$ th plane.

The quantities  $\alpha_i$  and  $\beta_i$  are the slopes of yield lines. A simple function simulates the best variation of this property during plastic flow, which has been represented as two hyperbolic functions as follows:

$$\eta_{xi} = \frac{(\tau_{xi}/\tau_i)\epsilon_{xt}^{pt} \tan(\phi_{xi})}{A_i + \epsilon_{xt}^{pt}} \quad (7)$$

$$\eta_{yi} = \frac{(\tau_{yi}/\tau_i)\epsilon_{yt}^{pt} \tan(\phi_{yi})}{A_i + \epsilon_{yt}^{pt}} \quad (8)$$

$\phi_{xi}$  and  $\phi_{yi}$  are peak internal frictional angles corresponding to xi and yi directions on the ith plane.  $A_{xi}$  and  $A_{yi}$  are soil parameters and  $\epsilon_{xi}^{pt}$  and  $\epsilon_{yi}^{pt}$  are current values of plastic shear strain on the ith plane.

A small elastic domain (defined by angle  $\phi_e$ ) is considered to provide elastic behavior of cohesionless material at the start of stress increment or whenever the direction of stress path changes. This domain as shown in Figure 2 is small and negligible. Therefore, the value of  $\phi_e$  for all sands is assumed to be the same. However, the minimum value of  $\eta_{xi}$  and  $\eta_{yi}$  are  $\tan(\phi_e)$  at the first loading process.

The usual plastic potential function employed in Reference.4 is used in this research. This function is stated in terms of  $\tau_{xi}$ ,  $\tau_{yi}$  and  $\sigma_{ni}$  for ith plane as follows:

$$\Psi_{xi}(\tau_{xi}, \sigma_{ni}) = \tau_{xi} + \eta_{cxi} \cdot \sigma_{ni} \cdot \log(\sigma_{ni}/\sigma_{nio}) \quad (9)$$

$$\Psi_{yi}(\tau_{yi}, \sigma_{ni}) = \tau_{yi} + \eta_{c yi} \cdot \sigma_{ni} \cdot \log(\sigma_{ni}/\sigma_{nio}) \quad (10)$$

$\eta_{cxi}$  and  $\eta_{c yi}$  are the slope of critical state lines for the plasticity in xi and yi directions and  $\sigma_{nio}$  is the initial value of effective normal stress on ith plane. Typical presentations of this function are shown in Figure 2. The gradient of this function in both directions represents the condition of contractancy and dilatancy behavior in the ranges as:

$$0.0 \leq \tau_{xi} \text{ or } \tau_{yi} \leq \sigma_{ni} (\eta_{cxi} \text{ or } \eta_{c yi}) \quad (\text{contractant}) \quad (11)$$

$$\tau_{xi} \text{ or } \tau_{yi} \geq \sigma_{ni} (\eta_{cxi} \text{ or } \eta_{c yi}) \quad (\text{Dilatants}) \quad (12)$$

Derivative of this function is found as:

$$\partial \Psi_{xi} / \partial \sigma_{ni} = \{1, \eta_{cxi} - \eta_{xi}\}^T \quad (13)$$

$$\partial \Psi_{yi} / \partial \sigma_{ni} = \{1, \eta_{c yi} - \eta_{yi}\}^T \quad (14)$$

Obviously, dilatancy is positive if  $\eta_{xi} > \eta_{cxi}$  or  $\eta_{yi} > \eta_{c yi}$  and vice versa. However, on critical state line of each direction,  $\eta_{xi}$  or  $\eta_{yi}$  will be equal to either  $\eta_{cxi}$  or  $\eta_{c yi}$  and there is no volumetric plastic strain.

Accordingly, the derivatives of the adopted plastic potential function which is based on the conception of energy equation [6], can only be expressed in terms of variable  $\eta_{xi}$  or  $\eta_{yi}$ , identify the components of plastic strain increment ratio as well as the position of no dilatancy/contractancy boundary. This aspect seems to be the most suitable form, which conforms to the constitutive formulation of sampling plane in the case of having a double hardening plasticity rule for granular media.

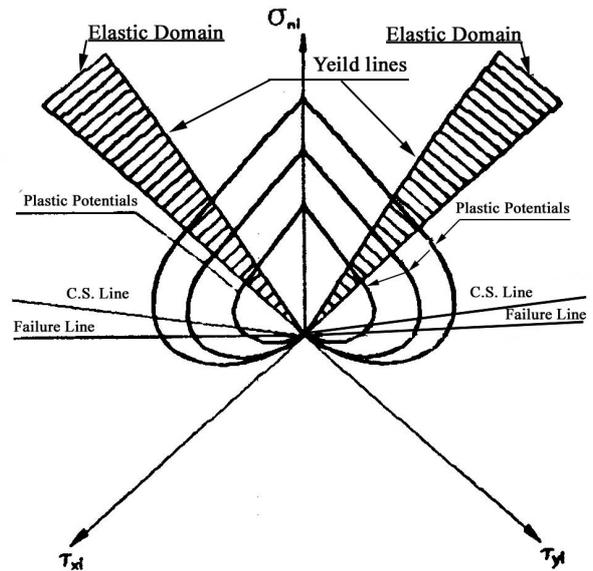


Figure 2. Plastic Potential and yield surfaces of one sampling plane in local coordinate.

Flow rule for both directions similarly is expressed as follows:

$$d\varepsilon_{\sim xi}^p = \lambda_{\sim xi} \cdot \frac{\partial \psi_{\sim xi}}{\partial \sigma_{\sim i}} \quad (15)$$

$$d\varepsilon_{\sim yi}^p = \lambda_{\sim yi} \cdot \frac{\partial \psi_{\sim yi}}{\partial \sigma_{\sim i}} \quad (16)$$

$\lambda_{\sim xi}$  and  $\lambda_{\sim yi}$  are proportionality scalar parameters and changes during plastic deformations.

In theory of plastic flow, consistency condition is a necessary condition, which requires that a yield criterion been overcome as far as the material is in a plastic state. Mathematically, this condition is stated as follows:

$$\frac{\partial F_{\sim xi}}{\partial \sigma_{\sim i}} \cdot d\sigma_{\sim i} + \frac{\partial F_{\sim xi}}{\partial \varepsilon_{\sim xi}^{pt}} \cdot d\varepsilon_{\sim xi}^{pt} = 0.0 \quad (17)$$

$$\frac{\partial F_{\sim yi}}{\partial \sigma_{\sim i}} \cdot d\sigma_{\sim i} + \frac{\partial F_{\sim yi}}{\partial \varepsilon_{\sim yi}^{pt}} \cdot d\varepsilon_{\sim yi}^{pt} = 0.0 \quad (18)$$

$\varepsilon_{\sim xi}^{pt}$  and  $\varepsilon_{\sim yi}^{pt}$  are the components of plastic shear strain on ith plane. From Equations 15, 16 and 17, 18 the followings are obtained:

$$d\lambda_{\sim xi} = - \frac{\begin{Bmatrix} \frac{\partial F_{\sim xi}}{\partial \sigma_{\sim i}} \\ \frac{\partial F_{\sim xi}}{\partial \varepsilon_{\sim xi}^{pt}} \end{Bmatrix}^T \cdot d\sigma_{\sim i}}{\begin{Bmatrix} \frac{\partial F_{\sim xi}}{\partial \sigma_{\sim i}} \\ \frac{\partial F_{\sim xi}}{\partial \varepsilon_{\sim xi}^{pt}} \end{Bmatrix} \cdot \begin{Bmatrix} \frac{\partial \psi_{\sim xi}}{\partial \sigma_{\sim i}} \\ \frac{\partial \psi_{\sim xi}}{\partial \tau_{\sim xi}} \end{Bmatrix}} \quad (19)$$

$$d\lambda_{\sim yi} = - \frac{\begin{Bmatrix} \frac{\partial F_{\sim yi}}{\partial \sigma_{\sim i}} \\ \frac{\partial F_{\sim yi}}{\partial \varepsilon_{\sim yi}^{pt}} \end{Bmatrix}^T \cdot d\sigma_{\sim i}}{\begin{Bmatrix} \frac{\partial F_{\sim yi}}{\partial \sigma_{\sim i}} \\ \frac{\partial F_{\sim yi}}{\partial \varepsilon_{\sim yi}^{pt}} \end{Bmatrix} \cdot \begin{Bmatrix} \frac{\partial \psi_{\sim yi}}{\partial \sigma_{\sim i}} \\ \frac{\partial \psi_{\sim yi}}{\partial \tau_{\sim yi}} \end{Bmatrix}} \quad (20)$$

These relations can also be expressed in another form as:

$$d\varepsilon_{\sim xi}^p = \left\{ \frac{1}{H_{pxi}} \right\} \left\{ \frac{\partial F_{\sim xi}}{\partial \sigma_{\sim i}} \right\} \left\{ \frac{\partial \psi_{\sim xi}}{\partial \sigma_{\sim i}} \right\}^T \cdot d\sigma_{\sim i} \quad (23)$$

$$d\varepsilon_{\sim yi}^p = \left\{ \frac{1}{H_{pyi}} \right\} \left\{ \frac{\partial F_{\sim yi}}{\partial \sigma_{\sim i}} \right\} \left\{ \frac{\partial \psi_{\sim yi}}{\partial \sigma_{\sim i}} \right\}^T \cdot d\sigma_{\sim i} \quad (24)$$

$H_{pxi}$  and  $H_{pyi}$  are defined as hardening modulus of ith plane corresponding to xi and yi directions and are obtained as follows:

$$H_{pxi} = - \left\{ \frac{\partial F_{\sim xi}}{\partial \varepsilon_{\sim xi}^{pt}} \right\} \left\{ \frac{\partial \psi_{\sim xi}}{\partial \tau_{\sim xi}} \right\} \quad (25)$$

$$H_{pyi} = - \left\{ \frac{\partial F_{\sim yi}}{\partial \varepsilon_{\sim yi}^{pt}} \right\} \left\{ \frac{\partial \psi_{\sim yi}}{\partial \tau_{\sim yi}} \right\} \quad (26)$$

$$H_{pxi} = \sigma_{ni} \cdot \frac{A_{\sim xi} \tan(\phi_{\sim xi})}{(A_{\sim xi} + \varepsilon_{\sim xi}^{pt})^2} \quad (27)$$

$$H_{pyi} = \sigma_{ni} \cdot \frac{A_{\sim yi} \tan(\phi_{\sim yi})}{(A_{\sim yi} + \varepsilon_{\sim yi}^{pt})^2} \quad (28)$$

however,

$$C_{\sim xi}^p = \frac{(A_{\sim xi} + \varepsilon_{\sim xi}^{pt})^2}{A_{\sim xi} \cdot \tan(\phi_{\sim xi}) \sigma_{ni}} \begin{bmatrix} 1 & -\eta_{\sim xi} \\ \eta_{\sim cxi} & \eta_{\sim xi}(\eta_{\sim xi} - \eta_{\sim cxi}) \end{bmatrix} \quad (29)$$

$$C_{\sim yi}^p = \frac{(A_{\sim yi} + \varepsilon_{\sim yi}^{pt})^2}{A_{\sim yi} \cdot \tan(\phi_{\sim yi}) \sigma_{ni}} \begin{bmatrix} 1 & -\eta_{\sim yi} \\ \eta_{\sim cyi} & \eta_{\sim yi}(\eta_{\sim yi} - \eta_{\sim cyi}) \end{bmatrix} \quad (30)$$

Therefore, plastic compliance matrix for ith plane is obtained as follows:

$$C_t^p = \begin{bmatrix} h_{\sim xi} & 0 & -h_{\sim xi} \eta_{\sim xi} \\ 0 & h_{\sim yi} & -h_{\sim yi} \eta_{\sim yi} \\ h_{\sim xi} \eta_{\sim cxi} & h_{\sim yi} \eta_{\sim cyi} & \begin{pmatrix} h_{\sim xi} \eta_{\sim cxi} (\eta_{\sim xi} - \eta_{\sim cxi}) \\ + h_{\sim yi} \eta_{\sim cyi} (\eta_{\sim yi} - \eta_{\sim cyi}) \end{pmatrix} \end{bmatrix} \quad (31)$$

$$h_{\sim xi} = \frac{(A_{\sim xi} + \varepsilon_{\sim xi}^{pt})}{A_{\sim xi} \cdot \tan(\phi_{\sim xi}) \sigma_{ni}} \quad (32)$$

$$h_{yi} = \frac{(A_{yi} + \epsilon_{yi}^{pt})}{A_{yi} \cdot \tan(\phi_{yi}) \sigma_{ni}} \quad (32)$$

$C_i^p$  as a whole, represent the plastic resistance corresponds to  $i$ th plane and must be summed up as the contribution of this plane with the others. Accordingly, the conceptual numerical integration of multi-laminate framework presents the summation given by equation 4 for computing  $C$ .

#### 4. DEFINITION OF PLANES WITH NEW COORDINATES

To satisfy conditions of applicability of the theory from the engineering viewpoint and also to reduce the extremely high computational costs, a limited number of necessary and sufficient sampling planes are considered.

The choice of 13 independent planes for the solution of any three dimensional problem based on getting a good distribution of plastic deformation through the media and avoiding huge computing time is a fair number. The orientation of the

sampling planes as given by their direction cosines and the weight coefficients for numerical integration rule are given in fist three and the last columns of Table 1. The coefficients  $W_i$  are simply calculated based on Gauss Quadrature numerical integration rule.

A coordinate system has been employed for each plane in such manner that one axis is perpendicular to the plane and the other two are on the plane. Plastic shear strain increments on each plane is considered as two component vectors on defined coordinate axes of plane. 13 sets of direction cosines of coordinate axes are presented in Table 1.

Figure 1 shows the orientation of all 13 planes in similar cubes. In order to clarify their positions, they have been presented in four cubes.

#### 5. MODEL PARAMETERS

In a general case, for the most anisotropic, non-homogeneous material, 13 sets of material parameters corresponding to plastic sliding of each sampling planes are required. However, any knowledge about the similarity of the sliding behavior of different sampling planes reduces the

TABLE 1. Direction Cosines and Weight Coefficients of Integration Points.

Direction Cosines of Integration Point									Weight $W_i$
$+\sqrt{1/3}$	$+\sqrt{1/3}$	$+\sqrt{1/3}$	$+\sqrt{1/6}$	$+\sqrt{1/6}$	$-\sqrt{2/3}$	$-\sqrt{1/2}$	$+\sqrt{1/2}$	0	27/840
$+\sqrt{1/3}$	$-\sqrt{1/3}$	$+\sqrt{1/3}$	$+\sqrt{1/6}$	$-\sqrt{1/6}$	$-\sqrt{2/3}$	$+\sqrt{1/2}$	$+\sqrt{1/2}$	0	27/840
$-\sqrt{1/3}$	$+\sqrt{1/3}$	$+\sqrt{1/3}$	$-\sqrt{1/6}$	$+\sqrt{1/6}$	$-\sqrt{2/3}$	$+\sqrt{1/2}$	$+\sqrt{1/2}$	0	27/840
$+\sqrt{1/3}$	$+\sqrt{1/3}$	$-\sqrt{1/3}$	$+\sqrt{1/6}$	$+\sqrt{1/6}$	$+\sqrt{2/3}$	$+\sqrt{1/2}$	$-\sqrt{1/2}$	0	27/840
$+\sqrt{1/2}$	$+\sqrt{1/2}$	0	$-\sqrt{1/2}$	$+\sqrt{1/2}$	0	0	0	1	32/840
$-\sqrt{1/2}$	$+\sqrt{1/2}$	0	$+\sqrt{1/2}$	$+\sqrt{1/2}$	0	0	0	1	32/840
$+\sqrt{1/2}$	0	$+\sqrt{1/2}$	$-\sqrt{1/2}$	0	$+\sqrt{1/2}$	0	1	0	32/840
$-\sqrt{1/2}$	0	$+\sqrt{1/2}$	$+\sqrt{1/2}$	0	$+\sqrt{1/2}$	0	1	0	32/840
0	$-\sqrt{1/2}$	$+\sqrt{1/2}$	0	$+\sqrt{1/2}$	$+\sqrt{1/2}$	1	0	0	32/840
0	$+\sqrt{1/2}$	$+\sqrt{1/2}$	0	$-\sqrt{1/2}$	$+\sqrt{1/2}$	1	0	0	32/840
1	0	0	0	0	1	0	1	0	40/840
0	1	0	1	0	0	0	0	1	40/840
0	0	1	0	1	0	1	0	0	40/840

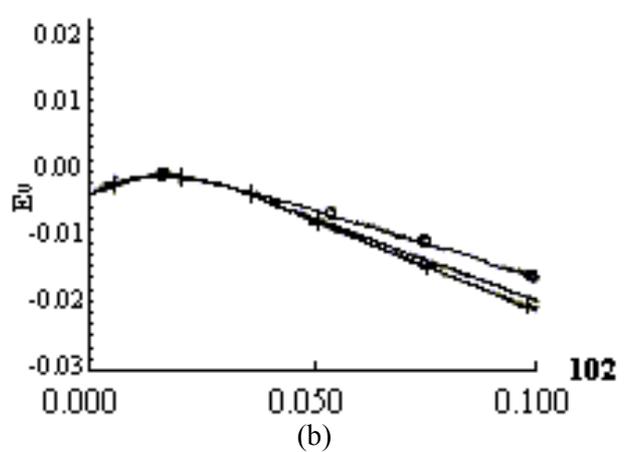
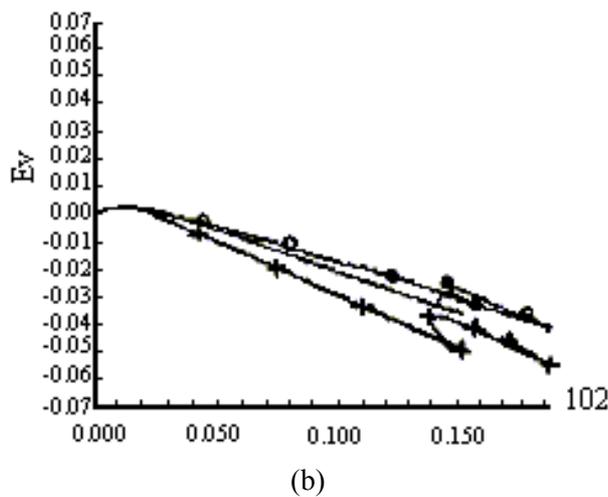
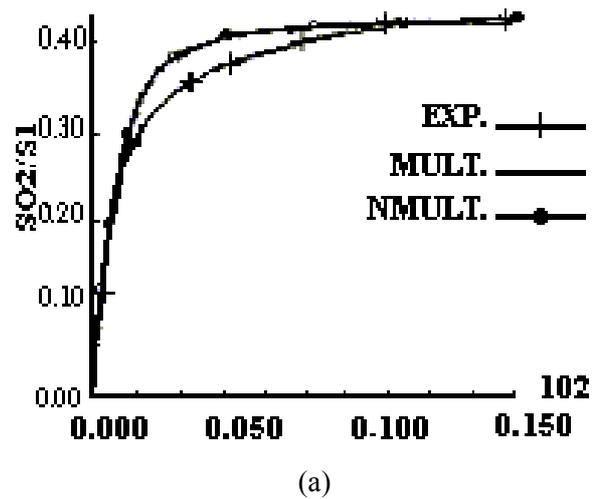
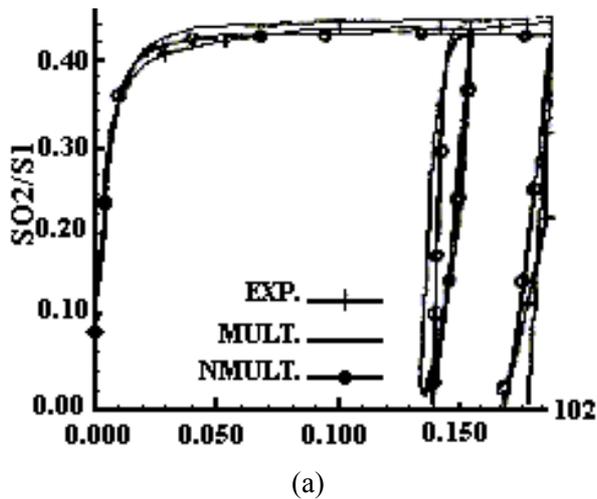


Figure 3. Test CH1.

Figure 4. Test CH2.

number of required parameters. Therefore, any fabric quality of each plane can be introduced as different parameters corresponding to two perpendicular orientations defined on that particular plane. To find these specific parameters, a test must be planned under special stress path to activate only the corresponding orientation of that plane. However, it may be thought that finding all parameters corresponding to both orientations of 13 planes is troublesome and too much, but this is the most possible anisotropic case of material property which is under consideration. Certainly, in most of the cases, where few planes contain anisotropy behavior or fabric, the required number of parameters is not much.

The number of parameters required to be used

in proposed model to obtain the behavior of an isotropic homogeneous porous medium, as sand is five. Two of these parameters correspond to elastic behavior of soil skeleton and the rest to plastic flow on each sampling plane. These parameters are listed as 1) Elastic modulus ( $E'$ ), 2) Poisson ratio ( $\nu'$ ), 3) Slope of critical state line ( $\eta_c$ ), 4) Constant value in hardening function ( $A$ ), 5) Peak angle of internal friction ( $\phi_i$ ).

$E'$  and  $\nu'$  are found in the usual way as for any other model. The other three parameters correspond to the plastic behavior of one plane. In this research, these three parameters have been assumed to be the same for all 13 defined planes because of initial isotropic conditions.

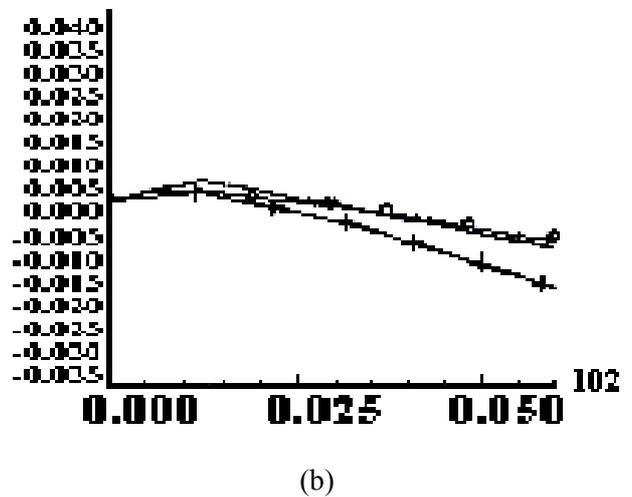
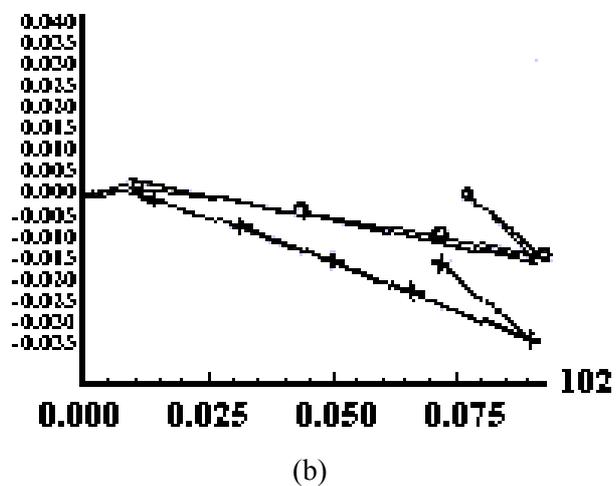
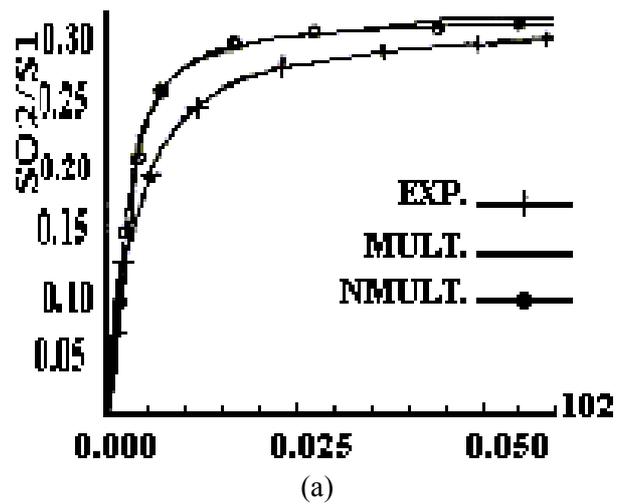
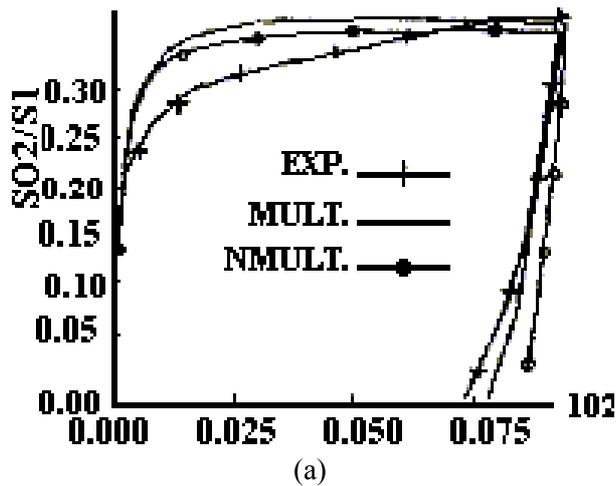


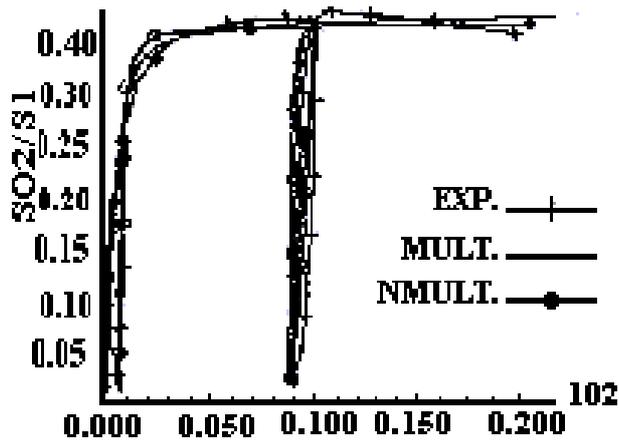
Figure 5. Test CH3.

Figure 6. Test CH4.

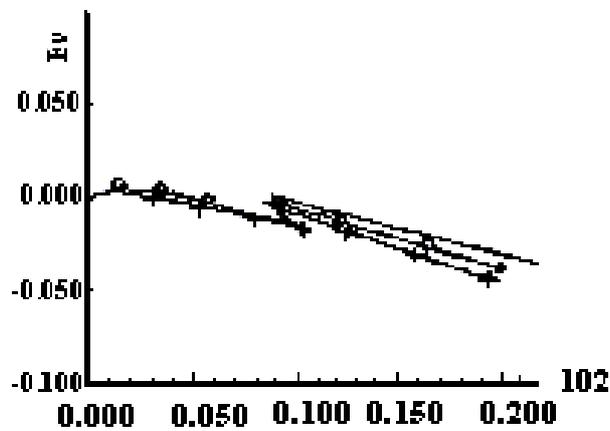
## 6. COMPARISON OF RESULTS

To show the capability of proposed plasticity model a set of cube test results on Hostun sand [7] named as CH1, CH2, CH3, CH4, CH5, and PHH3B are considered. The same mechanical properties as stated in Reference 5 employed. The first five test results are the variation of  $SD2/S1$  versus  $ID2$  and  $\epsilon_v$  versus  $ID2$ . The comparison of experiment with previous multi-laminate model result and new model are presented. Figure 3 shows the result of compression test CH1 upon initial mean stress equal to 200 kPa. The comparison shows a better result in drawing A, however, error of volumetric strain change is getting more. The differences in comparing the

results of extension test CH2 are shown in Figure 4. Figure 5 shows the compression test results upon 500 kPa. initial mean stress. CH4 is the extension test results upon 500 kPa. initial mean stress which are shown in Figure 6. Figure 7 shows the comparison of experiment with both theoretical upon two cycles, one early and the second while the rate of plastic flow is more. In review of five presented comparisons there are slightly change of new multi-laminate results mostly in drawing type. In all cases both of theoretical results are matched, however, both are a little far from experiment. This may represent the need for some change in functions employed through constitutive law. The last comparison concerns the variation of  $\tau$  and  $\epsilon_v$  versus  $\gamma$  (plastic



(a)



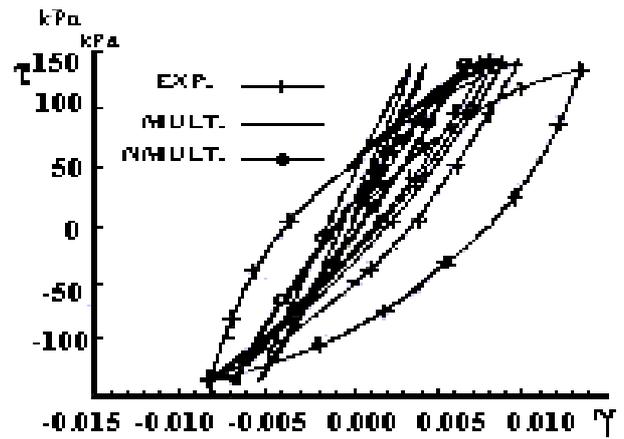
(b)

Figure 7. Test CH5.

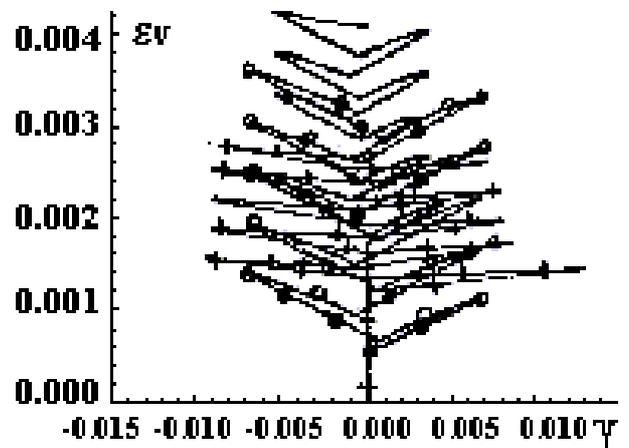
shear strain). Through these comparisons as shown in Figure 8, it may be concluded that the results of this new multi-laminate model are fairly better in such cyclic stress path. However, the hysteresis loop must be amended through change in function employed to define  $\eta$  parameter.

## 7. CONCLUSIONS

From this study a model capable of predicting the behavior of granular material on the basis of sliding mechanisms and elastic behavior of particles and double oriented hardenings has been presented. The concept of multi-laminate framework was



(a)



(b)

Figure 8. Test CH6.

successfully applied for induced anisotropic behavior of granular materials. However, this is achieved by the use of a generally simplified, applicable, effective, and easily understandable relations between micro and macro scales. These relations demonstrate an easy way to handle any heterogeneous material property as well as mechanical behavior of materials. Significantly, the stress-strain relations are primarily defined on the sampling planes. Therefore, there is no need to handle tensorial invariance requirements, which are a source of great difficulty in constitutive modeling. In this way, not only the tensorial invariance is subsequently ensured, but also some more effects, which in ordinary models are missed, are

additionally included. This inclusion is achieved by combining the responses from sampling planes of all orientations including sliding components on each plane through the material. Consequently, these results are a step closer to real plastic behavior of such particulate materials.

The predictions are actually achieved in such a way that the application of some difficult tasks such as induced two orientation anisotropies and rotation of principal stress and strain axes, which take place during plastic flow, are out of constitutive relations. Accordingly, the sampling plane constitutive formulations provide convenient means to classify loading event, generate history rules and formulate independent evolution rules for local variables.

The behavior of soil has also been modeled based on a semi-microscopic concept, which is very close to the reality of particle movement in soils.

Kinematics and isotropic hardening based phenomenological features of two perpendicular orientations on sampling planes are contributed and appropriately summed up. Therefore, the solution of any complexities involved in random cyclic loading can be obtained and presented.

In spite of producing the final results in macro scale, there is another significant feature that represents the ability of being informed of the semi-micro scales procedures during any transient monotonic or cyclic loading stress path. This feature is very fruitful in clarifying the history and rate of all local average micro scales variations through

the medium. The final point, which can be gained through this process, is the information about failure and corresponding orientation through the medium.

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