RESEARCH NOTE

THE RESPONSE OF TWO-DEGREE-OF-FREEDOM SELF-SUSTAINED SYSTEMS WITH QUADRATIC NONLINEARITIES TO A PARAMETRICEXCITATION

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Abstract In this study the interaction between self-excited and parametrically excited oscillations in two-degree-of-freedom systems with quadratic nonlinearities is investigated. The fundamental parametric resonance of the first mode and 3:1 internal resonance is considered, followed by 1:2 internal and parametric resonances of the second mode. The method of multiple time scales is applied to derive four first-order non-linear ordinary differential equations that describe the modulation of the amplitudes and phases of both modes caused by resonance. These equations are used to determine steady state amplitudes. To determine stability of the steady state solutions, small disturbances in the amplitudes and phases are superposed on the steady state solutions and the resulting equations are linearized. The eigenvalues of the corresponding system of first-order differential equations determine the stability of the steady state solutions. The instability modes are discussed and the amplitude and frequency response curves are presented by varying parameters of the system.

Key Words Nonlinear Vibration, Self-Excitation, Parametric Excitation, Perturbation Method

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INTRODUCTION

The response of two-degree-of-freedom systems with cubic nonlinearities to parametric excitation has been studied by Tso and Asmis [1] and Tezak et.al. [2]. Nayfeh [3] studied the response of a two-degree-of-freedom system with quadratic nonlinearities to a parametric harmonic excitation. Most recently, Natsiavas and Metallidis [4,5] analyzed the dynamic behavior of two-degree-of-freedom self-excited nonlinear systems, in the presence of internal and external resonances.

It seems so far, no work has been done on the dynamic behavior of quadratically two-degree-of-freedom nonlinear systems subject to both self-excitation and parametric excitation. This study attempts to do so. The method of multiple time scales is applied to derive a first-order uniform expansion of the solution consists of four first-order nonlinear ordinary differential equations. These equations describe modulation of the amplitudes and phases of the modes of oscillations. The steady state amplitudes and their stability are studied using these equations.

ANALYSIS

An analysis is presented of the nonlinear response of two-degree-of-freedom self-sustained systems with quadratic nonlinearities to a harmonic parametric excitation. Equations of motion are governed by

$$\ddot{u}_1 + \omega_1^2 u_1 + \varepsilon\,[2\cos\Omega t\,(\alpha_{11} u_1 + \alpha_{12} u_2) - \mu_1 \dot{u}_1 + \frac{\beta_{11}}{3}\dot{u}_1^3 + \frac{\beta_{12}}{3}\dot{u}_2^3 - (\lambda_1 u_1^2 + \lambda_2 u_1 u_2 + \lambda_3 u_2^2)] = 0 \quad (1)$$

$$\ddot{u}_2 + \omega_2^2 u_2 + \varepsilon\,[2\cos\Omega t\,(\alpha_{21} u_1 + \alpha_{22} u_2) - \mu_2 \dot{u}_2 + \frac{\beta_{21}}{3}\dot{u}_1^3 + \frac{\beta_{22}}{3}\dot{u}_2^3 - (\lambda_4 u_1^2 + \lambda_5 u_1 u_2 + \lambda_6 u_2^2)] = 0 \quad (2)$$

where w_n , W , a_{ij} , m_i , b_{ij} , l_i are constant parameters. The dimensionless parameter e is assumed to be a small positive quantity, and a dot denotes time derivative. Consider the multiple time scales [6,7]

$$T_n = \varepsilon^n t \quad , \quad n = 0,1,2,\ldots \quad (3)$$

Then

and $$\frac{d(\;)}{dt} = D_0 + \varepsilon D_1 + \ldots$$

$$\frac{d^2(\;)}{dt^2} = D_0^2 + 2\varepsilon D_0 D_1 + \ldots \quad (4,5)$$

where $D_n^m = \dfrac{\partial^m}{\partial T_n^m}$. The solution of Equations 1 and 2 may be expressed in the form of

$$u_n(t,\varepsilon) = u_{no}(T_0, T_1, T_2) + \varepsilon u_{n1}(T_0, T_1, T_2) + \ldots \quad , \quad n = 1,2 \quad (6)$$

Introducing Equation 6 into Equations 1 and 2, and equating terms of equal powers of e yields the following perturbation equations

$$\varepsilon^0: \quad D_0^2 u_{10} + \omega_1^2 u_{10} = 0 \quad , \quad D_0^2 u_{20} + \omega_2^2 u_{20} = 0 \quad (7a,b)$$

$$\varepsilon^1: \quad D_0^2 u_{11} + \omega_1^2 u_{11} = -2D_0 D_1 u_{10} - [\exp(i\Omega t) + \exp(-i\Omega t)](\alpha_{11} u_{10} + \alpha_{12} u_{20}) + \mu_1 D_0 u_{10} - \frac{\beta_{11}}{3}(D_0 u_{10})^3 - \frac{\beta_{12}}{3}(D_0 u_{20})^3 + \lambda_1 u_{10}^2 + \lambda_2 u_{10} u_{20} + \lambda_3 u_{20}^2 \quad (8a)$$

$$D_0^2 u_{21} + \omega_2^2 u_{21} = -2D_0 D_1 u_{20} - [\exp(i\Omega t) + \exp(-i\Omega t)](\alpha_{21} u_{10} + \alpha_{22} u_{20}) + \mu_2 D_0 u_{20} - \frac{\beta_{21}}{3}(D_0 u_{10})^3 - \frac{\beta_{22}}{3}(D_0 u_{20})^3 + \lambda_4 u_{10}^2 + \lambda_5 u_{10} u_{20} + \lambda_6 u_{20}^2 \quad (8b)$$

The solutions of Equations 7a,b can be written in the form

$$u_{no} = A_n(T_1)\exp(i\omega_n T_0) + c.c. \quad n = 1,2 \quad (9)$$

where c.c. represents the complex conjugate terms and A_n are unknown complex functions that are determined by eliminating the secular terms from the solution.

Substituting Equation 9 into Equations 8, the resulting equation will be considered from which the 3:1 internal resonance due to self-excitation with parametric resonance of the first mode will be investigated, followed by the case of 1:2 internal and parametric resonances of the second mode.

FUNDAMENTAL RESONANCE OF THE FIRST MODE AND 3:1 INTERNAL RESONANCE

The departure from exact internal and fundamental resonance is expressed by the detuning parameters s_1 and s_2 as

$$\Omega = 2\omega_1 + \varepsilon\sigma_1$$
$$3\omega_1 = \omega_2 + \varepsilon\sigma_2 \quad (10a,b)$$

Eliminating secular terms from resulting equations derived from Equations 8 and 9 leads to the following solvability equations

$$i\omega_1(\mu_1 A_1 - 2A'_1) - \alpha_{11}\bar{A}_1 exp(i\varepsilon\sigma_1 T_0)$$

$$-i\beta_{11}\omega_1^3 A_1^2 \bar{A}_1 = 0 \quad (11a)$$

$$i\omega_2(\mu_2 A_2 - 2A'_2) + \frac{i}{3}\beta_{21}\omega_1^3 A_1^3 exp(i\varepsilon\sigma_2 T_0)$$

$$-i\beta_{22}\omega_2^3 A_2^2 \bar{A}_2 = 0 \quad (11b)$$

To analyze Equations 11, it is convenient to write A_n in polar form as

$$A_n = a_n(T_1) exp[i\theta_n(T_1)] \quad , \quad n=1,2 \quad (12)$$

where both a_n and q_n are real. Substituting Equation 12 into Equations 11 and separating the result into real and imaginary parts leads to the following sets of equations

$$-2a'_1\omega_1 + \mu_1 a_1\omega_1 - \beta_{11}\omega_1^3 a_1^3 - \alpha_{11}a_1 sin\gamma_1$$

$$= 0$$

$$2a_1\omega_1\theta'_1 - \alpha_{11}a_1 cos\gamma_1 = 0 \quad (13a,b)$$

$$-2a'_2\omega_2 + \mu_2 a_2\omega_2 - \beta_{22}\omega_2^3 a_2^3 +$$

$$\frac{\beta_{21}}{3}\omega_1^3 a_1^3 cos\gamma_2 = 0$$

$$-2a_2\omega_2\theta'_2 + \frac{\beta_{21}}{3}\omega_1^3 a_1^3 sin\gamma_2 = 0 \quad (13c,d)$$

where g_1 and g_2 are new phase angles defined as

$$\gamma_1 = -2\theta_1 + \sigma_1 T_1$$

and $\gamma_2 = 3\theta_1 - \theta_2 + \sigma_2 T_1$ (14a,b)

By combining Equations 9, 13 and 14, the approximate solutions of Equation 6 may be found in the form of

$$u_1(t) = a_1(T_1) cos[(\Omega T_0 - \gamma_1)/2] + o(\varepsilon)$$

$$u_2(t) = a_2(T_1) cos[(\Omega T_0 - \gamma_1 - \frac{2}{3}\gamma_2)\frac{3}{8}] + o(\varepsilon)$$

(15a,b)

The steady state constant solution may be obtained from Equations 13 by setting $a'_1 = a'_2 = \gamma'_1 = \gamma'_2 = 0$. Consequently, it follows that

$$a_1(\mu_1\omega_1 - \beta_{11}\omega_1^3 a_1^2 - \alpha_{11} sin\gamma_1) = 0$$

$$a_1(\omega_1\sigma_1 - \alpha_{11} cos\gamma_1) = 0 \quad (16a,b)$$

$$\mu_2 a_2\omega_2 - \beta_{22}\omega_2^3 a_2^3 + \frac{\beta_{21}}{3}\omega_1^3 a_1^3 cos\gamma_2 = 0$$

$$-a_2\omega_2\sigma + \frac{\beta_{21}}{3}\omega_1^3 a_1^3 sin\gamma_2 = 0 \quad (16c,d)$$

where

$\sigma = 3\sigma_1 + 2\sigma_2$. Equations 16 admit the trivial solution $a_1 = a_2 = 0$. Also, the single-mode response $a_1 = 0$, a_2 Ú 0 is possible. Setting $a_1 = 0$, Equation 16c yields

$a_2 = \frac{1}{\omega_2}\sqrt{\frac{\mu_2}{\beta_{22}}}$. In the case of mixed-mode response, a_1 and a_2 are the solutions of the following equations, obtained by manipulating Equations 16

$$a_1^4 - 2\Gamma_1 a_1^2 + \Gamma_2 = 0 \quad \text{and} \quad a_2^6 - 2\Gamma_3 a_2^4 + \Gamma_4 a_2^2 - \Gamma_5 = 0$$

(17,18)

where

$$\Gamma_1 = \frac{\mu_1}{\beta_{11}\omega_1^2} \quad , \quad \Gamma_2 = \frac{\omega_1^2(\sigma_1^2 + \mu_1^2) - \alpha_{11}^2}{\beta_{11}^2\omega_1^6}$$

$$\Gamma_3 = \frac{\mu_2}{\beta_{22}\omega_2^2} \quad , \quad \Gamma_4 = \frac{\mu_2^2 + \sigma^2}{\beta_{22}^2\omega_2^4} \quad , \quad \Gamma_5 = \left(\frac{\beta_{21}\omega_1^3}{3\beta_{22}\omega_2^3}\right)^2 a_1^6$$

(19)

It follows from Equations 17 and 19 that real solution for a_1 exists when $\alpha_{11}^2 \geq \omega_1^2\sigma_1^2$. Then two cases are possible: $\alpha_{11}^2 < \omega_1^2(\sigma_1^2 + \mu_1^2)$ and $\alpha_{11}^2 \geq \omega_1^2(\sigma_1^2 + \mu_1^2)$. For the former case two real solutions exist and for the later case, only one real solution is possible. Equation 18 can be solved to obtain a_2. In this equation, the amplitude of the first mode appears as the forcing function. Using Cardan's method, it can be shown that if $b_{22} > 0$ and $m_2 > 0$, Equation 18 always have one positive real root.

Stability Analysis The stability of the steady state solutions are determined by assuming amplitudes and phases as

$$a_1 = a_{10} + \delta a_1 \quad , \quad a_2 = a_{20} + \delta a_2$$

$$\gamma_1 = \gamma_{10} + \delta\gamma_1 \; , \; \gamma_2 = \gamma_{20} + \delta\gamma_2 \qquad \text{(20a-d)}$$

The steady state amplitudes and phases of the modes are denoted by a_{n0} and g_{n0}, respectively, and symbol d represents small perturbation on these quantities. Substituting Equations 20 into Equations 13, keeping only the linear terms in da_1, dg_1, da_2 and dg_2, results in

$$\delta a'_1 = \left(\frac{\mu_1}{2} - \frac{3}{2}\beta_{11}\omega_1^2 a_1^2 - \frac{\alpha_{11}}{2\omega_1} sin\gamma_1\right)\delta a_1$$

$$-\left(\frac{\alpha_{11} a_1}{2\omega_1} cos\gamma_1\right)\delta\gamma_1, \delta\gamma'_1 = \left(\frac{\alpha_{11}}{\omega_1} sin\gamma_1\right)\delta\gamma_1$$

$$\delta a'_2 = \left(\frac{\beta_{21}}{2\omega_2}\omega_1^3 a_1^2 cos\gamma_2\right)\delta a_1 + \left(\frac{\mu_2}{2} -\right.$$

$$\left.-\frac{3}{2}\beta_{22}\omega_2^2 a_2^2\right)\delta a_2 - \left(\frac{\beta_{21}}{6\omega_2}\omega_1^3 a_1^3 sin\gamma_2\right)\delta\gamma_2$$

(21a-d)

$$\delta\gamma'_2 = -\left(\frac{\beta_{21}\omega_1^3 a_1^2}{2\omega_2 a_2} sin\gamma_2\right)\delta a_1 - \left(\frac{3\alpha_{11}}{2\omega_1} sin\gamma_1\right)\delta\gamma_1$$

$$+\left(\frac{\beta_{21}\omega_1^3 a_1^3}{6\omega_2 a_2^2} sin\gamma_2\right)\delta a_2 - \left(\frac{\beta_{21}\omega_1^3 a_1^3}{6\omega_2 a_2} cos\gamma_2\right)\delta\gamma_2$$

Solution of Equations 21 determines stability of the steady state responses. Let

$$\delta a_1 = \delta\overline{a_1}\, exp(\nu T_1) \quad , \quad \delta\gamma_1 = \delta\overline{\gamma_1}\, exp(\nu T_1)$$

$$\delta a_2 = \delta\overline{a_2}\, exp(\nu T_1) \quad , \quad \delta\gamma_2 = \delta\overline{\gamma_2}\, exp(\nu T_1)$$

(22a-d)

Then, for the first mode, using Equations 16 as the steady state solution the eigenvalues of the coefficient matrix becomes

$$\nu_1 = -\beta_{11}\omega_1^2 a_1^2 \text{ and } \nu_2 = \mu_1 - \beta_{11}\omega_1^2 a_1^2$$

(23a,b)

For the first mode to be stable, both eigenvalues must be negative. Then

$$\beta_{11} > 0 \text{ and } \mu_1 < \beta_{11}\omega_1^2 a_1^2 \qquad \text{(24a,b)}$$

The eigenvalues which govern stability of the second mode are given by

$$\nu_{3,4} = -\frac{1}{2}\left[2\beta_{22}\omega_2^2 a_2^2 - \mu_2\right]$$

$$-\frac{1}{2}\left[\pm\sqrt{\beta_{22}^2\omega_2^4 a_2^4 - \sigma^2}\right]$$

(25)

The conditions for stability of the second mode may be obtained as

$$(\beta_{22}\omega_2^2 a_2^2 - \mu_2)(3\beta_{22}\omega_2^2 a_2^2 - \mu_2) < \sigma^2$$

and

$$\mu_2 < 2\beta_{22}\omega_2^2 a_2^2 \qquad \text{(26a,b)}$$

where $S = 3S_1 + 2S_2$. The eigenvalues of the coefficient matrix for the second mode are real if $\sigma < \beta_{22}\omega_2^2 a_2^2$. It is easy to show that the trivial solution $a_1 = 0$ is always unstable. Also, the single-mode response a_2Ú0 is stable if $m_2 > 0$.

FUNDAMENTAL RESONANCE OF THE SECOND MODE AND 1:2 INTERNAL RESONANCE

This resonant case may be expressed as

$$\Omega = 2\omega_2 + \varepsilon\sigma_1 \quad \text{and} \quad 2\omega_2 = \omega_1 + \varepsilon\sigma_2$$

(27a,b)

Eliminating the terms which produce secular terms, yields

$$i\omega_1(-2A'_1 + \mu_1 A_1) - i\beta_{11}\omega_1^3 A_1^2 \overline{A_1}$$

$$+A_1 + \lambda_3 A_2^2 exp(i\varepsilon\sigma_2 T_0) = 0 \qquad \text{(28a)}$$

$$i\omega_2(-2A'_2+\mu_2 A_2)-i\beta_{22}\omega_2^3 A_2^2\overline{A_2}$$

$$-\alpha_{22}\overline{A}_2 exp(i\varepsilon\sigma_1 T_0)$$

$$+\lambda_5 A_1\overline{A_2}\, exp(-i\varepsilon\sigma_2 T_0)=0 \qquad (28b)$$

To study the effects of internal and parametric resonances, introducing Equation 12 into Equations 28 and separating real and imaginary parts, one gets

$$-2a'_1\omega_1+\mu_1 a_1\omega_1-\beta_{11}\omega_1^3 a_1^3+\lambda_3 a_2^2 sin\gamma_1=0$$

$$2a_1\omega_1\theta'_1+\lambda_3 a_2^2 cos\gamma_1=0 \qquad (29a,b)$$

$$-2a'_2\omega_2+\mu_2 a_2\omega_2-\beta_{22}\omega_2^3 a_2^3$$

$$-\alpha_{22}a_2 sin\gamma_2-\lambda_5 a_1 a_2 sin\gamma_1=0 \qquad (29c)$$

$$a_2(2\omega_2\theta'_2+\lambda_5 a_1 cos\gamma_1-\alpha_{22}cos\gamma_2)=0$$

$$\gamma_1=2\theta_2-\theta_1+\sigma_2 T_1\ ,\ \gamma_2=-2\theta_2+\sigma_1 T_1 \qquad (29d,30a,b)$$

Approximate solutions of Equation 6 are found as

$$u_1(t)=a_1(T_1)\cos(\Omega T_0-\gamma_1-\gamma_2)+o(\varepsilon)$$

$$u_2(t)=a_2(T_1)\cos\left[(\Omega T_0-\gamma_2)/2\right]+o(\varepsilon) \qquad (31a,b)$$

In the steady state, Equations 29 accept constant solutions. Hence, it follows that

$$\mu_1 a_1\omega_1-\beta_{11}\omega_1^3 a_1^3+\lambda_3 a_2^2 sin\gamma_1=0$$

$$2a_1\omega_1(\sigma_1+\alpha_2)+\lambda_3 a_2^2 cos\gamma_1=0 \qquad (32a,b,c,d)$$

$$a_2(\mu_2\omega_2-\beta_{22}\omega_2^3 a_2^2-\lambda_5 a_1 sin\gamma_1$$

$$-\alpha_{22}sin\gamma_2)=0$$

$$a_2(\omega_2\sigma_1+\lambda_5 a_1 cos\gamma_1-\alpha_{22}cos\gamma_2)=0$$

Equations 32 admit the trivial solution $a_1=a_2=0$. The single-mode response a_1Ú0, $a_2=0$ is also possible. In this case, Equations 32a,b give $a_1=\frac{1}{\omega_1}\sqrt{\frac{\mu_1}{\beta_{11}}}$. When a_1Ú0 and a_2Ú0, Equations 32 may be combined to obtain two nonlinear equations in the form

$$(\mu_1 a_1\omega_1-\beta_{11}\omega_1^3 a_1^3)^2+4a_1^2\omega_1^2(\sigma_1+\sigma_2)^2$$

$$=\lambda_3^2 a_2^4 \qquad (33a)$$

$$(-\lambda_3\beta_{22}\omega_2^3 a_2^4-\lambda_5\beta_{11}\omega_1^3 a_1^4+\lambda_3\mu_2\omega_2 a_2^2$$

$$+\lambda_5\mu_1\omega_1 a_1^2)^2$$

$$+\left[\lambda_3\sigma_1\omega_2 a_2^2-2\lambda_5\omega_1\sigma_1 a_1^2(\sigma_1+\sigma_2)\right]^2$$

$$=\lambda_3^2\alpha_{22}^2 a_2^4 \qquad (33b)$$

Using a two dimensional Newton-Raphson algorithm, Equations 33 may be solved to obtain a_1 and a_2.

Stability Analysis The stability characteristics of the steady state solutions may be obtained by a procedure similar to that in the preceding section. Imposing small perturbations on the steady state amplitudes and phases, and eliminating steady state and non-linear terms of the solvability conditions, a set of first-order differential equations may be obtained. The resulting variational equations are in the form $\dot{\underset{\sim}{a}}=\Gamma\underset{\sim}{a}$, where

$$\underset{\sim}{a}=(\delta a_1,\delta\gamma_1,\delta a_2,\delta\gamma_2) \qquad (34)$$

and

$$\Gamma=\left[C_{ij}\right]_{4\times4}\quad ,\quad i,j=1,2,3,4 \qquad (35)$$

where

$$C_{11}=\frac{A}{2}\ ,\ C_{12}=-a_1\sigma\ ,\ C_{13}=-B\frac{a_1}{a_2}$$

$$C_{14}=0\ ,\ C_{21}=\frac{\sigma(1+E)}{a_1}\ ,\ C_{22}=\frac{B(1-E)}{2}$$

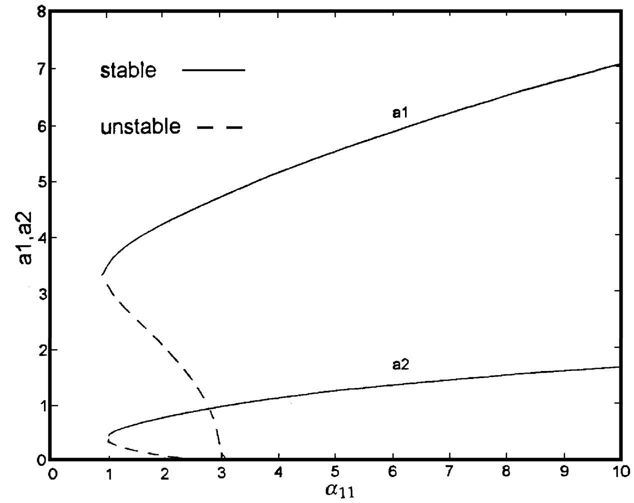

Figure 1. Amplitude-response curves, W@2w$_1$, 3w$_1$@w$_2$.

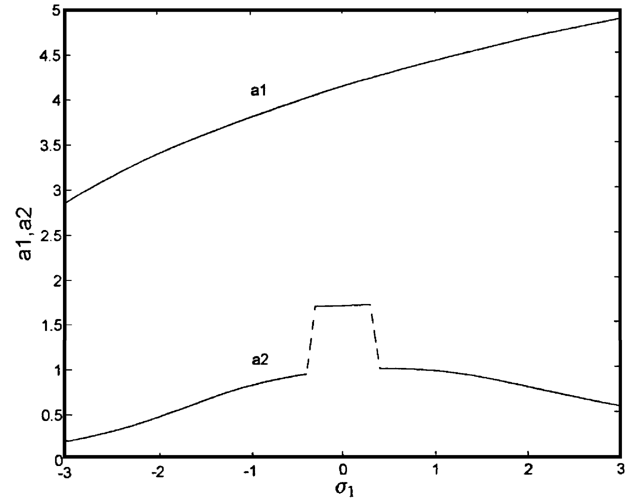

Figure 2. Frequency-response curves, W@2w$_1$, 3w$_1$@w$_2$.

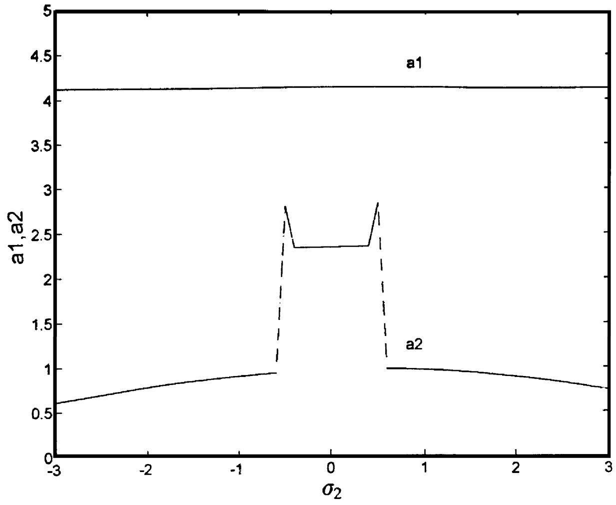

Figure 3. Frequency-response curves, W@2w$_1$, 3w$_1$@w$_2$.

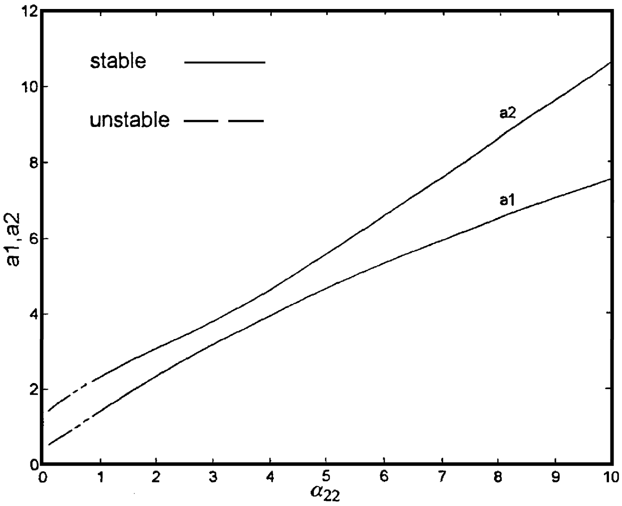

Figure 4. Amplitude-response curves, W@2w$_2$, 2w$_2$@w$_1$.

$$C_{23}=-2\frac{\sigma}{a_2}\ ,\ C_{24}=\frac{-(C+EB)}{2},\ C_{31}=\frac{a_2BE}{4}$$

$$C_{32}=a_2\sigma E\ ,\ C_{33}=\frac{(D-C)}{2}$$

$$a_2(\sigma E-\sigma_1)\ \ldots\ \frac{\sigma E}{a_1},\ C_{42}=\frac{EB}{2}$$

$$C_{43}=0\ ,\ C_{44}=C+\frac{EB}{2}$$

$$A=\mu_1-3\beta_{11}\omega_1^2a_1^2\ ,\ B=\mu_1-\beta_{11}\omega_1^2a_1^2$$

$$C=\mu_2-\beta_{22}\omega_2^2a_2^2,\ \ D=\mu_2-3\beta_{22}\omega_2^2a_2^2$$

$$E=\frac{2a_1^2\omega_1\lambda_5}{a_2^2\omega_2\lambda_3}\ ,\ \sigma=\sigma_1+\sigma_2 \tag{36}$$

The eigenvalues of matrix G determine stability of the steady state solutions. If the real part of all the eigenvalues are negative, the steady state solution is stable.

RESULTS AND DISCUSSION

The results for the case of fundamental resonance of the first mode and 3:1 internal resonance due to self-excitation are shown in

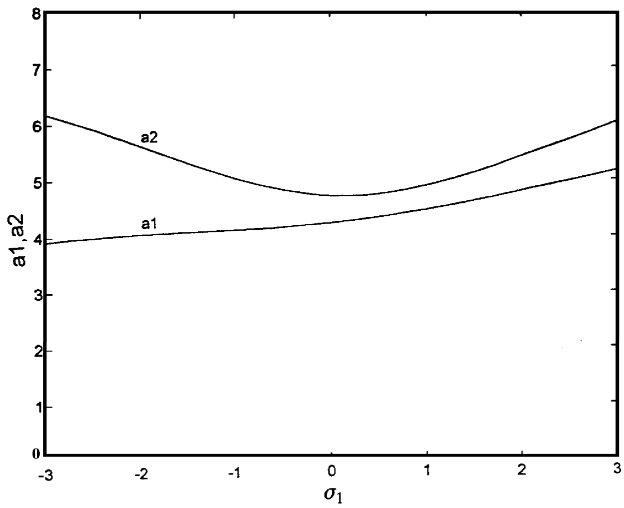

Figure 5. Frequency-response curves, W@2w_2, 2w_2@w_1.

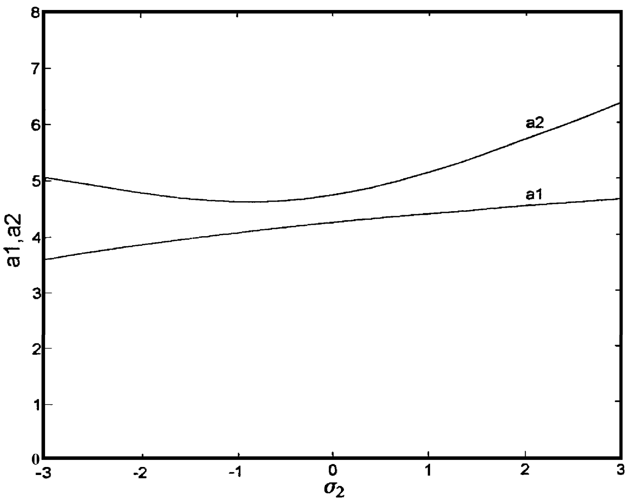

Figure 6. Frequency-response curves, W@2w_2, 2w_2@w_1.

Figures 1-3. All of the results are obtained for W= 2 and e= 0.1. Figure 1 shows amplitude-response curves for the case: s_1= s_2= 1, m_1= 3, m_2= 0.1, b_{11}= b_{21}= b_{22}= 0.3, and a_{11} is varied from 0 to 10. As a_{11} is increased from zero, no steady state solutions exist until a_{11} reaches 0.95. Two solutions for steady state amplitudes of a_1 and a_2 exist between a_{11}= 0.95 and a_{11}= 3. According to Equations 23 and 25 the higher branches of amplitudes correspond to the stable solutions. Beyond a_{11}= 3, the stable solutions are single valued. The stable steady state amplitudes a_1, the directly excited mode and a_2, the indirectly self-excited mode, increase as a_{11} increases. However, the rate of increase of a_2 is less than a_1. Figure 2 shows the frequency-response curves for s_2= 0 (a perfectly tuned internal resonance), m_1 = m_2 = 0.1, a_{11} = 5, b_{11} = b_{21} = b_{22} = 0.3, and s_1 is varied from -3 to 3. The first mode amplitude a_1 increases with increasing s_1. The second mode amplitude a_2 experiences jumps for s_1 near -0.4 and 0.4. In Figure 3, the frequency-response curves for the case of s_1=0 are plotted against s_2 which is a measure of the resonance due to self-excitation. All other parameters are the same as in Figure 2. The stable steady state value of the amplitude a_1 of the directly excited mode is independent of the detuning parameter s_2. This may be seen from Equation 17. Again, jump phenomenons exist on the amplitude of the second mode a_2, for s_2 near -0.6 and 0.6.

Figures 4-6 show results for the case of parametric resonance of the second mode and 2:1 internal resonance. The amplitude-response curves in Figure 4 are plotted for s_1 = s_2 = 1, m_1 = m_2 = 0.1, b_{11} = b_{22} = 0.3, l_3 = l_5 = 1, and a_{22} is varied from 0 to 10. Both steady state amplitudes increase as excitation amplitude a_{22} increases. For a_{22} between 0.5 and 0.9, no stable steady state solutions exist. In this range, the eigenvalues of matrix (35) have positive real parts. In Figures 5 and 6, a_1 and a_2 are plotted as functions of the detuning parameters s_1 and s_2, respectively, a_{22}=5 in both figures, s_2=0 in Figure 5 and s_1= 0 in Figure 6. All other parameters are the same as in Figure 4. As may be seen, the stable amplitudes are not very much dependent on the detuning parameters.

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