THRUST-LIMITED OPTIMAL THREE-DIMENSIONAL SPACECRAFT TRAJECTORIES

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Abstract Several optimal three-dimensional orbital transfer problems are solved for thrust-limited spacecrafts using collocation and nonlinear programming techniques. The solutions for full nonlinear equations of motion are obtained where the integrals of the free Keplerian motion in three dimensions are utilized for coasting arcs. In order to limit the solution space, interior-point constraints are used which proved to be beneficial in finding the optimal results by making initial estimates more sensible. The application of this methodology to the design of several three dimensional optimal trajectories is also investigated. The results indicate that the method is suitable for any 3D transfer between non-coplanar and non-coaxial orbits.

Key Words Optimal Trajectory, Spacecraft, Tree-Dimensional Maneuvers, Orbital, Transfer, Thrust-Limited

چکیده با استفاده از یک تکنیک جدید بهینه سازی مستقیم، تعدادی از مسائل مانورهای بهینه مداری سه بعدی سفینه های فضائی با نیروی رانش محدود مورد بررسی قرار گرفته اند. روش هم مکان سازی مستقیم و برنامه ریزی غیر خطی طوری استفاده شده اند که محاسبه تحلیلی ماتریسهای ژاکوبین و هسیان را امکان پذیر می سازند. حل معادلات حرکتی غیر خطی با در نظر گرفتن انتگرالهای حرکت کپلری در مسیرهای پرواز آزاد میانی بدست آمده است. به منظور محدود کردن فضای پاسخ، قیدهای میانی انتخاب شده اند. کاربرد روش فوق در طراحی ماموریتهای فضائی بررسی گردیده و نتایج مطلوبی برای مانورهای سه بعدی، بین مدارهای غیر واقع در یک صفحه بدست آمده است.

INTRODUCTION

In compliance with the utilization of space science and technology to gather information about the outer space, many challenging problems have developed. One of the main related problems is the optimal three-dimensional spacecraft maneuver. The importance of these maneuvers is realized in situations such as satellite maintenance missions (servicing Hubble space-telescope), time optimal rendezvous between the space shuttle and the space station, rendezvous with comets from outside of our solar system (Giotto's extended mission), three dimensional transfer trajectories to halo orbits (SOHO mission) and three dimensional gravity-assisted maneuvers needed to exit the ecliptic plane (Ulysses mission). To perform such maneuvers, energy is expended in controlled directions resulting a change in the velocity vector and causing the desired orbital changes. Traditionally, the impulsive thrust models have been utilized to produce the desired change in the velocity magnitude and direction for very short duration [1]. Due to several problems such as misdirected injection [2] and potential damage to spacecraft subsystems, because of high impulsive forces, low-thrust models have also become attractive and in some cases provide the only feasible alternatives [3,4,5]. Most practical work involving optimal orbital transfers has focused on two dimensions. These include two-impulse transfer between two coplanar elliptic orbits [6,7], transfer to synchronous orbits [8], Mars orbit insertion [9,10], multiple-impulse rendezvous problem in fixed time [11], optimal coplanar rendezvous problem using solar sails [5] and two dimensional orbital evasive maneuvers [12]. Due to the increasing importance of three-dimensional orbital maneuvers, this study focuses mainly on the optimal design of 3D trajectories.

The three dimensional equations of motion are highly nonlinear and one way to approach them is through linearization, which is appropriate for short orbital maneuvers. Cooperative rendezvous maneuvers between neighboring circular orbits [4] and the low-thrust rendezvous return after impulsive intercept [5] are good examples of this approach. In this study, however, the full nonlinear 3D equations are solved numerically. The optimal 3D maneuvers with intermediate coast and maximum-thrust (MT) arcs are obtained through a variation formulation. The optimal controls are defined so as they minimize the total burn time. In addition, several sensitivity analyses are performed based on initial MT level, orbital radii and inclination angles. Interior point constraints and discontinuity in state variables have also been taken into account in order to improve the design of maneuver structure. Optimal transfer into the recently discovered solar sail halo orbits [13,14] is another problem that is considered using the developed technique.

The optimal control of spacecraft results in a two-point boundary-value problem (TPBVP) for which two general approaches of solution exists, namely direct and indirect methods. In indirect approach, the state space equations are obtained using the calculus of variations for which various common methods of solution exist. They include steepest ascent, quasilinearization, gradient projection and variation of extremals. The difficulty with the mentioned techniques for the indirect approach, is their sensitivity to the initial guess. The second approach is to discretize the variation form of the original problem, approximating the optimal control problem with a discrete optimization problem that is referred to as the direct approach [15,16,17].

The present paper uses a hybrid approach in which the continuous variational equations of the system are discretized and a collocation scheme is utilized to satisfy the state and costate differential equations as well as any problem constraints. By this technique, a set of nonlinear algebraic equality/non-equality equations is obtained that is solved using nonlinear programming (NLP) methods. Hargraves and Paris [17] have investigated the application of direct method to the solution of aerospace flight mechanics problems. The problem of planar orbital transfers is another application performed by Enright and Conway through the direct collocation with nonlinear programming [10] and direct transcription [18]. As evidenced by Betts and Huffman, solution algorithms include sparse nonlinear programming [15] and sequential quadratic programming [16]. Most above-mentioned researchers have relied on numerical techniques for the evaluation of their Jacobian and Hessian matrices, and have studied issues of accuracy, speed and stability of solutions. Here we construct our Jacobian and Hessian matrices analytically, which allows us to control the performance of our NLP solver precisely.

THRUST-LIMITED SPACECRAFT MODEL IN THREE DIMENSIONS

The spherical coordinate system Figure1 is utilized to express the spacecraft motion [19]. The dynamical equations of the system are

$$\dot{x}_1 = x_4 , \tag{1a}$$

$$\dot{x}_2 = x_5/(x_1 \cos x_3) , \tag{1b}$$

$$\dot{x}_3 = x_6/x_1 , \tag{1c}$$

$$\dot{x}_4 = \frac{1}{x_1}(x_5^2 + x_6^2) - \frac{\mu}{x_1^2} + x_7 u_r , \tag{1d}$$

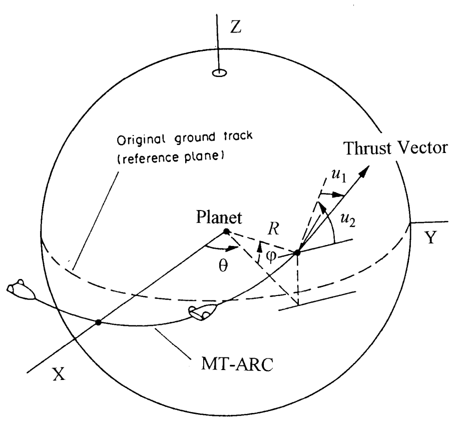

Figure 1. Trajectory Simulation in Spherical Coordinates.

$$\dot{x}_5 = \frac{x_5 x_6 \tan x_3}{x_1} - \frac{x_4 x_5}{x_1} + x_7 u_\theta , \tag{1e}$$

$$\dot{x}_6 = -\frac{x_4 x_6}{x_1} - \frac{x_5^2 \tan x_3}{x_1} + x_7 u_\varphi , \tag{1f}$$

$$\dot{x}_7 = x_7^2 / c , \tag{1g}$$

where $x_1 = R$, $x_2 = \theta$, $x_3 = \varphi$, $x_4 = \dot{R}$, $x_5 = R\dot{\theta}\cos\varphi$, $x_6 = R\dot{\varphi}$. The variable x_7 is the propulsive acceleration generated by the thruster. The variables u_r, u_θ and u_φ are the controls. The parameter c is the effective exhaust velocity and μ denotes the gravitational constant, which in canonical coordinate systems is set equal to unity. The control variables are related to the local spherical angles (u_1 and u_2) through the following relations:

$$u_r = \sin u_1 , \tag{2a}$$

$$u_\theta = \cos u_1 \cos u_2 , \tag{2b}$$

$$u_\varphi = \cos u_1 \sin u_2 , \tag{2c}$$

In Equations 2, u_1 is the angle between the thrust direction and the local horizontal and u_2 is the angle between the projection of the thrust vector on the local horizontal plane and e_θ. The maneuver structure usually consists of an alternating sequence of MT and coast arcs (in the absence of the thrust-vector, the free two-body motion takes place which is known as coasting) [18]. During the coasting interval, the state variables are related through the six constants of integration as follows:

$$Q_1(\mathbf{x}) = h_x = x_1 x_6 \sin x_2 - x_1 x_5 \sin x_3 \cos x_2 , \tag{3a}$$

$$Q_2(\mathbf{x}) = h_y = -x_1 x_6 \cos x_2 - x_1 x_5 \sin x_3 \sin x_2 , \tag{3b}$$

$$Q_3(\mathbf{x}) = h_z = x_1 x_5 \cos x_3 , \tag{3c}$$

$$Q_4(\mathbf{x}) = e_x = \cos x_2 \cos x_3 [\frac{x_1 x_5^2}{\mu} + \frac{x_1 x_6^2}{\mu} - 1] + \frac{x_1 x_4 x_5 \sin x_2}{\mu} + \frac{x_1 x_4 x_6 \sin x_3 \cos x_2}{\mu} , \tag{3d}$$

$$Q_5(\mathbf{x}) = e_y = \cos x_3 \sin x_2 [\frac{x_1 x_5^2}{\mu} + \frac{x_1 x_6^2}{\mu} - 1] - \frac{x_1 x_4 x_5 \cos x_2}{\mu} + \frac{x_1 x_4 x_6 \sin x_2 \sin x_3}{\mu} , \tag{3e}$$

$$Q_6(\mathbf{x}) = e_z = \sin x_3 [\frac{x_1 x_5^2}{\mu} + \frac{x_1 x_6^2}{\mu} - 1] - \frac{x_1 x_4 x_6 \cos x_3}{\mu} , \tag{3f}$$

where h_x, h_y, and h_z are the components of the angular momentum vector, **h**, and **e** = (e_x, e_y, e_z) is the well known Lenz vector [20]. No mass is expelled during a coast arc, therefore, the thrust acceleration x_7, at the end of the thrust arc, which precedes the coast arc, equals the thrust acceleration at the start of the proceeding MT arc.

OPTIMAL CONTROL PROBLEM BOUNDARY CONDITIONS AND PERFORMANCE INDEX

It is desired to guide the spacecraft from a given initial state to a target orbit, where the final state variables must satisfy some terminal conditions. The boundary conditions may be written as:

$$\mathbf{x}(t_0) = \{R_0, \theta_0, \varphi_0, \dot{R}_0, (R\dot{\theta}\cos\varphi)_0, (R\dot{\varphi})_0\}^T , \tag{4a}$$

$$\Phi(\mathbf{x}(t_f), t_f) = 0 . \tag{4b}$$

There are two basic types of propulsion systems. High-thrust systems in which the thrust duration is relatively short in comparison with the total mission time, and low-thrust systems in which the thrusting intervals are prolonged. In the current study the thrust-limited category of low-thrust systems has been used (there is another type referred to as power-limited systems). For the low-thrust systems an optimal trajectory is defined so that the amount of consumed propellant is minimized; in other words, the burn times (total duration of the MT arcs) must be minimized.

Assuming a maneuver structure consisting of N number of MT arcs, the cost function is defined as

$$J = \int_{t_0}^{t_1^-} dt + \int_{t_1^+}^{t_2^-} dt + \cdots + \int_{t_{N-1}^+}^{t_N^-} dt + \lambda^T \cdot \Phi + \nu^T \cdot \Delta, \quad (5)$$

where t_{i-1}^+ is the time at which the i-th MT arc begins and t_i^- is the time when the i-th MT comes to end. The constraint vector Δ_j takes into account the effect of the j-th coast arc of which the components are defined as

$$\Delta_j^m = Q_m(\mathbf{x}(t_j^-)) - Q_m(\mathbf{x}(t_j^+)) = 0, \quad (6)$$

$$m = 1,..,6;\ j = 1,2,..,N-1.$$

The definition of the cost function (5) does not necessarily minimize the total flight time. However, with only one MT arc, the maneuver will also be time-optimal.

HAMILTONIAN FUNCTION AND THE OPTIMALITY CONDITIONS

The Hamiltonian function (on the MT arcs) is given by

$$\begin{aligned} H = {} & 1 + p_1 x_4 + p_2 \frac{x_5}{x_1 \cos x_3} + p_3 \frac{x_6}{x_1} \\ & + p_4 [\frac{1}{x_1}(x_5^2 + x_6^2) - \frac{\mu}{x_1^2} + x_7(u_r)] \\ & + p_5 [\frac{x_5 x_6 \tan x_3}{x_1} - \frac{x_4 x_5}{x_1} + x_7(u_\theta)] \\ & + p_6 [-\frac{x_4 x_6}{x_1} - \frac{x_5^2 \tan x_3}{x_1} + x_7(u_\varphi)] + p_7 \frac{x_7^2}{c}, \end{aligned} \quad (7)$$

where p_i (i=1, ..., 7) are the adjoint variables. The necessary condition for optimality will be

$$\frac{\partial H}{\partial \mathbf{u}} = \mathbf{0}, \quad (8)$$

where the control vector $\mathbf{u}$ satisfies the following constraint

$$\mathbf{u}^T \cdot \mathbf{u} = 1. \quad (9)$$

Equations 8, 9, and the convexity condition (Legendre-Clebsch condition) result in the extremal controls in terms of the adjoint variables:

$$u_r = -\frac{p_4}{\sqrt{p_4^2 + p_5^2 + p_6^2}}, \quad (10a)$$

$$u_\theta = -\frac{p_5}{\sqrt{p_4^2 + p_5^2 + p_6^2}}, \quad (10b)$$

$$u_\varphi = -\frac{p_6}{\sqrt{p_4^2 + p_5^2 + p_6^2}}. \quad (10c)$$

Because of the nature of thrust vectoring and Equation 9, there are only two independent control components.

DYNAMICS OF THE ADJOINT VARIABLES

The adjoint variables are chosen to satisfy [21,22]

$$\dot{\mathbf{p}}^T = -\frac{\partial H}{\partial \mathbf{x}}, \quad t_{i-1}^+ < t < t_i^- \text{ (for the } i\text{-th MT arc)}, \quad (11)$$

i = 1,..,N;

$$\mathbf{p}^T(t_m^-) = \mathbf{v}_m^T \cdot \frac{\partial \mathbf{Q}(\mathbf{x}(t_m^-))}{\partial \mathbf{x}(t_m^-)}, \quad (12a)$$

$$\mathbf{p}^T(t_m^+) = \mathbf{v}_m^T \cdot \frac{\partial \mathbf{Q}(\mathbf{x}(t_m^+))}{\partial \mathbf{x}(t_m^+)}, \quad (12b)$$

$$\mathbf{p}^T(t_N^-) = \mathbf{p}^T(t_f) = \boldsymbol{\lambda}^T \cdot \frac{\partial \Phi(\mathbf{x}(t_f), t_f)}{\partial \mathbf{x}(t_f)}, \quad (12c)$$

with m = 1, .., N-1 and $\mathbf{Q}=\{Q_1,..,Q_6\}^T$. The vectors $\boldsymbol{\lambda}$ and $\mathbf{v}_m$ are the Lagrange multipliers and t_m must be chosen to satisfy the transversality condition

$$H(t_m^-) = H(t_m^+). \quad (13)$$

The transversality condition at the final time t_f

will be

$$H(t_f)+\frac{\partial}{\partial t_f}[\boldsymbol{\lambda}^T\cdot\Phi(\mathbf{x}(t_f),t_f)]=0. \qquad (14)$$

For the problem under consideration, the Hamiltonian function is not an explicit function of time, Howeres, using Equations 13 and 14, it can be shown that, along an optimal trajectory (if Φ does not include t_f explicitly), the following relations hold

$$H^*\equiv H^*(t_i^-)=H^*(t_i^+)=H^*(t_f)=0,\ i=1,..,\text{N-1}. \qquad (15)$$

INTERIOR-POINT CONSTRAINTS

In many problems it is suitable to constrain the manifold of solutions for a better prediction of the system performance and to get a good estimate of final results. The type and definition of the problem constraints depend on many factors that may be specified by trajectory designer.

Here, we impose a set of interior boundary conditions on the state variables as [21]

$$\mathbf{Z}[\mathbf{x}(T_j)]=0, \qquad (16)$$

where T_j is an intermediate time in the j-th MT arc, with

$$t_{j-1}^+<T_j<t_j^-,$$

and **Z** represents a vector function. The boundary conditions relevant to the adjoint variables, and the transversality condition are

$$\mathbf{p}^T(T_j^-)=\mathbf{p}^T(T_j^+)+\boldsymbol{\pi}^T\cdot\left.\frac{\partial\mathbf{Z}}{\partial\mathbf{x}}\right|_{t=T_j}, \qquad (17a)$$

$$H(T_j^-)=H(T_j^+), \qquad (17b)$$

where $\boldsymbol{\pi}$ is a vector of Lagrange multipliers.

NUMERICALLY SOLVED PROBLEMS

Transfer to a Non-Coplanar Orbit In this case a low-thrust spacecraft orbiting in a circular Keplerian orbit with radius one (canonical units) is desired to rendezvous with another spacecraft on a target circular orbit with radius 1.5 which has an inclination angle, φ_0, with the initial orbit. In the solution procedure, due to the relative closeness of two orbital radii, no coast arcs are allowed. The initial thrust acceleration and the exhaust gas velocity are taken at 0.15 and 1.5 respectively in canonical coordinates. fun the performance measure is taken to be the total maneuver time. Since there are no intermediate coast arcs, the time-optimal control problem and the minimum fuel problem would be identical for our low-thrust spacecraft. The boundary conditions for the state variables are given below using vector notation:

$$\mathbf{x}_0=\{1,0,\varphi_0,0,1,0,0.15\}^T,$$

$$\mathbf{x}_f=\{1.5,\mathit{free},0,0,0.816,0,\mathit{free}\}^T,$$

The problem is solved through a collocation scheme. The system differential equations are discretized using the trapezoidal transcription formula. The total maneuvering time to be determined is divided into 20 equal intervals which creates an NLP problem having 327 variables and 327 constraints. The solution process is repeated for three inclination angles: 5.7°, 16° and 21.2°. The numerical results for control components, spacecraft velocity components and spacecraft radial as well as angular positions are shown in Figures 2. A three-dimensional view of the trajectory is also shown for one of the inclination angles, φ_0 = 21.2°, in Figure 3.

A notable result is the increased oscillation of control components for larger inclination angles. Due to homogeneity of the boundary conditions on the two variables $\dot{R}$ and $R\dot{\varphi}$, their variation must possess some kind of symmetry that is verified by the numerical results. Deviation from this symmetry occurs when the thrust acceleration is changed. Since for the low inclination angles, the total flight time and the range of variations in the

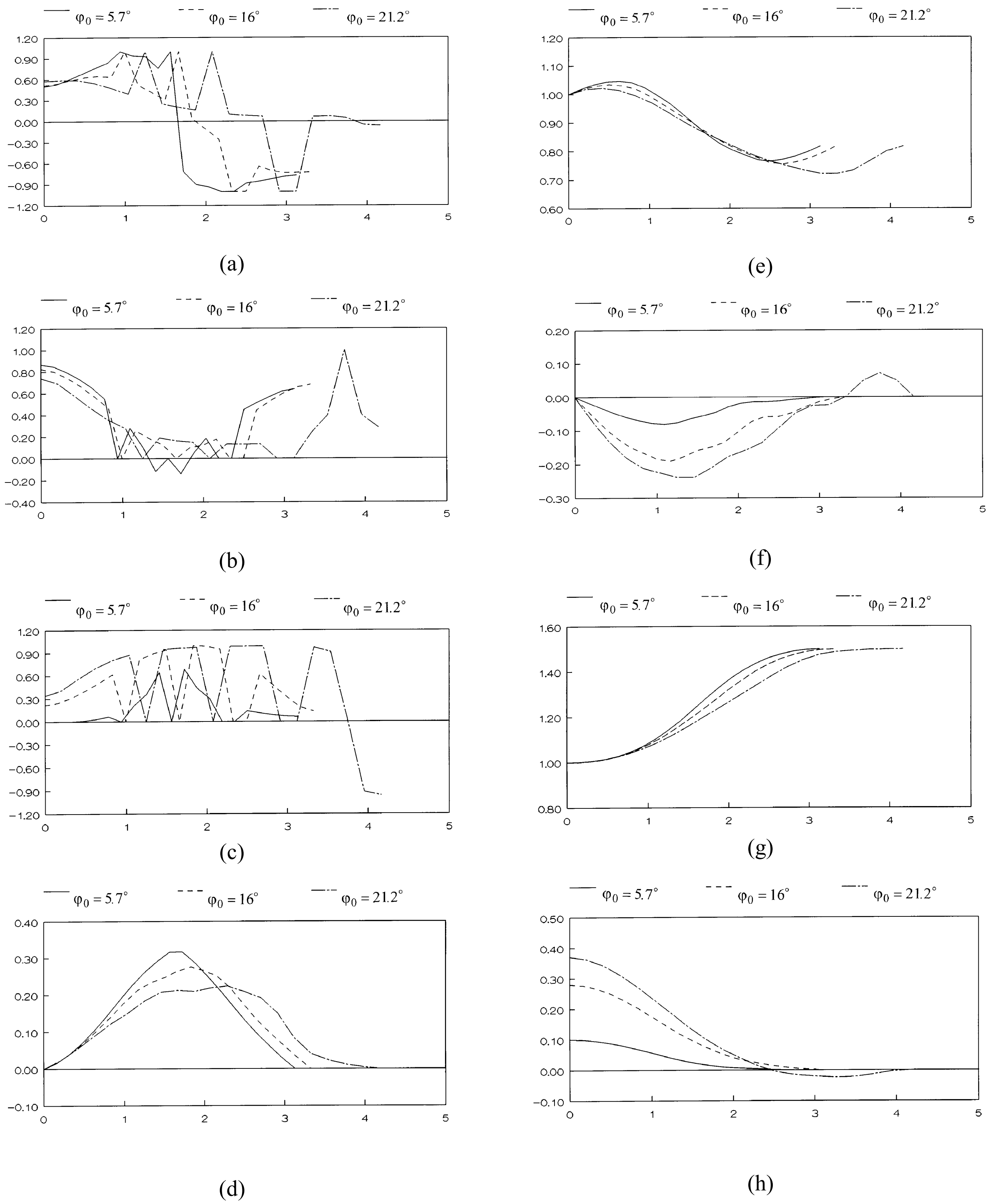

Figure 2. Control history vs. scaled time: (a). u_r, (b) u_θ and (c) u_φ, and variation of (d) x_4, (e) x_5 and (f) state component vs. scaled time.

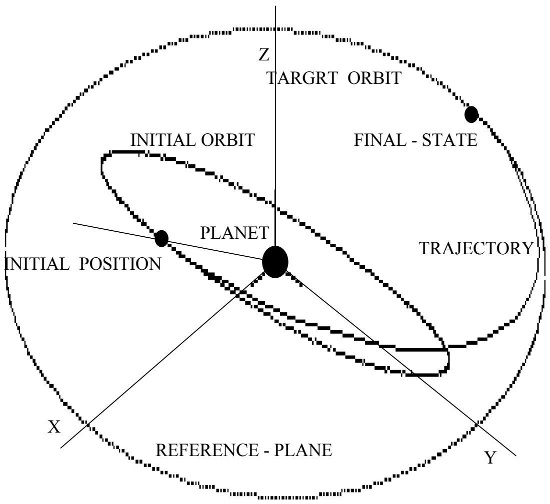

Figure 3. Three-dimensional View of the Two-Burn Transfer to a Noncoplanar Keplerian Orbit.

thrust acceleration decrease, the predicted symmetry becomes more noticeable.

Transfer to a Solar Sail "Halo" Orbit Halo orbits have recently been utilized using Solar radiation pressure that generates the required thrust power through large area, light structured, Solar sails. A recent application of such orbits was the Russia's space mirror deployed in 1993. Due to a wide variety of applications of halo orbits, such as real-time stereographic investigation of planetary surface, investigation of full three-dimensional structure of magnetotail, etc., the transfer to such orbits has considerable importance [13,14]. In this example the goal is to transfer a spacecraft from an initial circular Keplerian orbit with radius one to a parallel halo orbit h units away vertically. The linear velocity for the halo orbit is taken to be 0.816. The initial thrust acceleration and the exhaust gas velocity is taken as 0.2 and 1.5 respectively. The boundary conditions on the state variables are given as

$$\mathbf{x}_0 = \{1,0,0,0,1,0,0.2\}^T,$$

$$\mathbf{x}_f = \{x_{1f}, free, \varphi_f, 0, 0.816, 0, free\}^T.$$

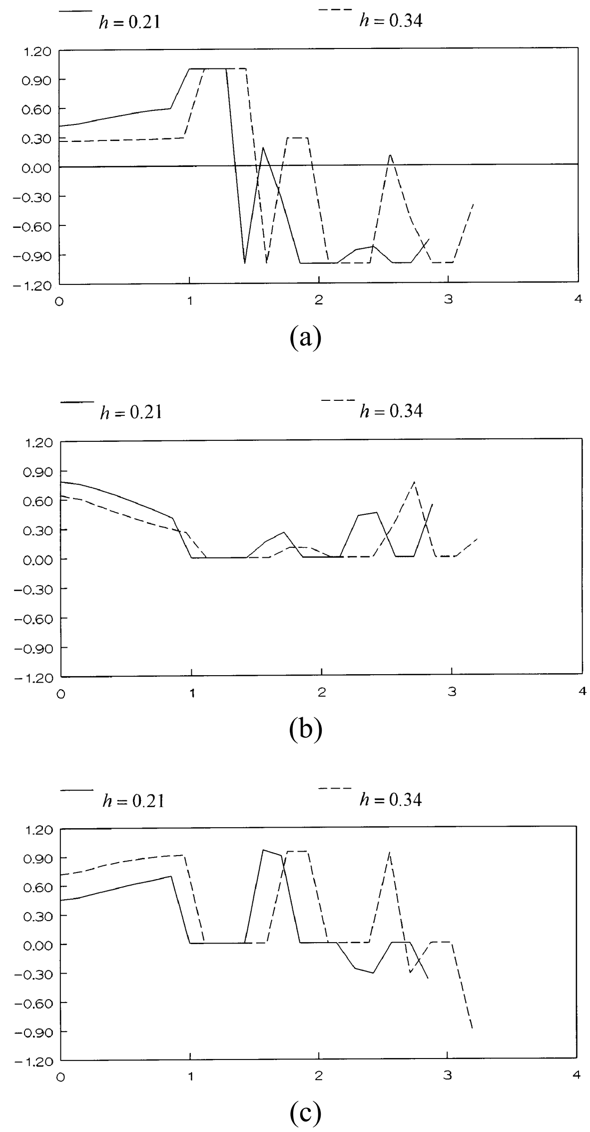

Figure 4. (a)u_r, (b) u_θ and (c) u_φ control history vs. scaled time.

The performance index is the total maneuvering time of the spacecraft. The final angle φ_f is computed from the following relation

$$\varphi_f = \arctan\left(\frac{h}{\sqrt{x_{1f}^2 - h^2}}\right).$$

The solution and discretization procedures are the same as those in the first example. This example is solved for two values of h, for which the control histories are shown in Figures 4. A three-dimensional transfer trajectory for $h = 0.34$ is shown in Figure 5. As it can be seen from the results, the control histories are totally different for the two chosen vertical displacements, where fluctuations in control components have been increased by increasing h. The initial thrust acceleration in such problems could not be arbitrary and its minimum allowable value for a meaningful solution is obtainable from the following relationship

$$x_7\big|_{t=t_0} > \frac{\mu}{x_{1f}^2}\sin\varphi_f .$$

After reaching the target orbit, the Solar sails become responsible for station keeping and the transfer maneuver is completed. The stability of halo orbits is of primary concern in their usage [13,14].

Two-Burn Transfer to a Non-Coplanar Orbit In this example, the problem of transfer between two non-coplanar non-intersecting circular orbits with large radii ratio is considered. The initial orbit is of radius one and the target orbit is of radius 4 (both in canonical units), where the two orbital planes make an inclination angle of 21.2°. Due to this large radii ratio of the two orbits, the maneuver phase consists of two MT arcs and one intermediate coast arc. The performance index Equation 5 represents that the total powered flight time is minimized. The constraints related to the intermediate coast arc are augmented to the performance index using Lagrange multipliers. Due to the special form of this proposed problem a well-chosen initial guess is needed to overcome the solution divergence. To alleviate this difficulty, an interior-point constraint is used that limits the solution space and thus makes it more predictable. With regard to Equation 16, the simple constraints on the two states x_3 and x_6 would be

$$Z_1(\mathbf{x}(T_1)) = x_3(T_1) = 0,$$

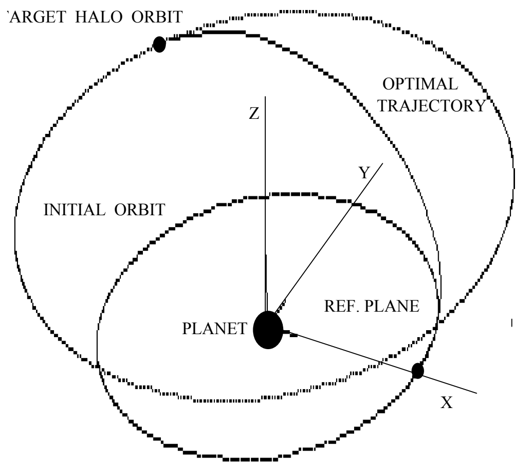

Figure 5. Three-Dimensional View of the Transfer to a Non-coplanar Halo Orbit.

$$Z_2(\mathbf{x}(T_1)) = x_6(T_1) = 0,$$

$$t_0^+ < T_1 < t_1^-,$$

where t_1^- is the moment when the first MT arc ends. With this form of the interior constraints, the spacecraft would tangentially enter the target orbit plane while it is in its first MT trajectory and continues the three dimensional maneuver most of which will remain within the target orbital plane. The discretized system equations are obtained using the trapezoidal rule. The first MT trajectory is subdivided into 30 equal intervals, while the last MT arc consists of 10 equal intervals. This makes an NLP problem with 579 variables and 579 constraints.

The solution has been generated for three values of initial thrust acceleration, and the resulting optimal control components have been shown in Figure 4. A three dimensional view of the entire trajectory has also been demonstrated in Figure 7. Due to the specified interior constraint, the control component u_φ approaches zero once the spacecraft enters into the final target plane. As realized from the results, lower initial thrust acceleration auses a reduction in the control

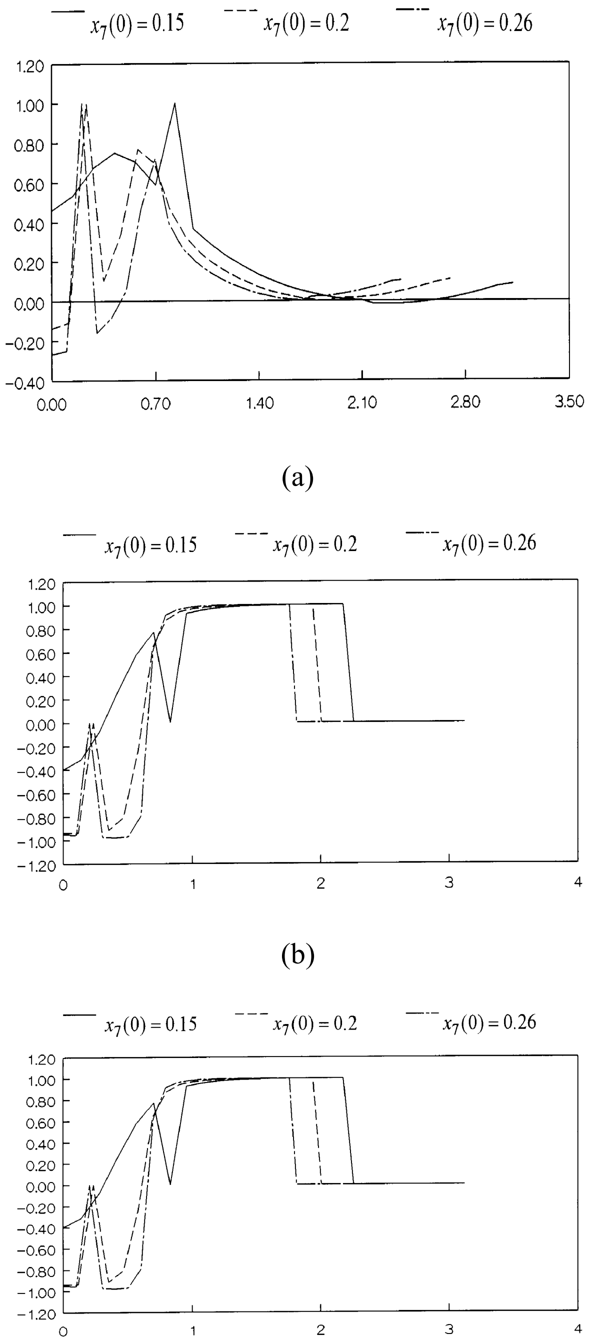

Figure 6. Control history vs. scaled time: (a) u_r, (b) u_θ.and (c) and u_φ.

fluctuations. Apart from a decrease in the controller effort, this is practically desirable, for it

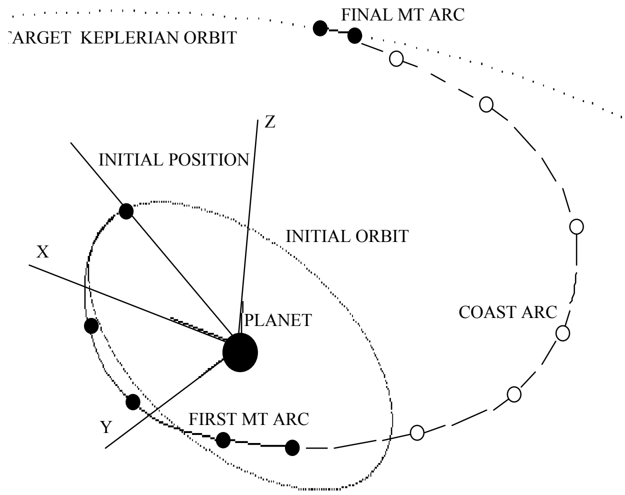

Figure 7. Three-Dimensional View of the Two-Burn Transfer to a Noncoplanar Keplerian Orbit.

increases the system robustness to the small variations in the system parameters (such as the inclination angle between two orbital planes).

CONCLUSION

Some problems in the optimal three-dimensional orbital transfers have been solved for thrust-limited spacecraft using a recently developed collocation and NLP techniques. The full nonlinear dynamical equations of motion for a point mass have been used and the integrals of free Keplerian motion in three dimensions have been utilized to determine the coast arcs. In order to better predict the final results, some interior-point constraints have been used which constrain the solution manifold and make the initial estimates more sensible. This method was successfully applied to the three dimensional transfer between non-coplanar Keplerian orbits and transfer to a Solar-sail "Halo" orbit from a Keplerian circular orbit. For the problem where the difference between the initial and target orbital radii is large, intermediate coast arc has been considered. This method appears to be suitable for any three dimensional transfer between non-coplanar and non-coaxial orbits.

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