A SIMPLE SPEED CONTROL TECHNIQUE AND PULSATING TORQUE ELIMINATION METHOD IN A BRUSHLESS DC MOTOR

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Abstract This paper presents a theoretical derivation and computer simulation of an optimal speed controller for a brushless dc motor using feedback from a linear model running in parallel with the inverter-fed model. The intent of the feedback from the linear model is to eliminate torque ripples from the inverter drive. A nonlinear model of such a motor, transformed into a linear model by a local dipheomorphism (defined in section 2) and a new model is introduced in order to eliminate the undesirable effects of the inverter harmonics.

Key Words Speed Control, Pulsating Torque, Brushless DC Motor

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INTRODUCTION

Rapid progress in microelectronic and power electronic devices have created a remarkable change in the control and applications of electrical motors. Permanent magnet synchronous motor (PMSM) drives progress have been due to the progress of power electronics. There are, of course, many advantages such as high efficiency and reliability in such motors. A high energy PMSM is ideal for a drive with a large torque, because of high efficiency and low cost. Using a rare earth permanent magnet material in PMSM leads to a large stator current with no demagnetization effect.

The present technological tendency is toward a full digital control of speed and position of brushless dc drives [1,2]. The advantages of microprossesor lie in computation ability and data storing capability which can be used for modification, and prediction of the feed-forward loops. A good dynamic performance and stability of PMSM can be acheived by a digital control technique.

In a low speed drive, torque pulsation may render difficult to control. In recent years more attention has been paid on eliminating the pulsating torque of drives. A method has been described in [3] in order to eliminate the harmonic torques of the motor design. Two methods have been proposed in [4]: the first is based on the state space and the second is based upon the theory of the variable structure systems.

This paper presents a speed control technique for a brushless dc motor which eliminates pulsating torque. The proposed technique is a simple and at the same time an efficient one. A nonlinear model of brushless dc motor is transformed into a linear model by a local dipheomorphism (defined in section 2) and state feed-back; then a voltage source inverter is added to the model. To eliminate the undesirable effects of the inverter harmonics, a new model is introduced. It will be realized that the torque ripples diminish and the speed approach steady state mode with optimum performance.

NONLINEAR MODEL OF STATE VARIABLES

A brushless dc motor (BDCM) consists of a PMSM itself, an inverter and a position sensor. Generally a BDCM is supplied by an ac source, its performance is similar to a shunt dc motor. A typical BDCM is shown in Figure 1. Three windings of the stator are arranged 120 degrees apart, each supplied from one phase of a three-phase supply. Interaction between the resulted rotating field and the rotor field developes electromagnetic torque on the shaft of the motor. The state equations of the BDCM are as follows [5,6]:

$dq_r/dt = w_r$

$dw_r/dt=(1/J)(P/2)[(3/2)(P/2)(\lambda_m i_{qs}^r+(L_d - L_q) i_{ds}^r i_{qs}^r) - B_m(2/p)w_r - T_1(t)]$

$di_{qs}^r/dt=[v_{qs}^r - r_s i_{qs}^r - L_d w_r i_{ds}^r - \lambda_m w_r]/L_q$

$di_{ds}^r/dt=[v_{ds}^r - r_s i_{ds}^r + L_q w_r i_{qs}^r]/L_d$ (1)

In order to transform the nonlinear equation, into exact linear equations, change of variables are required. The state equations are as follows:

$\dot{x}=f(x)+G(x)v$ (2)

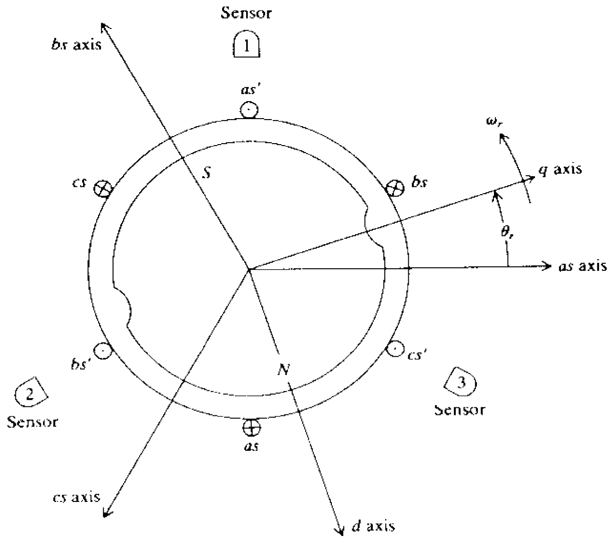

Figure 1. A typical BDCM.

To simplify it, the state variables are defined as x_1, x_2, x_3 and x_4, the inputs as u_1 and u_2 and coefficients as k_i:

$k_1=(1/J)(P/2)(3/2)(P/2)\ \lambda_m$

$k_2=(1/J)(P/2)(3/2)(P/2)(L_d - L_q)$

$k_3=(1/J)B_m$

$k_4=(1/J)(P/2)$

$k_5=r_s/L_q$

$k_6=L_d/L_q$

$k_7=\lambda_m/L_q$

$k_8=r_s/L_d$

$k_9=L_q/L_d$

$q_1=1/L_q$

$q_2=1/L_d$ (3)

The system of state equations will be:

$px_1=x_2$

$px_2=k_1x_3+k_2x_3x_4 - k_3x_2 - k_4T_1(t)$

$px_3=q_1u_1 - k_5x_3 - k_6x_2x_4 - k_7x_2$

$px_4=q_2u_2 - k_8x_4+k_9x_2x_3$ (4)

The exact linearization technique is now applied to the nonlinear state equations. There must be some sufficient and necessary conditions for transforming a nonlinear system into a linear system [7]. Before expressing the above mentioned conditions, some definitions from differential geometry are noted [8].

Definition 1: Let f and g be two vector fields on R^n, the Lie bracket of f and g is third vector field defined by:

[f,g]= êg.f - êf.g (5)

where êg and êf are Jacobian matrices of f and g.

Definition 2: A linearly independent set of vector fields $\{f_1,f_2,...,f_n\}$ is said to be involutive, if, there are scalar functions $a_{ijk}(x)$:R^nØR such that:

$[f_i,f_j]$= S $a_{ijk}(x)$ $f_k(x)$ "$_{ij}$ (6)

Definition 3: A function ä:R^nØR^n, defined in a region W, is called a dipheorphism if it is smooth and if its inverse $ä^{-1}$ exists and is smooth.

The following necessary and sufficient conditions must be satisfied for the existence of transformation [7,9,10]:

Theorem 1: The nonlinear system of Eq. 2 can be transformed into the following linear system:

$\dot{y}$=Ay+Bv (7)

where

$$A=\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&0\\0&1&0&0\end{bmatrix}\quad and\quad B=\begin{bmatrix}0&0\\0&0\\1&0\\0&1\end{bmatrix}$$

and the state variables x_1, x_2, x_3, x_4 lie in a sufficiently small open neighborhood U of the origin in R^4, if, and only if, the following conditions hold on U[10]:

a) The set C= $\{g_1,[f,g_1],(ad^2f,g_1),g_2\}$ has full rank.

b) The set $C_1=\{g_1,[f,g_1],g_2,[f,g_2]\}$ is involutive.

c) The span of C_1 is equal to the span of C_1ú C.

It can be proved that all three conditions of theorem 1 have been satisfied for BDCM. It is now possible to construct a non-unique transformation for the same variables using the second theorem.

Theorem 2: If the nonlinear system 2 is T-related to the system on U, then:

a) ÃT_j/Ãu_k=0, for j=1,2,3,4 and k=1,2.

b) the 2x2 matrix [ÃT_j/Ãu_k], j= 5,6 and k= 1,2 is nonsingular on U.

<dT_m, g_i>0, m=1,2 and i=1,2, where

<dT_m,g_i>= ÃT_m/Ãx_i - g_i

and <dT_m,f>=T_{m+1} , m=1,2

<dT_3,f+Gu>=T_5

<dT_4,f+Gu>=T_6

Solution of these equations leads to the vector field T, which can non-uniquely transform the nonlinear equations 4 into the linear equation 7:

$$T(x)=\begin{cases}x_1\\ x_2\\ \dot{x}_2\\ x_4\\ k_2x_3(q_2u_2-k_5x_4+k_9x_2x_3)+(k_1\\ \qquad +k_2x_3(q_1u_1-k_5x_3\\ -k_6x_2x_4-k_7x_2)-dT_1/dt-k_3x_2\\ -q_2u_2-k_8x_4+k_9x_2x_3\end{cases}\qquad(8)$$

The final linear equations of the motor are as follows:

$$\begin{aligned}\dot{y}_1&=y_2\\ \dot{y}_2&=y_3\\ \dot{y}_3&=v_1\\ \dot{y}_4&=v_2\end{aligned}\qquad(9)$$

The mathematics associated with the control theory may be left out with only final results (Eqs. 8 and 9) given as applicable to the motor. If this mathematical part of the paper is not interesting for some readers, they can leave it and keep in mind that Eqs. 8 and 9 are linearization of Eqs. 1.

SPEED CONTROL

The obtained system of linear equations 9 has four state variables and two inputs. To define the feedback loop gains, a 4x2 matrix must be determined. By poles placement, there are only four options closed-loop feedback gain matrix. Therefore a method must be chosen such that the eight elements of the matrix can be determined by a systematic and optimal technique. An optimal control theory method [11], which is applicable in linear systems, is used here in order to obtain the feedback gain. For such a system, a quadratic performance criterion in term of the states and inputs must be always minimized. Using this constraint, the feedback matrix can be uniquely defined. The performance criterion of the system is:

$$J = 0.5 \int_0^T (x' Qx + u' Ru) dt \quad (10)$$

where Q=diag $(q_1\ q_2\ q_3\ q_4)$ and R=diag $(r_1\ r_2)$. Using the elements of the two latter matrices, the locus of the poles of the closed-loop system can be varied. To control the speed, the largest weight is given to the first state variable. It means that the variations of the speed integral has been minimized, and the final response is achieved. The fourth variable of the matrix Q, weighted i_{ds}^r, is independent of remaining variables and its eigenvalue can be varied with any weight. Consequently the matrix Q is used to control the system:

$$Q=\text{diag}\ [10^7\ 1\ 1\ 10^3] \quad (11)$$

Matrix R is chosen as unit matrix. The reason is to limit inputs, in order to keep them in the reasonable and acceptable limits. Using these two matrices, the Riccate's matrix can be obtained for the optimal control.

$$-\dot{K}=KA+A'K+KBR^{-1}B'K+Q \quad (12)$$

From this matrix, the feedback gain matrix can be determined for optimal control of the system:

$$G= -R^{-1}B'K \quad (13)$$

TORQUE

Inverter-fed electrical motors contain various harmonics which produce ripple on the shaft of motor. Many have paid attention to eliminating this ripple. Here a new method is introduced for eliminating the ripple. As the following equation indicates:

$$T_e=J(2/P)p\ w_r+B_m(2/P)w_r+T_1 \quad (14)$$

The electromagnetic torque is the sum of angular acceleration, angular speed and load torque with constant coefficients. Performance put limits on the variations of the angular acceleration and speed of the motor and it is always required to minimize the range of these variations. Consequently the electromagnetic torque also has a limited variations.

SIMULATION RESULTS

Block diagram of Figure 2 is the base for simulation of the optimal speed control. The sampled speed and currents are multiplied by a gain G, obtained from the optimum control, and suitable inputs v_1 and v_2 of the system are determined to control the motor. As shown in Figure 2, coefficient x_1 in matrix Q, denoting the motor position, is chosen a large number which puts limit on the torque to be variations. By inverse transformation of this set of variables, the main variables of the motor control the speed.

In the block diagram the upper part is taken as a model without noise and the lower part as a real inverter-fed motor. The inner loop is the inherent feedback loop of the motor, the middle loop is the linearizing feedback loop and the outer loop is the speed control loop having gain G. Using such a system, the undesirable effects of the inverter harmonics upon the performance of the system are eliminated and an optimal system is obtained.

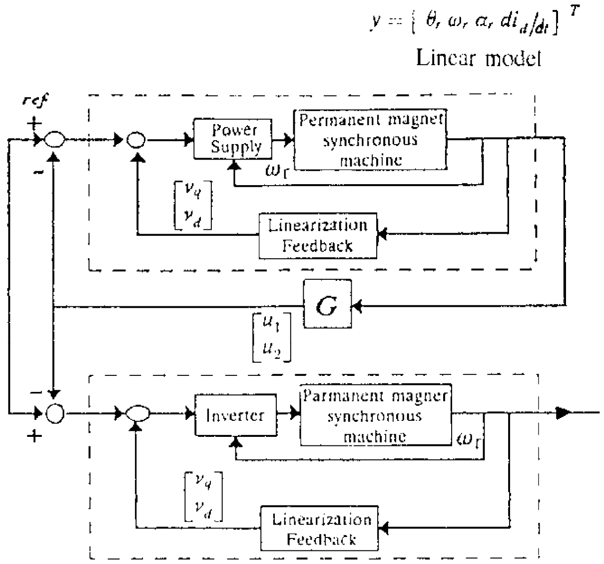

Figure 2. Block diagram of the optimal speed control of inverter-fed motor.

The BDCM, with the parameters given in Table 1, is simulated.

The three phase (as, bs and cs) output voltages waveforms of inverter for no-load condition have been presented in Figure 3. To control the speed, it is assumed that at steady-state mode of operation all state variables are zero. The aim is to obtain a speed equal to 160 rad/s. To achieve this the reference signal is set on the speed equal to 160 rad/s. The speed raises until a required level and a load torque equal to 0.14 Nm is then applied on the shaft of the motor. Applying the load motor reduces the speed up to the reference speed. Gain matrix, G, for the proposed motor is [12]:

$$G = \begin{bmatrix} 3194.9 & 453.67 & 38.23 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

For the sake of comparison, the output

TABLE 1. The Simulated BDCM Parameters.

R_s	L_a	L_b	L_{1s}	I_m	P	J	B_m
Ohm	mH	mH	mH	Wb	-	kgm^2	Nms/rad
3.34	7.33	0	1.10	0.0826	4	0.0001	0

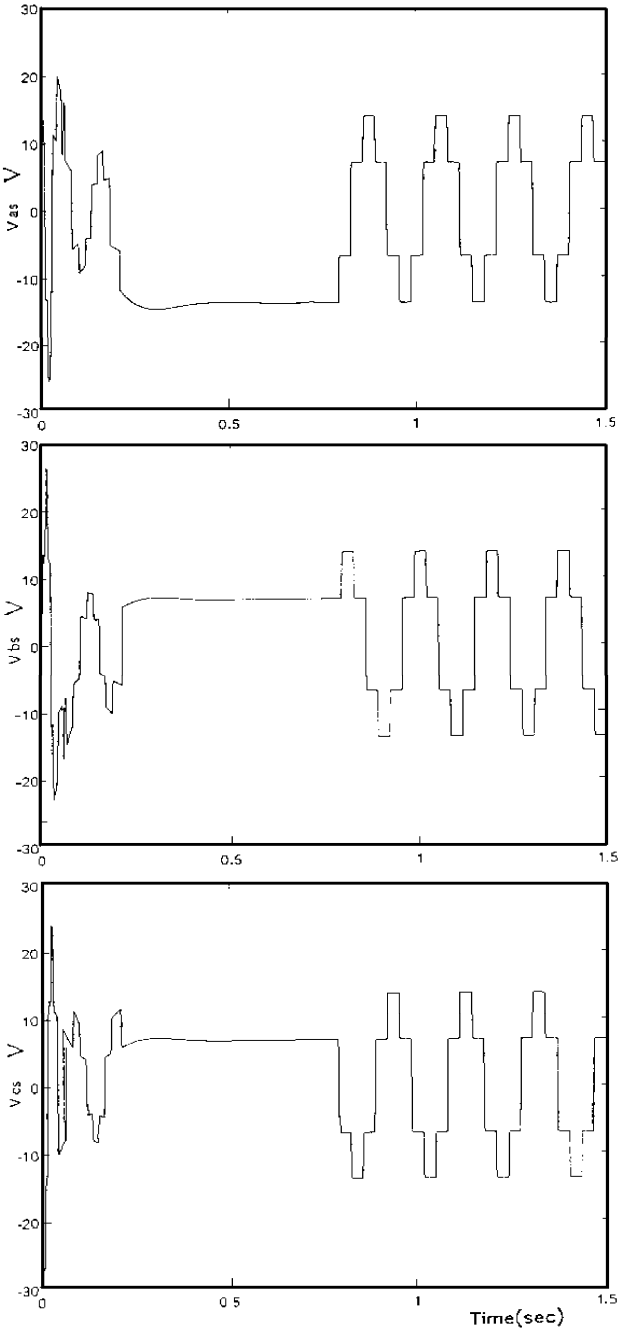

Figure 3. Output voltage waveforms of inverter for no-load condition.

characteristics of the open-loop system are also sketched on the same figures. It is clear that:

1. Applying a load torque on the open-loop system causes a large speed drop;
2. Variations of the torque amplitude and currents are larger than those of the similar

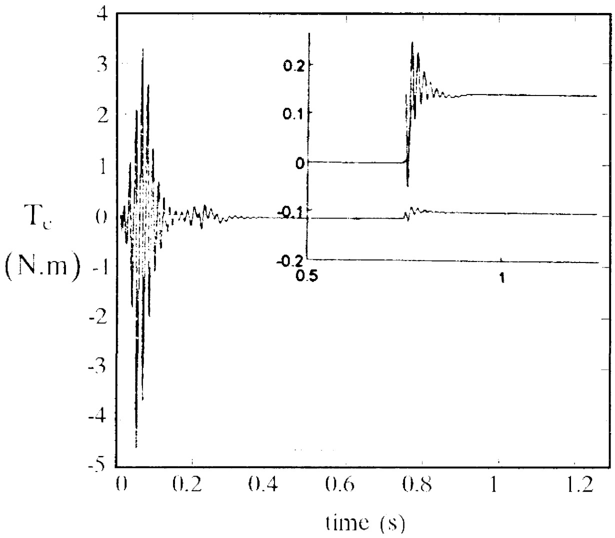

Figure 4. Torque/time characteristic for optimal speed control, including extended scale curve.

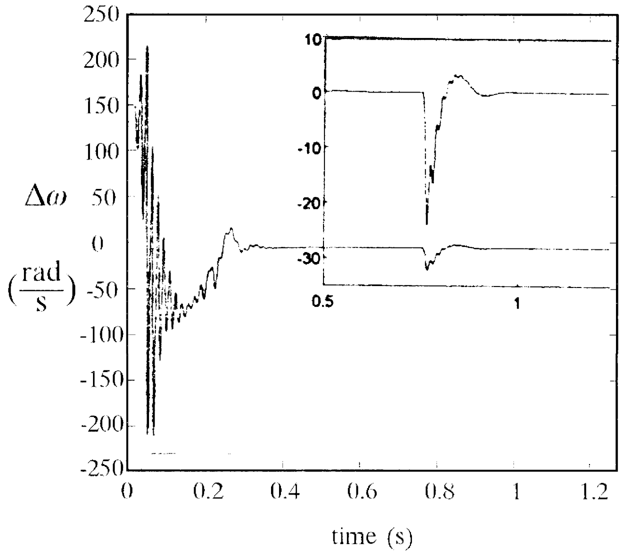

Figure 6. Speed error curve for optimal speed control, including extended scale curve.

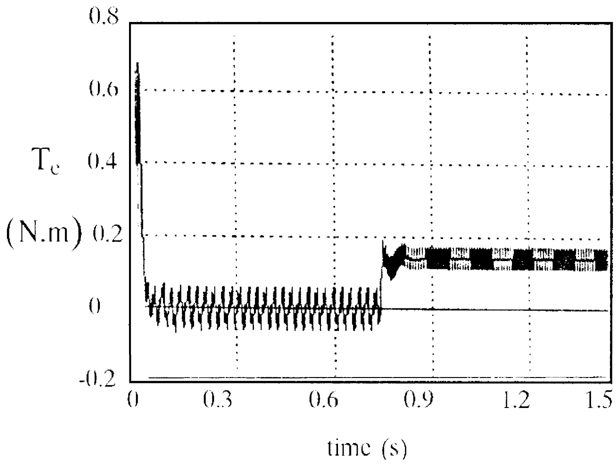

Figure 5. Torque/time characteristic for open-loop system.

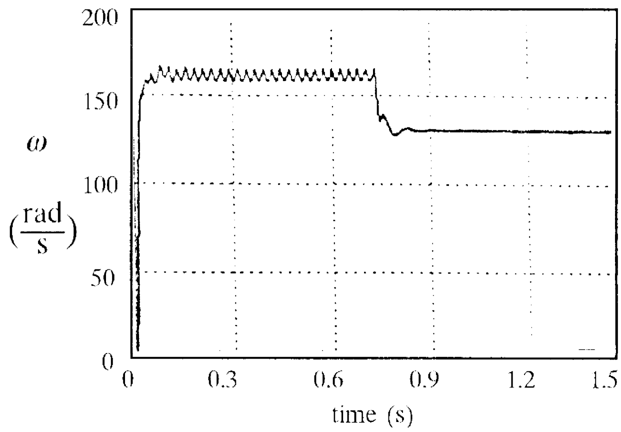

Figure 7. Speed/time characteristic for open-loop system.

variables in the closed-loop system.

Performance of the optimal system can be realized by comparing the two sets of the curves. Figure 4, electromagnetic torque/time for optimal control shows very large torque variation immediately after t= 0 up to t= 0.15 seconds, the extended scale figure is also shown in order to clarify this. Figure5, electromagnetic torque/time for open loop shows no such torque variation but a continuous torque ripple. This comparison between open- and closed-loop performance must be made between Figure 5 and the extended scale curve in Figure 4. It is evident that the controller offers overall operating improvement for starting and steady-state performance of the motor.

Figure 6 presents the speed error versus time. This error initially was 160 rad/s which must be diminished. In comparison with the speed waveform of the open-loop system, Figure 7, this curve approaches the final state slightly later, but there is no ripple. Variation of the currents due to the open-loop and closed-loop systems indicates the elimination of

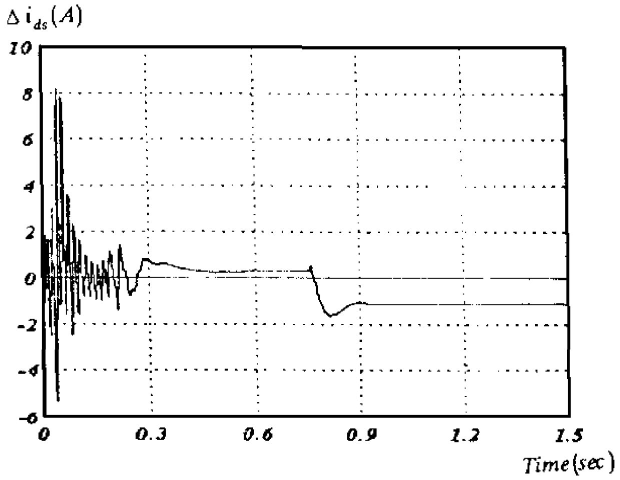

Figure 8. Variation of stator d-axis current for optimal control system.

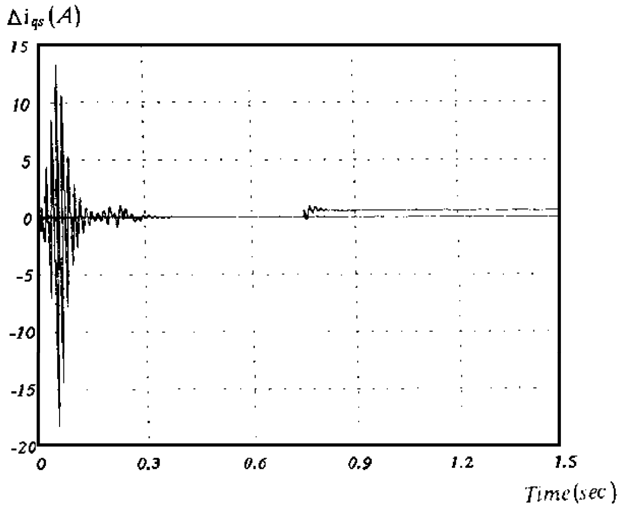

Figure 9. Variation of stator q-axis current for optimal control system.

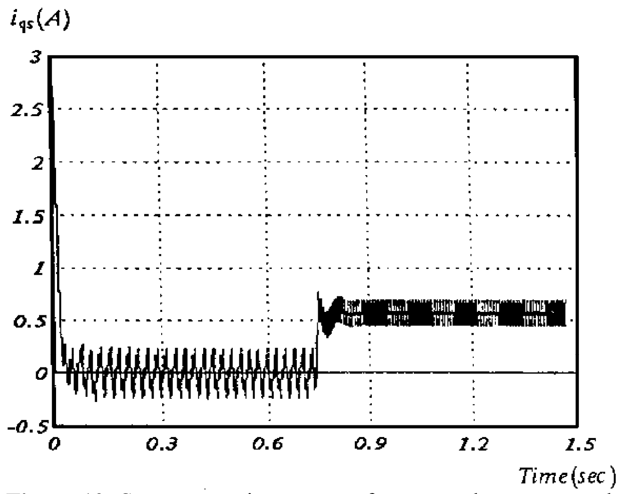

Figure 10. Stator q-axis current for open-loop control system.

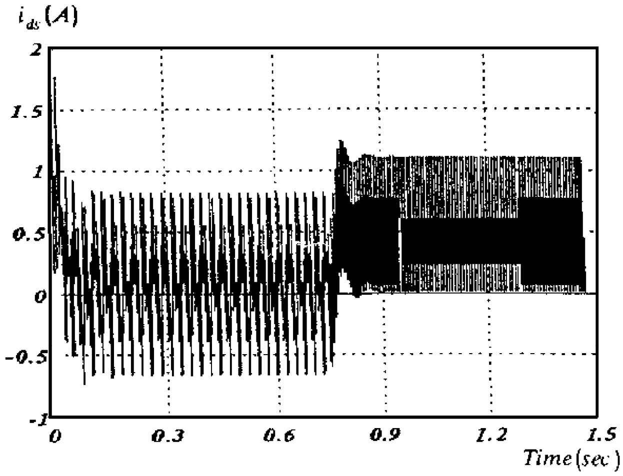

Figure 11. Stator d-axis current for open-loop control system.

the current ripple during the steady-state mode of operation which is a significant result obtained using this control system.

Figure 8-11 present the currents variations for optimal control and open-loop systems. Comparison of these figures indicates the advantage of the optimal closed-loop system from eliminating the ripple and approaching the required speed points of view.

Although it seems that a simplistic view of the torque ripple issues has been taken in BDCM, the obtained model can somehow follow a behaviour of the real machines. For instance such a model has been used in [10] and [13], and confirmed by the experimental results, to our best knowledge, eliminating the torque using an optimal control in BDCM has not been presented in the literature and comparison with others work is not possible.

CONCLUSIONS

The attempt has been made for an optimal control of a BDCM. The nonlinear system of the motor was transformed into a linear and controllable system using a local dipheomorphism and nonlinear control theory. The optimal control algorithm was then applied upon the linear system. To eliminate the effects of the inverter harmonics, a model without noise was used with an inverter-fed real motor model. This leads to the elimination of the harmonic effect on the system output. The ripple of the motor diminished and motor kept the constant speed.

ACKNOWLEDGEMENTS

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NOTATION

q_r	rotor angle
w_r	rotor angular speed
t	time
J	moment of inertia of motor
P	number of poles
$l\text{Å}_m$	magnetization flux-linkage
i_{qs}^r, ids^r	q and d axis current of stator
L_d	d - axis inductance
L_q	q - axis inductance
T_l	load torque
v_{qs} , v_{ds}	q and d axis voltage of stator respectively
r_s	stator resistance
B_m	damping coefficient
G	feedback gain matrix
T_e	electromagnetic torque
p	operator (d/dt)
L_{ls}	leakage inductance of stator

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