RESEARCH NOTE

OPTIMUM DESIGN OF INTERBASIN PIPE SYSTEM

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Abstract Pipe systems are often used to transfer moderate to medium water volumes between reservoirs or from well fields to consumers. Such systems consisting of an arbitrary number of reservoirs or other types of sources and sinks are considered here. The Lagrangean optimization principle is used for the determination of all pipe diameters giving minimum total cost of the system, when the individual discharges/recharges from/to reservoirs, elevations of the water surfaces or piezometric heads of all reservoirs, and the cost of pipes per unit length are known. Both the energy loss due to friction and the local energy loss due to gates, bends, etc. are taken into consideration. Turbulent flow in the rough flow range is assumed, i.e., the wall Reynolds number is in excess of seventy. The method may also be adapted to open channel systems, tunnel systems, or combined systems.

Key Words Optimum Design, Pipe System

چکیده انتقال آب بین مخازن ذخیره ای و به محل، ممکن است بوسیله لوله، کانال و یا تونل انجام گیرد. اجرای اینگونه پروژه ها باید با حداقل هزینه انجام پذیرد. دراین مقاله یک مدل ساده متشکل از چندین مخزن ذخیره کننده و تخلیه شونده با سطح ایستایی مشخص، درنظر گرفته شده است. آب از طریق لوله بین مخازن و به محل مصرف انتقال می یابد. براساس بهای واحد طول لوله از انواع مختلف و دبی ورودی و خروجی از هر مخزن، با بکارگیری اصول بهینه سازی لاگرانژ، قطر بهینه لوله های انتقال که هزینه را به حداقل ممکن کاهش می دهد، تعیین گردیده است. اتلاف انرژی ناشی از اصطکاک جدارهٔ لوله ها، دریچه های کنترل، زانوها و غیره مدنظر قرار گرفته است. جریان از نوع متلاطم بوده یعنی عدد رینولد از ۷۰ تجاوز می کند. چگونگی تعمیم مدل برای سیستم های کانال و تونل، پیشنهاد شده است.

INTRODUCTION

Christensen and Bush (1971) developed a computer program that determines the optimum sizes of tunnels transferring water from an arbitrary number of sub-reservoirs to a main reservoir, when discharges from the subreservoirs, the elevation of their water surface, the average construction cost per unit tunnel volume, and geometrical and hydraulic properties of the tunnels are known. The model solution involves the total cost of the tunnels as the objective function while energy equations involving the energy losses in the tunnels are chosen as constraints. The tunnel diameters are the decision variables.

In the following, the interbasin tunnel systems are replaced by pipes that essentially have different cost function. In the tunnel systems, the flow is strictly directed from subreservoirs to the main reservoir. In the pipe systems considered in this paper, the flow may be directed to or away from subreservoirs. While the earlier method for optimization of a system of tunnels is somewhat restricted to major projects transferring substantial water volumes between major drainage basins, the system consisting of commercially available pipes treated here is applicable to more systems handling moderate to medium water volumes. Figure 1 is an illustration of the pipe systems. Using an arbitrary surface as the datum, the elevation of the water surface in the main reservoir is denoted $h_0 > 0$ and the elevation of the water surface in reservoir No. i is denoted $h_i > 0$. The discharge leaving or reaching this reservoir is Q_i. Flows from the reservoirs are positive, while flows to the reservoirs are negative. This value is assumed to be constant or at least representing an average condition. The arbitrary pipe is numbered j and had the length l_j, hydraulic roughness k_j (Nikuradse's equivalent sand roughness) and is assumed to be circular. The unknown diameter of this pipe is d_j.

The objective function expresses the total cost of the system as a function of the pipe diameters using a cost function relating the cost (including installation) per unit pipe length c_j to the pipe diameter d_j. The form of this function may be found by a simple logarithmic regression analysis based on price lists available from pipe manufacturers, resulting in the relationship

$$c_j = a d_j^m \qquad (1)$$

where a and m are constants characteristic for pipe material, manufacturing process etc. In the previously analyzed tunnel systems m is of course equal to 2.

MATHEMATICAL FORMULATION

a substantial simplification of the analysis may be accomplished by using the energy losses in the individual pipes, rather than the pipe diameters as the decision variables. This is possible since a unique relationship exists for the relationship between diameter and energy loss in each pipe.

In order to establish the objective function in terms of the new decision variables (i.e., the energy losses), discharge and an expression relating energy loss to the pipe diameter must be known for all pipes.

DISCHARGES

From Figure 1 it is observable that the discharge in pipe No. j for this specific layout may be written as

$$Q_j = Q_{i=j/2} \qquad (2)$$

where j is even and

$$Q_j = \sum_{i=\frac{1+j}{2}}^{i=n} Q_i \qquad (3)$$

when j is odd. For other layouts of the pipe system similar expressions may be established.

ENERGY LOSSES

Turbulent flow in the rough flow range in any pipe may obey either of the following two formulas (Christensen, 1989; Davis and Sorensen, 1969):

$Q_j/A_j = m_j R_j^{2/3} S_j^{1/2}$; the conventional Manning formula (4)

with $M_j = 8.25\, g^{1/2}/K^{1/6}$ (or 1/n in SI-Systems or 1.49/n in English units, where n = Manning's n) when $R_j/K_j < 276$, or

$$Q_j/A_j = N_j R_j^{7/12} S_j^{1/2} \qquad (5)$$

with $N_j = 13.8 g^{1/2}/K_j^{1/2}$ when $R_j/K_j > 276$, i.e., for relatively large pipes with low roughnesses. In Equations 4 and 5, Q_j may be found from Equation 2 or 3 and A_j = cross-sectional area of the pipe, R_j is the hydraulic radius = A_j/P_j with P_j = wetted perimeter of the pipe, g is acceleration due to gravity, and $S_j = e_j/l_j$ in which e_j is the energy loss due to friction in the pipe j with length l_j. It is e_j that will be used as the decision variable representing pipe No. j.

Introducing the dimensionless shape factor,

$$a_j = d_j^{16/3} P_j^{4/3} / Aj^{10/3} \quad (6)$$

and expressing equations 4 and 5 in terms of e_j, the following equations result:

$$e_j = a_j Q_j^2 l_j / (M_j^2 d_j^{16/3}) \quad (7)$$

and

$$e_j = a_j Q_j^2 l_j / (4^{1/6} N_j^2 d_j^{31/6}) \quad (8)$$

for $R_j/K_j < 276$ and $R_j/K_j > 276$, respectively.
In pipes with circular cross-sectional area, $a_j = 10.29$. Only energy losses due to friction are included in Equations 7 and 8. In cases where local energy losses due to gates, bends, valves and so forth are of significance, these may be included by adding an equivalent length of the form

$$le_j = K_j^2 R_j^{2b} / 2g. \Sigma\zeta \quad (9)$$

where $K_j = M_j$ and b = 2/3 for $R_j/K_j < 276$, and $K_j = N_j$ and b = 7/12 for $R_j/K_j > 276$. $\Sigma\zeta$ represents the sum of the dimensionless local energy loss coefficients in pipe No. j.

Solving Equations 8 and 9 for d_j and including e_j we get

$$d_j = Z_j / e_j^{3/16} \quad (10)$$

and

$$d_j = Z_j / e_j^{6/31} \quad (11)$$

where in Equation 10

$$Z_j = [\, a_j (l_j + le_j)]^{3/16} Q_j^{3/8} / M_j^{3/8} \quad (12)$$

and in Equation 11

$$Z_j = [a_j (l_j + le_j)]^{6/31} Q_j^{3/31} / (4^{1/31} N_j^{3/31}) \quad (13)$$

It should be noted that le_j is a function of the pipe diameter d_j, but usually le_j is small compared to the physical length l_j of the pipe. Z_j, therefore, only varies slightly with d_j.

OBJECTIVE FUNCTION

The total cost of the pipe system C, may now be written as the sum of the cost of the individual pipes.

$$C = \sum_{j=1}^{j=2n} c_j l_j \quad (14)$$

or by using Equation 1

$$C = \sum_{j=1}^{j=2n} a l_j d_j^m \quad (15)$$

Introduction of Equations 10 and 11 to Equation 15 yield

$$C = \sum_{j=1}^{j=2n} a l_j (Z_j / e_j^{3/16})^m \quad (16)$$

and

$$C = \sum_{j=1}^{j=2n} a l_j (Z_j / e_j^{6/31})^m \quad (17)$$

respectively. Equations 16 and 17 are the non-linear forms of the objective function for the two flow regimes introduced by Equations 4 and 5, respectively.

CONSTRAINTS

With respect to the elevation of reservoirs and the

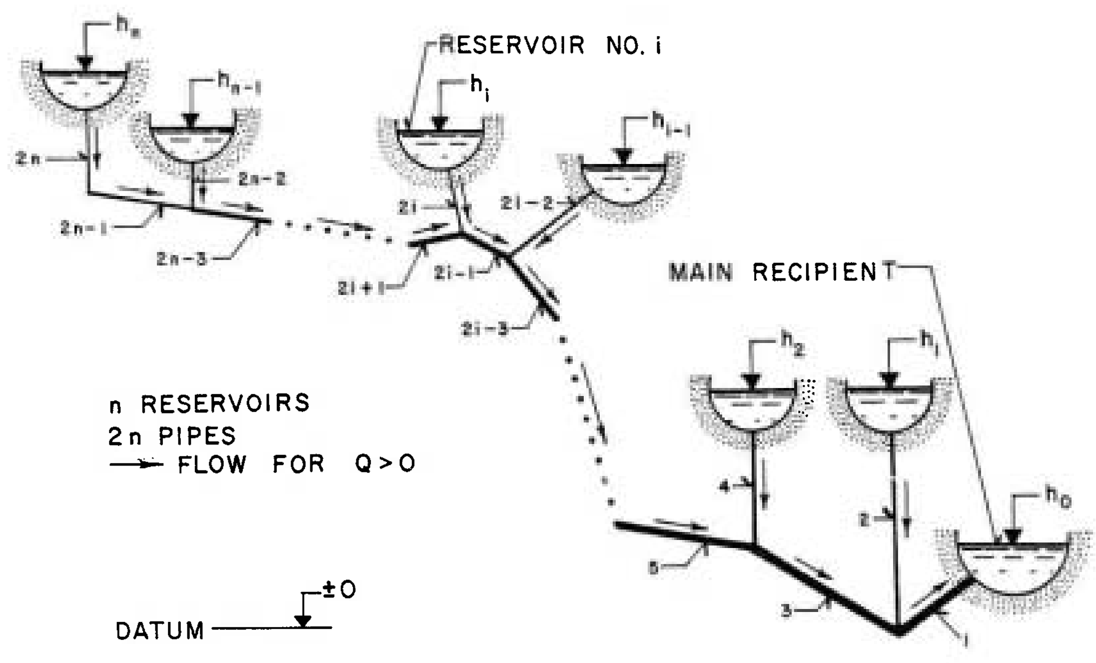

Figure 1. The Pipe System

direction of flow in the pipe systems shown in Figure 1, and assuming that any reservoir that contributes flow to the system has positive Q and any reservoir that gains flow from the system has negative Q, the following n constraints may be developed.

$$\phi_1 = \omega_2 e_2 + \omega_1 e_1 - h_1 + h_0 \quad =0 \quad (18.1)$$

$$\phi_2 = \omega_4 e_4 + \omega_3 e_3 + \omega_1 e_1 - h_2 + h_0 \quad =0 \quad (18.2)$$

$$\phi_3 = \omega_6 e_6 + \omega_5 e_5 + \omega_3 e_3 + \omega_1 e_1 - h_3 + h_0 \quad =0 \quad (18.3)$$

$$\vdots \qquad\qquad \vdots$$

$$\phi_{n-1} = \omega_{2n-2} e_{2n-2} + \omega_{2n-3} e_{2n-3} + \omega_{2n-5} e_{2n-5} + \ldots + \omega_5 e_5 + \omega_3 e_3 + \omega_1 e_1 - h_{n-1} + h_0 \quad =0 \quad (18.n-1)$$

$$\phi_n = \omega_{2n} e_{2n} + \omega_{2n-1} e_{2n-1} + \omega_{2n-3} e_{2n-3} + \omega_{2n-5} e_{2n-5} + \ldots + \omega_5 e_5 + \omega_3 e_3 + \omega_1 e_1 - h_n + h_0 \quad =0 \quad (18.n)$$

where ω_j (j = 1,2,... n) is equal to $|Q_j|/Q_j$ which accounts for the flow direction. As a result of the chosen numbering system, the pipe leaving reservoir No. n has been given two numbers (2n and 2n-1). Since these two numbers actually represent two different stretches of the same pipe, i.e., $d_{2n} = d_{2n-1}$ the following constraint must be included as well:

$$\phi_{n+1} = \omega_{2n} e_{2n} - \omega_{2n-1} e_{2n-1}\, l_{2n}/l_{2n-1} = 0 \quad (19)$$

Furthermore, all e_j - values must be positive in order to preserve the flow directions shown in Figure 1.

SOLUTION AND A NUMERICAL EXAMPLE

In order to find the set of positive e_j-values that satisfies the n + 1 constraints and minimizes the objective function, the Lagrangean optimization principle is used and programmed.

The system consisting of 8 subreservoirs, a main reservoir and 16 pipes is analyzed. The results of simulation are given in Table 1. It should be noted that 4 subreservoirs contribute water to the system

Table 1. Numerical Example of a Reinforced Concrete Pipe Systems with the Cost Function: $c = 89d^{1.62}$.

Subreservoir No.	Discharges (m^3/s)	Elevation from MSL(m)
1	0.6	122.0
2	-0.4	102.0
3	-0.5	105.0
4	-0.4	110.0
5	1.3	169.0
6	1.8	172.5
7	-0.4	122.0
8	1.5	195.0
Main Reservoir		100.0

Pipe No.	Length (m)	Discharge (m^3/s)	Optimum Diameter (m)
1	500	3.5	1.383
2	500	0.6	0.466
3	1000	2.9	1.307
4	100	-0.4	0.429
5	1000	3.3	1.360
6	1300	-0.5	0.666
7	1400	3.8	1.472
8	1500	-0.4	0.632
9	800	4.2	1.565
10	700	1.3	0.582
11	7000	2.9	1.404
12	2000	1.8	0.862
13	4000	1.1	1.049
14	1000	-0.4	0.414
15	10000	1.5	1.149
16	12000	1.5	1.149

and 4 subreservoirs gain water from the system. The objective function selected for the system is the function developed for the reinforced concrete pipes on the basis of 1992 prices in the USA markets.

CONCLUSION

A computer program has been developed that will determine the optimum diameters of pipes transferring water among an arbitrary number of subreservoirs and to a main reservoir when discharges/recharges from/to each individual subreservoir, the elevations of water surfaces in all reservoirs, and the hydraulic properties of pipes are known. The energy losses due to friction and the local energy losses due to gates, valves, etc., are selected as the decision variables which proved to simplify the analysis to a great extent. Introduction of the cost function in the objective function not only generalizes the analysis but has three advantages. First, if the optimum pipe diameters computed by the program for a particular brand of pipes are not in the range of pipes produced by the pipe manufacturers, commercially available larger pipes of different material with appropriate cost function can be considered. Second, the strength of pipes may be checked in regard to the hydraulic pressures in the system. If the pipe strengths are less than the hydraulic pressures, pipes of larger thickness (i.e., higher strength) can be suggested just by providing the computer program with a correct cost function. Finally, if the hydraulic pressure in the system and the calculated optimum pipe diameters exceed the strengths and the diameters of commercially available pipes, tunnel systems may be suggested as the last alternative conduits. The general model may also be applied in cases where transferring system consists of open channels or a combination of pipes, tunnels and open channels. In such cases again the correct and appropriate cost functions for different sections of the system should be provided for the model.

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