RESEARCH NOTE

ON-LINE IDENTIFICATION OF THE FIRST-MARKOV PARAMETER OF LINEAR MULTIVARIABLE PLANTS

A. Khaki-Sedigh

Department of Electrical Engineering
K. N. Toosi University of Technology
Tehran, Iran.

J. Kamali

Electric Power Research Center
Ministry of Energy
Tehran, Iran

Abstract In this paper three methods for on-line identification of first markov parameter of linear multivariable plants are presented. In these methods input-output data are used for the on-line identification of the first markov parameter.

Key Words Markov Parameter, On-line Identification, Multivariable Plants

چکیده دراین یادداشت، روشهای بلادرنگ شناسائی اولین پارامتر مارکوف سیستمهای چند متغیره، ارائه می گردند. دراین روشها، اطلاعات ورودی-خروجی جهت شناسائی اولین پارامتر مارکوف، بکار گرفته می شود.

INTRODUCTION

First markov parameter plays a key role in multivariable control systems analysis and design using high-gain and fast-sampling error-actuated controllers [1]. In designing multivariable adaptive controllers using such techniques, on-line identification of first markov parameter is of fundamental importance. In this paper three theorems are presented and proven which can be used effectively for on-line identification of first markov parameter of linear multivariable plants. Using on-line identification of first markov parameter of linear multivariable plants would circumvent the need for mathematical models of the plant either in transfer function or state space form in the design of such multivariable controllers. The plants are considered in continuous-time state space equations.

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

$$y(t) = Cx(t) \tag{2}$$

where $x(t) \subset R^n$, $u(t) \subset R^m$, $y(t) \subset R^m$ and the A, B and C are matrices of appropriate dimensions. The equations are also given in the discrete-time as:

$$x(k+1) = Gx(k) + Hu(k) \tag{3}$$

$$y(k) = Cx(k) \tag{4}$$

where

$$G = e^{AT} \quad (5)$$

$$H = \int_0^T e^{AT}\ B\ dt \quad (6)$$

The first markov parameter of the continuous-time system is CB and discrete-time system is CH.

The identification process used is Recursive Least Squares (RLS) [2]. The discrete-time behavior of the continuous-time system can be conveniently modelled by means of an autoregressive difference equation of the from [2]:

$$y_k + A_1 y_{k-1} + \ldots + A_N y_{k-N} = B_1 u_{k-1} + \ldots + B_N u_{k-N} \quad (7)$$

where the matrices $A_i \subset R^{m\times m}$ (i= 1, ... , N) and $B_i \subset R^{m\times m}$ (i= 1, ... , N) are the parameters of the Nth-order model. These parameters can be conveniently estimated using the RLS method [2] by implementing the parameter-estimation algorithm.

ON-LINE IDENTIFICATION OF FIRST MARKOV-PARAMETER

***Theorem* 1.** Consider the system described by state-space Equations 3 and 4 and the ARMA model given by Equation 7. Then the first markov parameter of the plant is the B_1 matrix in the ARMA model. That is

$$CH= B_1$$

Proof. For a unit impulse input $[1.0 \ldots 0]^T$ with zero initial condition Equations 3 and 4 give

$$y(1) = CBu(0) = \text{first column of CB.}$$

and also 7 gives

$$y(1)= B_1u(0)= \text{first column of } B_1.$$

continuing this process with unit impulse inputs on other channels gives the required result.

***Theorem* 2.** Consider the continuous-time system described by state-space Equations 1 and 2. Then in the limit $T\rightarrow 0$, where T is the sampling period, the first markov parameter of the plant is the B_1 matrix in the ARMA model given by Equation 7 divided by the sampling period T, that is

$$CB= B_1 / T$$

Proof. Equations (5) and (6) give

$$CH= C\int_0^T e^{AT}\ B\ dt = C\int_0^T (I + At + \ldots)\ Bdt$$

$$= CBT + 0(T^2)$$

and therefore, in the case $T\rightarrow 0$, we have

$$CB= B_1 / T$$

***Theorm* 3.** For a linear multivariable plant which can be expressed by a first order strictly proper multivariable system [3], the first markov parameter is given by $(T\rightarrow 0)$

$$CB= [[(I + A_1+A_2)^{-1} (B_1+B_2)]^{-1} - [(I+A_1e^{-T} +A_2e^{-2T})^{-1} (B_1e^{-T}+B_2e^{-2T})]^{-1}]^{-1}$$

where A_1, A_2, B_1 and B_2 are as in Equation 7 and T is the sampling period.

Proof. It can be shown that for a first order linear multivariable plant [3] represented by a first order strictly proper system

$$Ga(s)^{-1} = sC_0 + C_1$$

where $C_0 = (CB)^{-1}$ and $C_1 = Ga^{-1}(0)$; therefore,

$$(CB)^{-1} = Ga(1)^{-1} - Ga^{-1}(0)$$

It can also be shown that

$$Ga(1) = (I + A_1 + A_2)^{-1}(B_1 + B_2)$$
$$Ga(0) = (I + A_1 e^{-T} + A_2 e^{-2T})^{-1}(B_1 e^{-T} + B_2 e^{-2T})$$

and the result thus follows.

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