A SINGLE SUPPLIER PROCUREMENT MODEL WITH RANDOM YIELD

M. Modarres

Department of Industrial Engineering
Sharif University of Technology
Tehran, Iran

Abstract In this paper, we develop a procedure for selecting a supplier. Suppliers are characterized by their lead time, price and quality (random yield). Each purchased item is acceptable with a given probability and independent of the others. We assume the demands are deterministic with no set-up cost and backordering is allowed. For each supplier, an optimal ordering policy is developed. We prove the optimal policy is myopic if the value of items remains constant.

Key Words Supplier Selection, Random Yield, Production/Inventory, Dynamic Programming, Myopic Policy

چکیده دراین مقاله جهت انتخاب منبع تأمین قطعات موردنیاز مدلی ارائه می شود. قطعات از منابع مختلف قابل تأمین است. منابع مختلف ازنظر قیمت، مدت تحویل و کیفیت (درصد قطعات قابل قبول) از یکدیگر متمایز می شوند. هر قطعه ای که خریداری می شود، با احتمال مشخصی قابل قبول است و ضمناً این احتمال مستقل از وضعیت سایر قطعات است. فقط قطعات سالم قابل استفاده در تولید هستند و قبل از رسیدن و بازرسی امکان تشخیص سالم از معیوب وجود ندارد. تقاضا ثابت و غیر احتمالی فرض می شود، هزینه راه اندازی قابل صرفنظر کردن و تقاضای پس افت قابل قبول است. ابتدا برای هر منبع تأمین، سیاست سفارش بهینه را مشخص می کنیم. ثابت می شود که این سیاست از نوع «نزدیک بینی» (Myopic) است، به عبارت دیگر فقط بررسی اولین دوره کفایت می کند. سپس روش جهت انتخاب بهترین منبع ارائه می شود. هدف ازاین کار حرکت درجهت فلسفه JIT است که نیازها حتی الامکان از منبع یگانه تأمین میشود

INTRODUCTION

During the last decade, the concept of procurement management has evolved significantly. The main cause of this evolution is rooted in the popularity of the just-in-time philosophy. After the success of Japanese firms in international competition, their manufacturing system has been the focus of intensive study. To compete with them, many American and European manufacturers have implemented the principles of JIT.

Because of a heavy reliance on suppliers, purchasing is an important part of any manufacturing system. Although American original equipment manufacturers (OEM), fabricate more parts in-house than Japanese, fifty percent of the cost of goods sold by them goes on the procurement of raw materials, parts and components.[1]

In terms of the number of parts purchased by a typical OEM, the reliance is even heavier. Therefore, the role of supplier is very important, and two of the major issues of manufacturing strategy are ''how to select a supplier'' and ''the supplier's contract.''

In the Japanese system, the number of suppliers for each part is limited, and usually is only one. On the other hand, the number of deliveries made by a typical supplier is frequent, as much as one a day. A long-term steady relationship with suppliers is another important issue of that system.

For American firms, it is difficult or even impossible to

[1]About seventy percent of the cost of goods sold by a typical Japanese OEM is represented by purchasing, see [5].

adapt the Japanese principle of purchasing management completely, because of the difference in culture, business environments, historical and traditional relations between firms and even the existing laws. One difference is the location of the suppliers. In Japan, the suppliers have usually established their plant very close to their OEM, while the suppliers of American firms are scattered all over the country, and some of them are even located overseas. *Therefore, the American manufacturers are facing longer lead times*, and for them, safety stocks are more important. The second difference between the practice of purchasing management of these two systems is the number of suppliers for each part. Americans do not rely on a single supplier and usually purchase from multiple sources. In this way, they gain a better bargaining power to get more discounts or higher quality. By comparison, the Japanese have dealt with fewer suppliers, usually just one. By establishing a long-term relationship, a supplier is considered as part of the OEM's family. The supplier improves quality or reduces cost by cooperating and working closely with the OEM.

The main objective of this research is to develop a procedure for the selection of a single supplier with the lowest long-term cost. Although it is not obvious that outside of the Japanese business environment a single supplier always performs better than multiple ones, we restrict our attention to the case where the firm can select exactly one supplier and is willing to establish a long-term relationship with the supplier. Suppliers are characterized by their price, lead time and quality. Quality is measured in terms of the percentage of defective items in a supplied batch. After receiving an order, the items are inspected against certain standards, and some are labelled as defective. The defectiveness of each item is independent of the status of the other ones. The manufacturer can use only acceptable (non-defective) items and there is no way of knowing the quality of the items before receiving them. Sometimes, the suppliers cannot deliver all of the items that have been ordered by the due date. This may happen because of many uncertainties surrounding the manufacturing environment of the supplier. In this case, the undelivered parts are also considered as defective. One special case is when the parts are fabricated in-house, rather than ordering outside and clearly some parts are defective. A variety of reasons may be the cause of defectiveness [12].

Although the motivation for this research was to work on the strategic issue of selecting a supplier, it became clear that it was necessary to study the quality factor of a supplier first. Therefore, we have to deal with two different factors. The first one, an operational issue, is basically an inventory problem with an unreliable supplier (with random yield). Then the results can be used to select a supplier as well as to analyze the impact of supplier quality on production planning. As can be seen, the second issue is a strategic one.

In the first part of the paper, an ordering policy is developed for a single supplier when the quality of delivered items is not perfect. Although we handle this part independent of the strategic issue of selecting a supplier for the long-term relationship, our assumptions regarding the costs are based on the conditions of the main issue. In this part, by firstly applying stochastic analysis we develop a policy for a single period that depends on the number of on-hand and in-transit inventories. Then, by applying dynamic programming it is shown that for multiple periods as well as the infinite horizon problem, the optimal policy is a myopic one and it is the same as for the single period case. For this part dynamic programming is applied and the objective is to maintain the minimum average cost of operation.

In the second part of the paper, a procedure for the selection of a supplier among the different choices is developed. To compare the suppliers' long-term average costs, a Markov chain model is defined to determine the expected cost of the long-term operation over all different states. Then we show that under the optimal policy the state space is practically limited within a certain range. To obtain the limiting probabilities of different states, the resulting simultaneous linear equations are solved by a linear programming model.

Literature Survey

Although there has been a lot of work related to the first part of this research regarding random yield, not many researchers have workded on the second part of the problem. Gerchak *et al.* [4] explored properties of the solution structure of an inventory model with variable yield and uncertain demands. In their model, however, the lead time was zero and the planning horizon was finite. Henig and Gerchak [6] investigated the existence of some critical order points under similar assumptions. Only Ehrhardt and McClelland [3] consider positive lead times and come up with a heuristic procedure. Yano and Lee [15] review extensively the literature on lot-sizing with random yields.

Turning to the issue of supplier selection, a few researchers have looked into this matter, although from different angles from our own. The pros and cons concerning a single supplier versus multiple suppliers outside of the Japanese business environment, are discussed by Buffa [2] and Ouchi [9]. Tang [14] considers the impact of demand variation on the number of suppliers. Ahmadi and Tang [1] study dual provisioning. They develop a model for allocating production quantity among in-house and external suppliers. Lee and Zipkin [7] consider the make or buy decision. Moinzadeh and Nahmias [8] suggest a heuristic policy for an inventory model for two suppliers with different lead times. In their model, however, it is assumed the supplied items are all non-defective.

SINGLE SUPPLIER, SINGLE PERIOD

In this section, we consider a single period problem and develop an optimal policy to order from a single supplier with random yield. Subsequently, we will show that the same policy is also optimal in a dynamic environment. By a single period, we mean that we are concerned about the decision made in period one that has an impact on the cost of period $(T+1)$, where T denotes the lead time in terms of the number of periods, and $T \geq 1$.

Assumptions and Notation

We assume there exists only one supplier of a particular raw material, part or component of a product. Price and the lead time of the delivery are given. Demand is deterministic and constant. Each item delivered by the supplier is acceptable with a known probability. Backordering is allowed. The objective is to determine the size of order to minimize the total expected cost.

The following notation will be used:

d deterministic demand per period

h unit-holding cost per period, including the cost of capital

k unit-shortage cost per period

p probability that an item supplied is acceptable and $q= p$-1

c unit price of items delivered (defective or acceptable) so the unit value of an acceptable item is c/p

T lead time in terms of the number of periods

Y_n the total number of non-defective items of a batch of size n. Y_n is a binomial distributed random variable with parameters (p, n).

State of the System

At the beginning of the period, we define the state of the system as follows:

$$S \equiv (s, s_2, s_3, \dots, s_T) \tag{1}$$

where,

s the number of items on hand (positive or negative) after satisfying the demand of the first period

s_u the number of in-transit inventories expected to arrive at the beginning of period u (non-negative).

We also define,

- x — the size of the order
- i — the total number of items in-transit before the order, i.e., $i = \sum_{u=2}^{T} s_u$;
- I — the total number of items in-transit, after the order, i.e., $I = i + x$.

We assume, at any time during the planning period or at the end of it, the value of inventories remains unchanged. After the planning period, the extra item can be sold at the purchase price. That is, the expected value of any acceptable item on hand or the expected cost of any backordered item is (c/p). This assumption is justified by the fact that the objective of the model is to establish a long-term relation with a selected supplier of a particular part that will be needed for a long period of time. Therefore, it is not expected that particular items will be obsolete after this period.

Single Period Cost Function

The cost of any period consists of two components: the expected shortage and holding cost and the expected cost of depleted inventories.

a) The Expected Shortage and Holding Cost of a single period is represented by $L(S, x)$, and,

$$L(S, x) \equiv kW_s(S, x) + hW_h(S, x) \qquad (2)$$

where,

$W_s(S, x) \equiv$ the expected number of shortages at the end of period T, provided the state of the system is S at time zero and the size of the order at that time is x,
$W_h(S, x) \equiv$ the expected number of items exceeding the demand at the end of period T, provided the state of the system is S at time zero and the size of the order at that time is x.

By conditioning on the number of the acceptable items received out of $i + x$ in-transit items, (i.e., $Y_{(i+x)}$,) it can be shown, that

$$W_s(S, x) = \sum_{j=0}^{Td-s-1} (Td - s - j) P[Y_{(i+x)} = j] \qquad (3)$$

and,

$$W_h(S, x) = \sum_{j=Td-s}^{i+x} (j - Td + s) P[Y_{(i+x)} = j] \qquad (4)$$

Therefore, $(Td - s)$ indicates the upper bound of the number of items needed to satisfy the demand of the next T periods provided every in-transit item will be defective.

As can be seen, $L(S, x)$ depends only on s and $I = i + x$ and is independent of the individual component of $(s_2, s_3, \ldots, s_T, x)$. Therefore, we express the expected shortage and holding cost of a single period in terms of these two quantities only, represented by $l(s, I)$, i.e.,

$$l(s, I) \equiv L(S, x).$$

b) The Expected Cost of Depeleted Inventory

The price paid for the ordered items is not considered as a part of cost. The reason is the assumption made before, that the value of items on-hand or in-transit remains unchanged, even after the planning period. Therefore, when cx is paid for the price of an order of size x, then simultaneously, the value of the inventory will be increased by the same amount and the total asset remains unchanged.

On the other hand, during any period, d units of inventory are consumed, so the expected cost of the depleted inventory is cd/p.

Therefore, the single period cost function will be:

$$C(S, x) = \frac{cd}{p} + L(S, x) \qquad (5)$$

Since the first term is constant, minimizing the cost function is the same as minimizing the second term.

Optimal Policy

To determine an optimal policy, the following lemma and theorems are used:

Lemma 1: *$l(s, I)$ is a convex function of I, for every fixed s and so is the cost function $C(S, x)$.*
Proof: To show $l(s, I)$ is strictly convex, we prove $\Delta_I L = l(s, I+1) - l(s, I)$ is an increasing function of I. This can be done by conditioning on the outcome of the last item. If it is defective, with a probability of $1-p$, then $\Delta_I L = 0$. However, if the last item is acceptable, then the inventory level increases by one unit at the end of period T. In this case, we have to pay some extra holding cost for one unit if the inventory is positive and save the shortage cost for one unit if the inventory is negative. Probabilities of positive and negative inventories are $P(Y_I \geq Td - s)$ and $P(Y_I \leq Td - s - 1)$, respectively. Therefore, it can be shown that,

$$\begin{aligned} \Delta_I L &= hpP(Y_I \geq Td - s) - kpP(Y_I \leq Td - s - 1) \\ &= hp - (k+h)pP(Y_I \leq Td - s - 1) \end{aligned} \quad (6)$$

Therefore,

$$\begin{aligned} \Delta_{(I+1)} L - \Delta_I L = (k+h)p\,[P(Y_{I+1} \geq Td - s) \\ - P(Y_I \geq Td - s)] \end{aligned} \quad (7)$$

To complete the proof, we have to show that the right-hand side of the above equation is positive. This is true, because

$$P(Y_{I+1} \leq Td\text{-}s) - P(Y_I \geq Td\text{-}s) = pP(Y_I = a-1) \quad (8)$$

Convexity of the cost function is obvious when the convexity of $l(s, I)$ is given.

Theorem 1: *At the beginning of the period, if the number of on-hand inventory is s, then the optimal number of intransit inventory will be $I^*(s)$ which can be determined by the following two equations:*

$$P(Y_{I^*(s)} \geq Td - s) \geq \frac{k}{k+h} \quad (9)$$

$$P(Y_{I^*(s)} \geq Td - s + 1) \leq \frac{k}{k+h} \quad (10)$$

(This is the Newsboy result, so the proof is omitted).

As can be seen from the above theorem, the optimal policy is of the "Up to Level" type. "The Level", up to which an order should be placed, however, is not the shortage level, but it is calculated from (9) and (10). Therefore, if the number of in-transit inventroy is less than $I^*(s)$, order up to this level, i.e. $x = I^*(s) - i$. In this case, $I = I^*(s)$. Otherwise do not order, i.e. $x = 0$, so $I > I^*(s)$.
The existence of an optimal policy is guaranteed by the following theorem:

Theorem 2: *For any positive inventory on hand or any finite backorder, the optimal quantity of order size is finite.*
Proof: Suppose for a given s, no finite optimal I^* exists. This means, the optimal order size of x^* goes to infinity. In other words, for every order of size x, we have $I = i + x < I^*$ and the following relation always holds,

$$l(s, I+1) - l(s, I) < 0,$$

or from (6), the following inequality, for every I, always holds,

$$P(Y_I \leq a-1) > \frac{h}{k+h} \quad (11)$$

However, as I increases, the above inequality eventually does not hold because the left-hand side decreases sharply. For example, if $I > a/p$, then, for any integer $j > 0$,

$$[P(Y_{(I+j)} \leq Td\text{-}s) < (1-p)^j P(Y_I \leq Td\text{-}s)]$$

Then, as j increases the left-hand side of (11) will be less than the right side.

The Optimal Shortage and Holding Cost of One Period

The optimal shortage and holding cost of one period is expressed as $L^*(S)$, or $l(s,i)$, where

$$L^*(S) \equiv \min_x L(S, x) \equiv L(S, x^*)$$

or,

$$l^*(s,i) \equiv \min_I, l(s, I) \equiv l(s,i+x^*)$$

Definition 1: At the beginning of any period, we say the state of the system is an *ordering* state if, following the optimal policy, an order must be placed, i.e., if $i<I^*(s)$. Otherwise it is a *non-ordering* state.

Theorem 3: *At the beginning of any period, if the number of on-hand and in-transit inventories are s and i, respectively, then the optimal shortage and holding cost of a single period has the following properties:*
(a) For any "ordering" state, i.e. $i \le I^*$,

$$l^*(s,i) = l^*(s, I) \tag{12}$$

(b) For any ''non-ordering'' state, $l^*(s,i)$ is an increasing function of i.
Proof: Part(a) results from the definition of ordering state and optimality of I^*. Part(b) follows from the definition of non-ordering states and (6) and (9).

Theorem 4: *For every s, $l^*(s,i)$ is a convex function of i.*
Proof: To show $l^*(s,i)$ is convex, we show $l^*(s,i+1)-l^*(s,i)$ is a non-decreasing function of i, or the following term is non-negative,
for every s and i.

$$\Delta_i^2 = [l^*(s,i+2)-l^*(s,i+1)] - [l^*(s,i+1)-l^*(s,i)].$$

Suppose, for a particular s, I^* is the optimal number of items in-transit, Three possible case are considered:
a. $i \le I^*-2$, *then, by (12),*

$$l^*(s,i+2)= l^*(s,i+1)= l^*(s,i)= l^*(s, I^*)$$
so, $\Delta_i^2=0$.

b. $i= I^*-1$, then, by (12) and part (b) of theorem 3,
$$l^*(s,i+2) > l^*(s,i+1)= l^*(s,i)= l^*(s,I^*)$$

so $\Delta_i^2>0$.

c. $i \ge I^*$. Then by definition of non-ordering states, $l^*(s,i)= l(s,i)$. Therefore, it is convex by lemma 1.

SINGLE SUPPLIER, MULTIPLE-PERIOD

In this section, we will show the single-supplier, single-period policy is also optimal for finite as well as infinite horizons, when there is still only one supplier. Therefore, the optimal policy is a myopic one. First, we consider the multiple-period finite horizon and then the infinite horizon problem.

Markov Decision Process

To show the myopic policy is optimal, we set up the problem as a Markov Decision Process. The state of the system is S, as defined by (1). However, as was shown, S can be replaced by a vector of two dimensional (s,i), where, s and i are the number of on-hand and in-transit inventories (before the order), respectively. The cost of each period is $\frac{cd}{p} + L(S, x)$ and depends on S as well as on the decision variable x. As was mentioned before, $L(S, x)$ can be replaced by $l(s, I)$ and it is independent of the period number. It is evident the system has Markovian properties (see [11]). Let's define.
$V_n^*(s,i) \equiv$ the optimal expected total cost periods n through 1, if the state of the system is (s,i).
Then, for any $n \ge 1$.

$$V_n^*(s,i) = \frac{cd}{p} + \min_{I \ge i} \{ +l(s, I) + \sum_{j=0}^{S_2} P(Y_{S_2}=j)\, V_{n-1}^*(s+j-d, I-s_2)\} \tag{13}$$

where, I represents the number of in-transit inventory after the order is placed, S_2 has been defined by (1) and $V_0^*(s,i)= 0$, for all s and i.

Since the constant part of the cost does not have any impact on the decisions, it can be discarded in the model. Therefore, we defined,
$U_n^*(s,i) \equiv$ the optimal expected total shortage and handling

cost of periods n through 1, if the state of the system is (s,i) Clearly,

$$U_n^*(s,i)= \min_{I\geq i} \{l(s,I) + \sum_{j=0}^{S_2} P(Y_{S_2} =j)\, U_{n-1}^* (s+j-d,\, I-s_2)\} \tag{14}$$

It can be shown,

$$V_n^* (s,i)= U_n^* (s,i) + n\frac{cd}{p} \tag{15}$$

From this point on, we will be dealing with $U_n^* (s,i)$ rather than $V_n^* (s,i)$.

Theorem 5: *For every s, and n,*
(1) There exists an $I_n^*(s)$ that is the optimal number of in-transit inventory.
(2) $U_n^* (s,i)$ is a convex function of i,

Proof: The proof is by induction on n. For (n=1), part(1) follows from theorem 1 and part (2) from convexity of $l^*(s,i)$ in theorem 4.

To prove the theorem holds for n, assume it holds for n-1. Since both terms of (14) are convex functions of the number of in-transit inventory, then there must be an optimal number for this quantity, which we denote by $I_n^*(s)$. (Since $U_{n-1}^* (s,I)$ is a convex function of I, then the second term of (14) is also a convex function of I, because it is a convex combination of some convex functions.)

Now, assume the state of the system is (s,i). If $i<I_n^*(s)$, then the order size is $I_n^*(s)-i$ and $U_n^* (s,i) = U_n^* (s, I^*)$. However, if $i \geq I_n^*(s)$, then no order is placed and because of the convexity property, $U^*(s,i)$ is an increasing function of i.

Very similar to the proof of theorem (4), it can be shown that $U^*(s,i)$ is a convex function of i.

Theorem 6: *For any positive inventory on-hand or any finite backorder, the optimal quantity of order size is finite.*

Proof: Similar to the proof of theorem 2.

Now, we want to show the optimal policy is a myopic one. In other words, $I_n^*=I^*$, for every n. To do so, we take the following steps:

Suppose the state of the system at time zero is (s,i), the size of the first order is x and the order size of uth period is x_u, for $u\geq 2$; then the expected shortage and holding cost of the uth period can be calculated as follows:

$$k\sum_{j=0}^{a_u-1} (a_u-j)\, P(Y_{I_u}=j)+h \sum_{j=Td-s}^{I_u} (j-Td-s)\, P(Y_{I_u}=j) \tag{16}$$

where,

I_u is in-transit inventory for the uth period, on based on the information available at the beginning of the first period. i.e. $I_u=i + \sum_{j=1}^{u} x_j$, and,

$$a_u \equiv (T+u-1)\, d-s.$$

As can be seen again, this cost depends on s and I_u. Therefore, we define:

$l_u(s, I_u) \equiv$ the expected shortage and holding cost of the uth period, provided the number of on-hand inventory at the beginning of the first period is s and the total number of in-transit inventory up to the uth period is I_u.

Now it can be shown that the following relation holds:

$$U_n^X (s,i)= \sum_{u=0}^{n} l_u(s, I_u) \tag{17}$$

where,

$U_n^X (s,i)$= the total expected shortage and holding cost of the next n periods, if the policy $X = (x_1, x_2, \dots, x_n)$ is adopted, and x_u is the order size of the uth period and the state of the system at the beginning of the first period is (s,i).

Lemma 2: *Consider two policies $X = (x_1, \dots, x_u)$ and $Y= (y_1, \dots, y_u)$. The expected shortage and holding cost of the uth period of both policies is the same, if*

$$\sum_{j=0}^{u} x_j = \sum_{j=0}^{u} y_j$$

Proof: The cost of each policy for any particular period can be calculated from (16). For both policies I_u is the same, by definition. Similarly, a_u is the same for both policies.

Theorem 7: *The optimal policy is myopic*

Proof: The optimal policy of the single period is optimal for multiple-period cases. Suppose the state of the system can be represented by (s,i) and the optimal order quanity for single period is x^*. We will show in a multiple-period case this optimal order quantity of the first period is still x^*. If not, let the optimal policy be $\bar{Y} = (y_1, y_2, y_3, \ldots)$ and $y_1 \neq x^*$. In this case, we show at least one other policy exists which has a lower cost. Take a policy in which the decision variables are $\bar{X}\ (x^*, x_2, x_3, \ldots)$, where,

$$x_2 = y_2 + (y_1 - x^*) \text{ and } x_j = y_j, \text{ for } j \geq 3.$$

Then, following $\bar{Y}$ policy, we have,

$$U_n^{\bar{Y}}(s,i) = l(s,i+y_1) + \sum_{j=2}^{n} l_j(s, I_j) \tag{18}$$

However, following the second policy, the same cost will be,

$$U_n^{\bar{X}}(s,i) = l(s,i+x^*) + \sum_{j=2}^{n} l_j(s, I_j) \tag{19}$$

From optimality of x^* for a single period, it follows that $l(s,i+x^*) \leq l(s,i+y_1)$. The other corresponding terms of both policies are equal from lemma 2. Therefore, this contradicts the optimality of $\bar{Y}$ policy.

Convergence of the Average Cost

So far, we have shown the proposed myopic policy is optimal for finite horizon. Clearly, it is also optimal for the average cost of one period. To prove the policy is optimal for infinite horizon, we have to show the average cost of one period, or mathematically speaking, that the following limit exists:

$$\lim_{n\to\infty} \frac{\Sigma_{u=1}^{n} l_u^*(s,i)}{n} = L^* \tag{20}$$

Since the corresponding model Markov chain is ergodic, then the system will reach steady state and for every state of it, the limiting probability exists.

Let, $\pi(s, i)$ represent the limiting probability that the system will be in the (s, i) state, if the proposed myopic policy is adopted. Then the average shortage and holding cost of each period will be,

$$L^* = \sum_{s,i} \pi(s,i)\, l^*(s,i) \tag{21}$$

and since, by theorem (7) the average cost of any of the myopic policy is less than any other policies and the limit for that average exists, then this myopic policy is also optimal for infinite horizon.

MULTIPLE SUPPLIERS

In the previous sections, we developed an ordering policy from a single supplier with the random yield. In this section, we determine the strategic issue of how to select a supplier among N possible ones.

In the case of a single supplier, for each state of the system a separate decision is made. However, to compare the expected long-term costs of the suppliers, one should consider the average cost of all possible states. Therefore, in this section we show how to calculate the expected average cost for each supplier.

The Policy

If the *lead time*, *yield rate* and *price* of supplier i ($i = 1,2, \ldots, N$), are T_i, p_i, and c_i^1, respectively, then, from (5) and (21), the average cost of each supplier per period is as follows:

Total Average Cost of One Period

$$= \frac{c_i d_i}{p_i} + L_i^* \tag{22}$$

Calculation of The Average Cost, L_i^*

To calculate, the average shortage and holding cost of one period, from (21), the limiting probabilities as well as the optimal single-period cost of each state are needed.

[2] considering the discounts offered after establishing a long-term relation.

To see how to calculate the limiting probabilities of an ergodic Markov chain refer to Reference [10] for example. However, since in this model the transition probabilities of successive periods cannot be determined easily, we define a new Markov chain, in which the stages 0,1,2, ... are represented by periods *0,T,2T, ...* Therefore, $p_{s,s'}$ we mean the probability of transition from S to S' within T periods.

In this model, the number of states is not finite, However, we show that under the optiaml policy, with a very high probability (close to one), the number of states that the system visits is limited. With any desired probability, there exists a lower bound and an upper bound for the number of on-hand inventory, denoted by $\underline{s}$ and $\bar{s}$. Furthermore, since that upper bound is below (Td), we do not consider the non-ordering states. Then, considering only the ordering states, every state can be expressed by its first component of the state vector, as discussed before. In order to manipulate the expressions easily we apply the normal approximation, rather than using the binomal distribution directly.

Lemma 3: Let $\delta \equiv W_h(s, I^*(s)) - W_s(s, I^*(s))$, *then,*

a: $\delta = pI^*(s) - Td + s$.

b: *If* $Y_{I^*(s)}$ *is approximated by a normal distribution, then,*

$$[\Phi^{-1}(\frac{k}{k+h})]\sqrt{qpI^*(s)} \leq \delta \leq [\Phi^{-1}(\frac{k}{k+h})]\sqrt{qpI^*(s)} + 1$$

where, $\Phi(.)$ represents *the probability of standard normal distribution.*

c: If $k > h$ then, $pI^* \geq Td - s$.

Proof: Part(a) can be obtained from (3) and (4). Applying normal approximation for $Y_{I^*(s)}$ in (9) and (10) results in part. (b). If $k>h$, then, $k/(k+h) > .5$ and $\Phi^{-1}(\frac{k}{k+h}) \geq 0$ and part (c) holds.

Definition 2: For a desired probability of α, we define a lower bound and an upper bound for the number of on-hand inventory as follows:

$$\underline{s} \equiv max\ \{s \in I: pI^*(s) - Td \geq m\sqrt{qpI^*(s)}\} \qquad (23)$$

and

$$\bar{s} \equiv [m + \Phi^{-1}(\frac{k}{k+h})]\sqrt{qpI^*(\underline{s})} \qquad (24)$$

where $\Phi(m) = \alpha$,

Theorem 8: *If at the beginning of a period the number of on-hand inventory is s, then after T periods, with probability of α, the number of on-hand inventory, say s′, has the following property*

1) $s' \geq \underline{s}$,   *if* $s \geq \underline{s}$

2) $s' \geq s$,   *if* $s \leq \underline{s}$

Proof: By applying normal approximation for $Y_{I^*(s)}$ and considering lemma 3, it can be shown that with probability of α the lower bound of the number of on-hand inventory of the next stage is

$$[m - \Phi^{-1}(\frac{k}{k+h})]\sqrt{qpI^*(s)}\ ]$$

Since $I^*(s)$ is a decreasing function of s, the lower bound of the on-hand inventory of the next stage is an increasing function of s. Therefore, for any $s \geq \underline{s}$, this lower bound is at least as high as $\underline{s}$, by definition (2).

To prove part (2), we have to show $P[Y_{I^*(s)} \geq Td] \geq \alpha$, or

$$\frac{pI^*(s) - Td}{\sqrt{qpI^*(s)}} \geq m$$

This relation holds because of definition (2), and the fact that the left-hand side of the above relation is monotonically increasing function of $I^*(s)$ and $I^*(s)$ is decreasing function of s.

As theorem (8) shows if the system starts within the range of $[\underline{s}, \bar{s}]$, then with the desired probability, it will stay within that range in the next stage. Furthermore, if the system starts from below $\underline{S}$, then as part 2 of theorem (8) shows, it will move to that range gradually.

Theorem 9: *If at the beginning of a period the number of on-hand inventory is* $s \geq \underline{s}$, *then the probability* *that after T periods the number of on-hand inventory will be greater than Td is practically negligible. Precisely,*

$$P[Y_T^*(s) \geq 2Td\text{-}s] \leq 1\text{-}\Phi[\sqrt{\frac{Td}{q}} - \Phi^{-1}(\frac{k}{k+h})]$$

Proof: By applying the normal approximation for $I^*(s)$ and some algebraic manipulation, the above relation is obtained. It can be shown that

$$\underline{s} \geq -[m - \Phi^{-1}(\frac{k}{k+h})]\ \sqrt{qTd} - m^2q$$

Similarly, for any $s \geq \underline{s}$,

$$pI^*(s) \leq Td + m\ \sqrt{qTd} + m^2q$$

Now considering the above theorems, the state space of the Markov chain with an acceptable truncation, is limited. this Markov chain is obviously ergodic. then, the average shortage and holding cost of each period L^*, is determined by the following linear programming model:

$$min\ L^* = \sum_{s=\underline{s}}^{\bar{s}} \pi(s) l^*(s, I^*)$$

subject to,

$$\pi(s) = \sum_{s'=\underline{s}}^{\bar{s}} \pi(s') p_{s,s'} \qquad \underline{s} \leq s \leq \bar{s}$$

$$\sum_{s=\underline{s}}^{\bar{s}} \pi(s) = 1 \qquad (25)$$

$$\pi(s) \geq 0 \qquad \underline{s} \leq s \leq \bar{s}$$

where, $\pi(s)$ is the limiting probability that the number of on-hand inventory will be equal to s.

Summary of The Supplier Selection Procedure

1- Identify d, k and h. Select the desired probability (α), for truncation of state space and determine $m = \Phi^{-1}(\alpha)$ from the standard normal distribution table, (for example, α= .99, m= 2.33.)

2- For each supplier take the following steps:

1- Identify c_i, T_i and p_i,

2- Determine $\underline{s}$. Set $\underline{s} = -[m\text{-}\Phi^{-1}(\frac{k}{k+h})]\sqrt{qTd}$. If (23) holds then go to 2.3, otherwise set $\underline{s}=\underline{s}-1$ and repeat this step.

3- Calculate $\bar{s}$ from (24). For each $\underline{s} \leq s \leq \bar{s}$, determine $I^*(s)$ from (9) and (10) or by normal approximations.

4-For each $\underline{s} \leq s \leq \bar{s}$, determine $L(s, I^*(s))$ from (1), (2) and (3).

5- Solve the linear programming model of (25) and determine L^*.

6- Calculate the expected average long-term cost of (22).

3- Compare the costs and select the supplier.

Numerical Examples

Example 1: *For a particular item, assume the demand per period is 40, unit shortage and holding costs per period are 120 and 10, respectively. For truncation level, we set* α= .99, $m = \Phi^{-1}(.99)$= 2.33.

There are two suppliers with the following specification:

Supplier number	c_i	p_i	T
1	18	.9	4
2	16	.8	4

As can be seen the lead time as well as the price of acceptable items, i.e. (c_i/p_i), for both suppliers are the same. However, according to our procedure, the average long-term shortage and holding cost per period for supplier 1 and 2 are L_1^* =79.8, and L_2^*= 111.2, respectively. In fact the second supplier can be selected if he reduces his price as low as 15.3 dollars. After selecting supplier 1, then the ordering policy is as follows:

s	-4	-3	-2	-1	0	1	2
$I^*(s)$	189	188	187	186	185	184	183
$l(s,I^*)$	80.48	80.35	80.27	80.25	80.27	80.34	80.46
s	3	4	5	6	7	8	9
$I^*(s)$	182	180	179	178	177	176	175
$l(s,I^*)$	80.62	78.62	78.51	78.46	78.45	78.49	78.58
s	10	11	12	13	14	15	16
$I^*(s)$	174	173	172	170	169	168	167
$l(s,I^*)$	78.72	78.9	79.13	76.64	76.61	76.61	76.68

Example 2: *In example 1, assume the lead time of the first supplier is 7, but all other data are the same. Therefore, the first supplier is more reliable while the second one has shorter lead time.*

Following our procedure results in,

$$L_1^* = 104.8, \qquad L_2^* = 111.2.$$

The first supplier is selected, in spite of his longer lead time.

Example 3: *For demand and cost of the first example, assume there are three suppliers with the following specifications:*

supplier number	c_i	p_i	T
1	18	.9	7
2	17.1	.85	4
3	14.2	.7	2

The first supplier is the most reliable and the third one has the shortest lead time, while the second one is somewhere in the middle.

The average long-term costs of these suppliers are as follows:

supplier number	L_i^*	$\frac{C_i d_i}{p_i}$	Total Cost*
1	105.2	800	905.2
2	96.9	804.7	901.6
3	97.1	811.4	908.5

Therefore, the second supplier is selected.

The Effect of Different Parameters on the Policy and the Cost

Theorem 10: *The optimal quantity of in-transit inventory I^*, is a non-increasing function of p.*

Proof: *The above theorem follows from the fact that binomial random variables are getting stochastically larger as p increases.*

To illustrate the above theorem, suppose $p = 1$. Therefore, you order exactly as much as you need. In this case, no holding or shortage costs occur, i.e., $l(s, I^*) = 0$. As p decreases from one to some extent, then you have to order one unit more than you need to cover one possible defective item. As p continues to decrease, the order size will increase. Figure 1 shows the optimal value of order size with respect to p.

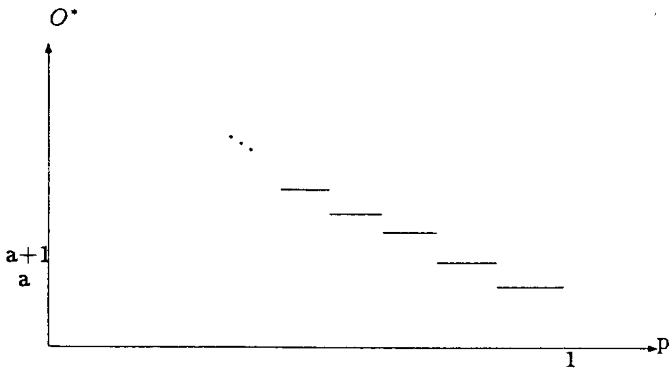

Figure 1. Optimal number of in-transit inventories vs. p

Theorem 11: *If for some inventory on hand, say s, the optimal quantity of in-transit inventory is $I^*(s)$, then $I^*(s)$ + s is a non-increasing function of s.*

Proof: *The above theorem follows from (9) and (10) and the following property of binomial random variables.*

$$P(Y_n \geq Td - s + 1) \leq P(Y_{n-1} \geq Td - s)$$

Impact of Quality on the Optimal Cost

Although we did not prove it mathematically, it seems trivial and our numerous numerical examples verify that the optimal shortage and holding cost of a single period, $I^*(s,i)$, is a non-increasing function of p. For any fixed lead time T, the general shape of L^* as a function of p is shown in Figure 2. Then as example 1 illustrates, if the lead times and the expected real prices of acceptable items of two suppliers are equal, i.e., $\frac{c_1}{p_1} = \frac{c_2}{p_2}$, the supplier with greater p, (better quality) will be selected. This means it is economical to pay higher prices for more reliability that results in less shortage cost. The trade-off between the price and reliability is determined by the proposed policy.

SUMMARY AND RESULTS

In this paper we consider two different but related problems. In the first part we develop an optimal ordering policy for an inventory system in which the supplier yield is random and some delivered items may be defective with a certain probability and are independent of the others. Assuming the value of an item is always equal to its purchase price, the policy happens to be myopic. In the second part of the paper, the strategic issue of the supplier selection is addressed. A procedure is developed to choose a long-term single supplier among the possible alternatives with different lead time, price and quality.

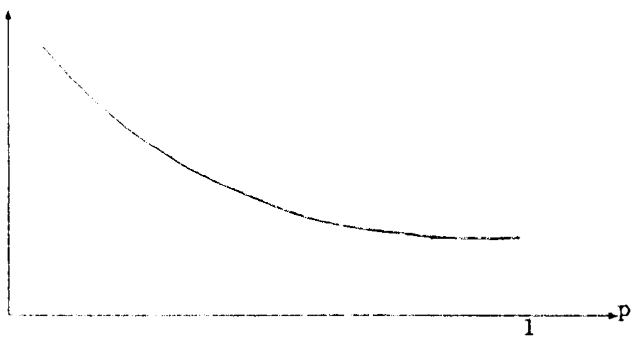

Figure 2. Optimal total average cost per period vs. p

ACKNOWLEDGMENT

The author would to thank Professor Chris Tang for introducing the problem and for his valuable suggestions. I am also grateful Professors Bruce L. Miller and Elwood Buffa, of Anderson Graduate School of Management, UCLA, for their constructive comments.

REFERENCES

1. R. Ahmadi and C. Tang: Production Allocation with Dual Provisioning, Working Paper, Anderson Graduate School of Management, UCLA (1991).
2. E. Buffa: "*Meeting the Competitive Challenge: Manufacturing strategy for U.S. Companies*," Richard Irwin Inc., (1984).
3. R. Ehrhardt and M. McClelland: Inventory management with Random Replenishment, paper presented at the ORS/TIMS Conference in New Orleans, (1987).
4. Y. Gerchak, R.G. Vickson and M. Parlar: Periodic Review Production Models with Variable Yield and Uncertain Demand, *IIE* Transactions, 20, 2, 144-150, (1988).
5. R.W. Hall, "*Zero Inventories*," Dow Jones-Irwin, Homewood, Illinois, (1983).
6. M. Henig and Y. Gerchak: The Structure of Periodic Review Policies in the Presence of Random Yield, *Ops. Res.*, 38, 4, 634-643, (1990).
7. S. Lee and P. Zipkin: A Dynamic Lot-Size Model with Make-or-Buy Decisions, *Management Sci.*, 447-458, (1989).
8. K. Moinzadeh and S. Nahmias: A Continuous Review Model for Inventory System with Two Supply Modes, *Management Sci.*, 34, 6, 761-770, (1988).
9. W.G., Ouchi: "*Theory Z: How American Business Can Meet the Japanese Challenge*, "Addisson, Weseley, (1981).
10. S.M. Ross: "*Applied Probability Models with Optimization Applications*," Holden-Day, San Francisco, (1970).
11. S.M. Ross: "*Introduction to Stochastic Dynamic Programming*," Academic Press, New York, (1983).

12. E.A. Silver: Establishing the Order Quantity when the Amount Received is Uncertain *INFOR*, 14, 32-39, (1976).
13. M.J. Sobel: Myopic Solution of Affine Dynamic Models, *Ops. Res.*, (1990).
14. C. Tang: Supply Contract and Supplier Selection, Working Paper, Anderson Graduate School of Management, UCLA, (1991).
15. C.A. Yano and H.L. Lee: Lot-Sizing with Random Yields: A review, Technical Report 89-16, University of Michigan, (1989).