IJE TRANSACTIONS B: Applications Vol. 39, No. 05, (May 2026) 1088-1100

International Journal of Engineering

Journal Homepage: www.ije.ir

Distribution Substation Allocation Considering Load Uncertainty based on Information Gap Decision Theory

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P A P E R I N F O

Paper history:
Received 10 March 2025
Received in revised form 24 May 2025
Accepted 11 June 2025

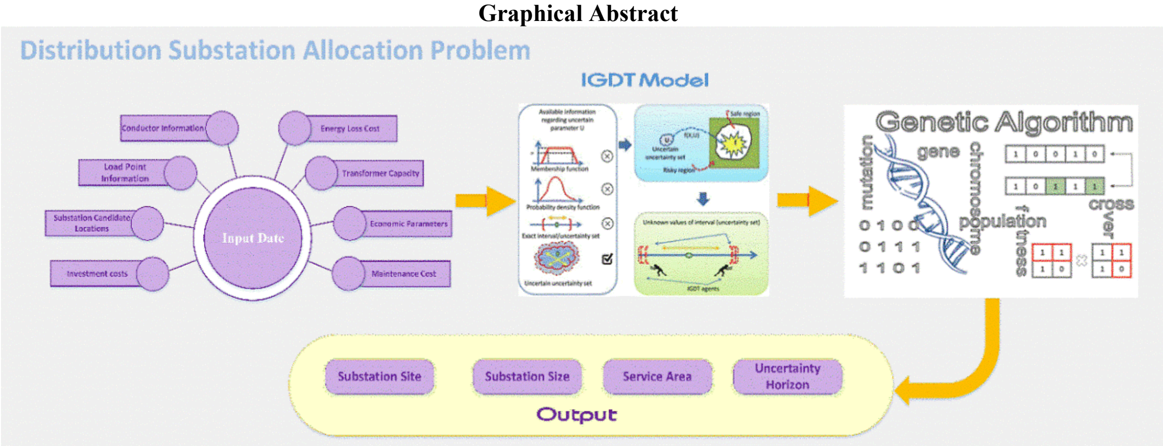
Keywords:
Distribution Substation
Load Uncertainty
Information Gap Decision Theory
Genetic Algorithm

A B S T R A C T

This paper presents a novel methodology for optimizing the location, sizing, and service area of distribution substations, explicitly incorporating load uncertainty. The escalating integration of renewable distributed generation and electric vehicles has introduced substantial uncertainties into distribution loads, exceeding conventional prediction errors. To mitigate this challenge, a robust optimization framework is employed, enabling risk modeling and ensuring secure operation under extreme uncertainty scenarios. Information Gap Decision Theory (IGDT) is utilized to effectively model load uncertainty, with a risk-averse strategy adopted to enhance robustness. The objective is to minimize the total cost of distribution system planning, encompassing investment, maintenance, and loss-related expenditures, while adhering to technical constraints. Initially, the distribution substation problem is solved using predicted load point values. Subsequently, a risk-averse IGDT approach is applied to identify robust solutions under load uncertainty. The efficacy of the proposed methodology is demonstrated using a test system. Results indicate that an average load increment of approximately 42.88% can be accommodated with a corresponding 50% increase in permissible cost. In such scenarios, if potential load uncertainty does not materialize, the average substation load would be approximately 55% of their nominal capacities.

doi: *10.5829/ije.2026.39.05b.04*

Graphical Abstract



NOMENCLATURE

Sets			
S	Set of selected substations	Cap_{n_i}	Rated capacity of substation i (MVA)
L	Set of load points	u	uncertain variable
DL	Set of downstream feeders	$\tilde{u}$	Predicted value of the uncertain variable u
		α	Uncertainty horizon

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Please cite this article as: Ramezani M, Falaghi H, Azimi A. Distribution Substation Allocation Considering Load Uncertainty based on Information Gap Decision Theory. International Journal of Engineering, Transactions B: Applications. 2026;39(05):1088-100.

Variables

C_{total}	Total cost ($)
C_{IC}	Investment cost ($)
C_{MC}	Maintenance cost ($)
C_{loss}^{S}	Substation energy loss cost ($)
C_{loss}^{LL}	Downstream network energy loss cost ($)
C_{CS_i}	Fixed cost of substation i
C_{FL}	Cost of downstream network ($/km)
$C_{MC_i}^{S}$	Annual maintenance cost of substation i ($)
$C_{MC_j}^{LL}$	Annual maintenance cost of downstream network ($)
D_{ij}	Distance between substation i and load point j (km)
x_i, y_i	Coordinates of node i
β_{ij}	Decision variable (0: load point has not been connected to Substation, 1: load point has been connected to Substation)
P_{nl_i}	Copper losses of substation i at the rated loading (kW)
P_{l_i}	Iron losses in substation *i* at the rated loading (kW)
$Cap_{l_i,y}$	Total power provided by substation *i* at year y (MVA)
β	Maximum allowable of objective function increase
Z_{base}	Base value of the objective function

Parameters

$Infr$	Inflation rate
$Intr$	Interest rate
n_y	Equipment life (y)
K_E	Cost of energy loss ($/kWh)
LSF	Loss factor
R	Conductor resistance (Ω/km)
V	Nominal voltage magnitude (v)
$S_{j,y}$	Predicted load of load point j at year y (VA)
$S_j, \bar{S}_j, \tilde{S}_j$	Load value j at horizon year /mean load/load variable at second level
n_s	Number of existing and potential substations
n_l	Number of load points
Z_{cap}	Maximum permissible loading percentage
$\Delta V_{\max}$	Maximum allowed voltage drop (V)

1. INTRODUCTION

The planning and operation of distribution networks face increasing challenges due to uncertainties associated with system participants. Deploying distributed generation, particularly with high penetration levels, and the growing adoption of electric vehicles introduce significant uncertainties into distribution networks, adversely impacting their efficient planning and operation (1).

The primary objective of distribution network planning is to meet future demand with the most cost-effective solution while adhering to technical constraints. The sub-transmission system can be divided into two main components: substations and primary feeders (2). These components require multiple variables to be considered during the planning stage. Various distribution planning models exist, which can be broadly categorized into subsystem and total system approaches. Subsystem approaches focus on planning either substations or feeders (2, 3), whereas total system techniques address both substations and feeders as a single, integrated problem (4, 5).

The complexity of distribution system planning is further compounded by its inherently nonlinear nature and the uncertain environment in which it operates. Consequently, many existing methods do not simultaneously account for both subsystems during the planning stage. A review of relevant studies reveals that most focus has been on allocating of distribution substations, transformer placement, feeder routing, and conductor sizing.

1. 1. Literature Review

Numerous papers have been published on optimal distribution substaccction placement, typically modeling the problem in deterministic settings to determine optimal substation siting, sizing, and timing, as well as service area definition.

Given the similarities between substation allocation and transformer placement, significant literature on transformer allocation has also been examined. Relevant literatures have discussed the subject from different points of view. It is worth to categorize them dependent to their innovations or bold features.

- **Optimization method**

Existing researches in this field can be categorized into three major groups:

1. Mathematical optimization (5-7)
2. Evolutionary optimization (1, 8, 9)
3. Heuristic optimization (3, 8, 10-12)

Mathematical optimization approaches often provide optimal solutions but are highly complex and impractical for large-scale case studies. Evolutionary optimization algorithms are more straightforward to implement across various problem types. However, these methods may struggle with local optima and can be time-consuming. Heuristic methods, while simple and easy to apply, lack guarantees for achieving globally optimal solutions.

Mazhari et al. (3) employed a modified fuzzy membership matrix and a memorable cost index vector to determine the substation service areas. Also, a hybrid heuristic and learning automata (LA)-based algorithm is used for substations sizing and defining the associated service area, simultaneously.

- **Study environment**

The substation allocation problem is solved considering vertical power system structure or deregulated power system. Some research has been conducted in traditional power system environments (8, 13, 14), while others address market-oriented frameworks (1, 15). Both of these environment are applicable and can be considered in power system studies depending on the environment of the case under study.

- **Scope of study**

The studies differ in their scope and context. Some researches just focus on distribution substation problem [1-3] and in some instances, substation allocation is integrated with the planning of other components, such as distributed generation units (4, 5, 15, 16) or overall distribution expansion planning (13, 14).

A dynamic bacterial foraging algorithm determines the optimal location, capacity, service area, and construction timing of substations under technical constraints, aiming to maximize profits from energy sales while minimizing associated costs (1).

Behzadi and Bagheri (5) addressed the siting and sizing of substations, feeder routing, and renewable-based distributed generation (DG) planning to create resilient and sustainable distribution systems. First, all problem relations are convexfied. Then, a mixed-integer quadratically constrained programming (MIQCP) algorithm is employed to minimize investment, operational, emission, and resiliency costs while accounting for renewable DG uncertainty through scenario-based modeling. Jalai et al. (15) Shayeghi and Bagheri (16) focussed on the expansion planning of distributed generation units and substations, incorporating energy prices for electricity received via substations or produced by distributed generators.

- **Static or dynamic modelling**

Determining the time of substation installation is one of the remarkable features that can be considered while categorizing studies. Distribution substation problem can be solved taking the whole planning period in optimization process (1, 17) or can be modeled considering different time intervals (2, 7, 8). It is obvious that solving the problem in dynamic procedure adds more accuracy to the results, on the other hand, makes the problem more complicated.

Temraz and Salama (2) used the General Interactive Optimizer (GINO) to solve the non-linear planning of problem. The model includes a continuous function that accurately simulates various cost components of a distribution substation. In this study besides size, location, and service area, time of substation installation is determined using pseudo dynamic procedure.

The Imperialist Competitive Algorithm (ICA) was utilized by Najafi et al. (8) to optimize the sizing, siting, and timing of medium-voltage substations (distribution transformers). This study introduces a loss characteristic matrix as a novel concept for the substation placement problem.

Lastly, a Mixed Integer Linear Programming (MILP) method is employed by El-Fouly et al. (7) to address the siting, sizing, and timing of distribution substations.

- **Incorporating geographical information system**

Several studies attempt to address real-world planning challenges, ranging from parameter uncertainty (18) to geographical characteristics (11, 19, 20). For instance, Yu et al. (11) proposed a method to determine the optimal location and size of substations based on GIS and semi-supervised learning. This method highlights the significance of land costs in substation allocation. By integrating actual land cost into the objective function, GIS tools enable precise substation location determination after load clustering using the k-means method.

Vahedi et al. (19), substation expansion planning is applied to a large-scale real network using GIS raster maps for analysis. To manage the complexity of large-scale networks, the planning area is divided into irregular mini-regions. A "system of systems" approach is introduced to facilitate interactions between these mini-regions, ensuring the overall integrity of the distribution network.

- **Reliability modelling**

One of the features that makes the proposed approaches more applicable to real distribution network is incorporating reliability modeling into the problem. Some researches consider the possibility of components failures in the study. A deterministic heuristic algorithm is used by Mazhari et al. (10) to approximate substation service areas, employing an expert selection strategy to enhance the likelihood of achieving a globally optimal solution. This study also examines supply interruptions caused by substation unreliability and feeder failures.

A related problem, distribution transformer planning, was addressed by Esmaeeli et al. (21), where reliability costs and load growth are considered. The paper allows transformer loading to exceed the nameplate rating during parts of the duty cycle, demonstrating that this does not significantly impact the relative thermal aging rate of transformers.

Smart substation allocation, aimed at restoring power to interrupted customers, was explored by Sun et al. (22). Interruption costs and reliability criteria are key factors in determining the number and location of smart substations. The problem is solved using Mixed Integer Linear Programming (MILP).

- **Uncertainty modelling**

Accounting for system parameter uncertainties is now essential to ensure practical solutions for real-world systems. Haghifam and Shahabi (23) proposed a pseudo-dynamic methodology for substation planning under load uncertainty modeled with LR fuzzy numbers.

Esmaeelia et al. (24), a Conditional Value at Risk (CVaR)-based method is applied for optimal planning of LV distribution networks and distribution transformer allocation. System uncertainties are modeled using scenarios, and the problem is solved with Mixed Integer Nonlinear Programming (MINLP).

Franco et al. (25) employed a robust model to address load uncertainty, using chance constraints to ensure substation capacity limits are not violated within a given robustness probability. Monte Carlo simulations validate constraint satisfaction under uncertain conditions.

Abedi et al. (18) used a genetic algorithm and heuristic clustering method to assign uncertain load centers to existing or candidate substations. The approach focuses on uncertain load centers with variability in both load values and locations. Genetic algorithms generate different load scenarios, while clustering provides potential sizes and locations for new substations. Fuzzy membership function also was used to model load uncertainty in distribution transformer allocation (17).

1. 2. Proposed Method

This paper addresses the challenges of substation siting and sizing under high uncertainty, considering the potential presence of electric vehicles and the high penetration of distributed renewable generation. These factors increase the uncertainty of electric load demand, surpassing traditional load forecasting errors. The random behavior of renewable generation and electric vehicle charging patterns makes it difficult to model the probability distribution function of loads at different load centers with sufficient accuracy.

To tackle this issue, the Information-Gap Decision Theory (IGDT) is applied as a robust solution for substation siting and sizing under uncertainty. IGDT is particularly advantageous for decision-making in uncertain environments as it enables risk-based decisions without requiring knowledge of the probability distribution functions of uncertain parameters (26). IGDT has been successfully applied to various power system problems, including self-scheduling of generation companies (27), day-ahead achedualing of micro grids (28), and bidding strategy problems (29).

The main contributions of this paper are as follows:

- Incorporation of significant uncertainty in electric load demand, considering the potential impact of electric vehicles and renewable energy sources.
- Application of IGDT to effectively manage and model uncertainties.
- Development of a robust solution framework that accommodates load points with varying levels of uncertainty.
- Economic analysis of the impact of prediction errors in uncertainty estimations on the final solution and overall system costs.

In the following, after introducing the principles of the problem in section 2, the objective function and constraints are discussed in section 3. After that, section 4 presents the IGDT-based decision-making methodology. Section 5 states the problem modeling using IGDT. The optimization algorithm is expressed in section 6. Finally, the numerical studies are explained in Section 7, showcasing the applicability and effectiveness of the proposed method.

2. SUBSTATION ALLOCATION PRINCIPLES

In distribution network planning and expansion, it is essential to determine the optimal location, size, and service area of distribution substations in order to meet demand at the minimum cost. To achieve this, the area under study is divided into smaller sub-areas, and load prediction is performed for each sub-area for the target year. Each sub-area is then modeled as a load point located at the center of gravity of that specific area. During the study period, annual load values (from the first year to the horizon year) are calculated using the load growth factor. Based on the load point values and their locations, as well as the candidate locations of distribution substations, the problem is formulated to obtain the following decision variables:

- The number of distribution substations
- The optimal location of distribution substations
- The size of distribution substations
- The service area of distribution substations

3. PROBLEM FORMULATION WITHOUT UNCERTAINTY

The substation planning problem is first formulated in a deterministic environment with the following objective function and constraints.

3. 1. Objective Function

As mentioned in the introduction, various studies have used different objective functions and constraints in distribution substation planning. These problems are often solved within a traditional, deterministic environment. Commonly, the objective functions include the investment cost of substations (including equipment and land cost) and feeders, maintenance costs, and energy loss costs. The problem formulation is as follows:

$$Min\ C_{total} = C_{IC} + C_{MC} + C_{loss}^{S} + C_{loss}^{LL} \tag{1}$$

$$C_{IC} = \sum_{i \in S} C_{CS_i} + \sum_{i \in S} \sum_{j \in L} C_{FL} \times D_{ij} \times \beta_{ij} \tag{2}$$

$$D_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \tag{3}$$

$$C_{MC} = \sum_{y=1}^{n_y} \left(\left(\frac{1+Infr}{1+Intr} \right)^y \times \left(\sum_{i \in S} C_{MC_i}^{S} + \sum_{i \in DL} C_{MC_i}^{LL} \right) \right) \tag{4}$$

$$C_{loss}^{s} = \sum_{y=1}^{n_y} \sum_{i \in S} \left(\frac{1+Infr}{1+Intr} \right)^y \times K_E \times 8760 \times \left(P_{nl_i} + P_{l_i} \times \left(\frac{Cap_{l_i,y}}{Cap_{n_i}} \right)^2 \times LSF \right) \tag{5}$$

$$C_{loss}^{LL} = \sum_{y=1}^{n_y} \sum_{i \in S} \sum_{j \in L} \left(\frac{1+Infr}{1+Intr} \right)^y \times K_E \times LSF \times D_{ij} \times \frac{8760 \times R}{V^2} \times S_{j,y}^2 \times \beta_{ij} \tag{6}$$

$$Cap_{l_i,y} = \sum_{j=1}^{n_l} \beta_{ij} \times S_{j,y} \tag{7}$$

The objective function is presented in Equation 1, where the first term represents the investment costs of the selected substations within the planning horizon. The second term accounts for the total maintenance costs of substations and feeders. The third term reflects the total cost associated with energy losses in substations, including both no-load losses and loading losses. The costs imposed by feeder losses are modeled in the fourth term of Equation 1.

Considering load growth during the planning period, an annual load growth rate of 2% is used. Transformer losses, feeder losses, and substation loads are updated based on this growth rate. Since the load center points are considered hypothetical, this study uses Euclidean distance to calculate the distance between the load points and candidate substations.

3. 2. Constraints To operate the distribution network within its allowable component limits, several constraints must be imposed during system planning. The problem constraints are as follows:

3. 2. 1. Load Supply Constraints The primary purpose of a power system is to meet all electricity demands. In this work, each load point must be connected to one selected substation.

3. 2. 2. Capacity Constraints To ensure sufficient reserve capacity for future load growth, and to extend the lifetime of transformers and other components, the maximum load for each distribution substation should be limited to a predefined value.

$$\sum_{j=1}^{n_l} \beta_{ij} \times S_j \le Z_{cap} \times Cap_{n_i} \qquad i = 1, \dots \tag{8}$$

3. 2. 3. Voltage Constraints As with other planning problems, permissible voltage magnitudes at load points are considered. A fixed voltage range is applied as follows:

$$\left| \frac{D_{ij} \times S_j}{V^2} \times R \right| \le \Delta V_{\max} \tag{9}$$

3. 2. 4. Radial-flow Constraints Each load point should be supplied by only one substation to reflect the practical operation of radial distribution networks.

4. INFORMATION GAP DECISION THEORY

IGDT is a risk control method that can be used for decision-making in the presence of uncertainties. Unlike other probabilistic methods, IGDT does not require probability density functions for uncertain parameters. Instead, IGDT only uses the predicted values of uncertain parameters (30). This method is particularly useful when initial information is limited, or when changes cannot be predicted with certainty.

IGDT creates a confidence interval by considering a prediction horizon for uncertain parameters, ensuring that if the uncertain parameter falls within this interval, the value of the objective function will not exceed the bounds determined by the planner (29).

Several models within the IGDT framework can represent parameter uncertainty. In this study, uncertainty is modeled using the envelope bound model. Equation 10 illustrates the uncertainty modeling function, where $\tilde{u}$ is the predicted value of the uncertain variable u, and α is the uncertainty horizon. Both parameters, u and α, are uncertain.

$$U(\alpha, u) = \{u : \left| \frac{u - \tilde{u}}{\tilde{u}} \right| \le \alpha \ , \alpha \ge 0\} \tag{10}$$

As previously mentioned, IGDT performs risk-based decision-making. Depending on company policies, either a risk-averse or risk-seeking approach can be adopted. In a risk-averse policy, the IGDT method seeks to maximize the uncertainty interval and provide a solution that guarantees a certain magnitude of the objective function. In this case, the decision maker minimizes risk by focusing on the most unfavorable deviations of uncertain parameters (such as reduced output from distributed generators or increased network load). By considering the worst-case scenario of uncertainty, the best decision is made. Thus, resilience against unfavorable deviations from the forecasted values of uncertain parameters is modeled using a robustness function (26). This robust function ensures that the objective function value never exceeds a predetermined critical value; thus, providing a safeguard against excessive risk. Equation 11 states the robustness function:

$$\tilde{\alpha} = max\{\alpha: \text{maximum} C_{Total} \text{ which is less than a target value.}\} \quad (11)$$

5. PROBLEM MODELING USING The IGDT METHOD

Distribution substations are installed at appropriate locations to meet customer's demands in the distribution network. Today, despite the strong emphasis on installing distributed renewable generation sources to supply part of the distribution network's load, there is a possibility of load reduction at certain nodes. Therefore, in addition to the inherent uncertainty of electric load, the uncertainty introduced by renewable energy sources also affects distribution substation planning.

Furthermore, in recent years, distribution networks have encountered new loads with uncertainty in location and magnitude. The increasing penetration of electric vehicles (EVs) has significantly altered the conditions and requirements of distribution networks. Therefore, special attention must be given to load uncertainty and its impact on the location, capacity, and service area of distribution substations.

The IGDT method is applied in this study to provide a solution that is robust against load uncertainty. The objective is to determine the maximum horizon of load variation while ensuring that the additional cost of substation placement does not exceed an allowable limit.

The base value of the objective function, denoted as Z_{base}, is first calculated in a deterministic framework using the forecasted values of load points. The maximum allowable load variations are then determined using Equations 12 to 17. A risk-averse policy is adopted to ensure that the solution remains valid despite unfavorable deviations in load.

$$\max \alpha \quad \text{Subject to:} \quad (12)$$

$$0 \le \alpha \le 1 \quad (13)$$

$$(1-\alpha)\bar{S}_j \le \tilde{S}_j \le (1+\alpha)\bar{S}_j \quad (14)$$

$$\{\min_{\max \alpha} C_{total}\} \le (1+\beta) \times Z_{base} \quad (15)$$

$$\sum_{j=1}^{n_l} \beta_{ij} \times \tilde{S}_j \le Z_{cap} \times Cap_{n_i} \quad (16)$$

$$0 \le \frac{D_{ij} \times \tilde{S}_j}{V^2} \times R \le \Delta V_{\max} \quad (17)$$

As seen in Equation 15, the objective function of the deterministic problem (Z_{base}) is incorporated as a limiting condition in the risk-averse strategy. Specifically, the highest level of load points is determined while ensuring that the total cost does not exceed β% more than Z_{base}.

The above problem is a bi-level optimization problem that can be transformed into a single-level problem. Since the maximum risk level and the objective function value occur at the highest level of load uncertainty, the values of load points can be defined using Equation 18.

$$\tilde{S}_j = (1+\alpha)\bar{S}_j \quad (18)$$

This transformation simplifies the problem by considering the worst-case scenario, eliminating the second level, and reducing it to a single-level problem as formulated in Equations 19 to 23.

$$\max \alpha \quad \text{Subject to:} \quad (19)$$

$$0 \le \alpha \le 1 \quad (20)$$

$$\{\min_{\max \alpha} C_{total}\} \le (1+\beta) \times Z_{base} \quad (21)$$

$$0 \le \frac{D_{ij} \times ((1+\alpha)\bar{S}_j)}{V^2} \times R \le \Delta V_{\max} \quad (22)$$

$$\sum_{j=1}^{n_l} \beta_{ij} \times ((1+\alpha)\bar{S}_j) \le Z_{cap} \times Cap_{n_i} \quad (23)$$

6. OPTIMIZATION ALGORITHM

Genetic algorithm is an evolutionary algorithm that has been used in many optimization studies (31-33). Sabattin et al. (31) introduced a parallel genetic algorithm to optimize a tree-based topology for large-scale electric power distribution networks. Mahmoodabadi et al. (32) solved AC/DC hybrid distribution system planning using genetic algorithm and Esmaeilzadeh et al. (33) introduced a new optimization algorithm based on particle swarm optimization, genetic algorithm, and sliding surfaces.

In this study, genetic algorithm has been used to solve the optimization problem. A two-dimensional chromosome structure is considered. In the base case study (deterministic problem), the number of columns corresponds to the number of candidate locations for distribution substations, while the number of rows represents the number of load points. Figure 1 illustrates the structure of the two-dimensional chromosome used in this study. Each element (gene) in the matrix can take values of either 0 or 1.

If the element in the i^{th} row and j^{th} column is equal to 1, it indicates that the i^{th} load point is supplied by the j^{th} substation. Given the radial structure of the system, each load center can only be supplied by one substation, ensuring that each row contains only one element with a value of 1.

For the IGDT-based problem, an additional column has been added to the chromosome (highlighted in gray in Figure 1) to represent the uncertainty horizon (α) for each load point.

Using this chromosome representation, the location, size, and service area of distribution substations can be determined. Any column with at least one element equal to 1 indicates a selected candidate location for substation installation. The service area is identified by determining which load points are connected to the selected substation. The required capacity is then chosen as the closest standard substation capacity to accommodate the total connected load. The substation load is calculated as the sum of the loads from all connected load points.

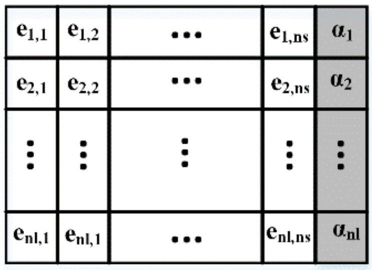

Figure 1. Proposed chromosome structure

7. TEST SYSTEM AND SIMULATION RESULTS

To evaluate the proposed model and verify its performance, a test system has been designed. This system consists of 36 load points and 14 candidate substation locations. The study area is assumed to be divided into 36 sub-areas, and load forecasting results for the horizon year are provided. Coordinates of the candidate locations for the distribution substation are provided in Table 1. The information regarding load points is available in Table 2, while the remaining required data can be found in Table 3. Three standard substation capacities of 30, 45, and 60 MVA are considered for all candidate locations. The construction costs for these substations are \$2 million, \$2.5 million, and \$3 million, respectively, covering the costs of substation infrastructure, buildings, and other necessary components.

In this study, the maintenance costs of equipment are assumed to be 3% of the initial investment cost. The simulations are based on a static planning approach, and the long-term planning period is set to 10 years.

The accuracy of the proposed method has been investigated using a test system introduced by Haghifam and Shahabi [24]. The study of substation allocation is performed for the following cases:

- **Case 1**: Deterministic substation allocation
- **Case 2**: Probabilistic substation allocation with a uniform uncertainty horizon
- **Case 3**: Probabilistic substation allocation with a variable uncertainty horizon

7. 1. Deterministic Substation Allocation-Case 1

In this case, it is assumed that there is no uncertainty in the load point values, and the study is conducted using predicted values. The loading factor of substations is set at 0.85. Figure 2 presents the results. The total planned cost is \$50.7 million, and eight locations have been selected for substation installation. Table 4 presents the selected substations' locations, sizes, and estimated loadings for the horizon year.

TABLE 1. Load point coordinates information

Load point	Load (MVA)	Coordinates		Load point	Load (MVA)	Coordinates		Load point	Load (MVA)	Coordinates	
		X(km)	Y(km)			X(km)	Y(km)			X(km)	Y(km)
1	10	2	2	13	10	3	13	25	10	4	24
2	10	7	3	14	6	7	12	26	10	۴6	21
3	10	12	4	15	2	13.5	12.5	27	5	14	22
4	5	17	1	16	6	19	11	28	5	17.5	23
5	5	21	3	17	2	22	14	29	6	22	21
6	5	28	2	18	5	28.5	13.5	30	5	26	23
7	10	2.5	7	19	10	2	16	31	5	2	26
8	6	6	9	20	10	7	17	32	10	8	27
9	6	11	8	21	1	14	17.5	33	10	12	28
10	1	16	9	22	6	18	19	34	10	18	29
11	5	24	6	23	5	21	16	35	5	24	26
12	5	27	6	24	5	27	18	36	6	28	27

TABLE 2. Coordinates information of Candidate points

Candidate point	Coordinates		Candidate point	Coordinates	
	X(km)	Y(km)		X(km)	Y(km)
1	5	6	8	28	22
2	4	11	9	3	36
3	11	10	10	9	20
4	12	2	11	11	29
5	16	5	12	21	23
6	20	12	13	16	16
7	25	8	14	27	19

TABLE 3. Required parameters information (2, 34)

Life time (Yr)	Power Loss Cost ($/kW)	Energy Loss Cost ($/kWh)
30	300	0.17
Inflation Rate	**Interest Rate**	**Loss Factor**
0.15	0.07	0.36
Maximum allowed voltage drop (%)	**Feeder construction cost ($/Km)**	**Resistance (Ω/Km)**
5	3000	0.0695

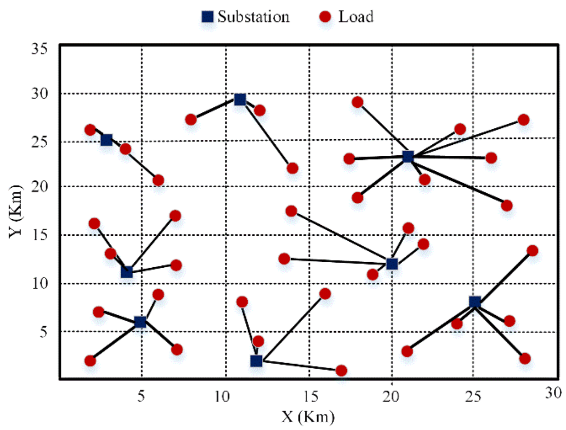

Figure 2. Substation allocation results for loading factor of 0.85 without uncertainty

7. 2. Probabilistic Substation Allocation-Common Uncertainty Horizon- Case 2

In this scenario, the uncertainty is assumed to be the same across all load points. The problem is solved to determine the maximum uncertainty horizon (α) for three different values of the β coefficient: 0.2, 0.3, and 0.5. In all cases, the maximum allowable substation loading factor is set to 85%.

When β=0.2, the maximum cost incurred by the planner to account for uncertainty is 20% higher than the base case value (Z_{base}). Numerical results indicate that a confidence interval of 12% is formed for each load point value. This means that by accepting a 20% increase in total cost, the results remain valid for up to a 12% increase in load values. The selected substations and their service areas are shown in Figure 3. Additionally, substation characteristics are provided in Tables 5.

TABLE 4. Selected substation features in Case 1

Selected substations	Capacity (MVA)	Loaded capacity (MVA)	Loading percentage (%)
1	45	36	80
2	45	36	80
4	30	22	73.3
6	30	16	53.3
7	30	25	83.3
9	30	25	83.3
11	30	25	83.3
12	60	48	80

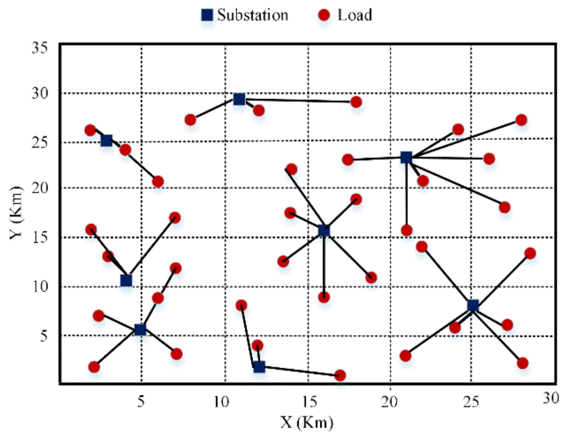

Figure 3. Substation allocation results for Case 2, β=0.2

TABLE 5. Selected substations features in Case 2, β=0.2

Selected substations	Capacity (MVA)	Loaded capacity (MVA)	Loading percentage (%)
1	60	47.04	78.40
2	45	33.60	74.67
4	30	23.52	78.40
7	45	30.24	67.20
9	45	28.00	62.20
11	45	33.60	74.70
12	60	41.44	69.07
13	30	23.52	78.40

In this case, 99.03% of the allowable budget covers the maximum possible uncertainty. When β=0.3, the uncertainty horizon (α) increases to 23%, utilizing 99.99% of the allowable budget. When β=0.5, α reaches 41%, and 98.06% of the allowable budget is used.

Figures 4 and 5 illustrate the selected substations and their service areas. Tables 6 and 7 present the characteristics of the selected substations for the cases where β equals 0.3 and 0.5, respectively. Cost details of case2 expriments are provided in Table 8.

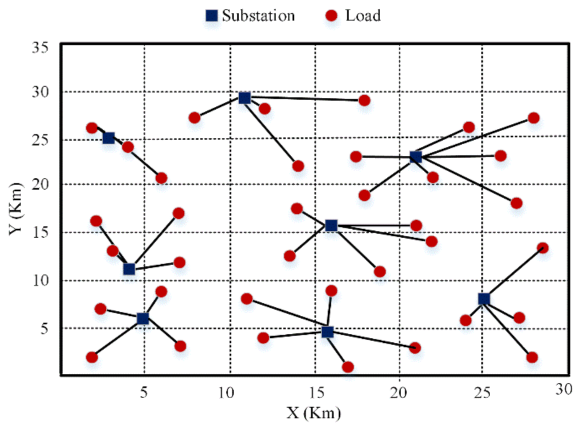

Figure 4. Substation allocation results for Case. 2, β=0.3

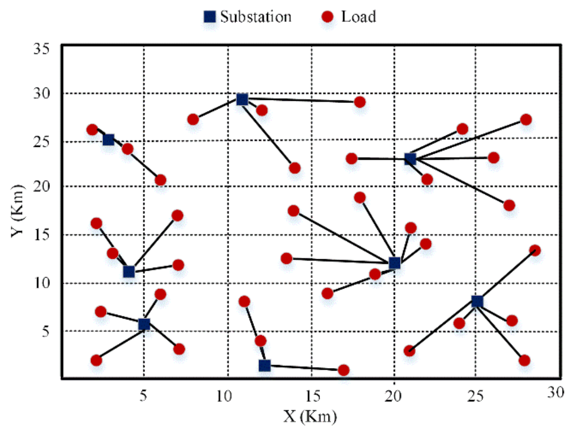

Figure 5. Substation allocation results for Case. 2, β=0.5

TABLE 6. Selected substation features in Case 2, β=0.3

Selected substations	Capacity (MVA)	Loaded capacity (MVA)	Loading percentage (%)
1	60	44.28	73.8
2	60	44.28	73.8
5	45	33.21	73.8
7	30	24.60	82.0
9	45	30.75	68.3
11	60	43.05	71.8
12	60	46.74	77.9
13	30	19.68	65.6

TABLE 7. Selected substation features in Case 2, β=0.5

Selected substations	Capacity (MVA)	Loaded capacity (MVA)	Loading percentage (%)
1	60	44.28	84.6
2	60	44.28	84.6
4	45	33.21	65.8
6	45	24.60	72.1
7	45	30.75	78.3
9	45	43.05	78.3
11	60	46.74	82.3
12	60	19.68	75.2

TABLE 8. Planning cost details in case 2

β	0.2	0.3	0.5
Investment cost (M$)	20.501	21.483	22.490
Maintenance cost (M$)	10.654	11.165	11.688
Loss cost (M$)	29.092	33.266	40.396
Total cost (M$)	60.247	65.914	74.575

7. 3. Probabilistic Substation Allocation-Variable Uncertainty Horizon- Case 3 In this scenario, the substation allocation problem is solved under a risk-averse approach. The uncertainty horizons of the load points vary and are determined through the optimization process. The aim is to determine the maximum allowable increase in load points, allowing the scheme's cost to be up to β% higher than the base case (Z_{base}). The study is conducted for β values of 0.2, 0.3, and 0.5. The maximum allowable substation loading factor remains at 85%.

The characteristics of the selected substations are listed in Tables 9, 10, and 11, while Table 12 provides a detailed breakdown of the costs. The selected distribution substations and their service areas for case 3, β= 0.2 are as case 2, β= 0.5. Substation allocation results for case3, and β of 0.3 and 0.5 are depicted in Figures 6, and 7.

Figure 8 illustrates the α values for different load points across the three cases studied in Case 3. As shown, the α values vary significantly, with some load points exhibiting a much higher allowable increase than others. The average α values for β= 0.2, β=0.3, and β=0.5 are 16.42%, 27.47%, and 42.88%, respectively. The results indicate that uncertainty values are anticipated to rise as β increases.

TABLE 9. Selected substation features in Case 3, β=0.2

Selected substations	Capacity (MVA)	Loaded capacity (MVA)	Loading percentage (%)
1	45	32.40	72.0
2	60	49.74	82.9
4	30	24.55	81.8
6	30	18.79	62.6
7	45	30.91	68.7
9	45	27.80	61.8
11	45	34.40	76.4
12	60	50.97	84.9

TABLE 10. Selected substation features in Case 3, β=0.3

Selected substations	Capacity (MVA)	Loaded capacity (MVA)	Loading percentage (%)
1	60	43.60	72.6
2	60	50.32	83.9
4	30	21.50	71.7
6	30	24.98	83.3
7	30	25.20	84.0
9	45	31.50	76.7
11	30	24.80	82.7
12	60	47.06	78.4
14	30	22.90	76.3

TABLE 11. Selected substation features in Case 3, β=0.5

Selected substations	Capacity (MVA)	Loaded capacity (MVA)	Loading percentage (%)
1	60	46.38	77.3
2	60	50.04	83.4
5	30	22.60	75.3
7	45	37.80	84.0
9	30	23.35	77.8
10	45	28.10	62.4
11	60	49.25	82.1
12	45	38.07	84.6
13	45	35.82	79.6

TABLE 12. Planning cost details in case 3

β	0.2	0.3	0.5
Investment cost (M$)	20.493	21.984	23.485
Maintenance cost (M$)	10.650	11.425	12.205
Loss cost (M$)	29.698	32.471	40.331
Total cost (M$)	60.841	65.881	76.022

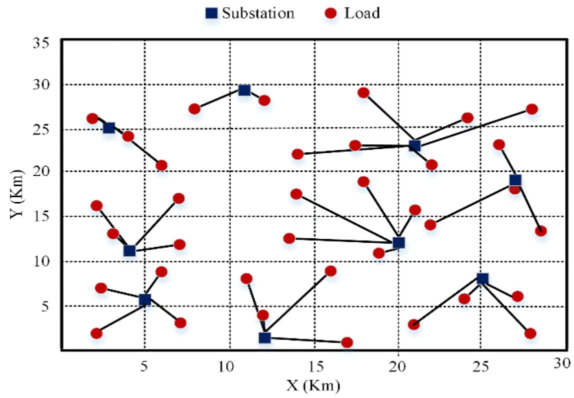

Figure 6. Substation allocation results for Case. 3, β=0.3

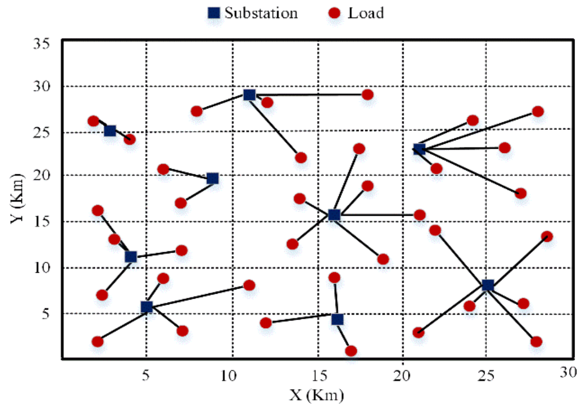

Figure 7. Substation allocation results for Case. 3, β=0.5

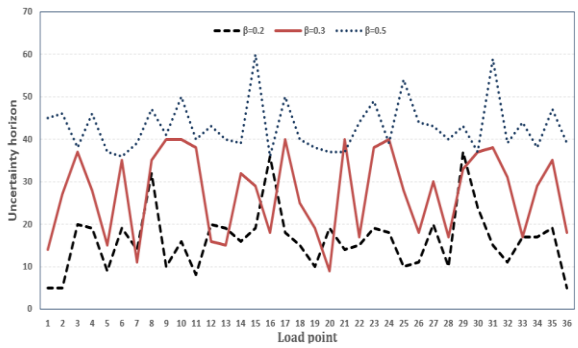

Figure 8. Uncertainty horizons (α) of load points in cases 3

To compare the proposed plans under deterministic and uncertain conditions, a study with β fixed at 0.5 and fixed α (Case 2- β=0.5) is compared with the base scenario (Case 1). When β is 0.5, 50% more financial resources are allocated to the proposed solution compared to the base scenario. In this case, the studies are conducted to cover the maximum uncertainty arising from load. However, if uncertainty does not materialize, in addition to the significant cost imposed on the system, many substations will operate at a low loading level.

Table 13 provides some suitable results in these cases. By considering perfect capacities for the selected substations of Case 2- β =0.5, 18 M$ would be saved. If the system encounters uncertainty, 76.65% of the substation capacity will be loaded in the design horizon year. But if uncertainty does not occur, the average transformer load will be 55% of their capacity, which indicates an unrealistic design. Therefore, before using this method to cover uncertainty, all influencing conditions such as social, economic, and political conditions must be considered.

TABLE 13. Comparing final solutions of case 2-β=0.5 with and without uncertainty

Selected substations	* Selected Capacity (MVA)	* Loading percentage (%)	** Loading percentage (%)	** Perfect Capacity (MVA)
1	60	84.6	60	45
2	60	84.6	60	45
4	45	65.8	46.7	30
6	45	72.1	51.1	30
7	45	78.3	55.55	30
9	45	78.3	55.55	30
11	60	82.3	58.33	45
12	60	75.2	53.31	45

*: Case 2- β=0.5
**: Not materializing of uncertainties
***: Perfect capacity proportional to substation loading in case of not materializing of uncertainties

8. CONCLUSION

Results indicate that an average load increment of approximately 42.88% can be accommodated with a corresponding 50% increase in permissible cost. In such scenarios, if potential load uncertainty does not materialize, the average substation load would be approximately 55% of their nominal capacities.

The findings reveal a clear correlation between increased permissible total cost and enhanced robustness against load uncertainty. Case 1 do not tolerate uncertainty by spending 50.7 M$ while case 3, β=0.5 can accommodate 42.88% uncertainty in average by spending M$ 76.022 for the provided solution.

The robustness is achieved through adjustments to substation's quantity, service area, and capacity, necessitating additional capital investment. Notably, in nearly all scenarios, the optimal selected substations capacities or service areas differ, demonstrating the increment substation deployment in vase of increasing budgetary allocation. The installed substation capacity reaches to 420 MVA in case 3, β=0.5 from 300 MVA in Case 1 without uncertainty. Also selected substation locations and service areas are considerably different.

When the uncertainty horizons fluctuated during the optimization process in Case 3, the final uncertainty values at load points exhibited significant variability. Specifically, the maximum α value was approximately seven times greater than the minimum α value, highlighting the non-linear relationship between β and α. Minimum and maximum load uncertainty horizons in case 3, β=0.5 are 36 and 60% while minimum and maximum load uncertainty horizons in case 3, β=0.5 are 5 and 37%.

The primary limitation of employing a fixed uncertainty horizon lies in its inflexibility. This constraint is effectively mitigated by adopting a variable uncertainty horizon, thereby enhancing the method's applicability to real-world scenarios. Conversely, the constant uncertainty horizon offers the benefit of a simplified design process, resulting in reduced computational time.

However, the risk-averse IGDT approach, while robust, suffers from a significant drawback: its conservative nature. By prioritizing the worst-case scenario, this methodology risks substantial financial losses if the actual uncertainty deviates from the assumed direction or if no uncertainty materializes. Therefore, a careful balance between robustness and cost-effectiveness must be considered when implementing this approach. In such scenarios, if potential load uncertainty does not materialize, the average substation load would be approximately 55% of their nominal capacities.

Possible directions for future research include: modeling uncertainties in additional parameters, such as the geographical coordinates of load points; performing simultaneous optimization of substation placement and medium-voltage feeder routing; and extending the problem formulation to incorporate considerations of a power market environment.

9. REFERENCES

1. Moradi SST. A probabilistic study on hole cleaning optimization. Записки Горного института. 2022;258:956-63. https://doi.org/10.1016/j.egypro.2018.11.109
2. Temraz H, Salama M. A planning model for siting, sizing and timing of distribution substations and defining the associated service area. Electric Power Systems Research. 2002;62(2):145-51. https://doi.org/10.1016/S0378-7796(02)00040-8
3. Mazhari SM, Monsef H, Falaghi H. A hybrid heuristic and learning automata-based algorithm for distribution substations siting, sizing and defining the associated service areas. International Transactions on Electrical Energy Systems. 2014;24(3):433-56. https://doi.org/10.1002/etep.1716
4. Alipour MA, Askarzadeh A. An efficient optimization framework for distribution network planning by simultaneous allocation of photovoltaic distributed generations and transformers. IET Renewable Power Generation. 2024;18(2):153-68. https://doi.org/10.1049/rpg2.12910

5. Behzadi S, Bagheri A. A convex micro-grid-based optimization model for planning of resilient and sustainable distribution systems considering feeders routing and siting/sizing of substations and DG units. Sustainable Cities and Society. 2023;97:104787. https://doi.org/10.1016/j.scs.2023.104787

6. Masud E. An interactive procedure for sizing and timing distribution substations using optimization techniques. IEEE Transactions on Power Apparatus and Systems. 2007(5):1281-6. https://doi.org/10.1109/TPAS.1974.293851

7. El-Fouly T, Zeineldin H, El-Saadany E, Salama M. A new optimization model for distribution substation siting, sizing, and timing. International Journal of Electrical Power & Energy Systems. 2008;30(5):308-15. https://doi.org/10.1016/j.ijepes.2007.10.002

8. Ravadanegh SN, Roshanagh RG. A heuristic algorithm for optimal multistage sizing, siting and timing of MV distribution substations. Electric power systems research. 2013;105:134-41. https://doi.org/10.1016/j.epsr.2013.07.019

9. Kiani Rad H, Moravej Z. Sub-transmission sub-station expansion planning based on bacterial foraging optimization algorithm. Journal of AI and Data Mining. 2017;5(1):11-20. https://doi.org/10.22044/jadm.2016.734

10. Mazhari SM, Monsef H, Romero R. A hybrid heuristic and evolutionary algorithm for distribution substation planning. IEEE Systems Journal. 2013;9(4):1396-408. https://doi.org/10.1109/JSYST.2013.2290973

11. Yu L, Shi D, Guo X, Jiang Z, Xu G, Jian G, et al. An efficient substation placement and sizing strategy based on GIS using semi-supervised learning. CSEE journal of power and energy systems. 2018;4(3):371-9. https://doi.org/10.17775/CSEEJPES.2017.00800

12. Wang S, Lu Z, Ge S, Wang C. An improved substation locating and sizing method based on the weighted voronoi diagram and the transportation model. Journal of Applied Mathematics. 2014;2014(1):810607. https://doi.org/10.1155/2014/810607

13. Salyani P, Salehi J, Gazijahani FS. Chance constrained simultaneous optimization of substations, feeders, renewable and non-renewable distributed generations in distribution network. Electric Power Systems Research. 2018;158:56-69. https://doi.org/10.1016/j.epsr.2017.12.032

14. Navidi M, Moghaddas Tafreshi SM, Anvari-Moghaddam A. Sub-transmission network expansion planning considering regional energy systems: a Bi-level approach. Electronics. 2019;8(12):1416. https://doi.org/10.3390/electronics8121416

15. Jalali M, Zare K, Tarafdarhagh M. Timing, Siting, and Sizing of Sub-transmission Substations and Distributed Generation Units Along Planning Horizon. Electric Power Components and Systems. 2016;44(10):1111-25. https://doi.org/10.1080/15325008.2016.1157107

16. Shayeghi H, Bagheri A. Dynamic sub-transmission system expansion planning incorporating distributed generation using hybrid DCGA and LP technique. International Journal of Electrical Power & Energy Systems. 2013;48:111-22. https://doi.org/10.1016/j.ijepes.2012.11.029

17. Ramezani M, Falaghi H, Moghaddam MP, Haghifam M-R, editors. Genetic based algorithm for optimal placement of distribution transformers. 2006 IEEE Power Engineering Society General Meeting; 2006: IEEE.

18. Abedi MH, Hosseini H, Jalilvand A. Sub-transmission substation expansion planning considering load center uncertainties of size and location. International Journal of Electrical Power & Energy Systems. 2019;109:413-22. https://doi.org/10.1016/j.ijepes.2019.02.032

19. Vahedi S, Banejad M, Assili M. GIS-based substation expansion planning. IEEE Systems Journal. 2020;15(1):959-70. https://doi.org/10.1109/JSYST.2020.3010565

20. Bosisio A, Berizzi A, Merlo M, Morotti A, Iannarelli G. A GIS-based approach for primary substations siting and timing based on Voronoi diagram and particle swarm optimization method. Applied Sciences. 2022;12(12):6008. https://doi.org/10.3390/app12126008

21. Esmaeeli M, Kazemi A, Shayanfar H, Haghifam M-R. Multistage distribution substations planning considering reliability and growth of energy demand. Energy. 2015;84:357-64. https://doi.org/10.1016/j.energy.2015.03.002

22. Sun L, You S, Hu J, Wen F. Optimal allocation of smart substations in a distribution system considering interruption costs of customers. IEEE Transactions on Smart Grid. 2016;9(4):3773-82. https://doi.org/10.1109/TSG.2016.2642163

23. Haghifam M, Shahabi M. Optimal location and sizing of HV/MV substations in uncertainty load environment using genetic algorithm. Electric Power Systems Research. 2002;63(1):37-50. https://doi.org/10.1016/S0378-7796(02)00087-1

24. Esmaeeli M, Kazemi A, Shayanfar H, Haghifam M-R, Siano P. Risk-based planning of distribution substation considering technical and economic uncertainties. Electric Power Systems Research. 2016;135:18-26. https://doi.org/10.1016/j.epsr.2016.03.013

25. Franco JF, Rider MJ, Romero R. Robust multi-stage substation expansion planning considering stochastic demand. IEEE Transactions on Power Systems. 2015;31(3):2125-34. https://doi.org/10.1109/TPWRS.2015.2443175

26. Sarhadi S, Amraee T. Robust dynamic network expansion planning considering load uncertainty. International Journal of Electrical Power & Energy Systems. 2015;71:140-50. https://doi.org/10.1016/j.ijepes.2015.02.043

27. Vahid-Ghavidel M, Shafie-khah M, Javadi MS, Santos SF, Gough M, Quijano DA, et al. Hybrid IGDT-stochastic self-scheduling of a distributed energy resources aggregator in a multi-energy system. Energy. 2023;265:126289. https://doi.org/10.1016/j.energy.2022.126289

28. Tostado-Véliz M, Kamel S, Hasanien HM, Turky RA, Jurado F. Uncertainty-aware day-ahead scheduling of microgrids considering response fatigue: An IGDT approach. Applied Energy. 2022;310:118611. https://doi.org/10.1016/j.apenergy.2022.118611

29. Liu J, Chen C, Liu Z, Jermsittiparsert K, Ghadimi N. An IGDT-based risk-involved optimal bidding strategy for hydrogen storage-based intelligent parking lot of electric vehicles. Journal of Energy Storage. 2020;27:101057. https://doi.org/10.1016/j.est.2019.101057

30. Moradi-Dalvand M, Mohammadi-Ivatloo B, Amjady N, Zareipour H, Mazhab-Jafari A. Self-scheduling of a wind producer based on Information Gap Decision Theory. Energy. 2015;81:588-600. https://doi.org/10.1016/j.energy.2015.01.002

31. Sabattin J, Fuertes G, Alfaro M, Quezada L, Vargas M. Optimization of large electric power distribution using a parallel genetic algorithm with dandelion strategy. Turkish Journal of Electrical Engineering and Computer Sciences. 2018;26(5):2648-60. https://doi.org/10.3906/elk-1801-26

32. Mahmoodabadi M, Nematia A, Daneshb N. A New Optimization Algorithm Based on Particle Swarm Optimization Genetic Algorithm and Sliding Surfaces. International Journal of Engineering Transactions C: Aspects. 2024;37(9):1716-35. https://doi.org/10.5829/ije.2024.37.09c.02

33. Esmaeilzadeh Z, Falaghi H, Ramezani M. Risk-Based AC/DC Hybrid Distribution System Planning. International Journal of Engineering, Transactions B: Applications. 2024;37(5):876-86. https://doi.org/10.5829/ije.2024.37.05b.06

34. Seifi H, Sepasian MS. Electric power system planning: issues, algorithms and solutions. Power Systems. 2011;49.

Persian Abstract

چکیده

این مقاله یک روش نوین برای بهینه‌سازی مکان، اندازه و ناحیه سرویس‌دهی پست‌های توزیع، با در نظر گرفتن عدم قطعیت بار، ارائه می‌کند. حضور رو به افزایش تولیدات پراکنده تجدیدپذیر و خودروهای الکتریکی، عدم قطعیت‌های قابل توجهی را به بارهای شبکه توزیع وارد کرده است که از خطاهای پیش‌بینی بار مرسوم فراتر می‌رود. برای کاهش این چالش، یک چارچوب بهینه‌سازی مقاوم به کار گرفته می‌شود که مدل‌سازی ریسک را ممکن ساخته و عملکرد ایمن را در سناریوهای عدم قطعیت شدید تضمین می‌کند. نظریه تصمیم‌گیری شکاف اطلاعات (IGDT) برای مدل‌سازی موثر عدم قطعیت بار به کار گرفته می‌شود، یک استراتژی ریسک‌گریز برای افزایش مقاومت در برابر عدم‌قطعیت اتخاذ می‌شود. هدف، حداقل کردن هزینه کل برنامه‌ریزی سیستم توزیع، شامل هزینه‌های سرمایه‌گذاری و نگهداری پست‌های توزیع و هزینه‌های مرتبط با تلفات، ضمن رعایت محدودیت‌های فنی است. در ابتدا، مسئله جایابی پست‌های توزیع با استفاده از مقادیر میانگین بار حل می‌شود. سپس، یک رویکرد IGDT ریسک‌گریز برای شناسایی راه حل‌های مقاوم در برابر عدم قطعیت بار اعمال می‌شود. اثربخشی روش پیشنهادی با استفاده از یک سیستم آزمون نشان داده می‌شود. نتایج نشان می‌دهد که با افزایش ۵۰ درصدی هزینه مجاز، می‌توان به طور متوسط افزایش بار حدود ۴۲/۸۸ درصد را مدیریت کرد. در چنین سناریوهایی، اگر عدم قطعیت بار احتمالی محقق نشود، میانگین بار پست حدود ۵۵ درصد از ظرفیت اسمی آنها خواهد بود.