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Realization of Statistical Models Based on Symmetric Unimodal Distributions

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ABSTRACT

The growing complexity of modern technical systems and increasing demands for operational efficiency necessitate more advanced numerical models and methods. This article addresses the development and analysis of statistical models, emphasizing a generalizing approach that derives the integral distribution function by reducing the degree of sub-integral expressions. The residual part is treated as modeling error, allowing pseudorandom value generation algorithms with accuracy limited only by computational power. A method is proposed for generating pseudorandom numbers through the sum of pseudorandom values, solving the problem of constructing a symmetric unimodal distribution with desired probabilistic properties. This is achieved via pairwise weighted summation of elements from standard sequences. Key analytical expressions underlying the method are derived and presented. Comprehensive computational experiments inform several improvements: enhanced decision-making in estimating probabilistic characteristics using symmetric unimodal distribution families; refined determination of confidence interval boundaries that reflect the accuracy of pseudorandom sequence reproduction in the Mathcad15 environment; improved procedures for forming and evaluating basic sequences of pseudorandom variables, leading to better generation precision.

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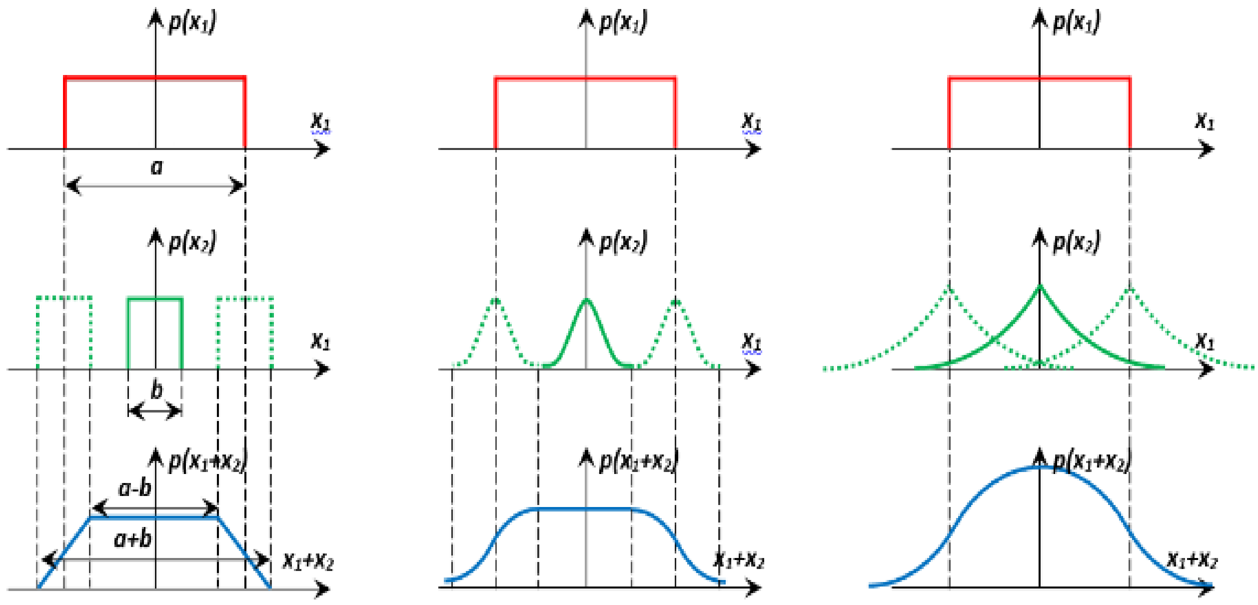
Graphical Abstract



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1. INTRODUCTION

When developing complex technical systems, especially those that implement new physical principles, one has to solve a number of specific problems related to the probabilistic properties of their digital counterparts, especially at the initial stages of their design (1, 2). They, as a rule, require the use of the method of statistical tests in their solution, which is caused by the necessity to take into account both actually observed random processes inside the system and in the environment, and the uncertainty of the situation of use. In this connection there arises a set of actual practical tasks on identification of probabilistic characteristics of random variables and vectors describing separate properties of complex technical systems (their digital doubles) and subsequent reproduction of various laws of their distribution (3-5). Moreover, the most acute is the issue of modeling of random variables in conditions of limited (insufficient) initial information.

Significant complication and appreciation of modern systems, high requirements to the efficiency of their functioning, laid at the level of conceptual models creation, toughen the requirements to the quality of created models and methods of their obtaining and research (6, 7). Moreover, modeling, as one of the driving forces of modern progress, is addressed by an increasing number of researchers, in most cases not possessing deep knowledge in the mathematical apparatus of statistical hypothesis testing, which, with all its methodological and instrumental armament, is often unable to help the analyst to reasonably choose one or another law of distribution in a situation where several hypotheses do not contradict the available experimental data.

In this regard, the solution of the problem of "direct" construction of statistical models of complex technical systems on the basis of limited initial data is relevant (8). There is a limited number of in-situ measurements, which does not allow to apply in full the classical apparatus of statistical hypothesis testing or leads to an information situation when several hypotheses put forward do not contradict the available experimental data (9-11). It is necessary to develop a modeling algorithm that allows us to obtain a sensor of random variables with specified probabilistic characteristics.

Recent research also highlights the growing role of hybrid optimization algorithms and advanced machine learning methods in the modeling of complex technical and infrastructural systems. For example, Widians et al. (12) proposed a hybrid Ant Colony and Grey Wolf Optimization algorithm to effectively balance exploitation and exploration in computational tasks, which may be applicable in optimizing statistical simulation processes. Similarly, Ladino-Moreno and García-Ubaque (13) demonstrated the use of K-BiLSTM combined with Monte Carlo dropout for uncertainty-aware leak detection in urban hydraulic systems—an approach that parallels the need for robust statistical inference under uncertainty. In the socio-economic domain, Vasyunina et al. (14) presented a new economic model aimed at improving the sustainability of regional budgets in Russia, showing the relevance of adaptive statistical tools for managing complex, data-driven systems. These studies underscore the necessity of integrating flexible statistical modeling strategies that operate effectively under conditions of incomplete data and complex structure—conditions often encountered in digital twin development and early-stage technical design.

The considered set of random variables is estimated as symmetric unimodal. This provision allows us to consider the information situation in a minimally narrow format, and then to extend the problem area of the study to the class of asymmetric unimodal, and, subsequently, to the class of multivariate distributions of unlimited dimensionality (15).

2. MATERIALS AND METHODS

The starting point of the research, started back in 1996, was the necessity to build statistical models of the processes of functioning of technical condition control systems and the discovery of an interesting family of exponential distributions, united by one analytical dependence and describing a large set of symmetric unimodal distributions of random variables can be emphasized (16, 17) that:

- the emphasis on the normality of the law of distribution of random variables of the error introduced into the measurement result is not confirmed by experimental data of studies of probabilistic characteristics of errors of electrical measuring devices (18);
- many widespread distributions - uniform, normal, Laplace, etc., - are discrete special cases of this continuous family, the distribution density of which has the following form Equation 1:

$$p(x) = \frac{\alpha}{2\lambda\sigma\Gamma\left(\frac{1}{\alpha}\right)} \exp\left(-\left|\frac{x-m_X}{\lambda\sigma}\right|^{\alpha}\right) \quad (1)$$

where $\lambda = \sqrt{\frac{\Gamma\left(\frac{1}{\alpha}\right)}{\Gamma\left(\frac{3}{\alpha}\right)}}$;

α- degree index;

σ- standard deviation;

m_X- mathematical expectation;

$\Gamma(z)$ - gamma function.

In the conditions of limited computational capabilities of that time, the presence of such a family of symmetric unimodal distributions eliminated the problems of fast reproduction of sequences of random variables used in statistical experiments (19).

The parameter uniting all these distributions is the parameter α of the degree of Equation 1, which is found by solving Equation 2:

$$\varepsilon = \Gamma\left(\frac{1}{\alpha}\right)\Gamma\left(\frac{5}{\alpha}\right)\left[\Gamma\left(\frac{3}{\alpha}\right)\right]^2 \quad (2)$$

It should be noted that the term "kurtosis" is not quite correct from the conceptual point of view. In the article we will use the term "kurtosis index", which will be defined by Equation 3 to characterize, as well as its "fellow", the islandarity of the distribution of random variables.

$$\varepsilon = \frac{\mu_4}{\sigma^4}, \mu_4 = \sum(\mathrm{x} - m_x)^4 \quad (3)$$

Some values of the kurtosis index corresponding to discrete special cases are presented in Table 1.

The modeling algorithm was obtained by traditional methods - inverse function combined with operator series of random variables (20, 21). To solve the problem in a general form (α - unknown exponent of degree), the value $x_0 = mx$ was used as a reference point, since it is the only point through which all curves of distribution laws of the family of symmetric unimodal distributions pass, defined by the ratio: F(x<x_0) = 0.5.

After a number of transformations, the final expressions for the quantiles of distribution Equation 1 in the form Equation 4 are obtained:

$$x_i = m_x + (\beta_i - 0.5)A\,exp|\chi|^\alpha + \frac{1}{2}(\beta_i - 0.5)^2(A^2\,exp(2|\chi|^\alpha)\alpha|\chi|^{\alpha-1})) + +\frac{1}{6}(\beta_i - 0.5)^3\{A^3\,exp(3|\chi|^\alpha)(2\alpha^2|\chi|^{2\alpha-1} + \alpha(\alpha - 1)|\chi|^{\alpha-2})\} + \cdots \quad (4)$$

where $\mathrm{A} = \frac{2\lambda\sigma\Gamma\left(\frac{1}{\alpha}\right)}{\alpha}$; $\chi = \frac{\mathrm{x}_0 - m_x}{\lambda\sigma}$.

Comparing the values of the components of dependence Equation 4 and examining the behavior of the function when changing the parameter α, we can conclude that the series converges rather quickly, which is confirmed by the results of computational experiment, which allows us to traditionally limit ourselves to the first three terms of series in Equation 4.

Then the expression for the generator of random variables realizing the probabilistic properties of the family of symmetric unimodal distributions x_i will take the form Equation 5:

TABLE 1. The most common distribution laws and their kurtosis indices

Type of distribution	Indicator value
uniform	ε = 1.5
normal	ε = 3
Laplace	ε = 6
close to Cauchy	ε = 28

$$x_i = m_x + \mathrm{T}_1 * \gamma_i + \mathrm{T}_2 * \gamma_i{}^2 + \mathrm{T}_3 * \gamma_i{}^3 \quad (5)$$

where $\mathrm{T}_1 = A\exp|\chi|^\alpha$;
$\mathrm{T}_2 = \frac{1}{2(A^2\exp(|2\chi|^\alpha)\alpha|\chi|^{\alpha-1})}$;
$\mathrm{T}_3 = \frac{1}{6}\{A^3\exp(|3\chi|^\alpha)(2\alpha^2|\chi|^{2\alpha-1} + \alpha(\alpha - 1)|\chi|^{\alpha-2})\}$, where $\gamma_i = \beta_i - 0.5$ - uniformly distributed in the interval [-0.5;0.5] of random variables.

Thus, the generation of a random number xi is reduced to obtaining a random number x_i distributed uniformly in the interval [0;1], with subsequent transformation by dependence Equation 5 (22-24).

The integral function of this distribution is obtained by the method of decreasing the degree of integrand expressions and is represented by dependence Equation 6:

$$F_1(\xi) = \sum_{i=1}^{n}\left[(-1)^{i+1}A^i e^{t^\alpha}\cdot\frac{(1-(i-1)\alpha)}{\alpha^2}\cdot t^{(1-i\alpha)}\right] - A\int\frac{\prod_{i=1}^{n}(1-i\alpha)}{\alpha^n}t^{-n\alpha}e^{t^\alpha} \quad (6)$$

where $A = \frac{\alpha}{2\lambda\sigma\Gamma(1/\alpha)}$.

The error in each case can be considered as the value remaining under the integral due to its smallness by means of the following expressions Equation 7:

$$\Delta_1(\xi) = \sum_{i=n+1}^{\infty}(-1)^{i+1}A^i e^{t^\alpha}\cdot\frac{(1-(i-1)\alpha)}{\alpha^{i+1}}\cdot t^{(1-i\alpha)} \quad (7)$$

where $\Delta_1(\xi)$ - modeling errors due to failure to take into account $\overline{n+1,\infty}$ terms of the integral function series Equation 6.

This approach made it possible to implement algorithms for generating pseudo-random values with the required, although not always high, accuracy, limited by the capabilities of modern.

Thus, at the "turn of the century" the technology of using the family of symmetric unimodal distributions to construct the required statistical models looked as follows:

On the basis of the available experimental data, the value of the kurtosis index ε was found using known statistical methods (25, 26).

The value of the degree index α was determined according to the pre-calculated tables ε =F(α). The most characteristic ones are presented in Table 2 by the maximum modeling error, usually given in %, the minimum number of series members n was determined;
generation of the required number of random variables obeying the law from the family of symmetric unimodal distributions (27, 28).

The introduction of a significant number of computer algebra computing systems allowed to significantly reduce the labor intensity of modeling, but did not solve the main problem of the researcher - how to obtain intermediate distributions, for example, trapezoidal (with the kurtosis index ε = 2.2...2.7) or intermediate between normal and Laplace distributions (with the kurtosis index ε = 4...5), also frequently encountered in practice (29). In addition, the proposed procedure had a low speed, which significantly depended on the given accuracy of random variables generation.

The above methodology and the modeling algorithms implementing them have played a positive role in the development, testing and operation of some complex technical systems (30-32). However, due to the qualitatively new level of development of modern computer technology, increasing requirements to the accuracy of modeling random variables, this approach has not been significantly developed.

The method of forming a pseudorandom number generator based on the sum of pseudorandom values. An alternative to the proposed method can be the realization of the calculation-analytical method of obtaining the generator of random variables with the given probabilistic characteristics described by the first four moments of the distribution, based on the transformation of the expression of the distribution excess of the sum of two independent random variables can be written as follows Equation 8:

$$\mu_4(y_\Sigma) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(y_1 + y_2)^4 p(y_1)\, p(y_2) dy_1 dy_2 = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}({y_1}^4 + 4{y_1}^3 y_2 + 6{y_1}^2{y_2}^2 + 4y_1{y_2}^3 + {y_2}^4)p(y_1)\, p(y_2) dy_1 dy_2 \rightleftarrows \mu_4(y_1) + 4\mu_3(y_1)\mu_1(y_2) + 6\mu_2(y_1)\mu_2(y_2) + 4\mu_1(y_1)\mu_3(y_2) + \mu_4(y_2) \quad (8)$$

where $\mu_r(t)$ is the central moment of random variables t.

TABLE 2. Dependence of the value of the distribution excess index on the degree index of the family of symmetric unimodal distributions

α	Index of excess	α	Index of excess
0,4	51,95	4	2,19
0,5	25,2	5	2,07
0,6	15,58	6	2
0,7	11,068	7	1,95
0,8	8,57	8	1,92
0,9	7,03	9	1,90
1	6	10	1,88
1,2	4,74	20	1,824442
1,4	4,02	30	1,811516
1,6	3,55	40	1,806683
1,8	3,23	50	1,804361
2	3	100	1,801135
2,5	2,63	500	1,800047
3	2,42	1000	1,800012

Accounting for the assumption of symmetric distributions, viz: $\mu_3(y_1) = 0, \mu_3(y_2) = 0$,
It allows us to simplify Equation 8 and get Equation 9:

$$\mu_4(y_\Sigma) \rightleftarrows \mu_4(y_1) + 6\sigma^2(y_1)\sigma^2(y_2) + \mu_4(y_2) \quad (9)$$

Value of the excess of the total distribution allows us to determine the form of the distribution law according to Equation 10.

$$\varepsilon_{y_\Sigma} = \frac{\mu_4(y_\Sigma)}{\sigma^4(y_\Sigma)} = \frac{\rightleftarrows \mu_4(y_1) + 6\sigma^2(y_1)\sigma^2(y_2) + \mu_4(y_2)}{\left(\sigma^2(y_1) + \sigma^2(y_2)\right)^2} \quad (10)$$

More practically significant, in the opinion of the authors of the article, is the problem of obtaining a symmetric unimodal distribution with the required probabilistic properties in the form of a sequence of random variables, each element of which is obtained on the basis of the procedure of pairwise weighted summation of two elements of standard sequences, the solution of which is of significant practical importance (33-35).

To solve it, we introduce the relation $s = \frac{\sigma(y_1)}{\sigma(y_2)}$, which allows us to reduce the dimensionality of the problem to be solved.

Then Equation 10 takes the form Equation 11:

$$\varepsilon_{y_\Sigma} = \frac{\varepsilon_{y_1}}{A^2} + \frac{6}{AB} + \frac{\varepsilon_{y_2}}{B^2} \quad (11)$$

where $A = 1 + s^2$; $B = 1 + \frac{1}{s^2}$ - certain coefficients.

The expressions for $\sigma(y_\Sigma)$ are similarly obtained Equation 12:

$$\sigma_{y_\Sigma} = \frac{\sigma_{y_1}}{\sqrt{A}} \quad or \quad \sigma_{y_\Sigma} = \frac{\sigma_{y_2}}{\sqrt{B}} \quad (12)$$

Analysis of Equation 11 shows that it is an analytic dependence of the four quantities $\varepsilon_{y_\Sigma}, \varepsilon_{y_1}, \varepsilon_{y_2}, s(A, B)$ and can be transformed to the form Equation 13:

$$\varepsilon_{y_\Sigma} = \frac{\varepsilon_{y_1}}{(1+s^2)^2} + \frac{6}{(1+s^2)\left(1+\frac{1}{s^2}\right)} + \frac{\varepsilon_{y_2}}{\left(1+\frac{1}{s^2}\right)^2} \quad (13)$$

whence by simple transformations we obtain $(1 + s^2)^2\varepsilon_{y_\Sigma} = \varepsilon_{y_1} + 6 \cdot s^2 + s^4\varepsilon_{y_2}$.

After substituting $s^2 = \rho, \rho \geq 0$, opening the parentheses and adding similar terms, the expression $\rho^2\left(\varepsilon_{y_2} - \varepsilon_{y_\Sigma}\right) + \rho\left(6 - 2 \cdot \varepsilon_{y_\Sigma}\right) + \left(\varepsilon_{y_1} - \varepsilon_{y_\Sigma}\right) = 0$

The roots of this quadratic equation are defined as Equation 14:

$$\rho_{1,2} = \frac{-(6 - 2\cdot\varepsilon_{y_\Sigma}) \pm \sqrt{(6 - 2\cdot\varepsilon_{y_\Sigma})^2 - 4\cdot(\varepsilon_{y_2} - \varepsilon_{y_\Sigma})(\varepsilon_{y_1} - \varepsilon_{y_\Sigma})}}{2\cdot(\varepsilon_{y_2} - \varepsilon_{y_\Sigma})} \quad (14)$$

The obtained expression allows to realize the proposed idea as an algorithm consisting of the following stages.

Stage 1. Input of initial data, which are understood as probability characteristics (mathematical expectation $\mu_1(y_\Sigma)$, standard deviation σ_{y_Σ} and excess ε_{y_Σ}) of the distribution of random variables for which it is necessary to create an approximation model. The asymmetry coefficient is equal to 0, as already indicated above.

Step 2. Selection of the underlying sensors. For this purpose it is necessary that the discriminant of expression D≥ 0. This condition is fulfilled in the case of Equation 15:

$$\left(6-2\cdot\varepsilon_{y_\Sigma}\right)^2 \geq 4\cdot\left(\varepsilon_{y_2}-\varepsilon_{y_\Sigma}\right)\left(\varepsilon_{y_1}-\varepsilon_{y_\Sigma}\right) \tag{15}$$

A series of simple transformations leads to the expressions Equation 16 and Equation 17:

$$\varepsilon_{y_\Sigma} \leq \frac{9-\varepsilon_{y_2}\varepsilon_{y_1}}{6-\varepsilon_{y_2}-\varepsilon_{y_1}} \tag{16}$$

$$\varepsilon_{y_\Sigma} \geq \frac{\varepsilon_{y_2}\varepsilon_{y_1}-9}{\varepsilon_{y_2}+\varepsilon_{y_1}-6} \tag{17}$$

Substituting specific values of $\varepsilon_{y_\Sigma}, \varepsilon_{y_1}, \varepsilon_{y_2}$ allows us to determine which sensors can be used to model the random variables.

For approximate estimation of possible combinations of base random number sensors, the value of the excess of the modeled distribution should be between the values of the excesses of the base random number sensors, because if this rule is satisfied, the subcorrected function of Equation 14 will be positive (36).

Step 3. Calculating the values of $\rho_{1,2}$. Exclusion of negative ones.

Step 4. Calculation of s values. Exclusion of negative ones.

Step 5. Calculation of $A=1+s^2$; $B=1+\frac{1}{s^2}$.

Step 6. Calculating $\sigma_{y_1}=\frac{\sigma_{y_\Sigma}}{\sqrt{A}}$ and $\sigma_{y_2}=\frac{\sigma_{y_\Sigma}}{\sqrt{B}}$.

Table 3 shows some values of ε_{y_1} and ε_{y_2} for the main combinations of basic random number sensors when varying the value of the excess of the modeled distribution in the interval [1.5; 28]. To simplify the calculations, the value of the standard deviation of the modeled distribution is taken σ_{y_Σ}=1.

Stage 7. Generation of random variables with calculated parameters.

For this purpose, random number sensors with calculated at the previous stage of standard deviation are generated, after which the desired distribution is modeled as random variables of the combination of basic random number sensors (37).

Stage 8. Verification of the values of probability characteristics.

The necessity of this stage is due to the fact that the probabilistic characteristics of real distributions modeled by the base random number sensors differ from those put into the model by the values of moments, especially by the third and fourth (38, 39). In this case, it is necessary to apply sample censoring or multiple repetition of the procedure of modeling of the basic random number sensor in order to select and "freeze" the sample closest to the reference sample.

Stage 9. Obtaining a given random variable by implementing the expression 18:

$$y_{\Sigma_i}=\sigma_{y_\Sigma}\cdot\eta_i+\mu_1(y_\Sigma) \tag{18}$$

where y_{Σ_i} - the final random variable with the given parameters; η_i- the normalized final random variable.

TABLE 3. Dependence of the index of excess of the final distribution on the indexes of excess of the initial distributions

$\boldsymbol{\varepsilon_{y_1}=1,5}$		$\boldsymbol{\varepsilon_{y_1}=1,5}$		$\boldsymbol{\varepsilon_{y_1}=1,5}$		$\boldsymbol{\varepsilon_{y_1}=3}$		$\boldsymbol{\varepsilon_{y_1}=3}$		$\boldsymbol{\varepsilon_{y_1}=6}$	
$\boldsymbol{\varepsilon_{y_2}=6}$		$\boldsymbol{\varepsilon_{y_2}=6}$		$\boldsymbol{\varepsilon_{y_2}=28}$		$\boldsymbol{\varepsilon_{y_2}=6}$		$\boldsymbol{\varepsilon_{y_2}=28}$		$\boldsymbol{\varepsilon_{y_2}=28}$	
$\boldsymbol{s}$	$\boldsymbol{\sigma_{y_\Sigma}}$	$\boldsymbol{s}$	$\boldsymbol{\sigma_{y_\Sigma}}$	$\boldsymbol{s}$	$\boldsymbol{\sigma_{y_\Sigma}}$	$\boldsymbol{s}$	$\boldsymbol{\sigma_{y_\Sigma}}$	$\boldsymbol{s}$	$\boldsymbol{\sigma_{y_\Sigma}}$	$\boldsymbol{s}$	$\boldsymbol{\sigma_{y_\Sigma}}$
0,05	1,51	0,05	1,51	0,05	1,51	0,05	3,00	0,05	3,00	0,05	5,99
0,10	1,53	0,10	1,53	0,10	1,53	0,10	3,00	0,10	3,00	0,10	5,94
0,15	1,57	0,15	1,57	0,15	1,58	0,15	3,00	0,15	3,01	0,15	5,88
0,20	1,61	0,20	1,62	0,20	1,65	0,20	3,00	0,20	3,04	0,20	5,81
0,25	1,67	0,25	1,68	0,25	1,76	0,25	3,01	0,25	3,09	0,25	5,74
0,30	1,74	0,30	1,76	0,30	1,91	0,30	3,02	0,30	3,17	0,30	5,70
0,35	1,81	0,35	1,85	0,35	2,11	0,35	3,04	0,35	3,30	0,35	5,68
0,40	1,89	0,40	1,94	0,40	2,36	0,40	3,06	0,40	3,48	0,40	5,71
0,45	1,96	0,45	2,05	0,45	2,67	0,45	3,09	0,45	3,71	0,45	5,78
0,50	2,04	0,50	2,16	0,50	3,04	0,50	3,12	0,50	4,00	0,50	5,92
0,60	2,19	0,60	2,40	0,60	3,94	0,60	3,21	0,60	4,75	0,60	6,37
0,70	2,32	0,70	2,65	0,70	5,03	0,70	3,32	0,70	5,70	0,70	7,05
0,80	2,44	0,80	2,90	0,80	6,25	0,80	3,46	0,80	6,81	0,80	7,92
0,90	2,54	0,90	3,14	0,90	7,55	0,90	3,60	0,90	8,01	0,90	8,92

1,00	2,63	1,00	3,38	1,00	8,88	1,00	3,75	1,00	9,25	1,00	10,00
1,20	2,75	1,20	3,79	1,20	11,46	1,20	4,04	1,20	11,71	1,20	12,21
1,40	2,83	1,40	4,14	1,40	13,79	1,40	4,32	1,40	13,96	1,40	14,30
1,60	2,88	1,60	4,43	1,60	15,81	1,60	4,55	1,60	15,93	1,60	16,16
1,80	2,92	1,80	4,67	1,80	17,51	1,80	4,75	1,80	17,60	1,80	17,77
2,00	2,94	2,00	4,86	2,00	18,94	2,00	4,92	2,00	19,00	2,00	19,12
2,50	2,97	2,50	5,20	2,50	21,55	2,50	5,23	2,50	21,58	2,50	21,64
3,00	2,99	3,00	5,42	3,00	23,24	3,00	5,43	3,00	23,25	3,00	23,28
3,50	2,99	3,50	5,56	3,50	24,36	3,50	5,56	3,50	24,37	3,50	24,39
4,00	2,99	4,00	5,65	4,00	25,14	4,00	5,66	4,00	25,15	4,00	25,16
4,50	3,00	4,50	5,72	4,50	25,70	4,50	5,72	4,50	25,70	4,50	25,71
5,00	3,00	5,00	5,77	5,00	26,11	5,00	5,77	5,00	26,11	5,00	26,12
6,00	3,00	6,00	5,84	6,00	26,67	6,00	5,84	6,00	26,67	6,00	26,67
7,00	3,00	7,00	5,88	7,00	27,01	7,00	5,88	7,00	27,01	7,00	27,01
8,00	3,00	8,00	5,91	8,00	27,24	8,00	5,91	8,00	27,24	8,00	27,24
9,00	3,00	9,00	5,93	9,00	27,39	9,00	5,93	9,00	27,39	9,00	27,39
10,0	3,00	10,0	5,94	10,0	27,51	10,0	5,94	10,0	27,51	10,0	27,51

3. RESULTS

Experiment 1.

The purpose of the computational experiment was the need to probabilistically evaluate the accuracy of generating sequences of pseudo-random values of standard distributions from the family of symmetric unimodal distributions, in particular, in the Mathcad15 computing environment (40-42).

For this purpose:

generation of the sequence of pseudorandom values of the studied generator of standard distributions consisting of N1 random variables was carried out:

N1= 10, 15, 20, 25, 30, 35, 40, 45, 50, 100, 1000.

For this sequence of pseudo-random values, probabilistic characteristics - mathematical expectation, standard deviation, skewness and kurtosis were calculated; the calculated values were saved;these procedures were repeated 1000 times, after which the probability characteristics were calculated again.

The experimental results presented in Tables 4-6 for uniform (ε = 1.8), normal (ε = 3) and Cauchy (ε = 25) respectively show that the following consistent trends are observed:

- with the increase in the number of generations in the sample - changes in the average value of the kurtosis (decrease - for the uniform law, increase - for the normal law) and reduction of its spread;
- there is a convergence of the values of the pseudorandom value generation kurtosis index to the steady-state value as the number of generations increases. For the uniform law this value is 1.8, for the normal law - 3.

TABLE 4. Estimation of the accuracy of the sensor of pseudo-random variables distributed according to the uniform law

Significance of probability characteristics	Number of random variables in the sample										
	10	15	20	25	30	35	40	45	50	100	500
Mathematical expectation	2,01	1,94	1,90	1,92	1,88	1,87	1,87	1,85	1,86	1,83	1,81
RMS deviation	0,55	0,43	0,34	0,29	0,25	0,23	0,22	0,20	0,19	0,12	0,05
Asymmetry	1,66	2,07	1,56	1,13	0,80	0,92	1,00	0,82	0,81	0,35	0,23
ε	7,18	12,9	8,10	5,40	4,25	4,58	5,27	4,12	3,91	3,27	3,12
Lower boundary	1,70	1,69	1,68	1,72	1,70	1,70	1,72	1,70	1,72	1,73	1,77
Upper boundary	2,32	2,19	2,12	2,12	2,06	2,04	2,02	2,00	2,00	1,93	1,85

TABLE 5. Estimation of the accuracy of the sensor of pseudo-random variables distributed according to the normal law

Significance of probability characteristics	Number of random variables in the sample										
	10	15	20	25	30	35	40	45	50	100	500
Mathematical expectation	2,46	2,63	2,65	2,78	2,81	2,79	2,91	2,87	2,87	2,95	2,99
RMS deviation	0,75	0,80	0,67	0,77	0,70	0,65	0,65	0,63	0,59	0,46	0,21
Asymmetry	1,48	1,71	1,34	1,77	1,73	1,97	1,17	1,98	1,46	1,42	0,64
ε	6,01	7,61	5,95	7,66	7,76	11,7	5,47	11,3	7,32	9,22	4,05
Lower boundary	2,15	2,38	2,43	2,58	2,63	2,62	2,76	2,72	2,73	2,85	2,95
Upper boundary	2,77	2,88	2,87	2,98	2,99	2,96	3,06	3,02	3,01	3,05	3,03

TABLE 6. Estimation of the accuracy of the sensor of pseudorandom variables distributed according to a law close to Cauchy

Significance of probability characteristics	Number of random variables in the sample										
	10	15	20	25	30	35	40	45	50	100	500
Mathematical expectation	4,80	7,25	9,74	12,6	15,3	16,8	19,9	22,2	24,7	50,9	250
RMS deviation	2,01	3,39	4,79	6,36	7,84	9,20	10,7	12,3	13,5	27,3	143
Asymmetry	0,21	0,33	0,34	0,24	0,24	0,38	0,31	0,38	0,34	0,29	0,36
ε	1,73	1,83	1,86	1,70	1,71	1,81	1,77	1,75	1,77	1,82	1,77
Lower boundary	4,49	7,00	9,52	12,4	15,1	16,6	19,75	22,05	24,56	50,80	249,96
Upper boundary	5,11	7,50	9,96	12,8	15,5	16,97	20,05	22,35	24,84	51,00	250,04

Regarding the accuracy of generation of pseudorandom values close to the Cauchy distribution by its properties, we can state that there is no convergence of the kurtosis value to the steady-state value when the number of generations increases.

The determination of the confidence intervals boundaries characterizing the accuracy of reproducing the sequences of pseudorandom values using the standard procedures of the computing environment was carried out using the standard procedures of the Mathcad15 computing environment at the significance level $\alpha = 0.05$ and is presented in the bottom two rows of each table. These results can be considered as generalized characteristics of the error of the Mathcad15 computing environment.

The conducted sample check of the accuracy of generation of random number sensors from the family of symmetric unimodal distributions by the first four moments of distribution in the intervals of 10...500 values, the results of which are presented above, has shown the possibility of using only generators of pseudo-random uniformly and normally distributed numbers in computational experiments requiring some repeatability of results.

In parallel and in a similar way, the probabilistic characteristics of a random generator realizing the properties of the normal distribution based on the sum of 12 random number sensors with a uniform distribution law of the kind were tested Equation 19:

TABLE 7. Estimation of the accuracy of the sensor of pseudo-random variables distributed according to the normal law obtained by Equation 19

Significance of probability characteristics	Number of random variables in the sample										
	10	15	20	25	30	35	40	45	50	100	500
Mathematical expectation	2,49	2,58	2,66	2,69	2,74	2,74	2,79	2,81	2,79	2,86	2,88
RMS deviation	0,74	0,76	0,73	0,62	0,63	0,61	0,59	0,55	0,52	0,40	0,19
Asymmetry	1,28	1,67	1,62	1,09	1,38	1,45	1,53	0,98	1,27	1,14	0,55
ε	4,82	7,24	7,01	4,49	6,92	6,32	6,92	4,13	6,15	5,85	3,84
Lower boundary	2,18	2,33	2,44	2,49	2,56	2,57	2,64	2,66	2,65	2,76	2,84
Upper boundary	2,80	2,83	2,88	2,89	2,92	2,91	2,94	2,96	2,93	2,96	2,92

TABLE 8. Evaluation of the accuracy of the implementation of the “gluing” procedure

Probabilistic characteristics		Experiment number	Probabilistic characteristics	Experiment number	Probabilistic characteristics	Experiment number	Probabilistic characteristics
		1	2	3	4	5	6
First follow N = 50	a_{y1}	-0,00733	0,03622	0,04242	0,01569	-0,01441	-0,00733
	ε_{y1}	16,00000	9,41600	12,50000	6,17800	12,07000	16,00000
Second follow N = 50	a_{y2}	0,05929	0,01569	0,01422	-0,03238	0,07934	0,08135
	ε_{y2}	10,24000	6,17800	7,74600	7,27300	7,17400	10,43000
Association N = 100	$a_{y\Sigma}$	0,02558	0,02555	0,02789	-0,00822	0,03196	0,03644
	$\varepsilon_{y\Sigma}$	13,12000	7,79700	10,12000	6,72500	9,62400	13,21000
Estimated values	$a_{y\Sigma}$	0,02598	0,02596	0,02832	-0,00835	0,03247	0,03701
	$\varepsilon_{y\Sigma}$	13,12000	7,79700	10,12300	6,72550	9,62200	13,21500

TABLE 9. Results of modeling the sum of elements of two distributions at s = 1 and и N = 1000

Values ε_{y_Σ}	Sensor Excesses					
Random number sensors 1	$\varepsilon_{y_1} = 1,5$	$\varepsilon_{y_1} = 1,5$	$\varepsilon_{y_1} = 1,5$	$\varepsilon_{y_1} = 3$	$\varepsilon_{y_1} = 3$	$\varepsilon_{y_1} = 6$
Random number sensors 2	$\varepsilon_{y_2} = 3$	$\varepsilon_{y_2} = 6$	$\varepsilon_{y_2} = 28$	$\varepsilon_{y_2} = 6$	$\varepsilon_{y_2} = 28$	$\varepsilon_{y_2} = 28$
Estimated	2,63	3,38	8,88	3,75	9,25	10,00
Experimental	2,63	3,41	8,69	3,73	9,09	9,67

$$n_j = \frac{1}{6}\sum_{i=1}^{12}\xi_i - 6 \quad (19)$$

where ξ - uniformly distributed random variables.

The results of the auxiliary experiment are presented in Table 7. Their comparative analysis with the results of Table 5 convincingly demonstrates that the pseudo-random variable generator presented in the Mathcad15 computing environment (Table 5) is preferable, since it is characterized by a higher rate of convergence of the excess indicator to a value of 3 with an increase in the number of generations in the analyzed sequence with other parameters being approximately the same (43, 44).

Experiment 2.

The purpose of the computational experiment was to evaluate the possibility and subsequent realization of the procedure of forming a system of “basic” sequences of pseudorandom values from a family of symmetric unimodal distributions (45-47). Preliminary experiments in this direction allowed us to recommend the following extensive “lego-technology”:

An array of pseudo-random values with dimensionality, for example, 50 x 1000 of one of the standard distributions (uniform, normal or Cauchy) is generated;

The first four moments of the distribution are calculated for them, after which the normalization and centering procedures are performed (48, 49).

The matrix of asymmetry and kurtosis values for the whole data set is used to select and save (in some sources - “freezing”) a sequence of 50 pseudorandom values with the minimum asymmetry value a ≤ 0.1. Such asymmetry accuracy is set taking into account the subsequent experiments. Otherwise, the asymmetry of the sequences of pseudorandom values introduces significant distortions in the obtained experimental results. The value of 50 is chosen for the reasons of ensuring a significant number of pseudorandom samples with a minimum value of asymmetry (50-52).

After finding the maximum possible number of sequences, limited only by the researcher's patience, the sequences are “glued” together using the standard matrix operator “stack (A, B)” in the Mathcad15 computing environment, which forms a matrix whose first rows contain matrix A and the last rows contain matrix B (matrices A and B have the same number of columns) (53, 54). It has been established that in a sufficiently wide range of information situations characterized by a scatter of excess values with minimal asymmetry for centered and normalized values of pseudorandom variables approximate expressions for analytical express estimation of the gluing result can be applied, which is confirmed by the results presented.

To reduce the asymmetry value of the unifying sequence, sequences with opposite in sign asymmetry values can be selected (or specially located), as shown in experiments 1 and 4 (55-57).

In the future, these basic sequences, like lego cubes, can be connected to obtain other sequences with the required generation accuracy and, most importantly, with the required dimensionality (58-60).

This procedure will be further automated and proposed as an independent technique for generating pseudorandom values.

The result of this experiment was some information array of sequences of pseudorandom value sequences saved as a separate file (61-63).

Thus, the results of the experiment indicate that obtaining a system of “basic” sequences of pseudorandom values from the family of symmetric unimodal distributions is possible, although it is a labor-intensive procedure without automation.

Experiment 3.
The results of the experiment convincingly demonstrate the possibility of obtaining sensors of pseudorandom variables from the family of symmetric unimodal distributions with the required probabilistic characteristics (64, 65).

To confirm the performance of the algorithm described by expressions 8-17, a computational experiment was conducted, the essence of which is as follows.

Two specially formed normalized sequences of pseudo-random variables (mathematical expectation $\mu_1(y_\Sigma)$= 0, standard deviation σ_{y_Σ}= 1) of the same dimension with known excess indices are selected. The "gluing" procedure can be used for this.

To reduce the computational procedures, the relation s = 1 is used.

Using the pairwise addition method, a new sequence of pseudo-random variables is obtained, for which the excess coefficient ε_{y_Σ} is calculated and compared with the calculated one obtained using expression 13.

Some results of the experiment to determine the probability characteristics of the sum of two random variables for the case of 1000 pseudo-random variables are given in Table 9.

The results of the experiment convincingly demonstrate the possibility of obtaining sensors of pseudo-random variables from a family of symmetric single-modal distributions with the required probability characteristics.

Experiment 4.
The purpose of this experiment was to evaluate the accuracy of the kurtosis index determination and compare it with the accuracy of applying the mathematical apparatus of the theory of hypothesis testing (66, 67).

This experiment consisted in testing the simple hypothesis of the normal distribution law for populations consisting of 100 pseudo-random variables with different distribution laws obtained during the previous experiment (68).

For its realization, the χ^2 Pearson S_{χ^2} concordance statistic was calculated for each population according to the method for 11 equal intervals (estimated value). The obtained results are summarized in Table 10. Their analysis shows that the boundary value of the statistic $S_{\chi^2}^{\text{ГР}}$= 18.34 is larger than the estimated value over a wide range of values of the kurtosis parameter $\varepsilon \in [1{,}8\ ;4{,}36]$.

As a result, even with 20 measurements of normally distributed random variables, the resulting confidence interval is much narrower than suggested by the mathematical apparatus of hypothesis testing theory for 100 generations.

Thus, the obtained results indicate that the accuracy of determining the type of distribution law by direct determination of the kurtosis index for a family of symmetric unimodal distributions is at least as good as that of using the mathematical apparatus of the theory of hypothesis testing (69).

The values of asymmetry of the initial standard sequences given in Table 10 show its influence on the obtained result, in particular the value of the statistic S_{χ^2}; in particular, it is noted that the distortion of the results

TABLE 10. Results of the experiment to test the simple hypothesis of the normal distribution law for a family of symmetric unimodal distributions

The meaning of asymmetry *a*	Value of the kurtosis index	Importance of statistics	Hypothesis Accepted +
0	1,59	34,95	-
-0,02	1,7	21,79	-
0	1,8	15,64	+
-0,01	1,91	13,62	+
0,01	2,01	12,87	+
0	2,12	9,87	+
-0,02	2,21	15,34	+
0,11	2,34	8,1	+
0,01	2,54	3,88	+
-0,09	2,75	3,36	+
0	3,06	7,02	+
0,04	3,61	6,37	+
-0,07	4,08	11,66	+
-0,02	4,36	14,63	+
0,07	4,66	57,02	-
-0,08	5,21	35,99	-
-0,04	5,69	52,42	-
0,06	6,3	92,66	-

of the computational experiment at *a*>0.01 becomes significant.

4. DISCUSSION

It should be noted that the fourth moment of distribution - excess and indicators on its basis in the statistical literature paid insufficient attention, most likely because of the labor intensity of its calculation at the initial stages of development of statistics as a science, when calculators, much less computers, did not exist (70, 71). For the modern stage of development of computer technology, a rather simple form of analytical definition of the kurtosis index and its availability as a standard in most application programs opens wide prospects for its application as some “statistical measure” of the form of distribution of a random variable.

The check of the accuracy of generation of random number sensors from the family of symmetric unimodal distributions, carried out in experiment 1, has shown the possibility of using this indicator as the main one in solving the problem of determining the type of distribution law even in the case of significant information limitation (N = 20).

This is essential in the context of the creation and implementation of the concept of digital twins, as well as the need for compact storage of information, since the modern concept of storing "big data" assumes that in the case of consistency of several hypotheses when determining the type of distribution law, the original sample is stored unprocessed - in the form of a "data lake". Identification of the distribution as a family of symmetric unimodal distributions allows to significantly reduce the storage volume to four numerical values of probability characteristics, and the developed procedures - to restore the sequence of pseudo-random variables with properties close to the original ones.

4. CONCLUSIONS

Since the decision-making procedure in most cases is accompanied by information uncertainty, the parameters of the distribution laws of random variables are determined only with a certain confidence probability, and the type of distribution laws - with significant discreteness determined by the application of the theory of testing statistical hypotheses.

Obtained during the development of the mathematical and algorithmic apparatus for the implementation of statistical models based on symmetric unimodal distributions

The use of a family of symmetric unimodal distributions makes it possible to determine the type of distribution law with higher accuracy than the algorithms of the theory of hypothesis testing, at least with respect to the family of symmetric unimodal distributions.

As a result of the computational experiment, the proposed procedures were substantiated and refined, and patterns were identified that contributed to the development of a new procedure for obtaining sequences of pseudo-random variables.

The results obtained in this article are the basis for further research and relevant publications; they allow continuing work in the chosen direction by conducting:

- computational experiments to assess the performance indicators of the obtained modeling algorithms and the possibility of their use in digital twins operating in real time;
- comparative analysis of performance (accuracy, speed, convergence) with standard methods, such as the Box-Muller method or sampling with deviation;
- checking the proximity of the generated data to theoretical distributions based on well-known statistical tests (Kolmogorov-Smirnov, Anderson-Darling, etc.).

This would allow us to expand the problem area of the study to the class of asymmetric unimodal and multivariate distributions of unlimited dimension, and subsequently - to enhance the applied significance of the work by creating an autonomous package of application programs or an embedded software package in modern statistical computing environments.

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Persian Abstract

چکیده

پیچیدگی روزافزون سیستم‌های فنی مدرن و افزایش تقاضا برای کارایی عملیاتی، مدل‌ها و روش‌های عددی پیشرفته‌تری را ایجاب می‌کند. این مقاله به توسعه و تجزیه و تحلیل مدل‌های آماری می‌پردازد و بر رویکردی تعمیم‌دهنده تأکید می‌کند که تابع توزیع انتگرال را با کاهش درجه عبارات زیر انتگرال استخراج می‌کند. بخش باقیمانده به عنوان خطای مدل‌سازی در نظر گرفته می‌شود و به الگوریتم‌های تولید مقادیر شبه تصادفی با دقت محدود تنها توسط توان محاسباتی اجازه می‌دهد. روشی برای تولید اعداد شبه تصادفی از طریق مجموع مقادیر شبه تصادفی، حل مشکل ساخت یک توزیع متقارن تک وجهی با خواص احتمالی مورد نظر پیشنهاد شده است. این از طریق جمع وزنی زوجی عناصر از توالی های استاندارد به دست می آید. عبارات تحلیلی کلیدی زیربنای روش مشتق شده و ارائه شده است. آزمایش‌های محاسباتی جامع چندین پیشرفت را نشان می‌دهند: افزایش تصمیم‌گیری در تخمین ویژگی‌های احتمالی با استفاده از خانواده‌های توزیع متقارن تک‌وجهی. تعیین دقیق مرزهای فاصله اطمینان که منعکس کننده دقت بازتولید توالی شبه تصادفی در محیط Mathcad15 است. رویه‌های بهبود یافته برای تشکیل و ارزیابی توالی‌های اساسی متغیرهای شبه تصادفی، که منجر به دقت تولید بهتر می‌شود.