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Competitive Opinion Influence Maximization in Social Networks

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A B S T R A C T

In today's world, social networks have become one of the most important communication tools where individuals and organizations can exchange information and opinions. This significantly affects social attitudes and behaviors, making it essential to understand the processes of information dissemination in this context. This study addresses the influence maximization problem in competitive opinion diffusion. Unlike prior heuristic approaches, we formulated a bi-level mathematical programming model based on game theory, leveraging a Stackelberg game framework to model leader-follower strategic interactions. The model is solved using a genetic algorithm to identify effective dissemination strategies. Findings show that key parameters – delta threshold, social influence, initial adopters, and transmission cost – significantly affect diffusion. The bi-level model optimizes message dissemination across threshold values, highlighting the role of content attractiveness for user engagement. Lower transmission costs boost participation, increasing active nodes. The involvement of influential users at the outset amplifies dissemination. This research demonstrates that optimizing key parameters and reducing costs improves diffusion strategies, enhancing message impact. These results generalize to similar networks and have practical marketing applications.

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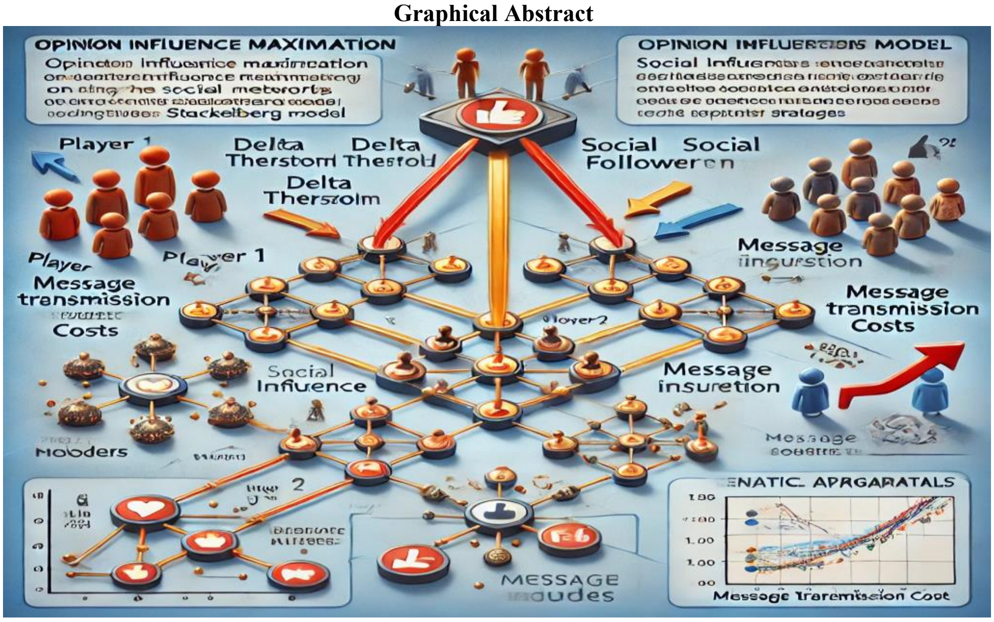
Graphical Abstract



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NOMENCLATURE			
i	Nodes from 1 to n	$p_{iv}(t)$	If at time t, node i disseminates the message of player v
j	Nodes from 1 to n	$s_{ijv}(t)$	If at time t, node i transmits the message of player v to node j
v	1 for leader and 2 for follower	α	Social influence
t	Time from 1 to T	δ	Influence threshold
Z1	First objective function	M	Message
Z2	Second objective function	$N[a_{ij}]$	Network matrix showing transmission thresholds
k	Number of active nodes at the beginning of the period	U	Large number
cv	Message transmission cost for player v	Bv	Available budget for player v
$\in$	Small number		

1. INTRODUCTION

In many social phenomena, competition emerges as individuals, ideas, or slogans spread across networks. Typically, one entity (or a group) initially disseminates its message before competitors enter the scene. Subsequently, other entities begin promoting their alternative options. This creates a competitive environment where individuals, as well as supporters of each option, must decide how to act. Consider, for example, a specific brand of mobile phone with a certain market share. When a new brand with different features enters the market, it directly competes with the existing brand. Similar competitive dynamics are observed in various fields, such as news dissemination, where different outlets may publish conflicting reports to influence public opinion.

These scenarios can be effectively modeled using the Stackelberg game framework, which involves a leader and a follower making decisions sequentially. The leader, often a well-established entity or provider of a superior product, has the first-mover advantage. The follower then determines its strategy after observing the leader's actions. Thus, players compete to improve their position and increase their gains. This framework is also applicable to modeling the spread of diseases or pollution (1).

Given the strategic interactions inherent in competitive diffusion in social networks, a game theory approach is highly significant. Game theory provides a mathematical framework for analyzing these interactions and predicting equilibrium outcomes (1). A key concept in game theory is the Nash equilibrium, where each player's strategy is optimal given the strategies of the other players. In Stackelberg games, the leader and follower must both act strategically to reach a Nash equilibrium.

To effectively model this competitive diffusion, the present study utilizes a mathematical model based on game theory and bi-level programming. Bi-level programming allows us to examine the impact of decisions made by two groups (leader and follower) in a hierarchical, competitive environment. This approach enables us to capture the dynamic interplay between the leader's initial dissemination strategy and the follower's subsequent reactions. Finally, for model analysis and optimization, GAMS software and a genetic algorithm are employed to enhance content diffusion in social networks, improve performance, and demonstrate increased influence in the network.

2. PROBLEM STATEMENT

In today's world, social networks are pivotal in information exchange, connecting individuals and groups in intricate patterns. These networks significantly impact information dissemination and influence maximization, playing a crucial role in various sectors. Analyzing social networks is essential due to their impact on information dissemination, sales enhancement in the economic sector, and the exchange of ideas and opinions for advertising. Social network analysis examines individual interactions at the micro level, relationship patterns (network structure) at the macro level, and the interplay between the two. Networks both shape and are shaped by individual behaviors; while network structures offer opportunities, they can also impose limitations depending on the relationships (2).

Identifying the most influential nodes to maximize information diffusion is a key optimization problem in social network analysis. Known as "influence maximization," this problem seeks to identify a small set of nodes that, when activated, maximize the number of activated nodes at the end of the diffusion process. This is critical for effective marketing campaigns, political mobilization, and public health initiatives, where maximizing reach and impact is paramount. Despite extensive research using heuristic, meta-heuristic, and approximate algorithms, these methods often fail to capture the competitive dynamics inherent in real-world scenarios. This study addresses this gap by analyzing the influence maximization problem in opinion diffusion under competitive conditions for the first time. Considering various factors, this study also develops a novel influence model and employs a bi-level mathematical programming model using game theory to analyze competitive opinion diffusion. This approach allows for a more realistic representation of competitive influence scenarios, enabling the identification of more effective strategies for maximizing influence in social networks compared to traditional methods.

3. LITERATURE REVIEW

The analysis of social networks has been of interest to social science researchers for several decades (e.g., (3-5)). It is noteworthy that the initial studies in this field focused on conducting research on small data sets. However, with the recent emergence of the internet and online social networks, as well as popular applications such as Telegram, WhatsApp, Instagram, etc., and the availability of large-scale data, research in the field of social network analysis has unprecedentedly grown (6). This unprecedented growth has led to the development of applied research in social network analysis to address various research questions. A vast volume of these studies focuses on analyzing and examining the problem of information and influence diffusion in these networks (6). Therefore, in this section, we first review the literature on network analysis and influence maximization and then specifically review previous research on optimizing the competitive diffusion problem and opinion propagation in social networks.

The problem of influence maximization in social networks is to select k nodes from the network as the initial nodes for diffusion, considering a specific influence model (which determines how network nodes affect each other), so that by the end of the diffusion or influence process, the number of influenced nodes is maximized (7). In this context, Kermani et al. (8), focused on an optimization method to find the optimal set of initial contacts in a social network to maximize the total network members influenced by a message. In this study, it is assumed that initial contacts are costly, and the goal is to simultaneously maximize the total network members while minimizing the number of initial contacts. To achieve this, a bi-objective probabilistic integer programming model was developed, assuming that actors are heterogeneous in the probability of transmitting messages across their networks. Given the complexity of solving the proposed model, it was reformulated as a pure integer programming model. This algorithm was demonstrated through the analysis of message transmission in a short message system among university students.

The influence maximization problem is primarily applied in marketing, where the aim is to promote a new product by activating influential individuals within a social network to maximize the spread of usage among potential customers. Given a limited budget, the goal is to optimally select initial nodes for diffusion, and since this problem can be reduced to the classic set cover problem, it is classified as NP-Hard, as proven by Lu et al. (9).

In next research, Luo et al. (10), addresses competitive opinion maximization in social networks, where multiple products compete to maximize activated user opinions, presenting a #P-hard challenge that lacks submodularity. We introduce ICOM (Iterative Competitive Opinion Maximization), a model that optimally responds to competitor strategies while limiting negative opinion spread, demonstrating improved performance over baseline methods in empirical studies (10). Another studies focused on competitive influence maximization which addresses the strategic allocation of resources in competitive influence maximization scenarios also, explores independent opinion dynamics in competitive influence maximization scenarios.

Also, the study of Meena et al. (11), addresses the influence maximization (IM) problem by proposing a framework that diversifies influenced nodes in dynamic social networks through the DCDIM algorithm, which identifies and maximizes influential communities. The objective function is shown to be monotonic, submodular, and NP-hard, and experiments demonstrate that our approach outperforms benchmark algorithms in maximizing community influence across four datasets (11). Related to community, focuses on influence maximization within communities, which can add a layer of granularity to our network analysis problem also, Zhuang et al. (12), considers the dynamic nature of social networks and how influence changes over time.

Another study Valizadeh et al. (13), analyzes competition in supply chains with a leader-follower structure involving manufacturers and retailers across three scenarios: decentralized leader-decentralized follower, integrated leader-decentralized follower, and decentralized leader-integrated follower. Findings reveal that higher price competition reduces the leader's profit while benefiting the follower, with the first scenario yielding the highest retail and wholesale prices and integrated chains delivering better service levels (13).

In Kermani et al. (14) research, a scenario-based robust optimization approach to influence maximization that focuses on maximizing the number of infected nodes while minimizing the number of costly seed nodes, accounting for the heterogeneity of nodes in their message-passing capabilities and activation thresholds. Experiments on a real text-messaging social network show that the proposed model significantly outperforms several well-known heuristic methods (14).

As mentioned earlier, influence maximization is a well-known problem in social network analysis literature, aiming to find a small subset of seed nodes to maximize the diffusion or spread of information. The primary application of this problem in the real world is in viral marketing. For instance, Kermani et al. (15), proposed a modified influence maximization approach called OAIM, which aims to maximize desirable opinions by optimizing message content to select the best seed nodes through a multi-objective nonlinear mathematical programming model. They convert this model into a single-objective linear format and achieve exact solutions for small datasets with the CPLEX algorithm, while a genetic algorithm is introduced for medium and large datasets, demonstrating the efficiency and applicability of the OAIM model through experimental results.

Liang et al. (16) examined the challenge of maximizing influence in competitive social networks, focusing on identifying key nodes that effectively enhance the spread of information. They introduced the MECIC model to account for data transmission uncertainties and node states, ultimately developing three algorithms that significantly increase influence spread in competitive networks, as confirmed by their experimental results (16). Next research, In their article, Liu et al. explored maximizing influence in online social networks by introducing the UDCLT model, which accounts for varying user preferences across different brands. They addressed the User-Driven Competitive Influence Maximization problem by proposing a new criterion called topological importance to identify key users, and developed a two-step algorithm, CDIA, which successfully enhances influence distribution in social networks, as demonstrated by experiments on real data (17). Also, Liang et al. demonstrated a method for maximizing influence in advertising and marketing within social networks by identifying a small group of individuals who can significantly promote a product. They formulated the problem as TIMC and employed the Independent Cascade Model to model influence spread, developing an algorithm called RRG that effectively identifies key influencers, especially in large, sparse networks facing high competition, as confirmed by their experimental results (18).

Huang et al. (19) discussed methods to enhance the influence of a new product in social networks by proposing a competitive and complementary independent cascade model, which identifies seed users who can best convey the product's message within an environment of competing or complementary products. They introduced a deep model to analyze user connections and influences, along with an approximation algorithm to determine optimal seed users, and their experimental results demonstrate improved prediction accuracy and efficiency compared to prior methods (19). Another study conducted in this field is the article by Chen et al. (20) which they focused on the influence maximization problem within the linear threshold model, proving its NP-hard classification. They developed a scalable algorithm for maximizing influence in social networks, demonstrating through extensive simulations that it delivers acceptable solutions across networks of all sizes, including those with millions of nodes, and outperforms the algorithm presented by Kempe et al. (21). Even-Dar et al. (22) examined the influence maximization problem in a network where diffusion operates according to the voter model. The main feature of this article is that in the case where activating each node at the start of the process has a fixed cost, they obtained an exact solution for their examined problem. For the more general case, where the activation costs of nodes differ, they developed an algorithm to find a good solution (22). They claimed that the simplest heuristic algorithm, which suggests selecting nodes based on their degrees, finds the exact optimal solution for the equal cost case. Zhu and Wang also explores using deep reinforcement learning to optimize influence maximization under budget constraints, which is relevant to the cost considerations.

Other categories of articles exist in the literature (23, 24) that address this issue in non-competitive conditions.

As mentioned, one of the applications of the influence maximization problem is in advertising and creating purchase incentives for individuals. This application of the problem was examined in the article published by Li and Shiu (25) considering the transformation of internet-based social networks like Facebook and Twitter into social media and their growing expansion, the issue of advertising diffusion through social media has been analyzed and the mechanism for maximizing it has been proposed by the authors. Additionally, the authors implemented their proposed mechanism on data from a network of internet blogs. They also compared the output of their mechanism with the output of methods for selecting initial nodes based on centralities and demonstrated the efficiency of the proposed mechanism. Additionally, some studies have used game theory concepts to model competitive conditions. For instance, Irfan and Luis (26) introduced "influence games" as a graphical game model to study strategic interactions in networked populations. The work explores computational problems, identifies influential nodes, analyzes complexity, and develops approximation algorithms.

Hu et al. (27) introduced a model competition among N players to maximize diffusion over an undirected graph, where each player selects one starting node and the nodes are homogeneous. It finds that if only two players compete and the network diameter is less than two, a pure Nash equilibrium exists and can be identified in polynomial time. Ohsaka et al. (28) used game theory to model the competitive diffusion problem in social networks through a simultaneous non-cooperative two-player game based on the linear threshold model. They explore the existence and computational complexity of Nash equilibrium, along with its efficiency, while addressing conflicts arising when two players attempt to capture the same homogeneous node, a scenario not previously examined in other studies.

Some studies on TAIM[1] (29-31) concentrate on maximizing influence on users related to specific inquiry topics, i.e., topic-relevant targets. Formally, these studies introduce the concept of utility to differentiate among users and then calculate the influence of σ as the expected sum of benefits for the activated users, which is also referred to as targeted influence. Accordingly, they present techniques to identify the seed set that maximizes impact based on the computed utility models. Table 1 classifies these research background and gaps at a glance.

TABLE 1. The Research Background and Research Gap

Row	Article/ Research	Year	Methodology	Focus/Main Goal	Strengths	Limitations/Gaps
1	Kermani et al. (8)	2016	Bi-level Probabilistic Integer Programming Model	Optimizing the initial contact set to maximize influenced members in the network (considering the cost of initial contacts)	Considers the cost of initial contacts and heterogeneity of actors	Limited to a student SMS system, does not consider competition
2	Luo et al. (10)	2019	ICOM (Iterative Competitive Opinion Maximization) Model	Maximizing user opinions in competitive conditions by limiting the spread of negative opinions	Considers competition and attempts to limit negative opinions	Does not offer solutions for situations where competitors have more complex strategies
3	Meena et al. (11)	2025	DCDIM (Diversified Community-based Diffusion Influence Maximization) Algorithm	Maximizing influence in dynamic social networks by identifying and maximizing influential communities	Focuses on the diversity of influenced nodes and considers dynamic networks	High computational complexity, does not address economic and strategic aspects
4	Valizadeh et al. (13)	2021	Supply chain analysis with a leader-follower structure (manufacturers and retailers)	Examining the impact of price competition on the profits of leaders and followers in three different scenarios	Analyzes different supply chain scenarios and the impact of price competition	Does not examine social networks and competition in information/influence diffusion
5	Kermani et al. (14)	2021	Scenario-based Robust Optimization Approach	Maximizing infected nodes and minimizing costly seed nodes (considering the heterogeneity of nodes)	Considers the heterogeneity of nodes and provides a robust optimization approach	Limited to SMS networks, does not consider competitor strategies
6	Kermani et al. (15)	2018	OAIM (Opinion-Aware Influence Maximization) Approach	Maximizing desirable opinions by optimizing message content and selecting the best seed nodes	Focuses on optimizing content and considering opinions	Converts the model to a single-objective, uses a genetic algorithm for large datasets
7	Liang et al. (16)	2025	MECIC Model	Maximizing influence in competitive social networks considering data transmission uncertainties and node states	Considers data transmission uncertainties and node states	Model complexity, requires specific algorithms
8	Liu et al. (17)	2024	UDCLT (User-Driven Competitive Influence Maximization) Model	Maximizing influence in online social networks considering varying user preferences across different brands	Considers user preferences across different brands	May not be applicable to all types of social networks
9	Liang et al. (18)	2023	RRG Algorithm with TIMC formulation	Maximizing influence in advertising and marketing within social networks by identifying a small group of influencers	Focuses on advertising and marketing applications	May not be effective in networks with different structures or influence models
10	Huang et al. (19)	2021	Competitive and Complementary Independent Cascade Model with a Deep Learning Model	Enhancing the influence of a new product by identifying seed users in an environment of competing or complementary products	Uses deep learning to analyze user connections and influences	Deep learning models can be computationally expensive and require large datasets
11	Chen et al. (20)	2010	Scalable Algorithm for Influence Maximization in the Linear Threshold Model	Maximizing influence in social networks with millions of nodes	Scalable for large networks	Focuses on the Linear Threshold Model only
12	Even-Dar et al. (22)	2007	Exact and Heuristic Solutions for Influence Maximization in the Voter Model	Maximizing influence in a network where diffusion operates according to the voter model	Provides exact solutions for specific cases	Limited to the Voter Model

[1] Topic-Aware Influence Maximization

13	Irfan & Ortiz (26)	2011	"Influence Maximization Game" Model	Analyzing competition among players to maximize diffusion through selecting a subset of nodes	Uses game theory to model competition	Does not always have a Nash equilibrium in pure strategies
14	Hu et al. (27)	2014	Game-Theoretic Model with N players and Homogeneous Nodes	Modeling competition among N players to maximize diffusion over an undirected graph	Provides conditions for the existence of a pure Nash equilibrium	Limited to homogeneous nodes and specific network structures
15	Ohsaka et al. (28)	2014	Simultaneous Non-Cooperative Two-Player Game based on the Linear Threshold Model	Modeling competitive diffusion in social networks through a simultaneous game	Addresses conflicts arising when two players attempt to capture the same node	Based on the Linear Threshold Model and may not be applicable to other influence models
16	Mahmoudi & Tofigh (32)	2012	Game Theory (Dynamic Competitive Pricing Model)	Modeling dynamic pricing strategies in a competitive market with a price leader and followers.	Provides a mathematical model for understanding pricing dynamics; considers the interdependence of firms.	May not fully capture the complexities of real-world markets (e.g., consumer behavior, brand loyalty). Indirectly related to social networks as pricing can influence choices within those networks.
17	Ebrahimi Gouraji, et al. (33)	2025	Optimization (Sustainable Vehicle Routing Problem with Social Utility Considerations)	Optimizing vehicle routes while considering social utility (e.g., minimizing negative impacts on communities).	Considers social factors in routing optimization; promotes sustainability.	The "social utility" aspect might be simplified and not fully capture all relevant social impacts. Could be extended to model delivery optimization in social networks (e.g., for online marketplaces).
18	Studies on TAIM (29-31)	2013 2015 2016	Utility-based models and techniques	Maximizing influence on topic-relevant targets by identifying seed sets that maximize impact based on computed utility models	Focuses on topic-relevant influence maximization	Relies on accurate utility models, which can be difficult to obtain
19	Zhu et al.	2020	Deep Reinforcement Learning	Influence maximization with budget constraints.	Adapts to complex network structures; learns optimal strategies over time; directly incorporates budget constraints.	Computationally expensive; requires careful reward function design; may not generalize well to unseen network structures or drastically different budget scenarios; exploration vs exploitation tradeoff.
20	Chen et al.	2021	Community Detection/Analysis	Community-aware influence maximization.	Leverages community structure to improve influence spread; potentially more efficient than global methods; captures localized influence.	Community detection algorithms can be sensitive to parameter settings; may not accurately reflect real-world community boundaries; assumes influence is primarily within communities.
21	Gao & Tao	2022	Strategic Resource Allocation	Budget allocation for competitive influence maximization.	Explicitly models competition for influence; provides insights into optimal resource allocation strategies; applicable to scenarios with multiple competing entities.	Assumes a specific competitive model (e.g., simultaneous vs. sequential moves); may not capture complex strategic interactions or alliances; sensitivity to the accuracy of influence prediction models.
22	Zhuang et al. (12)	2023	Dynamic Network Analysis	Influence maximization in dynamic social networks with time-varying influence strength.	Captures the temporal evolution of networks and influence; more realistic than static models; can adapt to changing user behavior.	Requires continuous monitoring of network dynamics; computationally demanding; may rely on simplifying assumptions about how influence strength changes over time; data availability challenges.

23	Sun et al.	2024	Independent Opinion Dynamics Modeling	Independent opinion dynamics in competitive influence maximization.	Models opinion formation explicitly; captures the effects of competing opinions; provides a more nuanced understanding of influence.	Assumes independence of opinions (which may not hold in reality); simplifies opinion dynamics; may not capture all relevant factors influencing opinion formation (e.g., external events, biases); calibration of the opinion dynamics model.
24	**Present Research**	2025	A combination of mathematical optimization, heuristic algorithms, simulation, and game theory	Maximizing influence under competitive conditions considering cost, opinions, uncertainty, and competitors' strategies; Improving existing models for influence maximization by considering competition and real-world constraints (e.g., budget)	Considering more realistic competition, combining influential factors, providing robust optimization approaches, using game theory	Computational complexity, need for accurate information, simplifying assumptions, lack of complete generalizability

❖ **Research Gap:**

Based on the Table 2, the research identifies the following gaps in the existing literature:

- ✓ **Limited consideration of realistic competition:** Many existing models simplify competition or don't consider it at all. They often rely on heuristic or approximate algorithms, which may not be optimal in competitive scenarios.
- ✓ **Lack of comprehensive modeling of influential factors:** Previous studies often focus on a subset of factors influencing diffusion (e.g., cost, opinions, and uncertainty) but rarely combine them.
- ✓ **Limited robustness:** Many models are tailored to specific network structures or influence models and may not generalize well.
- ✓ **Absence of strategic perspective:** Many approaches lack a game-theoretic framework to model the strategic interactions between competing entities trying to maximize influence.

❖ **Innovation:**

The research introduces the following innovations:

- ✓ **A bi-level mathematical programming model within a Stackelberg game framework:** This allows for modeling the strategic interaction between a leader and a follower in a competitive influence maximization scenario. This is a significant departure from heuristic or approximate algorithms used in previous studies.
- ✓ **Integration of multiple influential factors:** The model considers the delta threshold, social influence, initial node count, and message transmission cost, providing a more comprehensive view of the diffusion process.
- ✓ **Optimization of message dissemination strategies across different threshold values:** The model allows for tailoring strategies based on the attractiveness of message content and user engagement, which is often overlooked in simpler models.
- ✓ **Use of a genetic algorithm to solve the complex optimization problem:** This provides a practical approach to finding effective dissemination strategies.

❖ **Scientific Contribution:**

The research contributes to the field in the following ways:

- ✓ **Provides a more realistic model of competitive influence maximization:** By incorporating a Stackelberg game framework and considering multiple influential factors, the research offers a more nuanced and practical understanding of how influence spreads in competitive social networks.
- ✓ **Identifies the significant impact of key parameters on the diffusion process:** The findings highlight the importance of factors such as the delta threshold, social influence, initial node count, and message transmission cost in shaping the outcome of influence maximization efforts.
- ✓ **Offers actionable insights for designing effective information dissemination strategies:** The research provides guidance for optimizing message content, reducing transmission costs, and leveraging influential users to enhance the impact of messages in social networks.
- ✓ **Demonstrates the effectiveness of genetic algorithms for solving complex influence maximization problems:** This provides a valuable tool for researchers and practitioners working in this area.

In summary, this research addresses important gaps in the literature by providing a more realistic and comprehensive model of competitive influence maximization. Its innovations lie in the use of a game-theoretic framework, the integration of multiple influential factors, and the development of a practical optimization approach. The scientific contributions include valuable insights into the dynamics of influence diffusion and actionable guidance for designing effective dissemination strategies.

4. MATHEMATICAL MODEL

Mathematical modeling and linear programming are powerful tools for solving complex problems and making strategic decisions. They leverage mathematical principles and algorithms to enable optimal decision-making under constraints. In competitive diffusion processes within social networks, various actors (companies, individuals) strategically seek to maximize their influence. Game theory, a framework for analyzing strategic interactions, provides a powerful lens to model these competitive dynamics. By representing these interactions as games, we can analyze the complex patterns that emerge in competitive diffusion (34). Furthermore, bi-level programming, an optimization technique for hierarchical decision-making, allows us to model the leader-follower relationship inherent in many influence scenarios. Specifically, it enables us to capture how a central entity (leader) can optimize its strategy considering the reactions of other actors (followers) in the network. Mathematical modeling using game theory and bi-level programming offers individuals, companies, and organizations the tools to adopt optimal dissemination strategies, emphasizing strategic interactions, Nash equilibria, network dependencies, and complex structures.

4. 1. Bi-Level Programming Model A bi-level programming model consists of two levels of decision-making: the upper level (leader) and the lower level (follower). The leader makes decisions to optimize a common objective, anticipating the reactions of the followers. Each follower independently makes decisions to optimize their own objectives, considering the leader's actions and the decisions of other followers. In the context of influence maximization in social networks, we can use bi-level programming to model competitive opinion dissemination. For example, imagine a scenario where a political campaign (leader) strategically disseminates messages to influence voters. Competing interest groups or even individual users (followers) then react by spreading their own counter-messages to maximize their influence. This creates a dynamic where the leader must anticipate and counteract the followers' responses. This scenario aligns with a Stackelberg game (32), where the leader moves first, and the followers react accordingly.

In game theory modeling, a bi-level programming model can depict strategic interactions between entities. In this context, the upper level typically involves maximizing a common objective function that encompasses the outcomes of the lower-level decisions (35). For instance, in our influence maximization problem, the leader aims to maximize the spread of their message, considering how the followers will react and potentially dilute or counteract that message. By using bi-level programming, we can analyze these interactions with greater accuracy and identify more effective strategies for maximizing influence in the face of competition. This approach allows us to move beyond simple heuristic algorithms and develop more nuanced and effective strategies for influence maximization in social networks.

4. 2. Model Description

Objective Function 1:

1. The objective function aims to maximize the number of initial nodes for message diffusion for player 1 (the leader).

Constraints 1:

2. Zero-cost constraint for player 1.
3. In the first constraint, we want k nodes to be activated by the leader, and the rest to remain inactive.
4. In the second constraint, we want no messages to be exchanged between nodes for player 1 in the first step.
5. In the third constraint, we want the number of messages transferred between nodes at time t not to exceed a certain limit.
6. In the next constraint, we want inactive nodes to remain inactive if they have not received any messages.
7. In the following constraint, we want inactive nodes not to have the ability to send messages.
8. In the next constraint, we want a node to be able to send an accepted message to its neighboring nodes only one step after being activated.
9. In the final constraint, we want the node to remain active and not deactivate once it has received the message and been activated until the end of the game.

$$\text{Max } Z1 = \sum_{i=1}^{n} p_{i1}(t), t \to \infty \quad [1]$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{t=1}^{T} c_1(p_{i1}(t) + s_{ij1}(t)) \leq B_1, \quad [2]$$

$$\sum_{i=1}^{n} p_{i1}(1) = k \quad [3]$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} s_{ij1}(1) = 0 \quad [4]$$

$$s_{ij1}(t) \leq a_{ij},\ \forall i, j \in V, \forall t, \text{ and } N = [a_{i,j}] \quad [5]$$

$$p_{i1}(t) - p_{i1}(t-1) \leq \sum_{j \in V} s_{ji1}(t),\ \forall i \in V \text{ and } t \geq 2, \quad [6]$$

$$s_{ij1}(t+1) \leq p_{i1}(t),\ \forall i, j, t \quad [7]$$

$$\sum_{j\in V} s_{ij1}(t) \le U(p_{i1}(t-1) - p_{i1}(t-2)),\ \text{for } t \ge 3, \text{ and } i,j \in V \quad [8]$$

$$p_{i1}(t) \le p_{i1}(t+1),\ \forall t \text{ and } i \in V \quad [9]$$

Objective Function 2

10. In the second objective function, we want to maximize the nodes initiating message dissemination for player 2 (follower):

Constraints 2

11. Zero-cost Constraint for Player 2
12. In the first constraint, we want the nodes activated by player 2 to be zero initially.
13. In the second constraint, we want no messages to be exchanged between nodes for player 2 in the first step.
14. In the third constraint, we want no messages to be exchanged between nodes for player 2 in the second step
15. In the fourth constraint, we want the number of messages transmitted between nodes at time “t” not to exceed a certain limit.
16. In the fifth constraint, player 2 sends their message to k nodes and some of them accept it.
17. In the sixth constraint, we want the message to be sent when the node is active.
18. In the seventh constraint, a node can only send an accepted message to its neighboring nodes one step after activation.
19. In the eighth constraint, we want inactive nodes to remain inactive if they have not received any message.
20. In the ninth constraint, we want nodes to remain active for the rest of the game once they receive a message and are activated.
21. In the next constraint, in each step of the game, a node is either inactive or can only accept one of the messages.

$$\text{Max } Z2 = \sum_{i=1}^{n} p_{i2}(t),\ t \to \infty \quad [10]$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{t=1}^{T} c_2(p_{i2}(t) + s_{ij2}(t)) \le B_2, \quad [11]$$

$$\sum_{i=1}^{n} p_{i2}(1) = 0 \quad [12]$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} s_{ij2}(1) = 0 \quad [13]$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} s_{ij2}(1) = 0 \quad [14]$$

$$s(t) \le a_{ij},\ \forall i,j \in V, \forall t, \text{ and } N = [a_{i,j}] \quad [15]$$

$$\sum_{i=1}^{n} p_{i2}(2) \le k \quad [16]$$

$$s_{ij2}(t+1) \le s_{i2}(t),\ \forall i,j,t \quad [17]$$

$$\sum_{j\in V} s_{ij2}(t) \le U(p_{i2}(t-1) - p_{i2}(t-2)),\ \text{for } t \ge 3, \text{ and } i,j \in V \quad [18]$$

[1] https://en.wikipedia.org/wiki/General_algebraic_modeling_system

$$p_{i2}(t) - p_{i2}(t-1) \le \sum_{j\in V} s_{ji2}(t),\ \forall i \in V \text{ and } t \ge 3 \quad [19]$$

$$p_{i2}(t) \le p_{i2}(t+1),\ \forall t \text{ and } i \in V \quad [20]$$

$$p_{i1}(t) + p_{i2}(t) \le 1, \text{ for } t \ge 1, \text{ and } i \in V \quad [21]$$

In the following constraints, each node selects some of its neighboring nodes, which are eligible to receive the message, exactly one step after being activated. This occurs with the conditional expressions (22 and 23):

[22] IF $p_{i1}(1) = 1$ AND $\alpha_j\left(1 - \frac{\|M_k - \beta_j\|}{2\sqrt{2}}\right) > \delta$ THEN $s_{ij1}(2) = 1$

[23] IF $p_{i1}(1) = 0$ OR $\alpha_j\left(1 - \frac{\|M_k - \beta_j\|}{2\sqrt{2}}\right) \le \delta$ THEN $s_{ij1}(2) = 0$

Then:

[24] IF $p_{i1}(1) * \left(\alpha_j\left(1 - \frac{\|M_k - \beta_j\|}{2\sqrt{2}}\right) - \delta\right) > 0$ THEN $s_{ij1}(2) - 1 \ge 0$

[25] IF $-p_{i1}(1) * \left(\alpha_j\left(1 - \frac{\|M_k - \beta_j\|}{2\sqrt{2}}\right) - \delta\right) + \epsilon > 0$ THEN $-s_{ij1}(2) \ge 0$

With the definition of variables uu and ww, the linearized form is as follows:

[26] $-s_{ij1}(2) + 1 \le u * w_1,\ \forall i,j \in V$

[27] $a_{i,j} * p_{i1}(1) * \left(\alpha_j\left(1 - \frac{\|M_k - \beta_j\|}{2\sqrt{2}}\right) - \delta\right) \le u.(1 - w_1),\ \forall i,j \in V$

[28] $y_{ij}(2) \le u * w_2,\ \forall i,j \in V$

[29] $-a_{i,j} * p_{i1}(1) * \left(\alpha_j\left(1 - \frac{\|M_k - \beta_j\|}{2\sqrt{2}}\right) - \delta\right) + \epsilon \le u * (1 - w_2),\ \forall i,j \in V$

4. 3. Model Implementation For implementing the model, the GAMS (General Algebraic Modeling System) software was used. This software is an advanced system for mathematical modeling and optimization, first developed in 1987. The 2014 version of this software includes numerous updates and improvements that enhance its performance and capabilities[1].

GAMS 2014 is a powerful tool for modeling and solving complex linear and nonlinear optimization problems, mathematical programming issues, and simulations. This software allows users to define their mathematical models algorithmically and then optimize and analyze these models through graphical and text-based user interfaces. One of GAMS's prominent features is its ability to use various solvers for solving different types of optimization problems, including CPLEX, GUROBI, and IPOPT. The 2014 version of GAMS includes improvements in solver performance and enhanced modeling capabilities.

4. 4. Abrar University Dataset The network examined in this subsection was collected by Kermani et al. (8) included 163 students from Abrar Nonprofit University in Tehran. These students are studying Industrial Engineering and Computer Engineering. In this network, the relationship between students is defined as messaging links; if student j can send a message to student i, there is a connection. This network is represented as a weighted directed graph where students are the graph nodes, and the connections between them are the graph edges. For each node i, the sets Oi and Ii (according to the definitions in the previous section) are determined based on the graph structure. The weights defined on the network are randomly selected. In this network, players outside the network are also considered, with different options for diffusion within the network, competing with Abrar University students to maximize the diffusion of their options (Figure 1). To better explain the model, assume that two different mobile phone brands (e.g., iPhone and Samsung) are competing to increase their sales within the network. Before the diffusion process begins, each network member has a mental background and initial inclination towards the two mentioned brands. This mental background and inclinations might be the result of advertisements, personal experiences, or the influence of friends and acquaintances. Advertising within this network is conducted via a short message system (36).

4. 5. Model Results In this section, we present the results of model implementation in GAMS, focusing on 4 specific influential variables:

4. 5. 1. Delta After running the model in GAMS, the following results were recorded for different delta values. According to the definition of the delta parameter, it can be considered a measure of accuracy for the dissemination of incoming messages in the network. In other words, the higher the threshold values, the less inclined the message-sending node will be to transmit. Therefore, the number of active nodes at the end of the diffusion process should be inversely related to the threshold values. Figure 2 shows the ratio of engaged nodes for different delta values.

The results in Figure 3 shows that for all displayed decisions, an increase in the threshold leads to a decrease in the number of active nodes at the end of the process. On the other hand, it is also true that if the number of initial nodes increases, the number of active nodes at the end of the diffusion process also increases. By examining and analyzing the model's sensitivity to the desired parameters, it can be concluded that the presented model is accurate in terms of modeling. Also, Table 1 records the values of objective functions 1 and 2 for different delta values:

The results in Table 2 shows that for a delta value of 0.4, the optimal solution of the model indicates that the optimal node to start with for player 1 is node 25 and for player 2 is node 68. Given the combination selected by the players, the objective function value for the game leader is 40, and the second level objective function

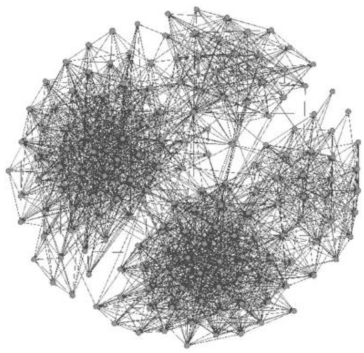

Figure 1. Social network of Abrar university [40]

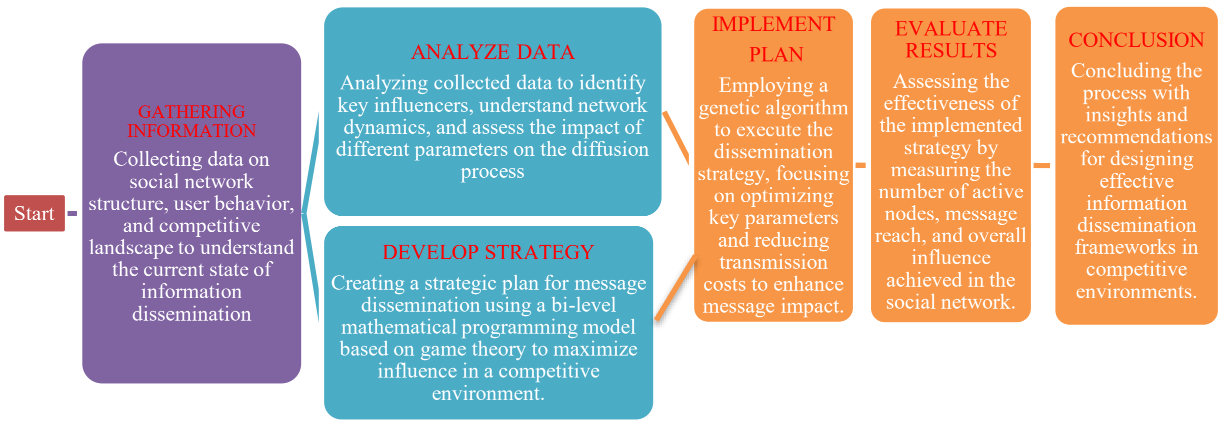

Figure 2. The flowchart of the research method

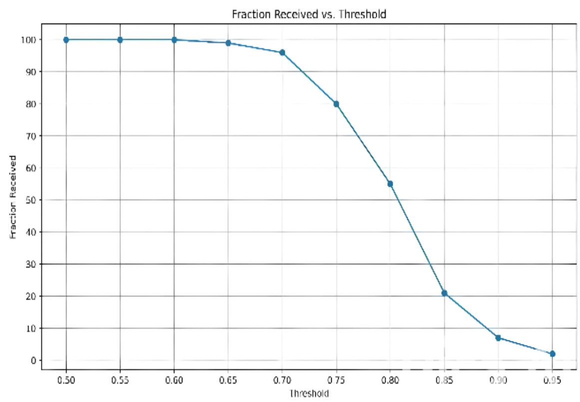

Figure 3. Number of engaged nodes for different delta values

value, which is the follower's outcome, is 33. These values indicate that at the end of the diffusion process, 40 nodes have accepted the leader's message and 33 nodes have accepted the follower's message. This solution corresponds to the pure Nash equilibrium of the Stackelberg game obtained under the examined conditions. According to Table 1, as the delta parameter value increases, the objective function values for both levels decrease. In other words, with an increase in this parameter, the number of activated nodes at the end of the diffusion process decreases. This is precisely what is expected, as increasing this parameter makes the message diffusion conditions more difficult, and network members act more cautiously in choosing destination nodes for message transmission. Additionally, the solutions obtained for the linear optimization problem correspond to the Nash equilibrium of the Stackelberg game and have the Nash equilibrium property.

Furthermore, the model's results suggest that as the intensity of nodes (individuals in the society) receiving messages increases, fewer changes occur in society. The intuitive interpretation of this statement is that the social level of individuals in a society directly impacts changes in cultural patterns and lifestyles. Another notable point is that for high threshold values, messages reach the entire network. However, as the threshold values increase (approximately above 0.6), the influence of messages on the network rapidly decreases and quickly approaches zero. This indicates the nonlinear behavior of diffusion concerning threshold changes.

4. 5. 2. Limit of Social Impact of Messages

The limit of social impact of a message in networks refers to the ability of a message to create significant changes in the behavior, beliefs, or decision-making processes of members within the network. This concept illustrates how messages are disseminated in a social network and the extent of their potential influence.

Factors Influencing the Limit of Social Impact

Several factors affect the limit of social impact of messages, including:

1. **Network Structure**:
   - Centrality of Nodes**:** Central or influential nodes, which have more connections with other nodes, have a greater impact on message dissemination.
   - Network Density: High density in a network can facilitate faster and more effective spreading of messages.
2. **Message Characteristics**:
   - The relevance and appeal of the message can significantly influence its impact on network members.
3. **Social Interactions**:
   - Stronger connections between members play a crucial role in message dissemination.
   - Shared norms and values within a network can enhance the effectiveness of messages.

Models for Analyzing Social Impact

Various models exist to analyze the limit of social impact of messages, including:

- **Information Dissemination Models**: These models describe the process of message spreading, either through cascading effects or based on thresholds of acceptance.
- **Influence Models**: These models consider the mutual influences between nodes and determine which individuals ultimately get selected for action.

Influence Model

The influence model helps identify which individuals are ultimately chosen. More specifically, if the network members are represented by V and a subset of k members is denoted as A, this model indicates which individual influences another at each stage of the dissemination process. One of the widely used models for studying how influences are disseminated is the cascading influence model. In this model:

- Each node vv, upon activation, gets a chance to "spark" its inactive neighbors.

TABLE 2. Values of the two objective functions for different delta values

Delta	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Value of Objective Function 1	92	70	64	40	24	15	8	3	1
Value of Objective Function 2	88	63	58	33	18	12	6	2	1

- This chance is assigned to each node only once during activation.

Consider w as an inactive neighbor of v. When v activates at time step t+1, w receives a spark from v. If this spark is successful, w becomes active at time t+1. The order in which w receives sparks does not affect the activation process. For instance, if multiple neighbors of w are active at time t, any sequence of received sparks can be considered valid. Once a node becomes active and sparks its neighbors, it will no longer influence the activation of other nodes in subsequent time steps.

Additionally, threshold models are among the first models proposed to illustrate dissemination in social networks. In this model:

- A threshold is defined for each node regarding its activation based on influence from its neighbors.
- Each node will activate after receiving a sufficient number of sparks from its neighbors.

Both cascading and threshold models provide valuable frameworks for understanding how information and influence spread in social networks. The cascading model emphasizes the significance of the sequence and chance of activation, while threshold models focus on the accumulated influence needed for activation. These insights can guide strategies for effective communication and engagement within social networks.

For example, in an online social network like Twitter, a user with a large number of followers can significantly impact other users by sharing an important message. If this message is engaging and is quickly retweeted, it can spread rapidly throughout the network and have a wide-reaching effect. Conversely, if this user posts irrelevant or unappealing content, the impact will be much less.

The results from Table 3 highlights the impact of the number of active nodes at the beginning of the period on the number of active nodes at the end of the period, which is a crucial aspect of social network analysis and information dissemination.

This influence can manifest in various ways, affecting the dynamics of the network and the patterns of message and information dissemination.

Following Figure 4, for instance, in online social networks such as Twitter or Facebook, if a number of popular and influential users start disseminating a specific message at the beginning of the period, the likelihood of that message being widely spread throughout the network is very high. These users, due to their large number of followers and extensive connections, can quickly spread messages and engage a substantial number of other users, prompting them to participate. Consequently, the number of active nodes at the end of the period can significantly increase. Despite these positive effects, it is important to acknowledge certain limitations and challenges associated with dissemination. For example, if the messages are not appealing or relevant, even with a substantial number of active nodes at the beginning, the dissemination of the messages may cease, leading to a decrease in the number of active nodes by the end of the period.

TABLE 3. Social impact of node i and values of two objective functions

Social Impact of Node i	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
Value of Objective Function 1	34	42	47	53	68	77	86	98	101
Value of Objective Function 2	31	37	41	49	62	73	82	92	97

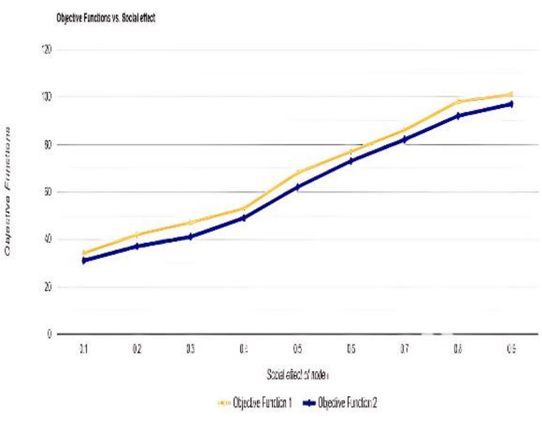

Figure 4. The social impact of node i and values on two objective functions

4. 5. 3. Number of Initiator Nodes for Dissemination Additionally, the network structure, the number of initiator nodes in the game, and the interaction dynamics between nodes can significantly affect the results.

As stated in Table 4, with an increase in the number of initiator nodes, the value of the objective function rises. This trend is also clearly illustrated in Figure 5, where the correlation between the number of initiator nodes and the objective function values is evident.

4. 5. 4. Message Transmission Costs Let us examine the impact of message transmission costs on the objective functions. The effect of transmission costs in social networks on the number of active nodes by the end

TABLE 4. Impact of the number of initiator nodes on objective functions

Number of initial Nodes	3	5	7	9	11	13	15	17
Value of Objective Function 1	78	86	92	94	98	101	104	108
Value of Objective Function 2	77	84	88	89	92	95	101	103

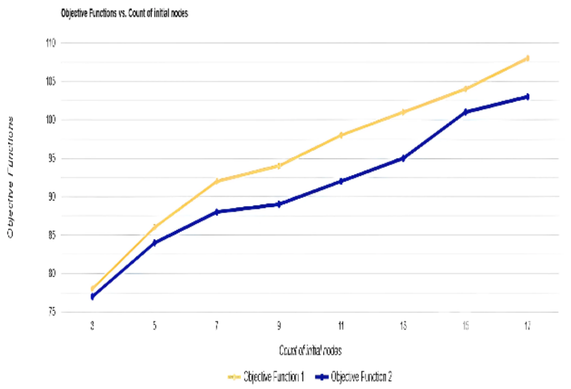

Figure 5. The impact of count of initial nodes and on two objective functions

of the period is a critical topic in analyzing social networks and the dynamics of information dissemination.

Transmission costs can be defined in various ways, including financial, temporal, energy consumption, or any other type of resource that users expend for sending and receiving messages. In online social networks, these costs may incorporate data usage, time required to compose and send messages, or even the psychological and social pressures experienced by users during the dissemination of messages.

For instance, in social networks like Twitter or Facebook, if sending messages incurs high costs (e.g., due to data limits or time constraints), users are less likely to send messages. This reduction in the number of messages transmitted can lead to a decrease in the number of active nodes by the close of the period. Moreover, only specific users and high-value messages may be disseminated within the network.

One way to increase the number of active nodes by the end of the period is to lower the costs associated with message transmission. Table 5 shows the Message Transmission Costs and Active Node Values at the End of the Period.

TABLE 5. Cost of message transmission and values of two objective functions

Cost of message transmission	3000	6000	9000
Value of the first objective function	105	71	58
Value of the second objective function	100	68	52

Based on the results presented in Table 6, the reduction in the number of messages sent may lead to a decrease in the number of active nodes by the end of the period. Additionally, it could result in only certain users and high-value messages being disseminated within the network. One effective strategy to increase the number of active nodes at the end of the period is to reduce the costs associated with message transmission.

Furthermore, as illustrated in Figure 6, alongside the increase in budgets, more nodes become engaged, resulting in higher values for the objective functions.

As illustrated in Figure 5, an increase in player budgets leads to a higher number of engaged nodes, resulting in an increase in the values of the objective functions. With the rising cost of message transmission, users' motivation to send messages also grows, leading to a greater willingness to share information and interact with others. This increased motivation can directly affect the number of active nodes within the network.

5. GENETIC ALGORITHM OVERVIEW

The use of genetic algorithms in issues related to dissemination in social networks has gained attention as an artificial intelligence approach. In these contexts, Genetic algorithms (GAs) are a powerful AI tool for optimizing information dissemination in social networks. They enhance message spread by refining strategies, targeting effective audiences, and aligning content with user preferences. GAs use chromosomes to encode dissemination strategies, with genes representing specific

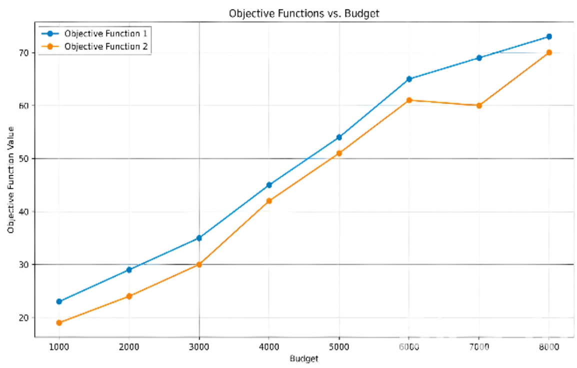

Figure 6. Increase in total cost with increasing player budgets

attributes. Mutation introduces diversity, fostering innovative solutions and preventing premature convergence. By optimizing behavioral models, network structures, and message attributes, GAs enable intelligent, data-driven dissemination strategies, maximizing impact and engagement in social networks (37).

In the context of social network dissemination, the genetic algorithm (GA) plays a pivotal role in optimizing strategies for information spread. The process begins with selection, where chromosomes (representing potential solutions) demonstrating the best performance are chosen to advance to the next generation. This selection is based on a rigorous evaluation of each chromosome's effectiveness in achieving the desired dissemination outcomes. The crossover (recombination) phase follows, where selected chromosomes are combined to generate new offspring with enhanced features. This step ensures the propagation of high-performing traits, improving the overall quality of solutions. Subsequently, evaluation is conducted to assess the performance of each chromosome, ensuring that only the most effective strategies proceed to further iterations (33).

By leveraging these core mechanisms—selection, crossover, and evaluation—the genetic algorithm enhances the dissemination process, maximizing the impact of content within social networks. This approach enables users to achieve their objectives through optimized strategies, ultimately increasing their influence on the network (Figure 7).

To simulate the dissemination process, a computational framework was developed, utilizing a genetic algorithm to refine strategies and optimize communication (Attachment A). The simulation employs a Node class to represent network entities, with attributes such as:

- Name: Identifier for the node.
- Opinions: Node's stance or beliefs.
- Status: Vulnerability or security level.
- Location: Spatial position within the network.
- Social Capability (τ): Ability to influence or be influenced.

Two key methods are defined within the Node class:

1. Adopts: Determines whether a node accepts a message based on the distance between its opinions and the message content, weighted by social capability and spatial proximity.

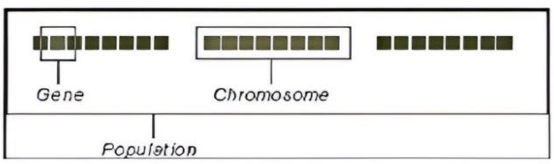

Figure 7. The components of Genetic algorithm

2. Sends: Manages message transmission to neighboring nodes, selecting recipients if the opinion distance falls below a predefined threshold.

The genetic algorithm is implemented through a series of steps, including selection, crossover, and mutation, to iteratively generate and refine dissemination strategies. A simulate function models the diffusion process, enabling nodes to send and accept messages dynamically (38).

The genetic algorithm's performance was evaluated through simulations, with convergence results visualized across multiple iterations. Figure 8 illustrates the algorithm's convergence trends for varying values of the threshold parameter (δ), demonstrating its effectiveness in identifying optimal dissemination strategies. Totally, this framework provides a robust approach to optimizing content dissemination in social networks.

By integrating genetic algorithms with network simulations, it offers actionable insights for enhancing the reach and impact of information within competitive environments. Next, we examine the variations in the content of the optimized messages concerning different dimensions of the message (Topics 1 to 3) for various values of h. Figure 9 illustrates this topic:

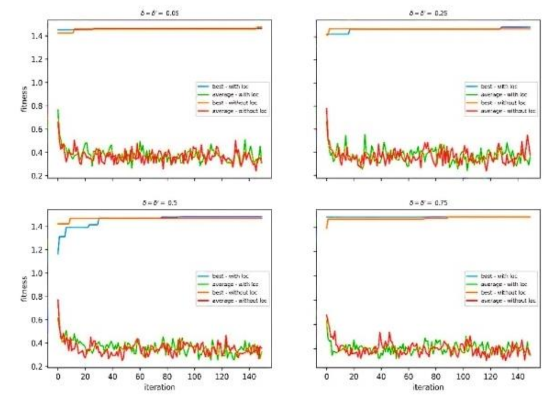

Figure 8. Convergence trends in the genetic algorithm for different delta values

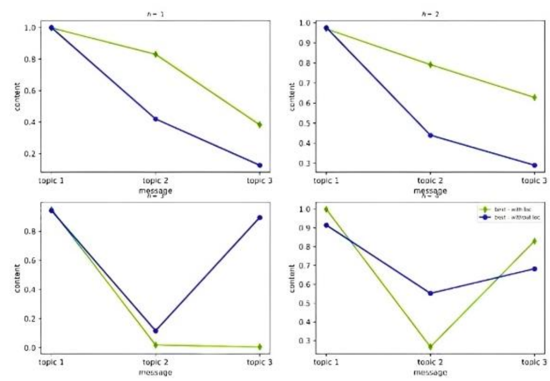

Figure 9. Variations regarding different topics for different h values

As Figure 8 shows, changing the topic associated with each message results in varying levels of influence. This indicates that the content of the transmitted message is also very important . The above genetic algorithm yields similar results concerning the increase or decrease in cost for different parameters.

6. MANAGEMENT RECOMMENDATIONS FOR OPTIMIZING COMPETITIVE INFLUENCE IN SOCIAL NETWORKS

Based on the findings of this study and the mathematical modeling employed, the following recommendations are offered to managers and practitioners in marketing and public relations. These suggestions aim to optimize strategies in social networks and increase influence within a competitive environment:

1. **Prioritize Content Attractiveness and Relevance to Target Audiences:**
   - **Recommendation:** Develop content that is not only engaging and entertaining but also aligned with the values, needs, and concerns of the target audience.
   - **Actionable Steps:**
     - Conduct market research and social network data analysis to gain a deeper understanding of the audience.
     - Personalize content based on different audience segments.
     - Utilize storytelling techniques, high-quality images, and engaging videos.
     - Continuously test and optimize content using A/B testing.
     - Pay attention to the appropriate tone in messaging, tailored to the values of the audience.
2. **Reduce Barriers to Participation and Optimize Message Transmission Costs:**
   - **Recommendation:** Create an easy and low-cost environment for users to participate in advertising campaigns and message dissemination. Cost refers not only to monetary expenses but also to barriers such as complexity, time consumption, and the need for significant effort to participate.
   - **Actionable Steps:**
     - Simplify the participation process (e.g., design short and engaging contests and surveys).
     - Provide incentives and rewards to encourage active user participation.
     - Use communication channels that are easily accessible to the audience.
     - Offer free content, discounts, and special offers to reduce the financial costs of participation.
3. **Strategically Select Early Adopters and Leverage Influencers:**
   - **Recommendation:** Identify and attract influential and highly credible individuals in social networks as early adopters of the message, in order to accelerate and expand the reach of dissemination.
   - **Actionable Steps:**
     - Identify influencers relevant to the field of activity and target audience.
     - Establish and build long-term relationships with influencers.
     - Collaborate with influencers in content production, organizing contests, and joint advertising campaigns.
     - Measure and evaluate the impact of influencer activities on campaign results.
4. **Utilize Mathematical Modeling and Data Analysis for Informed Decision-Making:**
   - **Recommendation:** Use data analysis tools and mathematical modeling to better understand user behavior, predict campaign results, and optimize resource allocation.
   - **Actionable Steps:**
     - Collect and analyze data related to user interactions, campaign performance, and other relevant factors.
     - Use social network analysis software and mathematical modeling.
     - Optimize message dissemination strategies based on data analysis results.
     - Test different strategies and evaluate their results to identify best practices.
5. **Continuously Monitor Competitors and Adapt to the Dynamics of Competition:**
   - **Recommendation:** Closely and continuously monitor competitors' activities in social networks and adapt strategies to changes in the competitive environment.
   - **Actionable Steps:**
     - Monitor the content strategies, communication channels, and advertising campaigns of competitors.
     - Anticipate competitors' reactions to new actions and initiatives.
     - Build flexibility into strategies to respond quickly to changes.
     - Emphasize innovation and offer creative ideas to maintain a competitive advantage.

By following these recommendations, managers can optimize their strategies in social networks and increase their influence in a competitive environment. Remember that social networks are dynamic and ever-changing environments, and success in this space requires continuous learning, adaptation to change, and constant innovation.

7. CONCLUSION

Social diffusion in networks is a critical and complex phenomenon, focusing on the spread of information, innovations, and social influence among individuals. Various models, such as the Independent Cascade Model and the Threshold Model, have been developed to analyze this process. In this study, we propose a mathematical model within the Stackelberg game framework to optimize the number of message-spreading nodes for two players (Leader and Follower) in a competitive setting.

Our findings highlight the significant impact of key parameters, including the delta threshold, social influence, initial node count, and message transmission cost, on the diffusion process. The application of genetic algorithms proves effective in identifying optimal strategies for information dissemination. Furthermore, adjusting threshold values and reducing transmission costs enhance message impact, significantly increasing the number of active nodes. The attractiveness of message content also plays a pivotal role in its spread, as engaging content boosts user willingness to share. Additionally, the involvement of influential users at the outset amplifies the likelihood of widespread dissemination, leveraging their extensive connections to rapidly propagate messages.

In conclusion, this research demonstrates that optimizing key parameters and reducing costs can significantly improve diffusion strategies, enhancing the overall impact of messages in social networks. These insights provide valuable guidance for designing effective information dissemination frameworks in competitive environments.

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Persian Abstract

چکیده

در دنیای امروز، شبکه‌های اجتماعی به یکی از مهم‌ترین ابزارهای ارتباطی تبدیل شده‌اند که در آن افراد و سازمان‌ها می‌توانند اطلاعات و نظرات خود را مبادله کنند. این امر تأثیر بسزایی بر نگرش‌ها و رفتارهای اجتماعی دارد و درک فرآیندهای انتشار اطلاعات در این بستر را ضروری می‌سازد. این مطالعه به مسئله حداکثرسازی نفوذ در انتشار رقابتی عقاید می‌پردازد. برخلاف رویکردهای ابتکاری پیشین، ما یک مدل برنامه‌ریزی ریاضی دو سطحی بر اساس نظریه بازی‌ها تدوین می‌کنیم و از چارچوب بازی استکلبرگ برای مدل‌سازی تعاملات استراتژیک رهبر–پیرو استفاده می‌کنیم. مدل با استفاده از یک الگوریتم ژنتیک برای شناسایی استراتژی‌های انتشار مؤثر حل می‌شود. یافته‌ها نشان می‌دهند که پارامترهای کلیدی – آستانه دلتا، نفوذ اجتماعی، پذیرندگان اولیه و هزینه انتقال – تأثیر قابل توجهی بر انتشار دارند. مدل دو سطحی، انتشار پیام را در مقادیر آستانه مختلف بهینه می‌کند و نقش جذابیت محتوا را در تعامل کاربران برجسته می‌سازد. کاهش هزینه‌های انتقال باعث افزایش مشارکت و افزایش تعداد گره‌های فعال می‌شود. مشارکت کاربران تأثیرگذار در ابتدا، دامنه انتشار را گسترش می‌دهد. این تحقیق نشان می‌دهد که بهینه‌سازی پارامترهای کلیدی و کاهش هزینه‌ها، استراتژی‌های انتشار را بهبود می‌بخشد و تأثیر پیام را افزایش می‌دهد. این نتایج به شبکه‌های مشابه قابل تعمیم هستند و کاربردهای عملی در بازاریابی دارند.