A New Approach for Seismic Damage Detection Based on Results of Pushover Analysis and Modal Based Damage Index

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PAPER INFO

1. INTRODUCTION

With the occurrence of an earthquake and damage to concrete structures, the members of these structures will be gradually damaged and will experience different levels of damage, including cracking, yielding and finally collapse. Therefore, information about the properties of the damage, i.e. determining the location and estimating its amount, is considered as an important issue that will be effective in the decision-making process regarding the repair and strengthening of the damaged structure. Structural damage causes changes in its physical properties, i.e. stiffness and damping of the structure, and these changes will ultimately affect the dynamic response of the structure and cause changes in its vibration characteristics [1]. A damage detection method should be able to detect damage in its early stages, determine its location, and provide an estimate of damage severity [1]. Extensive research has been done on the development of non-destructive damage assessment methods based on changes in the vibration characteristics of the damaged structure, and among the vibration-based methods, modal damage detection methods have received more attention. The primary research that has been done on modal information is generally about damage indexes, which are based on the comparison of changes in natural frequencies [2], the modal assurance criterion (MAC) [3] and the coordinate modal assurance criterion (COMAC) [4] as well as the multiple damage location assurance criterion (MDLAC) [5] were formulated between two states of intact and damaged structures. Achenbach [6], Hongnan and Tinghua [7], provided a comprehensive review of the history of damage detection methods in linear structures. Hearn and Testa [8]

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Abstract

The diagnosis of the location of structural damage and its extent after an earthquake using numerical methods is one of the ongoing research topics. After the occurrence of damage in a structure and a reduction in its stiffness, the dynamic characteristics of the structure change, and therefore, assessing the changes in its dynamic characteristics can be used as an indicator for detecting damage. In this article, an advanced technique called Direct Stiffness Calculation (DSC) and a new damage index based on flexural stiffness variations (SVI) are utilized for damage detection in structures. Initially, the proposed technique is examined on a steel beam with known specifications. Then, a reinforced concrete moment frame is modeled, and after extracting its dynamic characteristics, it is subjected to a pushover analysis to create a damage scenario without direct intervention. Based on the analysis results, the plastic hinge formation location at both ends of the beam is selected as the probable location of damage in the floor. By using the modal information of the damaged structure and calculating the SVI in the beams of the floors, it is determined that this index can accurately and significantly distinguish the location of damage only by knowing the first mode of the structure and with sufficient magnification compared to other points. Furthermore, the results demonstrate that with this method, it is possible to accurately determine the location of damage even without knowing the dynamic characteristics of the intact structure and solely with the information of the damaged structure.

suggested that changes in frequency ratios can be indicators of different levels of damage. Although the change in frequency may be a structural damage index, it is difficult to estimate the location of damage with this method. Also, the results of the studies showed that the changes in the natural frequency, as well as the comparison of the mode shape vectors of a structure in a healthy and damaged state, were not able to significantly highlight the damage location. Compared to the modal displacement vector, its derivatives, such as the mode shape curvature, are more sensitive to the existence of damages even in small amounts. Therefore, they can be used to detect damage [9, 10]. The issue of damage detection on beam structures has been carried out with the help of the mode shape curvature method and it was concluded that this method is very successful for detecting and locating damage in beams with different support conditions [11]. Also, this method was studied on a concrete bridge model located in Switzerland in the presence of several damage scenarios. It was concluded that modal curves are very sensitive to damage to bridges.

Foti [12] conducted two investigations on the finite element model of a simple span bridge with the help of techniques for changes in mode shapes and curvature of mode shapes. The damage was introduced as a reduction in bending stiffness. The results showed that the change in modal curvature is a suitable and usable index, while the MAC and COMAC indices did not show very good results. Dutta and Talukdar [13] investigated on changes in natural frequencies, mode shape, and mode shape curvature for a bridge that included several damaged sections at different points. In their article, the authors defined the damage factor based on the curvature to detect the damage. They showed that instead of using the mode vector itself, the damage location is better estimated by using the curvature of the mode shape. It was also shown that a sufficient number of mode shapes should be considered to detect the location of damage in the case of multiple damages in several positions. A detailed study was conducted by Whalen [14], to evaluate the changes in the mode shape and its four primary derivatives to identify the damage, and it clearly showed that the changes in the higher order derivatives of the mode shape are much more sensitive to damage.

Fayyadh and Razak [15] investigated the issue of crack location in a concrete beam by studying two damage indexes, one based on the curvature of the mode shape vector and the other based on the calculation of the fourth derivative of the mode shape vector. In this article, in addition to comparing the performance of the indicators, they provided suggestions to improve the performance of the used indicators. In the work of Maecck [16] in 1999, the method of direct stiffness calculation was introduced to locate and quantify damage on a simple beam. In this method, by using the basic relationships of material resistance, by dividing the amount of modal moment by the curvature of the beam, the amount of bending stiffness is obtained at each section, and then the location and amount of damage in the beam will be obtained with the help of this index [16]. Patel and Dewangan [17] presented a baseline-free method using roving mode shape response based, multiple damage localization in a cantilever beam. They considered the combined mass and stiffness damage, as well as only the mass change damage. From the results, it was found that the proposed method can reliably identify the damage and its position [17]. Ghasemi et al. [18] evaluated the location and severity of the damage combining two being-updated parameters of the flexibility matrix and the static strain energy of the structure using optimization technics and compared results.

A major challenge for civil engineers is to evaluate the amount of structural damage caused by moderate to severe earthquakes. Seismic damage estimation is an important task in the field of structural health monitoring and several methods to evaluate the seismic vulnerability of structures have been proposed in recent years. One method, the response-based damage index, has been critically evaluated for its applicability to seismic damage evaluation. This method uses parameters, such as stiffness, drift, rotation of an element and energy, to calculate the state of damage using mathematical functions [19]. There has been a lot of research on this method. Ozturk et al. [20] evaluated the use of dynamic analysis and fragility curves, precast industrial concrete buildings that were designed and built according to the building codes of Turkey. In another paper, based on the suggested method of damage assessment using the vulnerability index, they examined the damage in a school building. With the help of the results of this research, it is possible to provide a proper assessment of the structural damage without the need for expert observations [21]. Also, in another study, this new method was used for the seismic vulnerability assessment of two reference RC structures in Malaysia. The authors presented that the results of this damage index can be used as a guide for earthquake impact assessments in Malaysia and other countries [22]. Hait et al. [23] presented a new damage index based on a combination of different structural response criteria which provided accurate results compared to the Park&Ang damage index. Mazlooom and Fallah [24] introduced a new Stiffness Based Damage Index based on the pushover analysis output. By using this index that only uses the information of the capacity curve of the structure, the amount of damage to the whole structure can be properly estimated by performing a series of simple calculations. The evaluation of the effect of ductile details in reinforced concrete structures was investigated in another study. In this research, two ductile and non-ductile frames were modeled and using the results of pushover analysis, the effect of using ductile details on the performance of the models was investigated by Chiluka.
and Oggu [25].

The subject of seismic damage detection and the use of damage detection methods based on modal information in this regard, we do not have much research history. However, this issue has been pursued in some limited works. For example, in 2016, Yang et al. [26] investigated the seismic damage to reinforced concrete structures numerically and experimentally in an article using the direct stiffness calculation method. In this work, the reinforced concrete structure was subjected to earthquake records, and then based on the changes in dynamic parameters, the amount of damage in different nodes of the structure was determined. Samimifar and Massumi [27] evaluated the modal-based story damage index performance based on an analytical study on seismic responses of some RC frames subjected to a set of earthquake records. The results of the analysis were compared with Park-Ang and modal flexibility story damage indices. Also, Mazloom and Fallah [28] investigated the total damage and story-based damage of the numerical RC frame models, based on the results of the Pushover analysis, and finally developed a new story-based damage index.

Because there are few previous works regarding the use of this method in determining the location of structural damage in possible earthquake damage scenarios, in this research work, at first, a numerical model of a steel beam, which was previously investigated by other researchers, was built and with the direct stiffness calculation method, the location and amount of damage in different damage scenarios were examined. Then, a 4-story reinforced concrete frame was modeled with specific geometric details and mechanical specifications, and after performing the eigenvalue analysis and receiving the modal information of this undamaged structure, the Pushover analysis was performed on it to determine the damage scenarios based on the occurrence of yielding in the story beams. By receiving the output of the analysis, several damage scenarios were defined based on the occurrence of yielding in the beams, and the modal information of the damaged structure was extracted in these scenarios. Finally, by using the method of direct calculation of bending stiffness, the damage is located on the beam with appropriate accuracy compared to other points. The results showed that this method can identify the location of the damage properly and with appropriate accuracy in the story beams, by only having information about the first mode of the structure.

2. SVI INDEX BASED ON IMPROVED DIRECT STIFFNESS CALCULATION METHOD

For a Bernoulli-Euler beam with the shear deformation neglected, the bending stiffness EI can be expressed in the modal sense as follows [1]:

$$EI = \frac{M}{d^2\phi} = \frac{M}{k}$$

(1)

In this regard, M and k are respectively the modal moment and the modal curvature at the same cross-section and \( \phi \) is the mode shape function. According to Yang et al. [1], Equation (1) is valid for structures for which the deformation of each mode is considered small. Another assumption is that after damage occurs, the mass of the structure will remain unchanged and only the stiffness of the structure will decrease. The free vibration equation of a structure with mass \( M_0 \) and stiffness \( K_0 \) will be written as Equation (2):

$$M_0\dddot{\phi} + K_0\phi = 0$$

(2)

Assuming that the modal deformations are harmonic, the above equation can be rewritten in the form of Equation (3):

$$K_0\phi = \alpha_0^2M_0\phi$$

(3)

where \( \alpha_0 \) and \( \phi \) are the ith frequency and corresponding mode shape respectively. Let \( x_i \) and \( x_{i+1} \) denote two adjacent measurement sections of the beam, the bending moment \( M_{i+1} \) and shear force \( V_{i+1} \) at section \( x_{i+1} \) of the mth mode can be calculated (Figure 1):

$$M_{i+1} = \int_{x_i}^{x_{i+1}} \sigma_{l,i} \rho A \phi_m(x) (x_{i+1} - x) dx + V_i (x_{i+1} - x)$$

(4)

$$V_{i+1} = V_i - \int_{x_i}^{x_{i+1}} \sigma_{l,i} \rho A \phi_m(x) dx$$

(5)

Modal Curvature, k, is directly calculated from the measured Modal shapes using the central difference approximation as follows:

$$k_m = \frac{\phi_m(i + 1) - 2\phi_m(i) + \phi_m(i - 1)}{(V_i)^2}$$

(6)

Consequently, a damage index called the stiffness variation index (SVI) was introduced:

$$SVI = \frac{EI_{\text{damaged}} - EI_{\text{act}}}{EI_{\text{int}}}$$

(7)

3. RESEARCH METHODOLOGY

3.1. Description of Study Models

To evaluate the damage detection method, first, a simply supported steel
beam with known geometric and mechanical characteristics was modeled in Sap2000 software with the specifications in the literature reported data [1]. The studied beam with simple supports has a span of 2 meters and a rectangular section of 25 cm in width and 30 cm in height. The numerical model of this structure contains 20 elements (21 nodes) and it was assumed that the nodes are the places where measurement tools are installed (Figure 2). The nodes of the structure from the left support to the right support were numbered from 1 to 21 and between both nodes, a 10 cm long beam element was considered from 1 to 20, respectively. Other mechanical specifications of the desired beam model are shown in Table 1. The distribution of mass and stiffness along the beam is uniform and in all damage scenarios, the mass of the structure will not change after damage.

3. 2. Simply Supported Steel Beam Model

The steel beam was modeled in Sap2000 software with the specifications in the literature reported data [1]. The studied beam with simple supports has a span of 2 meters and a rectangular section of 25 cm in width and 30 cm in height. The numerical model of this structure contains 20 elements (21 nodes) and it was assumed that the nodes are the places where measurement tools are installed (Figure 2). The nodes of the structure from the left support to the right support were numbered from 1 to 21 and between both nodes, a 10 cm long beam element was considered from 1 to 20, respectively. Other mechanical specifications of the desired beam model are shown in Table 1. The distribution of mass and stiffness along the beam is uniform and in all damage scenarios, the mass of the structure will not change after damage.

3. 2. 1. Damage Detection Using Svi Index

First, by analyzing the eigenvalues on the intact beam, the mode shape displacements were extracted from the finite element model. Because measuring the higher modes of the structure is associated with great difficulty in practice, only the data of the first mode of vibration was used in this research. After receiving the modal shape (MS) of the healthy structure, the modal curvature (MC) was calculated using the central difference method. Also, by calculating the value of the modal moment (MM), the bending stiffness of the EI section was obtained for each element.

In the next step, by changing the modulus of elasticity of the beam, damage scenarios were created in the numerical model. According to literature [1] and Tables 2 and 4 damage scenarios were examined. In the first scenario, only one element (Element 6) was damaged. In the following scenarios, several elements were damaged at the same time. It is also necessary to remember that in all scenarios, for the entire length of the desired element, the stiffness reduction occurred equally.

According to Table 2, in the first damage scenario, only one beam element is damaged. Also, in the second and third, two elements were damaged at two different points of the beam with different severities. In the fourth scenario, damage with the same severity has occurred in three adjacent elements. In each of the damage scenarios, by receiving MS and then calculating MC, MM corresponding to it, EI was finally obtained for all beam elements. Then the SVI index was calculated and its change curve was drawn along the beam. In order to compare the results of the current research, with literature [1], the results of the fourth damage scenario, including the mode shape curve, modal curvature, modal moment and SVI index, in the first mode, were displayed in Figure 3. According to the details of damage scenario 4, only elements numbers 13, 14 and 15 were damaged by 4 different severities.

According to the literature reported data [1], around the damage zones, the value of the modal curvature is significantly increased compared to other points. This sensitivity to the occurrence of damage does not exist in the mode shape vectors. So it can be said that the value

![Figure 1. Sign convention of internal forces and modal displacements [1]](image)

![Figure 2. Geometric view of the simply supported beam [1]](image)

<table>
<thead>
<tr>
<th>TABLE 1. Material specification of the simply supported beam model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Properties</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>Young’s modulus (E)</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>Specific weight</td>
</tr>
</tbody>
</table>
The results also showed that in this scenario, the SVI index identifies the real damage location of the beam with a significant magnification without declaring any other point as the possible location of the damage. Other damage scenarios, i.e. scenario 1 (damage in element 6), scenario 2 (damage in elements 6 and 13) and scenario 3 (damage in elements 13 and 20) were also evaluated and the results are shown in Figures 5 to 7. As the results of the first scenario investigation showed, the value of MC as well as the value of the SVI index determined the location of the damaged element with a very appropriate magnification compared to other elements. Although there is a difference between the value of the SVI index and the exact value of the assumed damage, the accuracy of the results seems to be sufficient to identify the damaged element. In this scenario, this index was able to determine the occurrence of damage in the damaged element itself.

According to Table 2, element 6 is damaged in both the first and second damage scenarios. In the first scenario, this element is the only place where the damage of the SVI index has a strong dependence on the value of modal curvature.

Also, the results showed that in this scenario where adjacent elements are also damaged at the same time, the SVI index was able to identify the amount of damage with high accuracy. Due to the high dependence of the value of this index on the changes of the modal curvature in the damaged position, the value of this index has gradually changed from zero on the damaged elements to the real value on the damaged elements. For example, according to the results of this scenario in Figure 4, at the damage level of 70%, the SVI index started at 0.6 in node 12 and reached 0.7 in nodes 13 to 14, which is the place where elements 12 to 14 are connected. Finally, it continued to reach the value of 0.3 at the 15th node and after that, it reached zero again. Based on the graphs, it can be said that the SVI index has correctly determined the range of damage and its severity in elements 13 to 15.
Figure 4. Simulation results of damage detection in scenario 4

Figure 5. Simulation results of damage detection in scenario 1

Figure 6. Simulation results of damage detection in scenario 2

occurs in the beam, and in the second scenario, along with element 13, the damage locations in the beam are considered.

According to Figures 5 and 6, it should also be noted that in the second scenario, similar to the first, the value of the SVI index in element 6, without being affected by simultaneous damage in element 13, is still in the range of 10-70%. Also, no damage has been detected to any other element.
Finally, it can be said that the results of the current study are absolutely accurate and reliable compared to previous research [1]. According to the results obtained in different scenarios, the SVI index was able to accurately identify the location and amount of damage in the simply supported steel beam model and distinguish it from the healthy elements with noticeable magnification.

3.3. A 4 story Reinforced Concrete Moment Frame Model

A 4-story concrete frame model was selected based on the specifications reported in the literature [29] and modeled in the SAP2000 program. The number of floors is 4 and the number of frame spans is 3 (Figure 8). The height of all floors is 3 meters, and the length of each span is 5 meters [29]. The concrete used in the model has a compressive strength of 30 MPa, and the steel used has a yield strength of 414 MPa. All columns are fixed at the base. A uniform load of 20 kN/m is placed on the beams of all floors. The geometry of the two-dimensional model along with the beam and column details is shown in Figure 8. Beams and columns were modeled with frame elements with a length of 1 meter, and after eigenvalue analysis of the frame, the mode shape displacement of the beams was measured at half-meter intervals (Figure 9).

In this research, to examine the damage detection method, the first and second modes of vibration of the undamaged structure as well as the damaged structure were considered. The deformed shape of the frame and the corresponding period in different vibration modes are shown in Figure 10. Also, to properly address the beams on all floors, the left, middle, and right beams of each floor were named BL, BM, and BR, respectively (Figure 11).

3.3.1. Verification of The Analysis Results

To compare the results of the first three modes of vibration obtained from the eigenvalue analysis on the modeled concrete frame, a comparison was made between the results reported by Ferracuti et al. [29] and the results of the present research (Table 3):

As is clear based on the results, the numerical modeling in the literature and the current work are in very good agreement with each other. Therefore, according to
Damage detection based on the preliminary appropriate results, damage detection will be done based on two groups of intentional damage scenarios and unintentional damage to the beams. Damage detection based on the intentional damage scenario, which was also done in the first model of this article, is similar to most past research works. After examining the damage detection method in these scenarios, the results have been discussed. In the following, the possible damage scenarios for the frame during the earthquake will be created by performing the pushover analysis. Finally, the amount of damage and its location will be determined based on the comparison between the modal data of the damaged structure and the undamaged structure.

### 3.3.2. Damage Detection in Damage Scenarios

#### 3.3.2.1. 20% Damage on The First Story Beam

The damage to the left beam of story 1 (BL) was caused by reducing the moment of inertia of the element one meter to its left by 20% (Figure 12).

The assumed location of the damage was chosen according to the possible response of the frame due to the earthquake and the potential for the formation of plastic hinges in these zones. By creating this intentional damage to the structure, the stiffness of the structure is reduced a bit, which will change the vibration characteristics of the frame. Therefore, the eigenvalue analysis was performed on the damaged frame and the mode shape information of the first two modes was extracted. By obtaining the modal information of the healthy structure and the damaged one, their comparison has been done with a coordinate system according to Figure 13. As the graphs show, the comparison of the shapes of the first and second modes alone is not able to provide clear information about the location of the damage and its amount. Using the central difference method, the modal curvature was calculated for the first two modes, and finally, the value of the bending stiffness EI of the beam was obtained in two states of the healthy and damaged structure, and its results are presented in Figure 14.

**TABLE 3. Comparison between the results of the current study and reported data in literature [29]**

<table>
<thead>
<tr>
<th>Results</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref [29]</td>
<td>0.50</td>
<td>0.17</td>
<td>0.11</td>
</tr>
<tr>
<td>Current Paper</td>
<td>0.509</td>
<td>0.178</td>
<td>0.109</td>
</tr>
<tr>
<td>Difference %</td>
<td>1.8</td>
<td>4.7</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Figure 10. The first three mode shapes of the modeled frame

Figure 11. Naming floor beams based on their location

Figure 12. The location of the damaged element in the frame
According to Figure 15, the SVI index was able to determine both the damage location and its amount correctly. This index has reached its highest value at the location of the one-meter element adjacent to the connection to the column, which is the real damage location.

Also, the value of this index in the adjacent node dropped from 0.20 to 0.12 and then completely zero. This issue is due to the dependence of the value of this index on the modal curvature calculated between three adjacent points along the length of the beam. On the other hand, it is clear that there is no difference between the results in the first two modes, and in other words, it can be said that damage detection with this method can only be done by receiving the displacements of the first mode of the structure.

3. 3. 2. Damage Scenarios Based on The Pushover Analysis Results

After evaluating the performance of the SVI index in the considered frame based on an intentional damage scenario and obtaining a reliable response for this damage detection method, structural damage is induced using a probable realistic scenario. To create a probable damage scenario caused by an earthquake, the results of the pushover analysis are utilized. Through the pushover analysis, lateral forces are applied to the structure to gradually develop damage to its members.

Therefore, at a specific step of the analysis, the damage state in the structure is considered as a damage scenario, and the beams that experience damage earlier and to a greater extent than others are identified.
According to Figure 16, the plotted circles on the beams and columns of the frame indicate the location of plastic hinge formation and the development of failure mechanisms. In other words, the analysis results show that the beams and columns of the first-floor roof have the highest probability of damage, and therefore, the health monitoring system should be more concentrated on the members of the first floor. Based on the above descriptions, the desired reinforced concrete frame was subjected to a displacement control type pushover analysis with a uniformly distributed lateral load on the floors. The center of mass node on the last floor was considered as the control node. To determine the locations of damage mechanisms, automatic concentrated plastic hinges were used at the ends of the beams, which utilize performance levels based on measuring plastic rotation proposed by FEMA273.

The flexural automatic plastic hinge type M3 was used for the beams, and the interaction P-M3 type was used for the columns. The plastic hinge was applied at 0.05 and 0.95 of the length of these members. By performing the pushover analysis in the Sap2000 software and obtaining the structural capacity curve, in order to validate the analysis of results, the resulting curve from the analysis conducted in this study was plotted alongside the curve obtained from the literature [29] shown in Figure 17. Due to the good agreement of the two curves at different displacement levels, the necessary confidence in the modeling was achieved.

Next, in order to define the damage scenario, the occurrence of yielding in the beams at the floor levels was selected as the damage scenario using the results of the pushover analysis. According to the results and as shown in Figure 18, the onset of yielding has occurred in the vicinity of the right-side connection of beam BL in the first-floor roof.

In the second and third damage scenarios, yielding occurred in the second and third story beams of the structure, respectively. Also, in the second scenario, all the beams of the first story had already yielded and in the third scenario, the beams of the first and second stories had already yielded.

The vibration periods of the first three modes of the healthy structure and the damaged structure in three damage scenarios are presented in Table 4.
### TABLE 4. The first three vibration periods of the intact structure and the damaged structure

<table>
<thead>
<tr>
<th>Mode shape number</th>
<th>Intact</th>
<th>Damaged (Scenario 1)</th>
<th>Damaged (Scenario 2)</th>
<th>Damaged (Scenario 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.509</td>
<td>0.524</td>
<td>0.61</td>
<td>0.727</td>
</tr>
<tr>
<td>2</td>
<td>0.178</td>
<td>0.181</td>
<td>0.189</td>
<td>0.203</td>
</tr>
<tr>
<td>3</td>
<td>0.109</td>
<td>0.109</td>
<td>0.112</td>
<td>0.119</td>
</tr>
</tbody>
</table>

#### 3.3.2.1. First Damage Scenarios

Based on the results of the pushover analysis, the first yield location was in story 1 and the beam BL. Therefore, in two states of the intact and damaged structure, the modal displacement of the first and second modes of the beams of this floor was extracted. In the following, as an example, the changes in the modal displacement of the first and second modes of the beam BL are displayed in Figure 19. Using the modal displacement obtained from the previous step and the central difference method, the modal curvature of the first two modes was calculated. In the following, the changes of bending stiffness $EI$ on the beam were obtained in two states of intact and damaged structures, and finally, the SVI was calculated.

The results are shown in Figures 20 and 21. The value of the SVI index has reached its maximum value near the right support of the beam BL. This result is completely consistent with the initial guess of the yield location in the first scenario. To see the state of damage in other beams on the first floor, the SVI index value changes in story 1 are shown in Figure 22.

![Figure 19](image1.png)

**Figure 19.** Comparison between the a) first mode shape and b) second mode shape of the beam BL

![Figure 20](image2.png)

**Figure 20.** Comparison between calculated stiffness $EI$ in a) first mode shape and b) second mode shape of the beam BL (story 1)

![Figure 21](image3.png)

**Figure 21.** Comparison between Calculated SVI values in a) first mode shape and b) second mode shape of the beam BL (story 1)
3. 3. 2. 2. Second Damage Scenarios  In this scenario, the beam BR in the second story has yielded at one of its two end nodes. Also, in story 1, all beams have yielded at one of their two end nodes. Therefore, in this scenario, the value of the damage index has been calculated on both the first story and the second story. With a process similar to the first scenario, at half-meter intervals on the beams of story 2, the first and second modal displacements of the beams were extracted in two states of intact structure and damaged structure. In the following, the first and second modal displacements of the beam BL in story 2, as well as the changes in the bending stiffness EI of this beam, were drawn in two damage states. (Figures 23 and 24). In the following, at first, the SVI index value in the beams of the first story, all of which have yielded in this scenario, has been calculated based on the information of the first and second modes. (Figures 25 to 28). As it is clear, based on the results of this index, the yielding in the right node of the beams is known, and it is completely consistent with the results of pushover analysis.

Based on Figure 29, changes in the SVI index in the beams of the second story show that the beam BR has yielded at its right node, which is proven based on the value of the index in the first two modes. Also, at the two end nodes of the beam BM on the same floor, the value of the index has grown a lot. It indicates that this beam is ready to yield and will reach yielding with increasing lateral displacement of the structure, similar to the beam BR.

3. 3. 2. 3. Third Damage Scenarios  In scenario 2, in story 3, beam BL has yielded. Also, in the first and second stories, all the beams have yielded, and on the first floor, the column in the left corner of the frame has yielded. Similar to the process of evaluating the first and second damage scenarios, in damage scenario 3, the first and second mode shape displacements of the third story beams were compared with each other in the two states of the intact structure and the damaged structure (Figure 30).
Figure 25. Comparison between Calculated SVI values in a) first mode shape and b) second mode shape of the beam BL (story 1)

Figure 26. Comparison between Calculated SVI values in a) first mode shape and b) second mode shape of the beam BM (story 1)

Figure 27. Comparison between Calculated SVI values in a) first mode shape and b) second mode shape of the beam BR (story 1)

Figure 28. SVI variation in story 1 beams (scenario 2)

Figure 29. SVI variation in a) first mode shape and b) second mode shape of story 2 beams
The damage to the first and second story beams is shown in Figures 31 and 32. Also, changes in the SVI index on the beams of the third floor (Figure 33), in the first two modes of the structure, showed that the BL beam yielded at its right node.

Figure 30. Comparison between mode shapes of the beam BL (story 3) in a) mode 1 b) mode 2

Figure 31. Comparison between Calculated SVI values in a) first mode shape and b) second mode shape of story 1 beams

Figure 32. Comparison between Calculated SVI values in a) first mode shape and b) second mode shape of story 2 beams

Figure 33. SVI variation in a) first mode shape and b) second mode shape of story 3 beams

4. CONCLUSIONS

1. In this article, the SVI index was used to investigate the location and magnitude of damage in beams. Initially, a simple steel beam that had been previously studied by others was modeled and examined in several intentional damage scenarios. The results showed that this index was able to predict the location and magnitude of damage along the beam with appropriate magnification and acceptable accuracy. Additionally, the results demonstrated that in scenarios involving simultaneous damage occurrence in multiple elements along the beam, the SVI index was able to correctly identify the damaged beams.
locations and provide an accurate estimation of their magnitude. This investigation continued with the analysis of a reinforced concrete frame.

After validating the results of the frame model, an intentional and logical scenario was created in one of the beams of the concrete frame, and its location and magnitude were monitored. Then, based on the seismic scenarios according to the results of the pushover analysis, these evaluations were continued. Using this analysis method, an initial prediction of the vulnerable floor(s) was obtained, and as a result, the placement of the sensors in the field would be feasible based on the priority of the analysis results. In all of these analyses, the reading location of modal data was embedded at half-meter intervals along the beams, and the effects of environmental noise were not considered. In the first mode of the structure, the dominant displacement of the frame was in the transverse direction, and therefore, the greatest deformation occurred in the columns while the beams had less deformation. In this case, comparing the mode shape displacements of the intact and damaged structures alone did not provide tangible information regarding the location of damage in the beams.

2. The SVI index was able to accurately amplify the location of damage by utilizing the second derivative of the mode shape vector, which is highly sensitive to mode shape variations. The results also demonstrated that the use of the second mode in structural damage assessment did not significantly change the results, indicating that it is sufficient to have the first mode of the structure to identify the location of damage in the beams.

3. One common error among many modal-based damage detection methods is reporting damage in locations other than the actual damaged area on the member. This error was not observed in the performance of the studied index in this research. The SVI index was able to perform well in seismic damage scenarios where damage occurred simultaneously in multiple beams.

4. In order to better understand the changes in the flexural stiffness of the beam after damage, beam BL in the first story of the concrete frame, which had sustained damage in all scenarios, was selected, and the variations in its stiffness along its length were presented in different scenarios. As shown in the provided figure, the EI value of the beam in the damaged locations is clearly lower than in the undamaged locations. Therefore, even without access to information about the intact structure, the location of damage can be determined by comparing the EI values at different sections.

5. According to the results obtained from the analysis, the SVI index was able to accurately determine both the location and magnitude of damage in a scenario where intentional damage was created at a specific location on the beam. This is in contrast to the reported damage magnitude based on the received modal data after the seismic damage, which was a very large numerical value close to 100%, indicating significant amplification of damage in comparison to seismic damage indices. Considering the level of damage that occurred at the beam level due to yielding, the magnitude of the damage index shows significant amplification in determining the extent of damage.

5. REFERENCES


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