Mode Shape Change Based System Identification: An Improvement using Distributed Computing and Roving Technique

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**ABSTRACT**

Structure health monitoring is still a challenging issue despite continuous research efforts since a long time. Mode shape changes are a remarkable symptom of the damaged element location in structural system identification. In this paper, various mode shape based prediction techniques are applied to a common structural model. A cantilever beam model is formulated using the distributed mass and stiffness matrix based finite element modelling. Multiple damages are introduced to the above cantilever beam even with two, three and four member damage combinations. The results do not provide a concrete solution on the damage element location prediction. Further, in the computational part, the distributed computing technique using element-to-element matrix multiplications is applied. The Roving technique is also applied, which acts as a counter for self-automation. The proposed approach provides a better damage element location prediction even for the multiple damaged member combinations. The roving technique means an element scanning technique, which works with a computer clock speed. The novelty of the approach is that the method is simple and it could be applied to other structures. While scanning as automation no element is left out. Another beauty of the method is that no prior damage elements are assumed as many statistical based approaches assumed in prior. This approach could be a better way to the automation process, for the system identification and machine learning tools.

**NOMENCLATURE**

- **Greek Symbols**
  - $\phi$: Modal amplitude of the structural system
  - $\bar{\phi}$: Modified normalized modal amplitude of the structural system
  - $\lambda$: Square of the frequency (Hz)
  - $ud_{\phi}$: Modal amplitude of the undamage structural system
  - $dam_{\phi}$: Modified normalized modal amplitude of the damage structural system

- **Subscripts**
  - $d$: Damage case
  - $i$: Natural number to denote mode number
  - $j$: Natural number to denote mode number
  - $u$: Undamage case

1. **INTRODUCTION**

Mode shapes data are the significant field response data for any vibration based structural system identification [1]. These data are widely used for damage identification, damage location prediction, assessment of its severity, and the estimation of the future life of the structure [2].

Various methods have been proposed based on the mode shape response and its derivatives, such as the modal assurance criterion (MAC), cumulative damage factor method (CDF), mode shape based damage index method (MSDI), coordinate modal assurance criterion (COMAC), mode shape difference method (MSD), machine learning-based approaches, mode shape...
curvature method (MSC), and many more [3-15]. In addition, various other techniques are employed which are not modal parameter based, such as the wave propagation based approach, neural networking, machine learning and frequency response function (FRF) method [16-18]. The aforementioned techniques are limited to predicting and locating the damage. The mode shape based prediction techniques; MAC method and COMAC method are useful for finding the presence of damage [19-21]. The MSD method predicts the damage location and was noticed that, for the higher modes, the difference in modal curvature exhibits several peaks, which could result in a false indication of damage [22]. The mode shape curvature demonstrated that a useful indication of the location of the damage is the variation in the curvature mode between an undamaged and damaged structure [22]. Studies based on bridge assessment have been performed by Jayasundara et al. [23]. The CDF method [9, 24] and the MSDI [25] could identify 2, 3, and 4 number of damaged member combinations, the possibility of a false signal exists in cases of extremely small damage. Additionally, if the damage number is larger, there is a risk of a false alert. The reason behind the multiple member damage element location false output is that most of the algorithm has taken normal/regular matrix multiplications that lead the elements of the matrices to mix when computing the eigenvalue and eigenvector. For small damage, the element could be in order to catch the fine damage. The structural member is discretised to the different number of small finite elements. With an increase in the number of elements, during the formulation, the number of equations also increases hence its prediction will be more complex. For such a case, more research is necessary and it is covered in section 3.3. Machine learning-based approaches, such as neural networks and support vector machines, can also be used for mode shape extraction [26].

Currently, there is no reported literature on mode shape based system identification for multiple damage combinations using Distribute Computation Technique (DCT) and Roving Technique (RT). Hence, the primary goal of this research is to propose a new algorithm DCT and RT, and use it for an improvement in the current algorithm. In this paper, the damage caused by both changes in mass and stiffness as well as only the stiffness change, are considered. Another goal is to provide a common approach to improve the computational part of the existing techniques for mode shape based system identifications. Further, for the single damage case, many research works have been published. In order to identify the structural system with multiple damages, the RT means a scanning location technique which will work with a computer clock speed. The novelty of this paper is that the method is simple and automated and it provides a common algorithm for any mode shape based structural system identification. The method leads to a better way on automation process for system identification and for future machine learning techniques.

2. THEORETICAL DEVELOPMENT

2.1. Damage Assessment Methods

The damage assessment methods based on the modal amplitude response, are mentioned in this section. Although there are many other methods by various researchers, in this paper the authors have compared the developed method on these indexes hence only the following methods are discussed.

2.1.1. Modal Assurance Criterion (MAC)

This technique makes use of the baseline data, i.e., the mode shape response of the structure in the initial stage and of the damage condition. This method provides a single response for the whole system. The MAC is given by Equation (1) [19-21].

\[
MAC = \frac{\sum_{i=1}^{m} |\phi_i^1 \phi_i^2|}{\sum_{i=1}^{m} |\phi_i^1|^2 \sum_{i=1}^{m} |\phi_i^2|^2}
\]

2.1.2. Coordinate MAC (COMAC) Method

This method also makes use of the baseline data, i.e., the mode shape response of the structure in the initial stage and of the damage condition. This method provides response at the individual nodes. The COMAC is given by Equation (2) [21].

\[
COMAC = \frac{\sum_{i=1}^{m} \phi_i^1 \phi_i^2}{\sum_{i=1}^{m} (\phi_i^1)^2 \sum_{i=1}^{m} (\phi_i^2)^2}
\]

2.1.3. Mode Shape Difference (MSD) Method

This method also makes use of the baseline data, i.e., the mode shape response of the structure in the initial stage and of the damage condition. The absolute modal value of intact and damaged cases for the particular element will be used to determine the MSD value. The MSD is given by Equation (3) [22].

\[
MSD = \sum_{i=1}^{m} |\phi_i^1 - \phi_i^2|
\]

2.1.4. Mode Shape Curvature (MSC) Method

This method also makes use of the baseline data, i.e., the curvature of the mode shape of the structure in the initial stage and of the damage condition. The curvature of the mode shape is determined, by taking the second derivative of the mode shape. The damage could be located more precisely by this method. MSC for the structural system is given by Equation (4) [22, 23].

\[
MSC = \left| \frac{\phi_i^1 - 2\phi_i^2 + \phi_i^3}{h^2} - \left( \frac{\phi_i^2 - 2\phi_i^1 + \phi_i^0}{h^2} \right) \right|
\]
2. 1. 5. Cumulative Damage Factor (CDF) Method

This method also makes use of the baseline data, i.e., the curvature of the mode shape response of the structure in the initial stage and of the damage condition. CDF is the damage indicator whose value depends on the MSC. For any particular node of the structure, say the jth node, this damage indicator value is given by Equation (5) [24, 25].

\[
CDF_j = \frac{1}{m} \sum_{i=1}^{m} (MSC^d_j - MSC^s_j)
\]  \hspace{1cm} (5)

2. 1. 6. Mode Shape Damage Index Method (MSDI)

This method also makes use of the baseline data, i.e., the the derivative from the mode shape response of the structure in the initial stage and of the damage condition. This method is based on the comparison of the normalized modal amplitude of damage and the undamaged state of the structural system. The MSDI for a particular node is given by Equation (6) [26].

\[
MSDI = \frac{\phi_j^d - \phi_j^s}{\phi_j^s}
\]  \hspace{1cm} (6)

The above indices based methods are applied on a common cantilever beam model for the damage element location detection which is explained in section 3.2.

2. 2. Distributed Computing Technique (DCT)

A DCT is a method to perform computations over numerous matrices for superior results and fault tolerance. This method is based on the application of the vectorize operation for matrix multiplication [27] to modify the computational part of the system represented by the matrix form. While performing the regular/common matrix multiplication of any two matrices, let’s say K and Q, as demonstrated below:

\[
K = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2x2}
\]  \hspace{1cm} (7)

\[
Q = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}_{2x2}
\]  \hspace{1cm} (8)

On regular/common matrix multiplication, the product will come out as:

\[
R = K \times Q
\]  \hspace{1cm} (9)

\[
R = \begin{bmatrix} (a_{11} \cdot b_{11}) + (a_{12} \cdot b_{21}) & (a_{11} \cdot b_{12}) + (a_{12} \cdot b_{22}) \\ (a_{21} \cdot b_{11}) + (a_{22} \cdot b_{21}) & (a_{21} \cdot b_{12}) + (a_{22} \cdot b_{22}) \end{bmatrix}_{2x2}
\]  \hspace{1cm} (10)

Equation (10) illustrates that the matrix output at R(1,1) location represents the mix behaviour \( K(1,1), Q(1,1) + K(1,2), Q(2,1) \). Hence, element property is mixed completely in regular/common multiplication. On the other hand, when doing vectorize operation for matrix multiplication, the product will come out as:

\[
S = K \cdot Q = \begin{bmatrix} a_{11} \cdot b_{11} & a_{12} \cdot b_{12} \\ a_{21} \cdot b_{21} & a_{22} \cdot b_{22} \end{bmatrix}_{2x2}
\]  \hspace{1cm} (11)

Here, \( \cdot \) represents the term-by-term matrix multiplication.

By observing the S matrix properties from Equation (11), it can be easily observed that the element property doesn’t get mixed, hence the individual element property is not lost. Say, for example if \( K \) is a stiffness and Q is the response and if they are connected by a relationship like \( K \cdot Q = F \) and if F is a unit matrix. It will not mix the other locations elements property, rather it will be one to one means for the same element location. The benefit is that damage could be located, as the position of the individual element is not lost. For the division of matrix again the same element to element operation are considered in the computation process.

2. 3. Roving Technique (RT)  

The RT is, scanning each element one by one for the computation process. This technique is using an automatic counter which is starting from one to the number of elements (e) just like a for-end loop. For example for element_number i=1 to e for i = 1: e

\[ \text{The algorithm} \]

\[ \text{end} \]

3. METHODOLOGY

3. 1. Finite Element Modelling  

A cantilever beam was employed as the beam specimen for the study. The beam considered is 200 mm long, its width is 9 mm and its depth is 50 mm. The Young’s modulus (E) for the specimen material is 69.1 N/mm², Poisson’s ratio is 0.334 and the density of the specimen material is 2668.32 \( \times 10^{-10} \) N/mm³.

The finite element method is used for further analysis. The beam is discretized to 10 elements and the finite element model is shown in figure 1(a). The two noded beam elements are used for discretization as shown in figure 1(b). The nodal unknowns will be vertical deflection (u1 at node 1 and u3 at node 2) and rotation about the z-axis (u2 at node 1 and u4 at node 2) at each node. The degree of freedom then is 2 at each node.

For the study, a total of 20 damage cases are taken into account. For this, four locations were selected at random at a distance of 40 mm, 100 mm, 120 mm, and 160 mm. Figure 1(a) illustrates the specifics. These damage positions are represented by D1, D2, D3, and D4, respectively. The width of damage is considered insignificant, and its depth was measured in terms of percentage reductions in depth as 20
Table 1 considers all cases of damage with reductions of 0%, 30%, 40%, and 60% in mass. The reduction in mass to 40% and 60% has been examined. Figure 1(c) illustrates the damage considered. Three different damage combinations have been presented. Cases 1 to 12 illustrate the 2 members' damage at a time. Cases 13 to 17 represent the damage scenario for 3 members damages at a time, and Cases 19 and 20 represents for 4 member damage at a time. All these cases are taken as any random combination for the demonstration purpose. Also, the damage in terms of depth reduction only and combined mass and depth reduction is considered. All of these cases are mentioned in Table 1.

![Figure 1](image.png)

**Figure 1.** (a) Beam element (b) Vertical section of the crack damage region (c) Finite element model of a cantilever beam with damage positions

**TABLE 1.** The various damage cases for the cantilever beam

<table>
<thead>
<tr>
<th>Damage case</th>
<th>Location of damage</th>
<th>Depth reduction (%)</th>
<th>Mass reduction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>D1, D2</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td>Case 2</td>
<td>D1, D3</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td>Case 3</td>
<td>D1, D4</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td>Case 4</td>
<td>D2, D3</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td>Case 5</td>
<td>D2, D4</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td>Case 6</td>
<td>D3, D4</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td>Case 7</td>
<td>D1, D2</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Case 8</td>
<td>D1, D3</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Case 9</td>
<td>D1, D4</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Case 10</td>
<td>D2, D3</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Case 11</td>
<td>D2, D4</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Case 12</td>
<td>D3, D4</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Case 13</td>
<td>D1, D2, D3</td>
<td>0.3</td>
<td>-</td>
</tr>
<tr>
<td>Case 14</td>
<td>D1, D2, D4</td>
<td>0.3</td>
<td>-</td>
</tr>
<tr>
<td>Case 15</td>
<td>D1, D3, D4</td>
<td>0.3</td>
<td>-</td>
</tr>
</tbody>
</table>

| Case 16     | D1, D2, D3         | 0.4                 | 0.4                |
| Case 17     | D1, D2, D4         | 0.4                 | 0.4                |
| Case 18     | D1, D3, D4         | 0.4                 | 0.4                |
| Case 19     | D1, D2, D3, D4     | 0.2                 | -                  |
| Case 20     | D1, D2, D3, D4     | 0.6                 | 0.6                |

**TABLE 2.** Mode shape based prediction techniques

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Methods</th>
<th>Actual damage location (mm)</th>
<th>Identified damage location (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>MAC</td>
<td>40, 100, 120, 160</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>COMAC</td>
<td>40, 100, 120, 160</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>MSD</td>
<td>40, 100, 120, 160</td>
<td>32, 105, 129, 152</td>
</tr>
<tr>
<td>4</td>
<td>MSC</td>
<td>40, 100, 120, 160</td>
<td>39, 101, 122, 161</td>
</tr>
<tr>
<td>5</td>
<td>CDF</td>
<td>40, 100, 120, 160</td>
<td>38, 103, 121, 162</td>
</tr>
<tr>
<td>6</td>
<td>MSDI</td>
<td>40, 100, 120, 160</td>
<td>39, 101, 122, 161</td>
</tr>
</tbody>
</table>

3.2 Mode Shape based Prediction Techniques

Since the mode shape based methods taken into consideration for the study are baseline based methods, it is necessary to estimate the modal response for both the initial condition and after damage. These responses are determined using MATLAB programming. Additionally, the damage detection and localization indicators for the various methods have been determined using these responses, for the damage case 20 i.e. four members damages at a time considering damages in terms of both the depth and mass reduction.

Initially, various mode shape based prediction techniques mentioned in section 2.1, are applied to a common structural model a cantilever beam, for the different multiple damage combinations. It has been observed from Table 2 that the results do not provide a concrete solution for the identified damage element location prediction. The MAC method and COMAC method are useful for finding the presence of damage only. The methods such as MSD, MSC, CDF and MSDI could identify 2, 3, and 4 damages simultaneously. The possibility of a false signal exists in cases of extremely small damage. Additionally, if the damage number is larger, there is a risk of a false alert. Table 2 makes it clear that the identified damage location and the actual damage location are not precisely the same. The reason behind this is as follows:

a) In the above the normal/regular matrix multiplication rules are followed which leads the elements of the matrices in which the element properties of different elements get mixed while computing the eigenvalue and eigenvector. As a result, it gives the mixing of different properties of elements at the damage position as per Equation (10).
b) For the very small damage prediction, the number of elements could be increased theoretically for computation. Therefore, more research is necessary, which has been covered in Section 3.3.

3. 3. An Improvement using the Distribute Computing and Roving Technique In the following section, all the matrix operation follows the distributed computing as per Equation (12). In structural dynamics, for the linear structural system, the general form of the equation of motion is given by Equation (12):

\[ M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = 0 \]  

Equations (13) represents the stiffness matrix for damage localization as the difference between the stiffness matrix of the undamaged structure and the damaged structure, and Equation (14) represents the mass matrix for damage localization as the difference between the mass matrix of the undamaged structure and the damaged structure.

\[ R = K_u - K_d \]  
\[ M = M_u - M_d \]  

The global matrix of the system is used to determine the element location for inverse problems. In a way similar to the method by which the global stiffness matrix is created, the global mass matrix is also created, using a distributed type of mass matrix rather than a lumped method. Further, the solution to the problem for Equation (12) is reduced to the solution to Equation (15) for the multiple degrees of freedom system.

\[ (R - \lambda(u,i) M)\phi = 0 \]  

For the cantilever beam, the lumped mass matrix is used for global matrix formulation. The stiffness matrix for damaged and undamaged elements is given below:

\[
\begin{bmatrix}
987000 & 987000 & -987000 & 987000 \\
987000 & 131600000 & -987000 & 65800000 \\
-987000 & -987000 & 987000 & -987000 \\
987000 & 65800000 & -987000 & 131600000
\end{bmatrix}
\]

where, i = element numbers 1, 2, 4, 5, 8 and 10 are for undamaged elements.

\[
\begin{bmatrix}
123375 & 1233750 & -123375 & 1233750 \\
1233750 & 16450000 & -1233750 & 8225000 \\
-1233750 & -1233750 & 123375 & -1233750 \\
1233750 & 8225000 & -1233750 & 16450000
\end{bmatrix}
\]

where, j = element numbers 3, 6, 7 and 9 are for the damaged elements.

From the above element matrices, it has been observed that, when a structure is damaged, the stiffness value of the damaged element changes. Hence, after assembling the stiffness matrix, the value of the particular element in the global stiffness matrices changes; all other values remain unchanged. The regular matrix multiplication leads the elements of the matrices to mix when computing the eigenvalue and eigenvector. As a result, the damaged element’s position is lost. The DCT mentioned in subsection 2.2 is applied for calculating \( \lambda(u,i) \) and \( \phi ' \) to ensure that the position of the specific element is not lost and the damage could be located. The modal amplitude value \( \phi_i \) is used for each of the mode shape based prediction techniques addressed in subsection 2.1.

The RT is done by scanning each element one by one. Basically, the technique is using an automatic counter which is starting from one to the number of elements (e) just like a for-end loop. The RT for the MAC method is given below:

\[
\text{for } i = 1: m \\
\quad y_1 = \text{sqrt}(\text{mode}(ud, \phi ') \times (\text{dam, } \phi (i))) \\
\quad y_3 = \text{mode}(\text{dam, } \phi (i)) \times (\text{dam, } \phi '(i)) \\
\text{end}
\]

\[
\text{for } i = 1: n \\
\quad y_2 = \text{mode}(\text{ud, } \phi (i)) \times (\text{dam, } \phi '(i)) \\
\text{end}
\]

\[ MAC = y_1 / (y_2 \times y_3) \]

The MATLAB code for the RT of MAC method is given above. The Similar procedure has been followed for all the mode shape based prediction techniques addressed in subsection 2.1.

4. RESULTS

The system identification predictions based on mode shape changes have been improved for better damage location prediction. The algorithm applies DCT and RT. It has been observed that the damage existence and location predicted for the particular element could be visualized in Figures 2 to 7. A total of 20 damage cases are considered in this paper; the authors have shown the developed method for Case 20 in the following section.
With a significant change in the numerical value, the approach identifies a system with multiple damages, as well as it is very clear from Table 3 also. The results conclude that the modified method practice predicts better damage element locations even for the multiple damaged member combinations.

### Table 3. Multiple damage location identification by modified algorithms, for Damage Case 20

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Method</th>
<th>Actual damage location (mm)</th>
<th>Identified damage location (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Modified MAC</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>method</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>120</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td></td>
<td>160</td>
<td>160</td>
</tr>
<tr>
<td>2.</td>
<td>Modified COMAC</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>method</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>120</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td></td>
<td>160</td>
<td>160</td>
</tr>
<tr>
<td>3.</td>
<td>Modified MSD</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>method</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>120</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td></td>
<td>160</td>
<td>160</td>
</tr>
<tr>
<td>4.</td>
<td>Modified MSC</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>method</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>120</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td></td>
<td>160</td>
<td>160</td>
</tr>
<tr>
<td>5.</td>
<td>Modified CDF</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>method</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>120</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td></td>
<td>160</td>
<td>160</td>
</tr>
<tr>
<td>6.</td>
<td>Modified MSDI</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>120</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td></td>
<td>160</td>
<td>160</td>
</tr>
</tbody>
</table>
5. CONCLUSION

In this study, the mode shape change based system identification predictions improvement is performed for the better prediction of damage location using the mode shape data. The improvement has been done, using the DCT and the RT.

1. When a structure is damaged, the value of the mass and stiffness matrices of that particular element in the global matrix changes at that location while all the other elements’ parameter remains unchanged. In the case of normal/regular matrix multiplication, the matrix properties at the element location damaged/non-damaged elements get mixed during the eigenvalue and eigenvector computation, as a result, the damaged element’s position is lost.

2. Hence, the DCT is performed while computing the eigenvalue and eigenvector which is incorporated to the existing algorithms. The benefit of doing so is that damage could be located, as the position of the individual element is not lost.

3. The implementation of DTC and RT is capable of small damages cases also.

4. An improvement using the DCT and RT shows the better for finding locations even for the multiple members’ damage combinations which are taken as the random combination.

In the age of the forthcoming computer era, the RT means a scanning location technique which will work with a computer clock speed. Hence it will be a better solution for the automation process, of the machine learning technique. This paper will provide a landmark submission for an unknown element.

6. REFERENCES


چکیده
نظارت بر سلامت ساختمان هنوز یک موضوع چالش برانگیز است، علیرغم تلاش‌های تحقیقاتی مستمر از زمان طولانی تغییرات حالت بر عهده قرار گرفته که به همراه این بخش، محصول دقیقی از محل عنصر آسیب دیده در زمان ساختمان ساخته شده و با سرعت شده‌اند. الگوبرداری از محل عنصر آسیب دیده، نیازمند یک شناخت دقیق عنصر آسیب دیده و اعمال متعددی در زمان‌های مختلف است. در این مقاله، تعدادی از مدل‌های مختلف پیشنهادی شده تایید می‌شود که مدل تایید نیازمند افراد مناسب یک راه حل آماده‌تر و دقیق‌تر و برای بررسی واحد عضو بالاتر، آسیب‌های بینی در محل عنصر آسیب دیده را به‌طور دقیق ارائه می‌دهد. سه روش اصلی از مدل‌های پیشنهادی در این مقاله به‌عنوان روش‌های محاسباتی تایید می‌شود. روش‌های محاسباتی شامل روش‌های محاسباتی تایید شده، روش‌های محاسباتی تایید نیازمند و روش‌های محاسباتی تایید نیازمند می‌باشد. به‌طور کل، این مقاله به ارائه روش‌های بینی مقننه در زیر پنجره عنصر آسیب دیده و اعمال آن در زمان‌های مختلف اصلی است. این مقاله نیازمند بررسی دقیق عنصر آسیب دیده و اعمال آن در زمان‌های مختلف است. در این مقاله، روش‌های پیشنهادی به‌عنوان روش‌های محاسباتی تایید می‌شود.