



Thermal Analysis of Fluid Flow with Heat Generation for Different Logarithmic Surfaces

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ABSTRACT

This study investigated the effect of temperature changes on different logarithmic surfaces. One-dimensional heat transfer was considered. The heat generation source term is added to the governing equations. Most scientific problems and phenomena such as heat transfer occur nonlinearly, and it is not easy to find exact analytical solutions. Using the appropriate similarity transformation for temperature and the generation components causes the basic equations governing flow and heat transfer to be reduced to a set of ordinary differential equations. These equations have been solved approximately subject to the relevant boundary conditions with numerical and analytical techniques. According to the given boundary conditions, Collocation, Galerkin, and least squares methods were used to find an answer to the governing differential equations. The validation of the present techniques has been done with the fourth-order Runge-Kutta method as a numerical method. The temperature profiles for different values of β and α have been obtained. The results showed that the proposed methods could consider nonlinear equations in heat transfer. Therefore, the results accepted by current analytical methods are very close to those of numerical methods. Comparing the results provides a more realistic solution and reinforces the conclusions regarding the efficiency of these methods. Furthermore, changes in temperature profiles occur with decreasing and increasing β and α numbers.

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1. INTRODUCTION

Solving differential equations in mathematics helps to understand many physical concepts. Many phenomena can be expressed in engineering with differential equations. In many of these problems, the most widely used heat equations, it is complicated and impossible to obtain accurate solutions to the differential equations governing these problems. In recent studies, the methods developed by Jalili et al. [1, 2], Zangoee et al. [3], Ghadikolae et al. [4], Al-Sankoor et al. [5], Amouzadeh et al. [6] and Etbaitabari et al. [7] have solved a broad scope of issues. Also, the methods of weighted residuals, including accurate and straightforward trial functions, have been utilized to crack nonlinear differential equations. Least squares methods (LSM), Galerkin, and Collocation are examples of weighted residual methods presented by Ozisik [8] to solve samples related to heat

transfer. The collocation method is utilized to crack a third-order differential equation by Stern and Rasmussen [9]. Conductive and radiative heat transfer in a linear anisotropic cylindrical with the spectral position was investigated by couple of equations and in an unsteady flow by Sun et al. [10]. Basha and Sivaraj [11] investigated how to generate entropy in a porous tube containing nanofluid. Celik and Ozturk [12] investigated the speed and heat transfer in parallel circular surfaces. Nabati et al. [13] proposed the collocation method to solve the equation of thermal performance in a porous medium. Chandrakant et al. [14] proposed a numerical solution for a heat exchanger with helical flow channels. Recently, Biswal et al. [15] used the least-squares method to solve the governing equations of nanofluid flow in a semi-porous channel. Hatami and Ganji [16] discovered that LSM is more practical than other analytical and semi-analytical methods for cracking nonlinear heat

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transfer problems in many problems. Recently, several researchers have studied this issue and heat transfer [17-25]. Abbaszadeh et al. [26] have presented the Galerkin method to solve the Navier-Stokes equation in combination with the heat transfer equation. Numerical models for the analysis of unsteady heat transfer in PCM employing the Galerkin method have been carried out by Zhang et al. [27].

This article considers heat transfer on logarithmic levels with heat production. As the novelty of this study, the influence of some physical parameters such as the rate of effectiveness of temperature on non-dimensional temperature profiles is considered. They are mainly used in simple cases and special situations, so numerical methods solve differential equations in many problems. However, these methods can solve stress analysis, fluid flow, heat transfer, and **electromagnetic wave equations**. These methods approximate the solution of differential equations governing the environment. The present study uses the least-squares, collocation, and Galerkin methods to solve the nonlinear heat transfer issue. The validity of these methods is shown by comparing the outcomes with the numerical method.

2. GEOMETRY AND GOVERNING EQUATIONS

In this research, the level of heat transfer and heat production are indicated by $A(x)$ and $G(x)$, respectively, which change logarithmically. Also, heat transfer is investigated in one dimension. In addition, the coefficient of thermal conductivity k varies as a function of temperature.

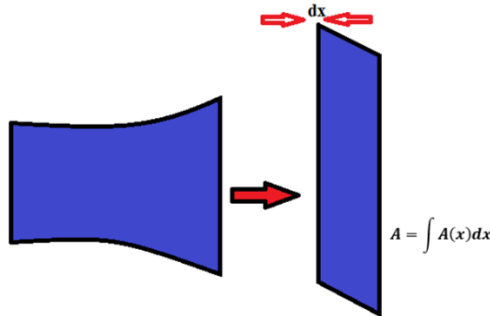


Figure 1. Geometry of the problem

In the following, the energy equation and boundary conditions related to the investigated geometry are given. By considering the governing equation for geometry of the problem (Figure 1):

$\frac{d}{dx} \left(k_0(1 + \beta T) \cdot A(x) \cdot \frac{dT}{dx} \right) + G(x) = 0$	(1)
$x = 0 \rightarrow T = T_0, \quad x = L \rightarrow T = T_L.$	(2)
$A(x) = A_0 e^x$	(3)

$G(x) = G_0 e^{-x}$	(4)
$k_T = k_0(1 + \beta T)$	(5)

β indicates the effective rate of temperature change with respect to thermal conductivity, and k_0 defines the fin's thermal conductivity with respect to the environment. T_0 simplify this equation [18]:

$\alpha \left(\frac{d}{dx} \theta(x) \right) + \alpha \cdot \beta \cdot T_0 \cdot \theta(x) \cdot \left(\frac{d}{dx} \theta(x) \right) + \beta \cdot T_0 \left(\frac{d}{dx} \theta(x) \right)^2 + \left(\frac{\partial^2}{\partial x^2} \theta(x) \right) + \beta \cdot T_0 \cdot \theta(x) \left(\frac{\partial^2}{\partial x^2} \theta(x) \right) + c \cdot e^{-\alpha x} = 0.$	(6)
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Here is the dimensionless temperature, θ

$\theta = \frac{T - T_0}{T_L - T_0}$	(7)
$c = \frac{G_0}{k_0 \cdot A_0 \cdot \beta \cdot T_0}$	(8)

By making the boundary conditions dimensionless in order to apply the desired methods, appropriate boundary conditions should be considered [18]:

$x = 0 \rightarrow \theta = 1, \quad x = L \rightarrow \theta = \frac{T_L - T_0}{T_L - T_0} = z$	(9)
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3. TECHNIQUES OF SOLUTION

3.1. Collocation method

In this technique the answer can be assumed as follows [13]:

$p(P) = \sum_{l=1}^N v_l \gamma_l(P).$	(10)
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Coefficients v_l , ($l = 1, \dots, N$) are the unknowns and the functions γ_l , are the test functions. A group of N nodes P_j of Γ chooses the collocation method. The equations are then noted at these nodes P_j , resulting in the following linear system of equations.

$\sum_{l=1}^N v_l k \gamma_l(P_j) = f(P_j). \quad \text{for } j = 1, \dots, N$	(11)
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This set of equations should be solved to compute the coefficient, so the answer p on Γ . The nodes P_j are named collocation nodes [13].

3.2. GM

The Galerkin technique was employed to equation (10) consists of selecting an approximate space of p . Besides,

p is reported as formerly (10) and, the function γ_m is the base of this space [16]. The equation defines the coefficient V_m .

$$\langle K_p, \gamma_p \rangle = \langle f, \gamma_p \rangle, \quad p = 1, \dots, M \quad (12)$$

Where $\langle \cdot, \cdot \rangle$ is the numerical product described in the approximate space. This makes the next linear approach:

$$\sum_{m=1}^N v_m \langle K \gamma_m, \gamma_p \rangle = \langle f, \gamma_p \rangle, \quad p = 1, \dots, M \quad (13)$$

The numerical procedure is similar to the procedure developed by the collocation method. Evaluate the vector B and the matrix A before solving the linear term.

3.3. LSM

Fakour et al. [28] represented that the least-squares technique is a kind of the weighted residual technique to make it the least the residuals of the test function presented in the nonlinear differential equation. To understand the basic concept of LSM, evaluate the derivative operator D, which operates on the v to develop the function h.

$$D(v(x)) = h(x), \quad (14)$$

v is supposed to be calculated by the function \tilde{v} , that is a linear mix of the base functions chosen from the linearly independent system.

$$v \cong \tilde{v} = \sum_{i=1}^n c_i \varphi_i, \quad (15)$$

By replacing equation (15) with D, the differential operator the consequence of the processes typically is not h(x), and a difference will occur. Thus a residual will exist as below:

$$R(x) = D(\tilde{v}(x)) - h(x) \neq 0, \quad (16)$$

The central idea of LSM is to move the residual to 0 in some moderate insight on the field. Therefore,

$$\int_x R(x) W_i(x) = 0, \quad i = 1, 2, \dots, n. \quad (17)$$

The number of weight functions and unknown coefficients is indicated by W_i and c_i , respectively, and their number equals each other.

3.4. Problem solving

By guessing the trial solution with undetermined coefficients and plugging it into the equation, the unknown coefficients are solved to obtain the particular solution. It should be mentioned that the trial answer must please the boundary conditions; therefore, the trial answer can be noted as follows [8]:

$$\theta(x) = \frac{e^{-\alpha L} - z}{-1 + e^{-\alpha L}} + \frac{(z - 1)e^{-\alpha x}}{-1 + e^{-\alpha L}} + C_1(x - x^2) + C_2(x - x^3) + C_3(x - x^4) + C_4(x - x^5). \quad (18)$$

The residual part will be seen by instructing equation (16). By replacing the residual amount with equation (18), a group of problems with five equations and five unidentified coefficients choice arise; coefficients C1–C4 will be acquired. After applying LSM, CM and GM when $\beta = 0$, $\alpha = 4$, $L = 1$, $T_0 = 10$, $z = 0.1$, $c = 2$ below equations will be obtained from the temperature profile on logarithmic surface.

$$\theta(x)_{LSM} = 0.424701244 x + 1.384653618 x^2 + 1.523432162 x^3 + 0.5634797880 x^4 \quad (19)$$

$$\theta(x)_{Galerkin} = 0.415091516 x + 1.329124398 x^2 + 1.483241831 x^3 + 0.5692089490 x^4 \quad (20)$$

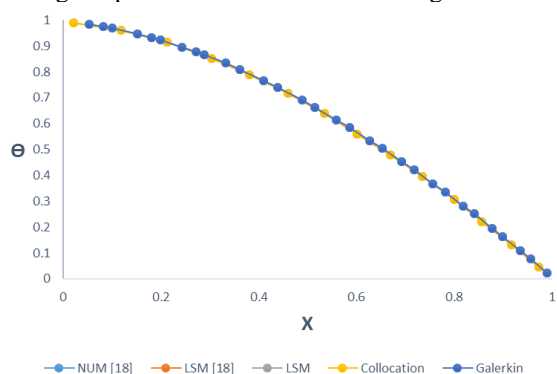
$$\theta(x)_{Collocation} = 0.499816241 x + 1.597444202 x^2 + 1.999655684 x^3 + 0.9020277229 x^4 \quad (21)$$

These equations were obtained by the LSM, Galerkin, and Collocation methods, respectively.

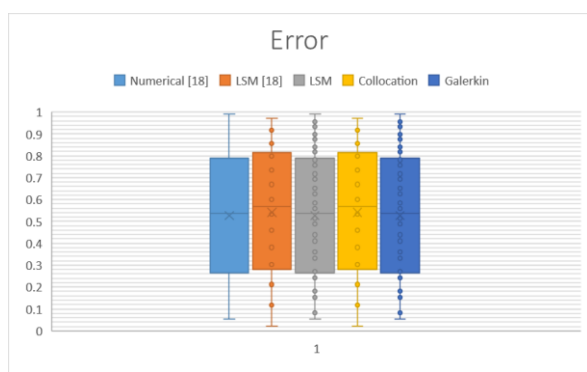
4. RESULTS AND DISCUSSION

This investigation desired to use the weighted residual methods named LSM, CM, and GM to define an analytical explanation for logarithmic area shapes of the heat transfer equation in Figure 1. According to Figure 2, a particular case indicates the efficiency of suggested techniques, and the outcomes are evaluated with the numerical and analytical methods conducted by Vahabzadeh et al. [18]. According to the obtained results, the percentage error of the present study compared to reported data in literature [18] is equal to 1.7%. This paper's approximate solution to the governing equation is obtained by applying the WRMs: the CM, LSM, and GM. The approximate solutions accepted that provided the WRMs are reliable and effective methods. A good agreement has been achieved by comparing the numerical solution obtained using the 4th-order Runge-Kutta method explained by Vahabzadeh et al. [18] and the proposed methods. It can be concluded from the figures and table that the maximal error remainder is negligible by these suggested methods. Moreover, the LSM provided the best approximate solution with less error and the Galerkin method reduces the dimensionality of the problem hence it is much faster. The collocation method also reaches convergence with

more calculations. Finally, this research found that selecting the parameters influenced convergence as well.



(a)



(b)

Figure 2. part (a) analogy between LSM solution and the numerical outcomes obtained from [18] and LMS, CM, and GM in the present study for $\theta(x)$ when $c = 2, \beta = 0, \alpha = 2, L = 1, T_0 = 10, z = 0.1$.

Part (b) Comparing percentage error between different methods

For numerical explanation, Vahabzadeh et al. [18] utilized a fourth-order Runge-Kutta approach to solve the nonlinear boundary value problem. The exactness of LSM obtained from literature [18] and the three methods offered in this research are displayed in Table 1.

TABLE 1. Comparison between LSM and NUM from literature [18] and LMS, GM and, CM from present study for $\theta(x)$ when $c = 2, \beta = 0, \alpha = 2, L = 1, T_0 = 10, z = 0.1$.

X	LSM [18]	NUM [18]	LSM	Galerkin	Collocation
0.0	1.0000000000	1.0000000000	1.0000000000	1.0000000000	1.0000000000
0.1	0.8648242717	0.8648242719	0.8645772366	0.8542549152	0.8562025261
0.2	0.7393106703	0.7393106705	0.7485718835	0.7682940431	0.7710213056
0.3	0.6243979822	0.6243979837	0.6268611235	0.6767081250	0.6802767791
0.4	0.5203671664	0.5203671665	0.5214787627	0.5814893015	0.5859442644
0.5	0.4270489850	0.4270489840	0.4341732679	0.4843450838	0.4895862861
0.6	0.3439779949	0.3439779952	0.3563945577	0.3866851428	0.3923393643
0.7	0.2705054957	0.2705054971	0.2792821640	0.2896094716	0.2949021784
0.8	0.2058812577	0.2058812581	0.1936546291	0.1938977815	0.1975249622

0.9	0.1493116429	0.1493116431	0.1482167345	0.1483782398	0.1481126567
1.0	0.1000000000	0.1000000000	0.1000000000	0.1000000000	0.1000000000

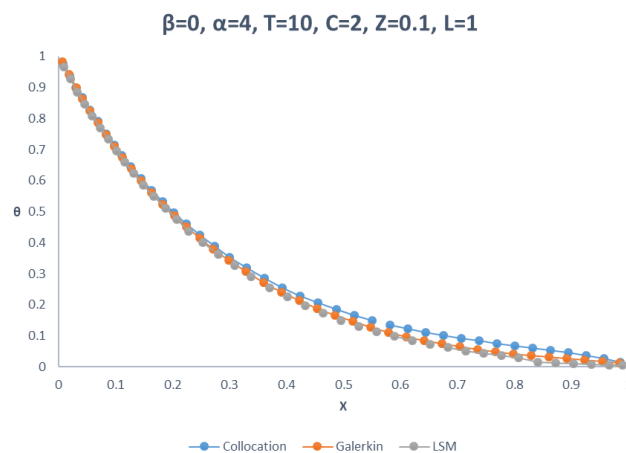


Figure 3. Impact of α on θ where $T_0 = 10, \beta = 0, \alpha = 4, L = 1, c = 2, z = 0.1$, for Collocation, Galerkin and LSM methods

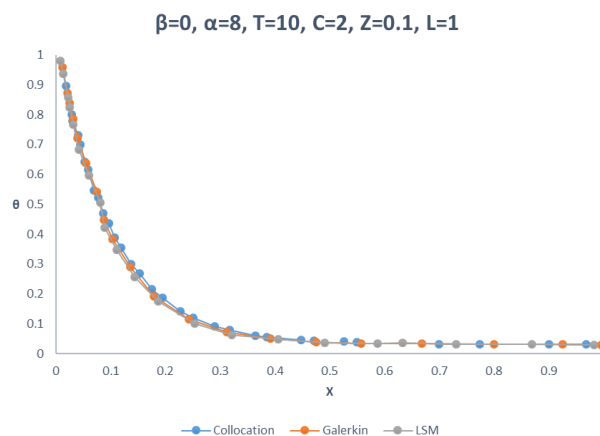


Figure 4. Impact of α on θ where $T_0 = 10, \beta = 0, \alpha = 8, L = 1, c = 2, z = 0.1$, for Collocation, Galerkin and LSM methods

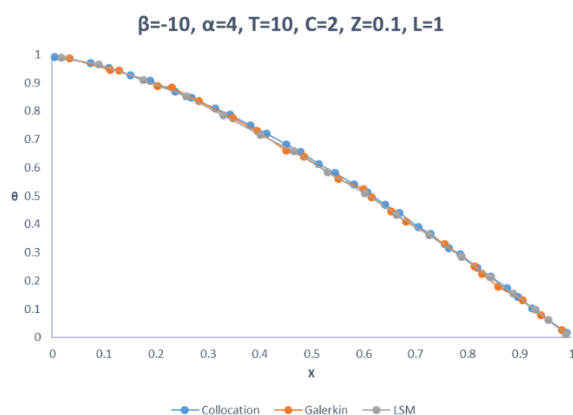


Figure 5. Impact of α on θ where $T_0 = 10, \beta = -10, \alpha = 8, L = 1, c = 2, z = 0.1$, for Collocation, Galerkin and LSM methods

Figures 3 and 4 show the effect of α on temperature characteristics. As α increases, the temperature profiles

in the range $0 < \alpha < 1$ decrease. This trend is established in Figure 3 for $\alpha = 4$ for the proposed methods. On the other hand, as α increases, the temperature profiles for $\alpha = 1$ decrease. Moreover, the variation of temperature profiles for $\alpha = 8$ is shown in Figure 4. Furthermore, the dimensionless temperature profile along the fin shell is displayed in Figures 5 and 6. If $\beta > 0$, the temperature profile rises with growing x . In the subject of $\beta < 0$, the temperature grows as x rises. (Figure 6)

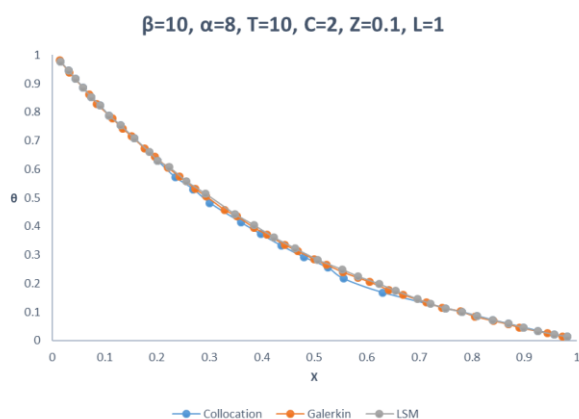


Figure 6. Impact of α on θ where $T_0 = 10$, $\beta = 10$, $\alpha = 8$, $L = 1$, $c = 2$, $z = 0.1$, for Collocation, Galerkin and LSM methods

5. CONCLUSION

In this study, Galerkin and Collocation methods have been proposed. These techniques have been successfully used for governing differential equations of specified geometries with various logarithmic surfaces. The results were compared to the solution solved using the numerical solution and LSM. The results indicate that these procedures transform complex problems into simple, fast-solvable ones. The fundamental goal of this analysis is to explore the convergence of the Galerkin method and the collocation method. The comparison of the results here provides a more realistic solution and reinforces the conclusions regarding the efficiency of these processes. Thus, the Galerkin and collocation methods are effective mathematical mechanisms and can involve extensive types of linear and nonlinear equations in the field of heat transfer issues. Also, differences in temperature profiles appear with reducing and raising β and α numbers. Future research should consider the potential effects of geometry more carefully, for example investigation of heat transfer in logarithmic curve. Also, in future work, investigating heat transfer in presence of porous media in logarithmic surface might prove important.

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Persian Abstract

چکیده

این مطالعه به بررسی تأثیر تغییرات دما بر سطوح مختلف لگاریتمی پرداخته است. انتقال حرارت یک بعدی در نظر گرفته شد و منبع تولید گرما به معادلات حاکم اضافه می شود. اکثر مسائل و پدیده های علمی مانند انتقال حرارت به صورت غیرخطی رخ می دهند و یافتن راه حل های تحلیلی دقیق آسان نیست. استفاده از تبدیل تشابه مناسب برای دما و مولفه های دیگر باعث می شود که معادلات اساسی حاکم بر جریان و انتقال حرارت به مجموعه ای از معادلات دیفرانسیل معمولی کاهش یابد. این معادلات با توجه به شرایط مرزی مربوطه با تکنیک های عددی و تحلیلی بصورت تقریبی حل شده اند. با توجه به شرایط مرزی داده شده، از روش های کالوکیشن، گلرکین و حداقل مربعات برای یافتن پاسخ معادلات دیفرانسیل حاکم استفاده شد. اعتبار سنجی تکنیک های حاضر با روش رانگ-کوتا مرتبه چهارم به عنوان یک روش عددی انجام شده است. پروفیل های دما برای مقادیر مختلف α و β به دست آمده است. نتایج نشان داد که روش های پیشنهادی قابلیت حل معادلات غیرخطی در انتقال حرارت را دارند. بنابراین، نتایج پذیرفته شده توسط روش های تحلیلی فعلی بسیار نزدیک به نتایج روش های عددی است. مقایسه نتایج راه حل واقعی تری ارائه می دهد و نتیجه گیری در مورد کارایی این روش ها را تقویت می کند. علاوه بر این، تغییرات در پروفایل های دما با کاهش و افزایش اعداد α و β رخ می دهد.