



Comprehensive Analysis of Stress-strain Relationships for Recycled Aggregate Concrete

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ABSTRACT

There is a growing demand of suitable substitute materials of concrete ingredients especially fine and coarse aggregates in order to achieve sustainable development in the era of rapid urbanisation. Therefore, the concrete making process by utilisation of aggregates that recycled from construction and demolition (C&D) debris has emerged as a primary objective for many government agencies. Consequently, the utilisation of recycled aggregate concrete (RAC) in structural applications become essential aspect. However, RAC can be employed in structural applications only if effective stress-strain relationship is available. The stress-strain models developed for natural aggregate concrete (NAC) are not fully suitable for RAC. Hence, the selection of good model which has precise prediction capacity plays a crucial role. Moreover, the stress-strain models provide the basis for the analysis and modern design procedures especially in FEA packages. In the present study, the stress-strain models for RAC have been selected from the literature and critically reviewed in order to evaluate their predictive efficacy. The test samples in the form of measured stress-strain relations—hips derived from literature have been compared with the predictions of each selected model. Besides the comparison of measured and predicted stress-strain profiles, the output of selected models in terms of normalized toughness and ductility index was assessed. The consistency of output of models are further evaluated by employing statistical tools such as coefficient of variance and root mean square error. The outcomes of the model in the form of polynomial expression was relatively more accurate to that of other counterparts.

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NOMENCLATURE

σ	Stress in concrete corresponding to specified strain level ϵ	ϵ	Strain in concrete corresponding to specified stress level σ
σ_{max}	Peak stress of concrete	n, k	Coefficients
f_c	the prism compressive strength	f_{cm}	Average cylinder compressive strength at 28 days
σ_c	Compressive stress	FEA	Finite Element Analysis

1. INTRODUCTION

Since the beginning of the urbanisation, there has been rapid growth in the building industry, and as a result, the amount of construction waste produced has increased year by year [1, 2]. Every year, around 2.01 billion tonnes of municipal debris are produced across the globe [3]. Due to rapid urbanization, the appropriate utilization of C&D waste has become very important. The demand of coarse and fine aggregate has been in an ever increasing rate, but as they are natural resources, their supply is limited [4]. On one side, there is exploitation of the

natural resources, on the other hand, debris produced by demolition and other construction activities is increasing. A process to extract fine and the coarse aggregate from C&D waste that could be replaced with the natural aggregates in construction is very crucial as it would relieve the pressure on natural resources. It is high time that the entire C&D waste is properly recycled and utilized as the natural building materials are becoming scarce. In the recent decade, due to the rapid population growth there has been a big boom in the construction industry as the expansion of cities and redevelopment of the existing cities have gained a big momentum. Due to

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this rapid development, there have been a massive increase in C&D waste generation. In order to promote sustainability and alleviate the stress on the environment from the excessive use of natural aggregates, recycled aggregates are increasingly used in concrete manufacturing [5].

Plenty of studies [6-8] on the utilisation of recycled aggregate from C&D waste are available in the literature, but the studies on structural application of RAC are comparatively limited in their number. Emphasis on the utilization of recycled concrete aggregate (RCA) in structural applications may accelerate sustainable development programmes. As a result, a thorough knowledge of stress-strain relations of this substitute material is inevitable in order to assess its structural potential. Moreover, if the stress-strain relationship accurately depicts the behaviour under loading, it might serve as a foundation for structural applications of RAC. Numerous compressive behaviour indicators, such as peak-stress (referred as compressive strength), peak-strain corresponds to peak stress, ultimate stress where the failure is defined in terms of certain percentage of peak stress in the post-peak region, ultimate strain and secant modulus are derived from the stress-strain relationship of concrete, making it crucial for theoretical and numerical assessments of structures and engineering designs.

There are different theories of constitutive modelling are available in the literature [9]. However, it can be categorised into two major groups: Empirical models that derived from the regression analysis of measured data and models based on continuum mechanics theories. The continuum mechanics theories like elasticity, plasticity, continuum damage mechanics, plastic fracturing, endochronic theory, microplane models, etc. are too dependent of experimental data for their validation. The group of empirical models itself is huge and consists variety of constitutive models for uniaxial, biaxial, triaxial, impact and cyclic loading. Under the class of uniaxial stress-strain models, many models for NAC are available, however, very limited models are available on the RAC. Moreover, due to various influencing factors such as shape and size of test specimens, aspect ratios, loading rate, duration of loading, type of testing machine etc., these models give different results especially in post peak region. The form of mathematical formulation also affects the prediction of test results. Therefore, it is evident to carry out comparative theoretical analysis of these models in order to ascertain the level of accuracy.

In this study, the empirical stress-strain relationships for RAC proposed by Xiao et al. [10], Du et al. [11], Belén et al. [12], and Suryawanshi et al. [13] have been considered for comparison of predictive efficacy. Besides comparison of graphical representation of measured and predicted stress-strain relationships, Normalised toughness (A), Ductility index (μ) have been

evaluated. Further, the accuracy of the predictions of selected models in terms of A and μ is verified by employing statistical tools such as Root Mean Square Error (RMS) and Coefficient of Variance (V). It has been observed that all the models yield reasonably similar results in pre-peak region but the predictions of post-peak region are deviating from their respective counterparts may be due to the formulation approach. However, the model of higher degree polynomial yields comparatively better results and its applicability in differential and integral calculus is also simple and easy.

2. SUMMARY OF STRESS-STRAIN MODELS

A small number of stress-strain models are available for RAC with the different replacement ratios of recycled coarse aggregates as a function. A brief and critical review of these models reported in different studies is as follows. The basic forms of these mathematical formulations for rising and falling branches of complete stress-strain curve (SSC) have also been reviewed.

Guo and Zhang [14] defined the stress-strain relationship for NAC and subsequently, it was modified by Xiao et al. [10] to fit it for RAC. The formulation consists of two equations to model ascending and descending portion of SSC and is the function of replacement level of natural aggregate.

$$\bar{\sigma} = \begin{cases} a\bar{\epsilon} + (3 - 2a)\bar{\epsilon}^2 + (a - 2)\bar{\epsilon}^3, \text{ for } \bar{\epsilon} < 1, \\ \frac{\bar{\epsilon}}{b(\bar{\epsilon}-1)^2 + \bar{\epsilon}}, \text{ for } \bar{\epsilon} \geq 1 \end{cases} \quad (1)$$

In Equation (1), $\bar{\sigma}$ is the normalised stress with respective to peak stress (σ/f_c) whereas $\bar{\epsilon}$ is the normalised strain with respective to strain corresponds to peak stress (ϵ/ϵ_0). The coefficients, a and b are outcomes of regression analysis, dependent on replacement level of natural aggregate in percentage (r), and to be determined as follows:

$$a = 2.2(0.748r^2 - 1.231r + 0.975) \quad (2)$$

$$b = 0.8(7.6483r + 1.142) \quad (3)$$

This model gives reasonably more accurate predictions for the ascending portion of the SSC, however, the descending portion is not predicted as accurately.

Du et al. [11] tried to model the falling branch of SSC after peak load, relating it to the area falling under the experimentally measured SSC. The expression for rising branch of SSC is similar to that proposed by Xiao et al. [10].

$$y = ax + (3 - 2a)x^2 + (a - 2)x^3, (0 \leq x < 1) \quad (4)$$

where, y is the normalised stress with respective to peak stress (σ/σ_{max}) and x is the normalised strain with respective to strain corresponds to peak stress (ϵ/ϵ_0).

a is represented by the ratio of initial tangent modulus and secant modulus correspond to peak point and the origin.

The expression for falling branch of SSC is as follows:

$$y = \alpha e^{\frac{(x-1)^2}{2\beta_1^2}} + (1 - \alpha)e^{\frac{(x-1)^2}{2\beta_2^2}}, (1 \leq x < +\infty) \quad (5)$$

where, α, β_1, β_2 are the parameters may be referred to in the respective reference [11].

The recommended model by Du et al. [11] is not fit for routine applications due to associated relative complexity in the formulation, especially for descending branch of SSC. Moreover, the transformation of α, β_1, β_2 parameters with the replacement level of natural aggregate and concrete grade involves comparatively more compound approach.

Linear regression analysis of test data was used by Belén et al. [12] to determine elastic modulus, ultimate strain and peak strain transformation coefficients and modified the stress-strain model recommended in Eurocode 2 [15], to outfit the RAC. An analytical expression of the SSC that considers the percentage of replacement was established by making use of the findings of the experiments. Moreover, the stress-strain expression recommended in Eurocode 2 [15], is also adhered by the Spanish code, EHE-08 [16]. The modified expression for RAC was generated for the ascending branch and subsequently, descending branch limited to the strain level of 0.0035.

$$\frac{\sigma_c}{f_{cm}} = \frac{k*n-n^2}{1+(k-2)n} \quad (6)$$

where,

$$n = \frac{\varepsilon_c}{\varepsilon_{c1}} \text{ and } k = 1.05 * E_{cm} * |\varepsilon_{c1}| / f_{cm}$$

The Eurocode 2 [15] recommends the following expressions to determine the secant modulus of elastic and peak strain, respectively as follows:

$$E_{cm} = 22 * (f_{cm}/10)^{0.3} \quad (7)$$

$$\varepsilon_{c1} = 0.7 * (f_{cm})^{0.31} \quad (8)$$

According to Belén et al. [12], the Eurocode 2 [15] expression for conventional concrete could be used for recycled concretes by employing transformation coefficients for secant modulus (E_{cm}), peak strain (ε_{c1}) and the ultimate strain (ε_{cu1}) and $\varepsilon_{cu2}(\beta_{cu}^{rec})$. The following modified coefficients are the products of regression analysis of the test data.

$$(\varphi_{cm}^{rec}) = -0.0020 x \%RCA + 1 \quad (9)$$

$$(\alpha_c^{rec}) = 0.0021 x \%RCA + 1 \quad (10)$$

$$(\beta_{cu}^{rec}) = 0.0022 x \%RCA + 1 \quad (11)$$

Although the model exhibits good efficacy for ascending branch, yet it is not that much effective in predictions for descending branch.

The theoretical relationship for RAC proposed by Bhikshma and Kishore [17], is the modified version of the model evolved by Saenz [18] for conventional concrete. This model has limited predictive efficacy for RAC; hence the results of the analysis are not reported here.

A fourth-degree polynomial expression was proposed by Suryawanshi et al. [13] to model the experimentally observed behaviour in normal-strength RAC. The model incorporated the influence of RCA replacement level respectively on elastic modulus, peak strain, peak stress and ultimate strain. This single equation works for both the pre-peak and post-peak parts of the SSC with the limiting value of normalised strain is equal to 2.

$$\bar{\sigma} = a(\bar{\varepsilon}) + b(\bar{\varepsilon})^2 + c(\bar{\varepsilon})^3 + d(\bar{\varepsilon})^4 \quad (12)$$

where $\bar{\sigma}$ is normalised stress (σ/f'_c) and $\bar{\varepsilon}$ is the normalised strain.

Coefficients a, b, c and d are derived by doing regression analysis of the measured test data and function of replacement level of natural aggregate in percentage. The coefficient 'a' implies the ratio of initial tangent modulus and the secant modulus (E_{im}/E_p). The secant modulus is the slope of line joining the origin of the SSC and the point corresponding to peak stress

$$a = (1.8242 - 0.0076R) \quad (13)$$

$$b = (4.67 - 2.86a) \quad (14)$$

$$c = (2.51a - 5) \quad (15)$$

$$d = (1.33 - 0.65a) \quad (16)$$

The compressive stress (f'_c) and the peak strain (ε_o) of RAC are the inputs required to handle the model. In absence of values of these input parameters for RAC, following equations may be referred to generate them from the test results of NAC.

$$f'_{c,RAC} = (1 - 0.0012R)f'_{c,NAC} \quad (17)$$

$$\varepsilon_{0,RAC} = (0.6 + 0.002R)10^{-3} (f'_{c,RAC})^{0.33} \quad (18)$$

3. ASSESSMENT OF STRESS-STRAIN MODELS

The relative performance of the analytical models in terms of their ability to reproduce the experimentally observed SSC on selected test specimens have been examined. Table 1 reveals the details of typical test specimens and measured values of compressive stress along with the corresponding values of peak and ultimate

TABLE 1. Summary of specimen details and experimentally obtained results

Author	Specimen type and size	Specimen Id	Replacement ratio	Compressive strength of concrete (MPa)	Peak strain	Ultimate strain
Xiao et al. [10]	Prism (100x100x300) mm	NC	0%	26.90	0.0018	0.0028
		RC-50	50%	23.60	0.0019	0.0026
		RC-100	100%	23.80	0.0022	0.0028
		H-0.65-0%	0%	31.92	0.0017	0.0037
		H-0.65-50%	50%	32.35	0.0019	0.0042
Belén et al. [12]	Cylinder (150x300) mm	H-0.65-100%	100%	30.13	0.0022	0.0045
		H-0.50-0%	0%	44.81	0.0019	0.0036
		H-0.50-50%	50%	37.45	0.0019	0.0040
		H-0.50-100%	100%	40.54	0.0022	0.0044
Suryawanshi et al. [13]	Cylinder (150x300) mm	R00	0%	38.00	0.0019	0.0036
		R100	100%	34.50	0.0024	0.0037

strains. The stress and the strain values have been derived from the reported information and subsequently the SSC models except the model proposed by Du et al. [11] are compared with each measured stress-strain relationship. Du et al. [11] proposed the stress-strain model explicitly for concrete containing 100% recycled aggregates [19].

Besides the graphical comparison, the area under curves of the measured SSC and predicted SSC has been calculated and compared. Moreover, root have been reproduced. The predictions of all the selected mean square error (*RMS*) value and coefficients of variance (*V*) are calculated in order to decide the efficacy of models on statistical grounds. The *RMS* error is the standard deviation of the prediction errors (residuals). Residuals are the distances between the data points and the regression line. It refers to the propagation of these residuals. Thus, it is the indicative of the concentration of the cluster of the data points around the line of best fit. This is one of the effective techniques in regression analysis to verify experimental results. The effectiveness of prediction model may be evaluated in terms of variance of the predicted data. It is defined as extent of spread from the average value. Variance is the dependent quantity of the standard deviation of the data. The value of variance is directly correlated to scatter of data. Therefore, it is the indicator of a data spread with respect to mean. More the value of variance, the more the data scatter and vice a versa. The quality of fit of the predicted SSC to the experimental SSC is investigated in terms of "*RMS*" and "*V*" respectively in order to quantify the predictive evaluation of various analytical models, according to procedure reported by Khan et al. [20] and Ayub et al. [21]. The stress values from the experiments and the predicted stress values correspond to the same strain values were used to figure out these parameters. "*RMS*" and "*V*" have been estimated using Equations

(19) and (20), respectively; which are reported in Tables 2-4:

$$RMS = \sqrt{\frac{\sum |t_i - o_i|^2}{n}} \quad (19)$$

$$V = 1 - \frac{\sum |t_i - o_i|^2}{\sum |o_i|^2} \quad (20)$$

where, "*t_i*", "*o_i*" and "*n*" indicate the measured outcomes (used as target), predicted outcomes (used as output) and the total amount of observations, respectively.

4. DISCUSSION

The relative efficacy of selected stress-strain models with their ability to predict the stress-strain relationships have been appraised. Figures 1, 2 and 3 reveal the comparison of measured stress-strain relationship by Xiao et al. and the predicted stress-strain relationships of Xiao's model along with the predictions of other selected models. It is quite obvious that the predictive efficacy of the model expected to be comparatively more accurate against own measured data as the respective model itself built on the regression analysis of that data. However, there are considerable amount of deviations in measured and predicted relations of descending portions especially for concrete containing 100% RCA. Figures 4-9 depict the measured stress-strain relationships of Belén et al. [12] against the predictions of selected models. There is reasonable match in the ascending branch except Figures 5 and 6. On the contrary, there is significant discrepancy in the measured and predicted relationships in the descending branches of SSCs. In Figures 10 and 11, the comparison of measured stress-strain relationship of Suryawanshi et al. [13] and predictions of selected

TABLE 2. Summary of measured and estimated areas under stress-strain curves

Specimen Id	Measured				Xiao et al. [10]				Belén et al. [12]				Suryawanshi et al. [13]			
	A ₁	A ₂	A	μ	A ₁	A ₂	A	μ	A ₁	A ₂	A	μ	A ₁	A ₂	A	μ
NC	0.70	0.86	1.56	0.55	0.68	0.85	1.53	0.56	0.70	0.81	1.51	0.54	0.65	0.77	1.42	0.540
RC-50	0.61	0.64	1.24	0.51	0.60	0.64	1.24	0.51	0.66	0.60	1.26	0.48	0.63	0.64	1.27	0.507
RC-100	0.61	0.65	1.26	0.52	0.59	0.53	1.12	0.47	0.67	0.71	1.38	0.51	0.62	0.62	1.24	0.500
H-0.50-0%	0.62	0.76	1.39	0.55	0.68	0.86	1.54	0.56	0.64	0.50	1.14	0.44	0.65	0.77	1.43	0.543
H-0.50-50%	0.61	0.81	1.42	0.57	0.60	0.64	1.23	0.51	0.66	0.64	1.30	0.49	0.62	0.78	1.41	0.557
H-0.50-100%	0.62	0.80	1.42	0.56	0.58	0.60	1.19	0.51	0.65	0.58	1.23	0.47	0.61	0.69	1.31	0.531
H-0.65-0%	0.67	0.89	1.56	0.57	0.68	0.89	1.57	0.57	0.68	0.76	1.44	0.53	0.65	0.79	1.44	0.548
H-0.65-50%	0.67	0.82	1.49	0.55	0.60	0.65	1.24	0.52	0.68	0.75	1.43	0.52	0.64	0.70	1.34	0.525
H-0.65-100%	0.68	0.85	1.52	0.56	0.59	0.87	1.46	0.60	0.69	0.78	1.47	0.53	0.62	0.62	1.24	0.500
R00	0.69	0.81	1.51	0.54	0.68	0.53	1.21	0.44	0.66	0.63	1.29	0.49	0.65	0.77	1.42	0.542
R100	0.59	0.66	1.25	0.53	0.59	0.53	1.12	0.47	0.67	0.71	1.38	0.51	0.62	0.60	1.22	0.493

TABLE 3. Comparison of Root Mean Square (RMS) of the all predictive models

Specimen Id	Xiao et al. [10]	Du et al. [11]	Belén et al. [12]	Suryawanshi et al. [13]
NC	0.023	--	0.199	0.053
RC-50	0.021	--	0.077	0.030
RC-100	0.098	0.136	0.085	0.030
H-0.65-0%	0.021	--	0.135	0.096
H-0.65-50%	0.144	--	0.102	0.113
H-0.65-100%	0.088	0.226	0.076	0.193
H-0.50-0%	0.076	--	0.174	0.056
H-0.50-50%	0.105	--	0.138	0.040
H-0.50-100%	0.122	0.196	0.149	0.074
R00	0.208	--	0.219	0.074
R100	0.103	0.152	0.098	0.071

TABLE 4. Comparison of Coefficient of Variance (V) of the all predictive models

Specimen Id	Xiao et al. [10]	Du et al. [11]	Belén et al. [12]	Suryawanshi et al. [13]
NC	0.9989	--	0.9941	0.9793
RC-50	0.9989	--	0.9874	0.9979
RC-100	0.9702	0.9577	0.9853	0.9978
H-0.65-0%	0.9992	--	0.9603	0.9800
H-0.65-50%	0.9415	--	0.9779	0.9689
H-0.65-100%	0.9837	0.8745	0.9881	0.8985
H-0.50-0%	0.9878	--	0.9165	0.9927
H-0.50-50%	0.9685	--	0.9546	0.9963
H-0.50-100%	0.9551	0.8952	0.9446	0.9862
R00	0.8847	--	0.8893	0.9886
R100	0.9680	0.9457	0.9798	0.9873

models have been presented. Perusal of all these figures clearly indicates that the predictions of the model in the form of polynomial expression proposed by Suryawanshi et al. [13] gives comparatively more accurate results irrespective of pre-peak or post-peak regions.

In order to verify the predictivity of selected models, the areas under the curves have been calculated by developing the program in Microsoft-Excel. The program employed the Trapezoidal rule to calculate the area. The area under the curve up to the peak is designated as A_1 and area after the peak is nomenclatured as A_2 . The summation of A_1 and A_2 is treated as a normalised toughness (A), since the SSC is in non-dimensional form. The predictive efficacy of the selected models have been checked against the ductility index (μ). The ductility index may be defined as the ratio of A_2 and A . The measured and estimated values of A_1 , A_2 , A and μ are compiled together in Table 2. It has been observed that the values of normalised toughness predicted by Suryawanshi et al. [13] are in good agreement compared to the predictions of other models. As stated in section 3, the quality of fit of predictions of stress-strain relationships with that the experimentally obtained, is investigated in terms of the values of root mean square error, RMS , and the coefficient of variance, V , respectively. The comparison of values of RMS and V are depicted in Tables 3 and 4, respectively. The model that conceives the lowest value of RMS and the unit value of V is considered as the most effective model. On the parameters of RMS and V , it is seen that the predictions of Suryawanshi et al. [13] are reasonably acceptable compared to the outputs of counterpart models.

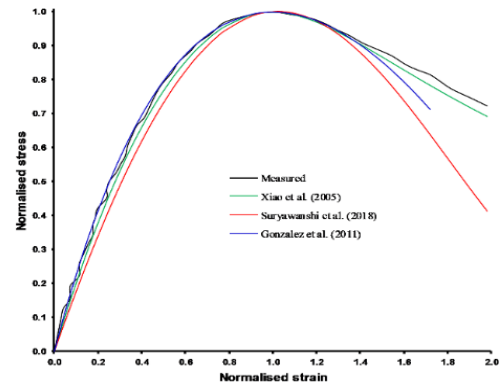


Figure 1. Comparison of measured (NC, Xiao et al. [10]) and predicted Stress-strain relationships

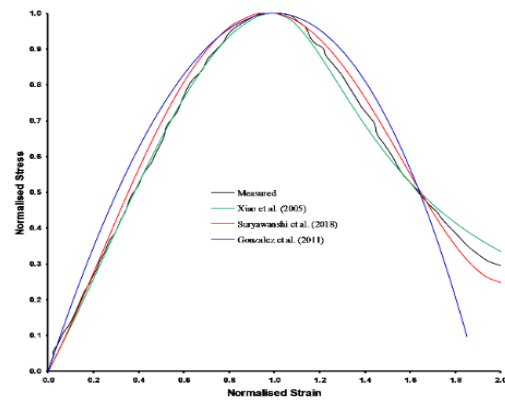


Figure 2. Comparison of measured (RC-50, Xiao et al. [10]) and predicted Stress-strain relationships

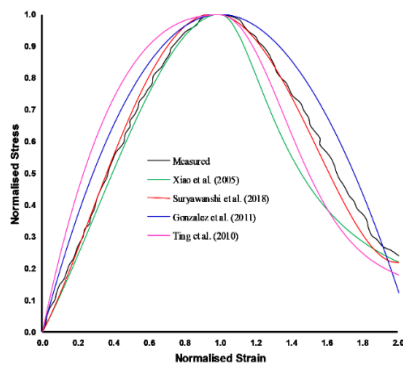


Figure 3. Comparison of measured (RC-100, Xiao et al. [10]) and predicted Stress-strain relationships

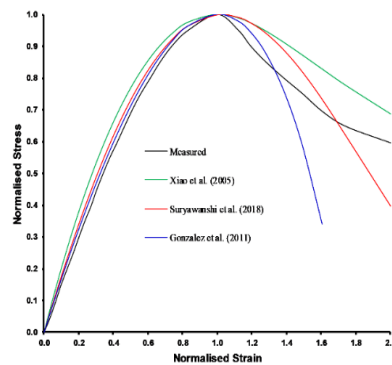


Figure 4. Comparison of measured (H-0.50-0%, Belén et al. [12]) and predicted Stress-strain relationships

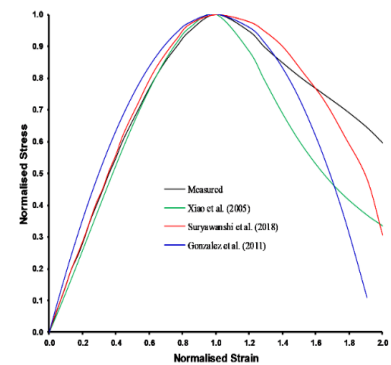


Figure 5. Comparison of measured (H-0.50-50%, Belén et al. [12]) and predicted Stress-strain relationships

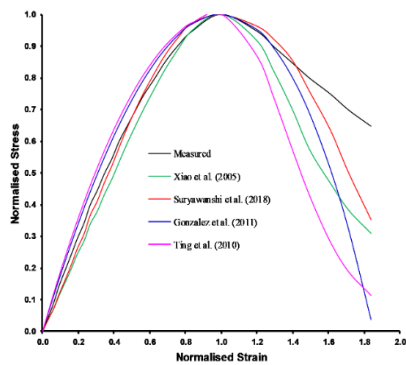


Figure 6. Comparison of measured (H-0.50-100%, Belén et al. [12]) and predicted Stress-strain relationships

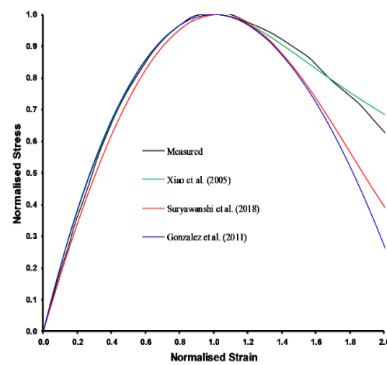


Figure 7. Comparison of measured (H-0.65-0%, Belén et al. [12]) and predicted Stress-strain relationships

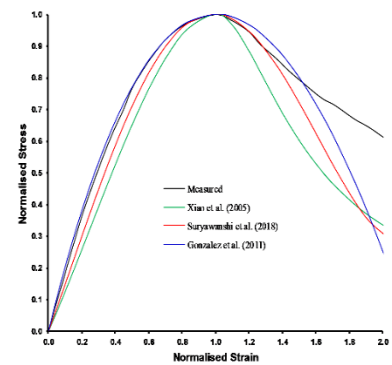


Figure 8. Comparison of measured (H-0.65-50%, Belén et al. [12]) and predicted Stress-strain relationships

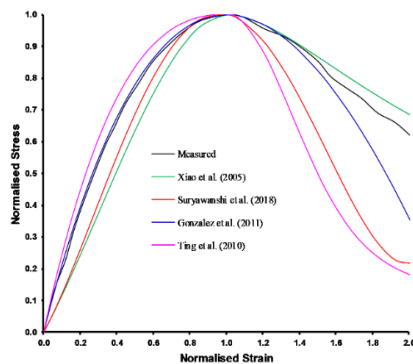


Figure 9. Comparison of measured (H-0.65-100%, Belén et al. [12]) and predicted Stress-strain relationships

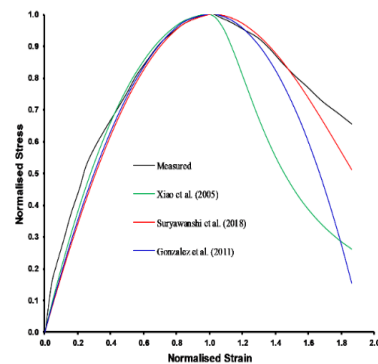


Figure 10. Comparison of measured (R00, Suryawanshi et al. [13]) and predicted Stress-strain relationships

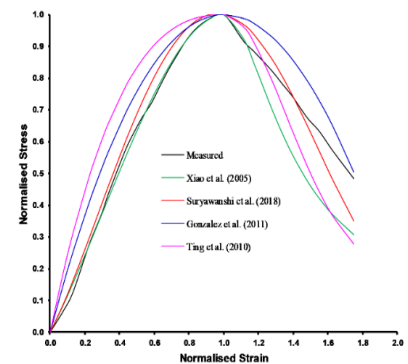


Figure 11. Comparison of measured (R100, Suryawanshi et al. [13]) and predicted Stress-strain relationships

5. CONCLUSION

In this work, the published models to define stress-strain relationships for RAC have been calibrated against the experimentally measured outcomes which are reported in the literature. The measured value of peak compressive stress and associated peak strain value were utilized as input parameters to operate prediction models in order to reproduce entire SSC. The findings of this study may be summarised as follows:

- The predictive efficacy of selected models is more or less similar for the ascending portion while it is found that the level of prediction accuracy is of varying degree for descending portion of the SSCs. This may be due to the approach adopted in the formulation of the model. Nevertheless, the stress-strain generated by polynomial expressions are more relatable.
- The predictive efficiency of the models has also been evaluated in terms of normalised toughness and ductility index. The predictions of models proposed and reported in literature are found closer to measured values.

- The accuracy of the outputs of the considered models in terms of normalised toughness and ductility index are further evaluated on statistical grounds. This was revisited by comparing the values of the *RMS* and *V*. The model that conceives the lowest value of *RMS* and the unit value of *V* is considered as the most effective model. On the parameters of *RMS* and *V*, it is seen that the predictions of polynomial expression are reasonably acceptable compared to the outputs of counterpart models.
- The use of commercial packages has become an integral part of the modern analysis of concrete structures. Some of the commercial computer programs offer freedom to the users to define the materials through the user-defined material models in the form of laboratory evaluated stress-strain relationships. The use of the stress-strain relationship in the form of polynomial expression may play crucial role in the simulation of the concrete structures and may yield near practical results.

6. REFERENCES

- Poon, C.S., Shui, Z. and Lam, L., "Effect of microstructure of itz on compressive strength of concrete prepared with recycled aggregates", *Construction and Building Materials*, Vol. 18, No. 6, (2004), 461-468. <https://doi.org/https://doi.org/10.1016/j.conbuildmat.2004.03.005>
- Fathifazl, G., Razaqpur, A.G., Isgor, O.B., Abbas, A., Fournier, B. and Foo, S., "Creep and drying shrinkage characteristics of concrete produced with coarse recycled concrete aggregate", *Cement and Concrete Composites*, Vol. 33, No. 10, (2011), 1026-1037. <https://doi.org/https://doi.org/10.1016/j.cemconcomp.2011.08.004>
- Kaza, S., Yao, L., Bhada-Tata, P. and Van Woerden, F., "What a waste 2.0: A global snapshot of solid waste management to 2050, World Bank Publications, (2018).
- Yasin Mousavi, S., Tavakkoli, A., Jahanshahi, M. and Dankoub, A., "Performance of high-strength concrete made with recycled ceramic aggregates (research note)", *International Journal of Engineering, Transactions C: Aspects*, Vol. 33, No. 6, (2020), 1085-1093. <https://doi.org/10.5829/ije.2020.33.06c.05>
- Masne, N. and Suryawanshi, S., "Analytical and experimental investigation of recycled aggregate concrete beams subjected to pure torsion", *International Journal of Engineering, Transactions A: Basics*, Vol. 35, No. 10, (2022), 1959-1966. <https://doi.org/10.5829/ije.2022.35.10A.14>
- Kou, S.-C., Poon, C.-S. and Wan, H.-W., "Properties of concrete prepared with low-grade recycled aggregates", *Construction and Building Materials*, Vol. 36, (2012), 881-889. <https://doi.org/https://doi.org/10.1016/j.conbuildmat.2012.06.060>
- Kou, S. and Poon, C.S., "Enhancing the durability properties of concrete prepared with coarse recycled aggregate", *Construction and Building Materials*, Vol. 35, (2012), 69-76. <https://doi.org/https://doi.org/10.1016/j.conbuildmat.2012.02.032>
- McNeil, K. and Kang, T.H.-K., "Recycled concrete aggregates: A review", *International Journal of Concrete Structures and Materials*, Vol. 7, No. 1, (2013), 61-69. <https://doi.org/10.1007/s40069-013-0032-5>
- Raveendra, B.R., Benipal, G.S. and Singh, A.K., "Constitutive modelling of concrete: An overview", (2005).
- Xiao, J., Li, J. and Zhang, C., "Mechanical properties of recycled aggregate concrete under uniaxial loading", *Cement and Concrete Research*, Vol. 35, No. 6, (2005), 1187-1194. <https://doi.org/https://doi.org/10.1016/j.cemconres.2004.09.020>
- Du, T., Wang, W., Liu, Z., Lin, H. and Guo, T., "The complete stress-strain curve of recycled aggregate concrete under uniaxial compression loading", *Journal of Wuhan University of Technology-Mater. Sci. Ed.*, Vol. 25, No. 5, (2010), 862-865. <https://doi.org/10.1007/s11595-010-0109-9>
- Belén, G.-F., Fernando, M.-A., Diego, C.L. and Sindy, S.-P., "Stress-strain relationship in axial compression for concrete using recycled saturated coarse aggregate", *Construction and Building Materials*, Vol. 25, No. 5, (2011), 2335-2342. <https://doi.org/https://doi.org/10.1016/j.conbuildmat.2010.11.031>
- Suryawanshi, S., Singh, B. and Bhargava, P., "Equation for stress-strain relationship of recycled aggregate concrete in axial compression", *Magazine of Concrete Research*, Vol. 70, No. 4, (2018), 163-171. <https://doi.org/10.1680/jmacr.16.00108>
- Dacheng, G.Z.Z.X.Z. and Ruqi, W., "Experimental investigation of the complete stress-strain curve of concrete", *Journal of Building Structures*, Vol. 3, No. 01, (1982), 1.
- Européen, C., "Eurocode 2: Design of concrete structures—part 1-1: General rules and rules for buildings", London: British Standard Institution, (2004).
- Rosa, G., González, G., Sánchez, P. and Prieto, B., "Review of the quality control of concrete reinforced with steel fibers according to the spanish structural concrete code ehe 08", in 2nd Building and Management International Conference. Proceedings., (2019).
- Bhikshma, V. and Kishore, R., "Development of stress-strain curves for recycled aggregate concrete", (2010). <https://www.sid.ir/en/Journal/ViewPaper.aspx?ID=185482>
- Desayi, P., Sundara Raja Iyengar, K. and Sanjeeva Reddy, T., "Equation for stress-strain curve of concrete confined in circular steel spiral", *Matériaux et Construction*, Vol. 11, No. 5, (1978), 339-345.
- Peng, J.-L., Du, T., Zhao, T.-S., Song, X.-q. and Tang, J.-J., "Stress-strain relationship model of recycled concrete based on strength and replacement rate of recycled coarse aggregate", *Journal of Materials in Civil Engineering*, Vol. 31, No. 9, (2019), 04019189. [https://doi.org/10.1061/\(ASCE\)MT.1943-5533.0002847](https://doi.org/10.1061/(ASCE)MT.1943-5533.0002847)
- Khan, S., Ayub, T. and Rafeeqi, S., "Prediction of compressive strength of plain concrete confined with ferrocement using artificial neural network (ANN) and comparison with existing mathematical models", *American Journal of Civil Engineering and Architecture*, Vol. 1, No. 1, (2013), 7-14. <https://doi.org/10.12691/ajcea-1-1-2>
- Ayub, T., Shafiq, N. and Nuruddin, M.F., "Stress-strain response of high strength concrete and application of the existing models", *Research Journal of Applied Sciences, Engineering and Technology*, Vol. 8, No. 10, (2014), 1174-1190.

Persian Abstract

چکیده

تقاضای فزاینده ای برای مواد جایگزین مناسب از مواد تشکیل دهنده بتن به ویژه سنگدانه های ریز و درشت به منظور دستیابی به توسعه پایدار در عصر شهرنشینی سریع وجود دارد. بنابراین، فرآیند ساخت بتن با استفاده از سنگدانه های بازیافت شده از زباله های ساخت و ساز و تخریب به عنوان یک هدف اولیه برای بسیاری از سازمان های دولتی پدیدار شده است. در نتیجه، استفاده از بتن سنگدانه های بازیافتی (RAC) در کاربردهای سازه ای جنبه ضروری می شود. با این حال، RAC را می توان در کاربردهای ساختاری تنها در صورتی به کار برد که رابطه تنش-کرنش موثر موجود باشد. مدل های تنش-کرنش توسعه یافته برای بتن سنگدانه طبیعی (NAC) برای RAC کاملاً مناسب نیستند. از این رو، انتخاب مدل خوب که دارای ظرفیت پیش بینی دقیق است، نقش مهمی ایفا می کند. علاوه بر این، مدل های تنش-کرنش مبنایی را برای تحلیل و رویه های طراحی مدرن به ویژه در بسته های FEA فراهم می کنند. در مطالعه حاضر، مدل های تنش-کرنش برای RAC از ادبیات انتخاب شده و به منظور ارزیابی کارایی پیش بینی کننده آن ها مورد بررسی انتقادی قرار گرفته اند. نمونه های آزمایشی در قالب روابط تنش-کرنش اندازه گیری شده به دست آمده از ادبیات، با پیش بینی های هر مدل انتخاب شده مقایسه شده اند. علاوه بر مقایسه پروفیل های تنش-کرنش اندازه گیری شده و پیش بینی شده، خروجی مدل های انتخابی از نظر چقرمگی نرمال شده و شاخص شکل پذیری مورد ارزیابی قرار گرفت. سازگاری خروجی مدل ها با استفاده از ابزارهای آماری مانند ضریب واریانس و ریشه میانگین مربعات خطا ارزیابی می شود. نتایج مدل در قالب بیان چند جمله ای نسبتاً دقیق تر از سایر همتایان بود.