International Journal of Engineering

Journal Homepage: www.ije.ir

Design of Open Pit Mines using 3D Model in Two-element Deposits under Price Uncertainty

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PAPER INFO

Paper history:
Received 17 May 2022
Received in revised form 09 June 2022
Accepted 11 June 2022

Keywords:
Binomial Tree
Pricing Uncertainty
Pyramid Tree
Two-element Deposits
Three-dimensional Tree

ABSTRACT

When it comes to evaluating mining projects, uncertainty plays a significant role, particularly in the analysis of mining economic characteristics, which makes the assessment of a mining project erroneous and untrustworthy. The volatility of mineral prices is a major cause of economic ambiguity and concern. Economic uncertainty has extensively been examined in mining production project planning, but the majority of the study has focused on single-element deposits, with little emphasis devoted to the significance of pricing uncertainty in two-element deposits. Using a three-dimensional tree model, this study investigates how design could be affected by the pricing uncertainty of two different elements. In this model, not only annual volatility but also monthly volatility were considered due to momentary changes in the price of several elements. To authenticate the proposed model, a numerical example was resolved using discounted cash flow, binomial tree, pyramid tree, and three-dimensional modeling techniques. The results of each approach were compared to those of real-world data. Following the findings of the current investigation, it can be concluded that the values derived from the suggested model (a net present value of $ 324.2 thousand) are more precise than the values acquired from other approaches, and that they are just 8% out of step with reality. Other methods, on the other hand, come up with results that are at least 17% and at most 39% different from those that come from real data.

doi*: 10.5829/ije.2022.35.10a.10*

1. INTRODUCTION

Design for open pit mines is a complex and significant issue that has been addressed by many researchers. The design process usually starts with a geological block model consisting of a group of imaginary regular blocks covering the surrounding ore and host rock resources. Then, a set of characteristics, including the grade, specific weight, and coordinates, are estimated or attributed to each block using drilling sample data. The geological features are combined with technical and economic parameters in the next step to determine the economic value of each block, forming the economic block model, which is a necessary input for the production planning.

Generally, production planning for an open-pit mine involves finding a sequence of blocks for optimized annual plans, which lead to the highest net present value (NPV) for the project cash flow while satisfying the technical limitations such as extraction capacity, processing capacity, block derivation sequence, and pit slope [1].

Design in mines can be categorized into deterministic and stochastic-based approaches. Deterministic open-pit production was first addressed in 1968 [2] and developed in many methods, such as integer programming [3,4], complex integer programming [5,6], dynamic programming [7], and metaheuristic approaches (e.g., genetic algorithms [8], particle swarm optimization [9], and ants colony algorithm [10,11]. The main issue of this approach is the input parameter assumptions. The deterministic parameter assumption might lead to unrealistic and incorrect production planning because these parameters are associated with a significant uncertainty [12-14]. Most studies considered single-element deposit, and there have been few studies regarding the role of economic parameter uncertainty for two-element deposits [15-24]. To address the shortcomings of previous studies, the present study

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Please cite this article as: G. H. Kakha, Design of Open Pit Mines using 3D Model in Two-element Deposits under Price Uncertainty, ***International Journal of Engineering, Transactions A: Basics***, Vol. 35 No. 10, (2022), 1123-1132

investigates the design of two-element deposits under price uncertainty using the proposed 3D tree model.

2. EXPERIMENTAL PROCEDURE

Figure 1 depicts the stages that this article aims to take in order to accomplish the goals of this study.

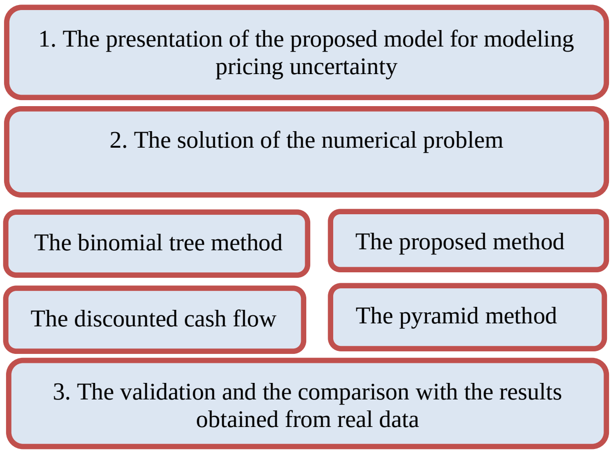

Figure 1. The process diagram

2.1. The model proposed for modeling pricing uncertainty

The binomial tree model is one of the most often used models for analyzing the discontinuously fluctuations of stock price. This model was first developed by Cox and Ross [25] to estimate the pricing stock uncertainty. Flexibility, accuracy, and speed in calculation are some of the advantages of the binomial tree model [26]. The structure of a binomial tree is formed of different branches and nodes. This model depicts all conceivable ways in which mineral prices might fluctuate throughout the project lifecycle. For each pricing node, it is seen how much the mineral was valued at that point in time. An illustration of a binomial tree is shown in Figure 2.

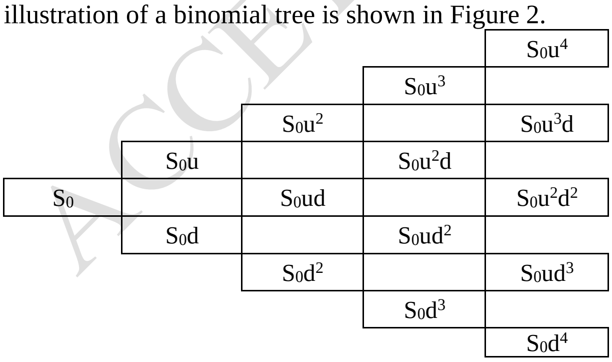

Figure 2. The schematic view of a binomial tree

As can be seen, the number of nodes in each layer corresponds to the number of layers. These branches indicate various routes from one node to the next one, and every single one of them has its own probability and rate of rise or decline of related nodes. Ascending branches have a probability of realization of P_r, whereas descending branches have a chance of realization of $1-P_r$. If a node is linked to the ascending branch, the value of that node is multiplied by u to get the node's value. By the same token, the value of the nodes linked to descending branches is derived by multiplying the value of the preceding node by d. For the purpose of illustration, if the value of the node in layer No. 1 of Fig. 2 is S_0, the value of the node linked to the ascending branch and its probability of occurrence will be S_0u and P_r, respectively. Moreover, the value of the node connected to the descending branch and its probability of occurrence will be S_0d and $1-P_r$ in that order. The equations below show how to figure out u, d, and the probability of P_r [25].

$$u = \exp\left(\sigma\sqrt{\delta_t}\right) \quad (1)$$

$$d = \frac{1}{u} = \exp\left(-\sigma\sqrt{\delta_t}\right) \quad (2)$$

$$P_r = \frac{(1+r_f)-d}{u-d} \quad (3)$$

where σ is the Instability (unpredictability), u is the increasing rate of each node's value, r is the risk-free discount rate, d is the decreasing rate of each node's value, T is the life expectancy of a project in terms of time periods, and N is the number of time periods of a tree.

The binomial tree approach has a major limitation when it comes to analyzing the impact of many uncertainties simultaneously [27]. Using a pyramid tree model, Dehghani et al. [28] were able to remove this problem in the binomial tree technique. In their investigation, they looked at the impact of price and cost uncertainty on the evaluation of mining ventures. In the pyramid model, all possible prices and operating costs for minerals are taken into account (see Figure 3).

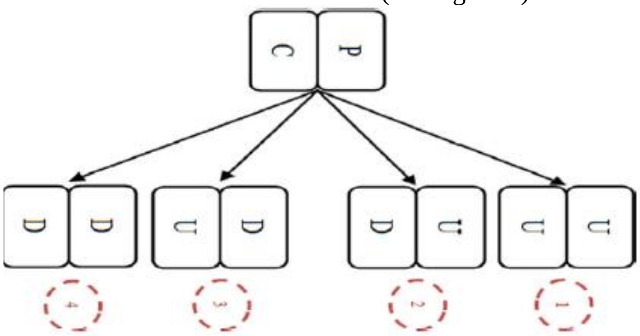

Figure 3. The pricing and operational cost variations (U: increasing and D: decreasing) [28]

The pyramid tree model is capable of modeling and estimating both uncertainties simultaneously. Figure 4 illustrates a view of the pyramid tree.

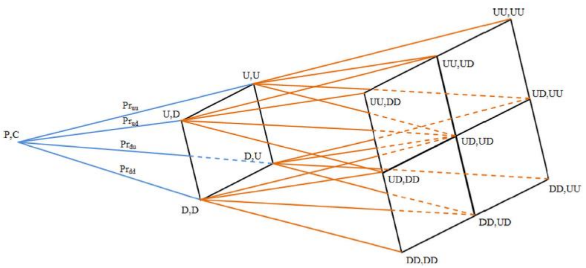

Figure 4. pyramid tree model (U: increasing and D: decreasing) [28]

It is made by multiplying the nodes of the economic value tree and the tree of probabilities, and then discounting them using Eq. (11). The net present value (NPV) will then be found by subtracting this tree from the other two trees and multiplying them.

$$Pr_{uu} = \frac{1}{4}\frac{(\Delta x_p \Delta x_c + \Delta x_c \vartheta_p \Delta t + \Delta x_p \vartheta_c \Delta t + \rho\sigma_p\sigma_c\Delta t)}{\Delta x_p \Delta x_c} \quad (4)$$

$$Pr_{uu} = \frac{1}{4}\frac{(\Delta x_p \Delta x_c + \Delta x_c \vartheta_p \Delta t - \Delta x_p \vartheta_c \Delta t - \rho\sigma_p\sigma_c\Delta t)}{\Delta x_p \Delta x_c} \quad (5)$$

$$Pr_{uu} = \frac{1}{4}\frac{(\Delta x_p \Delta x_c - \Delta x_c \vartheta_p \Delta t + \Delta x_p \vartheta_c \Delta t - \rho\sigma_p\sigma_c\Delta t)}{\Delta x_p \Delta x_c} \quad (6)$$

$$Pr_{uu} = \frac{1}{4}\frac{(\Delta x_p \Delta x_c - \Delta x_c \vartheta_p \Delta t - \Delta x_p \vartheta_c \Delta t + \rho\sigma_p\sigma_c\Delta t)}{\Delta x_p \Delta x_c} \quad (7)$$

$$Pr_{uu} + Pr_{ud} + Pr_{du} + Pr_{dd} = 1 \quad (8)$$

In the abovementioned relations, σ_P and σ_C denote the price and cost unpredictability, respectively. Moreover, Δt is the ratio of the life expectancy of a project to the number of time periods, ρ is the correlation between the price and cost data. It is worthy to note that ΔXp and ΔXc are calculated through the multiplication of volatilityin Δt [28].

$$\vartheta_p = \mathrm{r} - \frac{1}{2}\sigma_p^2 \quad (9)$$

$$\vartheta_c = \mathrm{r} - \frac{1}{2}\sigma_c^2 \quad (10)$$

$$DCF_{n,k} = BEV_{n,k} + (V/(1+i)) \quad (11)$$

$$V = Pr_{uu}.DCF_{n+1,uu} + Pr_{ud}.DCF_{n+1,ud} + Pr_{du}.DCF_{n+1,du} + Pr_{dd}.DCF_{n+1,dd} \quad (12)$$

As previously stated, the binomial tree model was only capable of investigating one parameter under uncertainty and made the assumption that all other essential values remained constant. Dehghani et al. [28] employ just unpredictability, the yearly increasing and decreasing coefficients for prices in their pyramid tree, but the changes in mineral prices are on the spot and should be established upon the quantity of the blocks extracted in a year, unpredictability, and monthly increasing and decreasing coefficients to solve the model. Figure 5 shows an overview of the three-dimensional tree model.

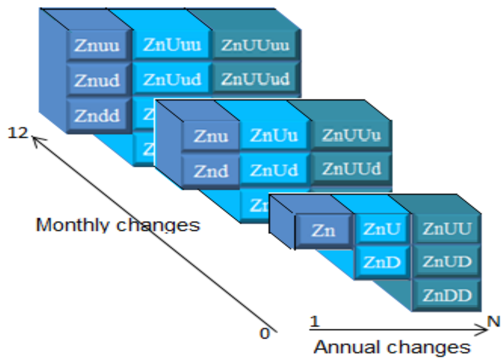

Figure 5. The 3D tree model for the price of zinc (ZN: The base price of zinc, U: The annual increasing coefficient, D: The annual decreasing coefficient, u: The monthly increasing coefficient, d: The monthly decreasing coefficient)

The priority of the proposed model over the other two models is to simultaneously consider the monthly and annual unpredictability. In the proposed model, the price 3D trees are first created for two elements, as shown in Figure 5. A three-dimensional economic value tree is then generated for all price change situations utilizing pricing trees. Beginning in the second year, for example, there are two prices for every single element. As a result, four value possibilities may be found in the economic value tree. They include: 1) an increase in both elements' prices, 2) an increase in the price of the first element and a decrease in the price of the second element, 3) a decrease in the price of the first element and an increase in the price of the second element; and 4) a decrease in both elements' prices.

Following the construction of a three-dimensional economic value tree using Eqs. (4) to (10), a three-dimensional tree of probabilities is constructed, and in the following step, the corresponding multiplication tree is constructed by correspondingly multiplying the nodes of the two trees of economic value and probabilities. In the next phase, you can use Eq. (11) to figure out how much the project will be value in the future by not taking into account the tree that you bought in the first phase.

2.2. Numerical example

The blocks of a lead and zinc mine are shown in the following hypothetical cross-section where the lead and zinc cutoff grades are indicated by a number on the left and right side, respectively. Furthermore, the supplementary data is given in Table 1, in order to calculate the economic value of every single block.

The hypothetical grades model of a lead and zinc mine is shown in Figure 6.

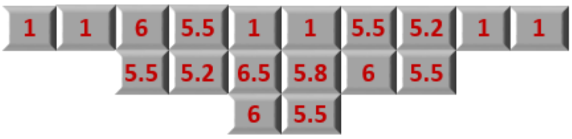

Figure 6. The Hypothetical Grades Model of a Lead and Zinc Mine

Osanloo and Ataei [29] examined the equivalent cutoff grade in multi-element deposits. The equations initiated from this research for two-element deposits are as follows:

$$BEV = TO * [\bar{G}_1 R_1 (P_1 - r_1) + \bar{G}_2 R_2 (P_2 - r_2) - C_r] - (TR * C_m) \quad (13)$$

where C_r is the cost of condensing and processing, C_m is the cost of the extraction of each ton ore, TO_i is the mineral tonnage in blocks, TR_i is the block tonnage, including tailings and minerals, P_2 is the price of the second element, r_2 is the cost of purifying and selling the second metal, R_2 is the total retrieval of the second metal, P_1 is the price of leading metal, r_1 is the cost of purifying and selling the leading metal, R_1 is the total retrieval of the leading metal, g_1 is the mean cutoff grade concerning the leading metal, and g_2 is mean cutoff grade concerning the second metal [30].

Eq. (4) is initiated by factoring $R_1(P_1-r_1)$ in Eq. (3).

$$BEV = TO * \left[R_1 (P_1 - r_1) \left(\bar{G}_1 + \bar{G}_2 \frac{R_2 (P_2 - r_2)}{R_1 (P_1 - r_1)} \right) - C_r \right] - (TR * C_m) \quad (14)$$

$$f_{eq} = \frac{R_2 (P_2 - r_2)}{R_1 (P_1 - r_1)} \quad (15)$$

Equivalent factor f_{eq} is used to show the economic valueof the blocks for the two-element deposits as Eq. (6).

$$BEV = TO * [R_1 (P_1 - r_1)(\bar{G}_1 + f_{eq} \bar{G}_2) - C_r] - (TR * C_m) \quad (16)$$

On the basis of relations (13) and (17), Table 1 and considering zinc as the leading metal and lead as the second metal, one can turn the supposable cutoff grade model into the cutoff grade model equivalent to Figure 7.

TABLE 1. The information required for the problem.

Description	Amount for lead	Amount for zinc	unit
Total retrieval	80	85	%
Block dimension	10*10*5	10*10*5	Meter
Cut off limit	1.4	1.48	%
Density	10	7	Ton/m^3
Price in the beginning of 2013	2224.5	1986.21	Dollar/ton
The cost of extraction	1	1	Dollar/ton
Processing cost	63	63	Dollar/ton
Risk-free rate	7	7	%
F_{eq}	1.05	Equivalent cutoff grade	2.9

Figure 7. The equivalent grade model

Based on the data available in Table 1, the materials' mean density (3 ton/m^3), and the equivalent grade model, the block economic value model is in the form of Figure 8.

In this case, it is anticipated that it will take one year to extract each of the three blocks. As a result, the duration of this project will be three years. In Figure 9, Roman shows how to use his "dynamic planning" method to plan mining in this part.

Figure 8. The block economic value model(1000 dollars)

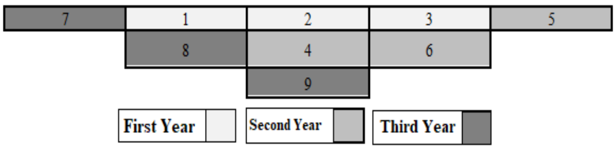

Figure 9. The order of mining

Based on Figures 8 and 9 and Eq. (17), a net present value of $291.53 thousand was derived using the discounted cash flow approach from the extraction of this cross-section.

$$NPV = \sum_{n=1}^{N} \frac{BEV_n}{(1+i)^n} \quad (17)$$

Where BEV_n is the sum of economic value of the blocks in year n, i is the discount rate, and N is the project life.

- **The binomial tree model**

Utilizing price data from 1990 to 2013, the binomial tree approach affecting by pricing uncertainty will be used to obtain the parameters needed to solve the numerical example.

TABLE 2. The historical price data on zinc and lead between 1990 and the beginning of 2013 and the calculation of volatility [31]

Year	The price of lead	LN(Pn+1-pn)	The price of zinc	LN(Pn+1-pn)
1990	809.5	0.185207	1517.92	-0.0872
1991	557.8	-0.37242	1121.36	-0.3028
1992	543.51	-0.02595	1241.84	0.102052
1993	407.34	-0.2884	963.96	-0.2533
1994	548.72	0.29794	998.22	0.034924
1995	629.3	0.13702	1031.09	0.032398
1996	774.13	0.207132	1024.97	-0.00595
1997	623.06	-0.2171	1314.9	0.249097
1998	526.92	-0.16759	1024.29	-0.24976
1999	501.77	-0.04891	1075.8	0.049065
2000	454.17	-0.09967	1127.7	0.047116
2001	476.36	0.047702	886.82	-0.24029
2002	452.25	-0.05194	778.9	-0.12976
2003	514.21	0.128397	827.97	0.061094
2004	881.95	0.539504	1048.04	0.2357
2005	974.37	0.099656	1380.55	0.27556
2006	1288.42	0.279381	3266.18	0.861139
2007	2579.12	0.694032	3249.73	-0.00505
2008	2593.32	0.005491	1884.83	-0.54473
2009	1719.44	-0.41094	1658.39	-0.12799
2010	2148.19	0.222627	2160.36	0.264428
2011	2400.71	0.111139	2195.53	0.016149
2012	2063.56	-0.15133	1950.02	-0.11858
2013	0.0751	2224.50	0.018389	1986.21

TABLE 3. The information required to create binomial tree

The parameters of binomial tree	The price of zinc	The price of lead
Volatility	27.2%	26.1%
Increasing coefficient	1.60%	1.58
Decreasing coefficient	0.62%	0.63%
The probability of increase	45%	46%

TABLE 4. The tree of changes of zinc price

2013	2014	2015
1986.21	3182.399	`5098.989
	1239.64	1986.21
		773.686

TABLE 5. The tree of changes of lead price

2013	2014	2015
2224.5	3521.376	5574.327
	1405.246	2224.50
		887.7127

The zinc and lead pricing trees are generated for 2013-2015 after calculating the binomial tree's necessary parameters and the base year price (Tables 4 and 5). According to Eq. (13) about the economic value of the block and the trees concerning the price of the two elements, the economic value tree has the following form, as shown in Table 6.

TABLE 6. The economic value tree for each year ($1000)

2013	2014	2015
128.2807	240.4564	335.0613
	90.8831	128.2807
		46.65893

TABLE 7. The economic value discount tree ($1000)

2013	2014	2015
419.8029	468.5665	335.0613
	110.0699	128.2807
		46.65893

Eventually, after discounting the economic value tree using Eq. (18), the amount of net present value is determined.

$$DCF_{n,k} = BEV_{n,k} + \frac{P_r \times DCF_{n+1,k} + (1 - P_r) \times DCF_{n+1,k+1}}{(1 + r)} \quad (18)$$

This section's net present value will be $419.80 thousand dollars if it is extracted using the binomial tree approach to account for zinc and lead pricing uncertainty.

- **The pyramid tree model**

In light of what has been mentioned so far, a numerical example will be solved using the pyramid tree model in this part. Similar to the parameters of a two-dimensional binomial tree, these parameters already listed in Table 1 are needed to create trees. Afterwards, using price binomial trees for the two elements of lead and zinc (Tables 4 and 5), the economic value tree is created (Table 8).

TABLE 8. The 3D economic value tree for each year

2013	2014	2015
128.2807	240.4564	335.0613
	174.9909	241.0051
	156.3486	204.3673
	90.8831	222.337
		128.2807
		91.6496
		177.3531
		83.29677
		46.65901

TABLE 9. The 3D tree of probabilities for each year

2013	2014	2015
1	0.472517	0.237791
	0.092562	0.068924
	0.087438	0.042277
	0.347483	0.16664
		0.182204
		0.013352
		0.051086
		0.111076

		0.12665

TABLE 10. A intermediate binomial tree obtained by multiplying the corresponding nodes of economic value and probabilities

2013	2014	2015
128.2807	113.6197	79.67441
	16.19749	16.61101
	13.67082	8.640079
	31.58033	37.05025
		23.37326
		1.223611
		9.060326
		9.252265
		5.909376

In the next step, given the price historical data of lead and zinc in addition to Eqs. (4) to (10), the multivariable tree of probabilities is created (see. Table 9).

In the end, the discounted binomial tree for the model presented by Dehghani et al. will be shown in Table 11. As can be seen, using the pyramid tree developed by Dehghani et al., the net present value resulted from the extraction of the desired cross-section is equal to 215.23 thousand dollars when considering the pricing uncertainty of zinc and lead.

TABLE 11. The discounted binomial tree for pyramid model.

2013	2014	2015
215.2318	184.9074	104.9255
	72.65671	61.64712
		24.22197

- **The three-dimensional recommended model**

This study's model incorporates not just yearly volatility but also monthly volatility owing to the rapid shifts in the price of several elements in recent years, as previously mentioned. The suggested model is used to solve a numerical problem in this section. On average, one block is extracted every four months because the yearly extraction capacity in the given example is equal to three blocks. In order to make the suggested model trees, four-month data as well as yearly data must be used.

It is important to develop a pricing tree for the proposed model after computing the parameters associated with the model. For the suggested model in Tables 13 and 14, the lifetime of lead and zinc pricing trees and the mining capacity are assumed to be three years and three blocks each year.

TABLE 12. The information required to create the proposed model

The parameters of annual binomial tree	The price of zinc	The price of lead
Volatility	27.2%	26.1%
Increasing coefficient	1.60%	1.58
Decreasing coefficient	0.62%	0.63%
The probability of increase	45%	46%
The parameters of four-month binomial tree	**The price of zinc**	**The price of lead**
Volatility	14.2%	14.8%`
Increasing coefficient	1.152%	1.158
Decreasing coefficient	0.86%	0.86%
The probability of increase	0.71%	0.7%

TABLE 13. The zinc pricing tree for the proposed model (ZN: The base price of zinc, U: The annual increasing coefficient, D: The annual decreasing coefficient, u: The 4-month increasing coefficient, d: The 4-month decreasing coefficient)

Zinc pricing tree for the first year			Zinc pricing tree for the second year			Zinc pricing tree for the third year		
BLOCK1	BLOCK2	BLOCK3	BLOCK1	BLOCK2	BLOCK3	BLOCK1	BLOCK2	BLOCK3
Zn	Znu	Znuu	ZnU	ZnUu	ZnUdd	ZnUU	ZnUUu	ZnUUuu
	Znd	Znud	ZnD	ZnUd	ZnUud	ZnUD	ZnUUd	ZnUUud
		Zndd		ZnDd	ZnUdd	ZnDD	ZnUDd	ZnUUdd
					ZnDdd		ZnDDd	ZnUDdd
								ZnDDdd

Table 14. The Lead pricing tree for the proposed model (Pb: The base price of Lead, U: The annual increasing coefficient, D: The annual decreasing coefficient, u: The 4-month increasing coefficient, d: The 4-month decreasing coefficient)

Lead pricing tree for the first year			Lead pricing tree for the second year			Lead pricing tree for the third year		
BLOCK1	BLOCK2	BLOCK3	BLOCK3	BLOCK3	BLOCK3	BLOCK3	BLOCK3	BLOCK3
Zn	Znu	Pbuu	PbUdd	PbUdd	PbUdd	PbUUuu	PbUUuu	PbUUuu
	Znd	Pbud	PbUud	PbUud	PbUud	PbUUud	PbUUud	PbUUud
		Pbdd	PbUdd	PbUdd	PbUdd	PbUUdd	PbUUdd	PbUUdd
				PbDdd	PbDdd	PbUDdd	PbUDdd	PbUDdd
							PbDDdd	PbDDdd

In light of what has been discussed regarding the suggested model, the hypothetical scenario using the described model will now be resolved. The pricing tree for zinc and lead has been built for the proposed model, as shown in Tables 3, 10 and 11.

TABLE 15. The zinc pricing tree for the proposed model.

Zinc pricing tree for the first year			Zinc pricing tree for the second year			Zinc pricing tree for the third year		
BLOCK1	**BLOCK2**	**BLOCK3**	**BLOCK1**	**BLOCK2**	**BLOCK3**	**BLOCK1**	**BLOCK2**	**BLOCK3**
1,986.21	2288.268	2636.261	3182.39	3666.37	4223.942	5098.989	5874.43	6767.798
	1724.025	1986.21	1239.64	2762.31	3182.399	1986.21	4425.909	5098.989
		1496.449		1076.00	2397.681	773.6886	1724.025	3841.677
					933.9689		671.5596	1496.449
								582.9119

TABLE 16. The lead pricing tree for the proposed model

Lead pricing tree for the first year			Lead pricing tree for the second year			Lead pricing tree for the third year		
BLOCK1	**BLOCK2**	**BLOCK3**	**BLOCK1**	**BLOCK2**	**BLOCK3**	**BLOCK1**	**BLOCK2**	**BLOCK3**
2,224.5	2578.172	2988.074	3521.376	4081.237	4730.111	5574.327	6460.585	7487.75
	1919.345	2224.5	1405.246	3038.316	3521.376	2224.5	4809.644	5574.327
		1656.05		1212.475	2621.522	887.7127	1919.345	4149.861
					1046.149		765.9368	1656.05
								660.8661

After establishing pricing trees for lead and zinc, it is time to develop a value tree. In this regard, for each price determined for a block associated with the first element, all prices associated with that block for the second element are included in the computation of the block's economic value, and the overall structure of the block's economic value tree is produced (see Table. 17). Following the computation of the economic value tree for the desired example using the given model, a probability tree for this model should be generated. This is accomplished by the usage of Eqs. (4) to (10). Table 18 depicts the probability tree for each block, which corresponds to the economic value tree. Then, the intermediate binomial tree is made by multiplying the relevant nodes from the two trees of economic value and probability.

TABLE 17. The economic value tree for the proposed model ($1000)

First year			Second year			Third year		
BLOCK1	**BLOCK2**	**BLOCK3**	**BLOCK1**	**BLOCK2**	**BLOCK3**	**BLOCK1**	**BLOCK2**	**BLOCK3**
69.78	-1.5	82.001	130.375	-1.5	152.301	-1.5	181.538	243.3131
	-1.5	72.731	95.905	-1.5	135.684	-1.5	160.88	216.6879
	-1.5	65.746	84.962	-1.5	123.164	-1.5	122.347	196.628
	-1.5	70.269	50.493	-1.5	99.811	-1.5	107.337	159.2108
		60.998		-1.5	130.199	-1.5	156.171	144.6356
		54.014		-1.5	113.581	-1.5	135.513	208.3248
		61.535		-1.5	101.062	-1.5	96.98	181.6996
		52.264		-1.5	77.709	-1.5	81.97	161.6396
		45.279		-1.5	113.744	-1.5	111.762	124.2225
					97.127		91.104	109.6473
					84.607		96.98	182.2774
					61.254		37.561	155.6522
					84.938		94.039	135.5923
					68.32		73.381	98.17508
					55.8		34.848	83.59995
					32.447		19.838	136.6763
								110.0511
								89.99116
								52.57397
								37.99885
								118.4786
								91.85344
								71.79351

								34.37632
								19.8012

TABLE 18. The tree of probabilities for the proposed model

First year			Second year			Third year		
BLOCK3	**BLOCK3**	**BLOCK3**	**BLOCK3**	**BLOCK3**	**BLOCK3**	**BLOCK3**	**BLOCK3**	**BLOCK3**
1	0.602733	0.359092	0.472517	0.359092	0.2533	0.237791	0.2533	0.194949
	0.096754	0.134216	0.092562	0.134216	0.014405	0.068924	0.014405	0.113992
	0.108246	0.040448	0.087438	0.040448	0.07406	0.184499	0.07406	0.080236
	0.192267	0.13931	0.347483	0.13931	0.095065	0.06664	0.095065	0.046479
		0.176649		0.176649	0.129678	0.111124	0.129678	0.059922
		0		0	0.150683	0.013352	0.150683	0.064091
		0.050663		0.050663	0.024188	0.179943	0.024188	0.042274
		0.09962		0.09962	0	0.011076	0	0.048318
		0		0	0.079806	0.126652	0.079806	0.014561
					0.027062		0.027062	0
					0.048067		0.048067	0.083913
					0		0	0.050157
					0.103685		0.103685	0.0636
					0		0	0
					0		0	0
					0		0	0.051995
								0.018239
								0
								0
								0
								0.067278
								0
								0
								0
								0

The intermediate binomial tree will be discounted in the following phase. This is done by converting the tree to a standard binomial tree, as shown in Table 20. The probability value of the major element (zinc element) as well as the 4-month rate (2 percent) for 4-month periods will then be utilized to discount the standard binomial tree based on the main element (zinc element) and 4-month extraction periods for each block. They show the trees that were discounted each year and how much they were discounted at the end of each year, in Tables 21 and 22.

TABLE 19. The intermediate binomial tree obtained through multiplying the corresponding nodes of economic value and probabilities

First year			Second year			Third year		
BLOCK3	**BLOCK3**	**BLOCK3**	**BLOCK3**	**BLOCK3**	**BLOCK3**	**BLOCK3**	**BLOCK3**	**BLOCK3**
69.78	-0.9041	29.44605	61.60437	-0.53864	45.98363	-0.35669	45.98363	47.43364
	-0.14513	9.761619	8.877194	-0.20132	2.317477	-0.10339	2.317477	24.70068
	-0.16237	2.65929	7.428917	-0.06067	9.061012	-0.27675	9.061012	15.77664
	-0.2884	9.789181	17.54531	-0.20897	10.204	-0.09996	10.204	7.399957
		10.77529		-0.26497	20.25199	-0.16669	20.25199	8.666857
		0		0	20.41953	-0.02003	20.41953	13.35174
		3.117536		-0.07599	2.345754	-0.26991	2.345754	7.681168
		5.206542		-0.14943	0	-0.01661	0	7.810104
		0		0	8.919254	-0.18998	8.919254	1.808803
					2.465443		2.465443	0
					4.661541		4.661541	15.29544
					0		0	7.807047
					9.750485		9.750485	8.623668
					0		0	0
					0		0	0
					0		0	7.106483
								2.007222

0
0
0
7.971005
0
0
0
0

TABLE 20. The intermediate standard binomial tree obtained through multiplying the corresponding nodes of economic value and probabilities

First year			Second year			Third year		
BLOCK3	BLOCK3	BLOCK3	BLOCK3	BLOCK3	BLOCK3	*BLOCK3*	*BLOCK3*	*BLOCK3*
69.78	-1.42983	68.48029	70.48156	-1.3081	128.1587	*-0.73682*	*147.9918*	*191.8403*
	-0.07017	28.12907	24.97423	-0.63542	59.58691	*-0.39329*	*70.69794*	*103.0542*
		2.803223		-0.06833	17.11281	*-0.47651*	*16.51065*	*51.88862*
					8.440671		*9.345174*	*13.95744*
								7.53524

TABLE 21. The discounted binomial tree (4-month) for every single year

First year			Second year			Third year		
BLOCK3	BLOCK3	BLOCK3	BLOCK3	BLOCK3	BLOCK3	BLOCK3	BLOCK3	BLOCK3
113.38	54.2778	68.4802	156.5804	104.914	128.1587	260.5559	310.9210	191.8403
	20.3335	28.12907	60.90663	45.7519	59.58691	125.266	157.2383	103.0542
		2.803223		14.2524	17.11281	45.01529	56.63752	51.88862
					8.440671		21.20979	13.95744
								7.53524

As can be seen, the net present value resulted from the extraction of the desired cross-section to consider the pricing uncertainty of zinc and lead will be equal to 324.27 thousand dollars using the proposed model.

TABLE 22. The final discounted binomial tree

324.275	331.2886	260.5559
	137.1659	125.266
		45.01529

2.3. Validation

Using real prices from 2013 to 2015, the numerical example in this work was solved to test the model (see Table 23).

TABLE 23. The real pricing data for zinc and lead

Years	Real price for Zinc ($/ton)	Real price for Lead ($/ton)
2013(4-month 1)	1986.21	2224.50
2013(4-month 2)	1851.01	2088.10
2013(4-month 3)	1893.28	2106.66
2014(4-month 1)	2026.64	2097.84
2014(4-month 2)	2206.17	2158.59
2014(4-month 3)	2250.10	2029.95
2015(4-month 1)	2113.07	1859.16
2015(4-month 2)	2043.05	1821.98
2015(4-month 3)	1638.91	1682.32

Figure 10. The block economic value model based on real prices ($1000)

Based on the real prices of zinc and lead for the years 2013 to 2015, the net present value of the extraction of the indicated section is 355.14 thousand dollars.

3. CONCLUSION

This paper adopted discounted cash flow, binomial tree, Pyramid tree, and our proposed method to predict the price in the future years. The results are presented in Table 24, and the following conclusions can be drawn.

- The proposed method is a practical and suitable approach to account for the price uncertainty of two-element deposits with the closest-to-reality output (8.6%).
- The second best method is the discounted cash flow, with a 17.9% difference from the real data. If the uncertainty is accounted for, the results will improve.
- The third and fourth-best methods are the binomial tree (18.2%) and Pyramid tree (39.4%), respectively.
- The result can be improved, provided that adequate methods are used to include the uncertainty.
- It is recommended that geological and economic uncertainties should be considered simultaneously in future research.

TABLE 24. The comparison of the evaluation results of different models

Column	Method	Net present value ($1000)	Difference from the real amount (D_i) ($1000)	Difference from the real amount (P_i) (%)
1	Real price DCF	355.14	0	0
2	Constant price DCF	291.53	63.61	17.9
4	Binomial tree	419.80	64.66	18.2
6	Pyramid tree	215.23	139.91	39.4
7	**Proposed model**	**324.27**	**30.87**	**8.6**

REFERENCES

1. Dagdelen, K., 2001. Open pit optimization-strategies for improving economics of mining projects through mine planning. ***In 17th International Mining Congress and Exhibition of Turkey*** (pp. 117-121).
2. Johnson, T.B., 1968. Optimum open pit mine production scheduling. University of California, Berkeley.
3. Dagdelen, K. and Johnson, T.B. 1986. Optimum open pit mine production scheduling by Lagrangian parameterization. ***Proceedings of the 19th International Symposium on Application of Computers and Operations Research in the Mineral Industry*** (APCOM '86). SME, Littleton, CO. pp. 127-142. USA.
4. Caccetta, L. and Hill, S.P. 2003. An application of branch and cut to open pit mine scheduling. ***Journal of Global Optimization***, vol. 27, no. 2-3. pp. 349-365. doi.org/10.1023/A:1024835022186.
5. Boland, N., Dumitrescu, I., Froyland, G. and Gleixner, A.M., 2009. LP-based disaggregation approaches to solving the open pit mining production scheduling problem with block processing selectivity. ***Computers & Operations Research***, 36(4), pp.1064-1089. doi.org/10.1016/j.cor.2007.12.006.
6. Elkington, T. and Durham, R. 2011. Integrated open pit pushback selection and production capacity optimization. ***Journal of Mining Science***, vol. 47, no. 2. pp.177-190. doi.org/10.1134/S1062739147020055.
7. Wang, Q. and Sevim, H. 1992. Enhanced production planning in open pit mining through intelligent dynamic search. Proceedings of the 23rd International Symposium on Application of Computers and Operations Research in the Mineral Industry, Tucson, AZ. ***Society for Mining, Metallurgy and Exploration, Littleton***, CO. pp. 461-471. doi.org/10.1007/s11771-018-3841-5.
8. Denby, B. and Schofield, D. 1994, Open-pit design and scheduling by use of genetic algorithms. ***Transactions of the Institution of Mining and Metallurgy. Section A. Mining Industry***, vol. 103. pp. A21-A26.
9. Khan, A. and Niemann-Delius, C. 2014. Production scheduling of open pit mines using particle swarm optimization algorithm. ***Advances in Operations Research***, 2014, no. 1. pp. 1-9. doi.org/10.1155/2014/208502.
10. Sattarvand, J. and Niemann-Delius, C. 2009. Long-term open-pit planning by ant colony optimization. PhD thesis, ***Institut fur Bergbaukunde III, RWTH Aachen University***.
11. Shishvan, M.S. and Sattarvand, J. 2015. Long term production planning of open pit mines by ant colony optimization. ***European Journal of Operational Research***, vol. 240, no. 3. pp. 825-836. doi.org/10.1016/j.ejor.2014.07.040.
12. Abdel Sabour, S.A. and Poulin, R. 2010. Mine expansion decisions under uncertainty. ***International Journal of Mining, Reclamation and Environment***, vol. 24, no. 4. pp. 340-349. doi.org/10.1080/17480931003664991
13. Dimitrakopoulos, R., Farrelly, C., and Godoy, M. 2002. Moving forward from traditional optimization: grade uncertainty and risk effects in open-pit design. ***Mining Technology***, vol. 111, no. 1. pp. 82-88. doi.org/10.1179/mnt.2002.111.1.82
14. Marcotte, D. and Caron, J. 2013. Ultimate open pit stochastic optimization. ***Computers & Geosciences***, vol. 51. pp. 238-246. doi.org/10.1016/j.cageo.2012.08.008
15. Yazdani, M., Kabirifar, K., Fathollahi-Fard, A.M. and Mojtahedi, M., 2021. Production scheduling of off-site prefabricated construction components considering sequence dependent due dates. ***Environmental Science and Pollution Research***, pp.1-17. doi.org/10.1007/s11356-021-16285-0
16. tahernia, T., ataee-pour, M. (2015). 'A Model for Determination of Block Economic Value in Underground Mining', ***Iranian Journal of Mining Engineering***, 10(28), pp. 43-51.doi.org/ 20.1001.1.17357616.1394.10.28.6.1
17. Samis, M., Davis, G.A., Laughton, D., Poulin, R., 2006. "Valuing uncertain as set cash flows when there are no options– a real options approach". ***Resources Policy*** Vol. 30, No. 4: 285–298.doi:10.1016/j.resourpol.2006.03.003.
18. Shafiee, S., Topal, E., Nehring, M., 2009. "Adjusted real option valuation to maximize mining project value – a case study using century mine". ***In: Project Evaluation Conference***, pp.125–134.
19. Dehghani, H., Ataee-pour, M., 2012. "Determination of the effect of operating cost uncertainty on mining project evaluation". ***Resources Policy,*** Vol. 37, No. 1: 109–117. doi.org/10.1016/j.resourpol.2011.11.001
20. Dehghani, H., Ataee-pour, M., 2012. "The role of economic uncertainty on the block economic value – a new valuation approach". ***Archives of Mining Sciences,*** Vol**.** 57, No. 4: 991–1014. doi.org/10.2478/v10267-012-0066-6
21. Mokhtarian Asl, M., & Sattarvand, J. 2018. "Integration of commodity price uncertainty in long-term open pit mine

production planning by using an imperialist competitive algorithm". ***Journal of the Southern African Institute of Mining and Metallurgy***, Vol 118, No. (2), 165-172. doi.org/10.17159/2411-9717/2018/v118n2 a10

22. Souza, F.R., Câmara, T.R., Torres, V.F.N., Nader, B. and Galery, R., 2019. "Optimum mine production rate based on price uncertainty". ***REM-International Engineering Journal***, Vol. 72, No. (4), pp.625-634. doi.org/10.1590/0370-44672018720093
23. Rimélé, A., Dimitrakopoulos, R. and Gamache, M., 2020. "A dynamic stochastic programming approach for open-pit mine planning with geological and commodity price uncertainty". ***Resources Policy***, Vol. 65, p.101570. doi.org/10.1016/j.resourpol.2019.101570
24. Jamshidi, M., and M. Osanloo., 2018 "Multiple destination influence on production scheduling in multi-element mines." ***International Journal of Engineering, Transactions A: Basics,*** Vol**.** 31, no. 1: 173-180. doi: 10.5829/ije.2018.31.01a.23
25. Cox, J.C. and Ross, S.A., 1976. "The valuation of options for alternative stochastic processes". ***Journal of Financial Economics***, Vol. 3, No. (1-2), pp.145-166. doi.org/10.1016/0304-405X(76)90023-4
26. Cox, J.C., Ross, S.A. and Rubinstein, M., 1979. "Option pricing: A simplified approach". ***Journal of Financial Economics***, Vol. 7, No. (3), pp.229-263. doi.org/10.1016/0304-405X(79)90015-1
27. Copeland, T.E., Antikarov, V. and Copeland, T.E., 2001. "Real options: a practitioner's guide". (p. 4). ***New York: Texere LLC***.
28. Dehghani, H., Ataee-pour, M. and Esfahanipour, A., 2014. "Evaluation of the mining projects under economic uncertainties using multidimensional binomial tree". ***Resources Policy***, Vol. 39, pp.124-133. doi.org/10.1016/j.resourpol.2014.01.003
29. Osanloo, M., & Ataei, M. 2003. "Using equivalent grade factors to find the optimum cut-off grades of multiple metal deposits". ***Minerals Engineering***, Vol. 16, No. (8), 771-776. doi.org/10.1016/S0892-6875(03)00163-8
30. Kakha, G.H. and Monjazi, M., 2017. "Push Back Design in Two-element Deposits Incorporating Grade Uncertainty". ***International Journal of Engineering, Transactions, B: Applications,*** Vol. 30, No. (8), pp.1279-1287. doi: 10.5829/ije.2017.30.08b.22
31. Lead and Zinc Statistical Bulletin, (1990-2015). https://www.ilzsg.org/static/statistics.aspx.

Persian Abstract

چکیده

امروزه، عدم قطعیت‌ها نقش موثری در ارزیابی پروژه های معدنی بخصوص در بررسی پارامترهای اقتصادی معدنی ایفا می‌کنند، به گونه‌ای که ارزیابی یک پروژه معدنی بدون در نظر گرفتن عدم‌قطعیت‌های موجود غیرقابل اعتماد و نادرست است. یکی از مهمترین منابع عدم‌قطعیت‌های اقتصادی می‌توان به عدم‌قطعیت قیمت ماده معدنی اشاره نمود. محققین بسیاری به مطالعه بررسی نقش عدم‌قطعیت‌های اقتصادی در فرآیند برنامه‌ریزی تولید پروژه معدنی پرداخته‌اند اما بیشتر تحقیق‌های انجام شده در ذخایر تک عنصره بوده و کمتر به بررسی نقش عدم‌قطعیت قیمت در ذخایر دو عنصره توجه شده است. در این تحقیق به منظور لحاظ کردن همزمان عدم‌قطعیت قیمت دو عنصر در طراحی معادن، مدل درخت سه بعدی ارائه شده است. برای اعتبارسنجی مدل پیشنهادی یک مثال عددی با روش‌های جریان‌نقدی تنزیل یافته، درخت دوجمله‌ای، درخت هرمی و مدل سه بعدی حل شد و نتایج حاصل از همه روش‌ها با نتایج حاصل از داده‌های واقعی مقایسه گردید. نتایج تحقیق حاضر نشان می‌دهد، مقادیر حاصل از مدل پیشنهادی (ارزش خالص فعلی برابر با 324/2 هزار دلار)، نسبت به نتایج روش‌های دیگر از دقت بیشتری برخوردار بوده و فقط 8٪ با واقعیت فاصله دارد. این در حالیست که نتایج حاصل از روش‌های دیگر حداقل 17 و حداکثر 39 درصد با نتایج حاصل از داده‌های واقعی اختلاف دارد.