



Objective Functions for the Optimization of an Ultra Wideband Antenna

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ABSTRACT

This work proposes an objective function to optimize an ultra wideband antenna for adjusting the bandwidth and coupling with other elements, based on the performance comparison of several objective functions from the literature. The optimal dimensions of a printed rectangular monopole antenna were obtained with the Particle Swarm Optimization method to compare such functions. In the results of the comparison, the linear functions had a mean value of S_{11} magnitude near the threshold, but they presented a smaller standard deviation than the rest of the functions. The logarithmic and cubic functions showed a mean value of S_{11} magnitude higher than the double of the threshold, but they had superior standard deviation values, which did not happen with the quadratic function. Hence, the proposed function is the mean of a logarithmic expression with the quadratic argument. With this function, a bandwidth adjustment of 130%, a mean S_{11} magnitude of -22.1 dB and a standard deviation equal to 6.7 dB were obtained on the resonant band for the designed antenna. In this way, the proposed function can be used to avoid interference with other wireless systems and to obtain a uniform coupling of the antenna.

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1. INTRODUCTION

The Ultra Wideband (UWB) antennas have achieved relevance in current wireless communication systems because numerous devices need an antenna to operate at different frequencies for several applications. UWB antennas can substitute multiple narrow band ones, reducing the number of antennas, consequently decreasing costs and power consumption [1-2].

However, the design of UWB antennas presents challenges as spatial limitations, interference, and the gain performance for multiple wireless applications [3]. Moreover, these antennas can turn up physical phenomena as resonance and coupling between components because they have sizes similar to the wavelength [4].

The mentioned challenges can be resolved with the optimization in the performance of UWB antennas to improve the results according to the specific requirement of the systems [5]. Many optimization methods have been implemented in the different shapes of UWB antennas, such as Particle Swarm Optimization (PSO),

Genetic Algorithm (GA), Surrogate Based Optimization (SBO) and Fractional Factorial Design (FFD) [6].

These optimization methods have been used because the antennas have a complex model that makes it difficult to apply traditional mathematical methods of optimization. Likewise, objective functions are used as surrogate models for improving the parameters selected by the designers. The most used optimization parameter in antennas is the resonant bandwidth [7], the objective functions mainly depend on S_{11} magnitude and many of them are nonlinear [8-14]. Nevertheless, there is not a comparison between the performance of the objective functions and how these adjust the antenna dimensions for improving its fundamental parameters.

In this work, the performance of several objective functions, which have been employed in the optimization of an UWB antenna, are compared for matching the responses in the adjustment of bandwidth and S_{11} magnitude. In this way, the results support the proposal of an objective function that allows improving such parameters to avoid interference with other wireless systems and to obtain a uniform coupling of the antenna.

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2. METHOD OF OPTIMIZATION

In this work, the PSO method was selected because it has been applied in multiple optimization processes with antennas for reducing the computation time through the prevention of next particle generations equal to previous [15]. The PSO belongs to the family of evolutionary algorithms and it is one of the fastest heuristic approaches for finding the optimal solution of a complex problem [16].

Figure 1 shows the PSO method algorithm, which was programmed in Visual Basic for invoking a Finite Elements Methods (FEM) software, the High-Frequency Structure Simulator (HFSS) from Ansoft. The antenna dimensions are imported to HFSS and the results are exported for each particle. With the results, the objective functions were evaluated and the minimum search mechanism was applied.

Step 1 comprises the initialization of the PSO method parameters, which are: N the number of dimensions, M the number of particles, N_{iter} the number of total iterations, w the inertial weight, c_1 and c_2 cognitive and social parameters, η_1 and η_2 random values between 0 and 1 for each particle and iteration, and V_{max}^n the maximum velocity for each dimension that is calculated with the following equation:

$$V_{max}^n = 0.1(X_{max}^n - X_{min}^n) \quad (1)$$

where X_{max}^n is the maximum position and X_{min}^n is the minimum position of the dimension n .

In step 2, the dimensions of the antenna are modified for the simulation through FEM software, and the objective function is evaluated with the results. Then, the value of the objective function is compared with the best response of the particle ($P_{best,i}^{mn}$) and the swarm (G_{best}) for each iteration i , with the decisions of step 3.

In step 4, the velocity and position are updated. The particle velocity is a real variable that must be between a maximum value (V_{max}^n) and a minimum value ($-V_{max}^n$) [17]. The velocity for the next iteration is determined as follows:

$$V_{i+1}^{mn} = wV_i^{mn} + c_1\eta_1(P_{best,i}^{mn} - X_i^{mn}) + c_2\eta_2(G_{best} - X_i^{mn}) \quad (2)$$

The position must be in a confinement interval, hence it must be verified that its value is in the range $\{Lim_{min}^n, Lim_{max}^n\}$ [18]. The position for the next iteration is calculated as follows:

$$X_{i+1}^{mn} = X_i^{mn} + V_{i+1}^{mn} \quad (3)$$

Finally (in step 5), the convergence criteria are evaluated. These criteria are that the number of iterations is equal to the maximum number of total iterations (N_{iter}), that the difference between the solutions of the best particle is less than a determined value, and that the best value of particle does not change in a consecutive number of iterations (N_{iter}^{cons}). When any of these three conditions is accomplished, the process of optimization ends, and the best position of the swarm is obtained [17].

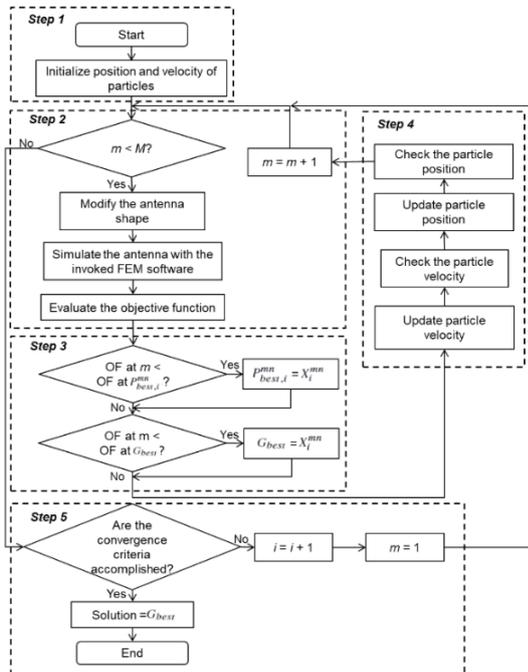


Figure 1. Flow diagram of the PSO method

3. OBJECTIVE FUNCTIONS

There are diverse objective functions to optimize an antenna, most of them search the optimal value of dimensions for improving S_{11} magnitude and bandwidth. In the following, different functions that have been applied in the last five years are described. In this work, linear and nonlinear functions for the optimization of UWB antennas in communication systems with single or multiple bands were selected.

3. 1. Linear Function with the Mean of S_{11} Magnitude

This function calculates the average of a set of values for each frequency component, which are the S_{11} magnitude or the value K , and the equation is [8, 9]:

$$OF1 = \min \mu \{m(f_k)\}$$

$$m(f_k) = \begin{cases} |S_{11}(f_k)| & \text{if } |S_{11}(f_k)| \geq |S_{11}|_{th} \\ K & \text{if } |S_{11}(f_k)| < |S_{11}|_{th} \end{cases} \quad (4)$$

where $\mu\{m(f_k)\}$ is the mean value of $m(f_k)$, $|S_{11}|$ is the S_{11} magnitude for the frequency sample f_k , $|S_{11}|_{th}$ is the threshold of the S_{11} magnitude, and K is equal to -10 dB.

3.2. Linear Function with Bandwidth and Penalty Factor

This function considers the bandwidth plus a discrete variable that depends on the S_{11} magnitude surpassing a threshold value [10]. It is determined as follows:

$$OF2 = \max A + 0.5B$$

$$A = \begin{cases} 1 & \text{if } |S_{11}(f_k)| \leq |S_{11}|_{th} \\ 0 & \text{if } |S_{11}(f_k)| > |S_{11}|_{th} \end{cases} \quad (5)$$

$$B = \begin{cases} \frac{f_M - f_m}{1GHz}, & f_M > f_{max} \quad f_m < f_{min} \\ 0 & \text{otherwise} \end{cases}$$

where B is the bandwidth, A is a discrete value that depends on S_{11} magnitude, f_M is the maximum frequency and f_m is the minimum frequency.

3.3. Nonlinear Function with Sum of Logarithms of the S_{11}

A nonlinear function with the sum of logarithms of the S_{11} magnitude with several logarithm bases was evaluated by Chen [11]. While, obtaining that high base values improved the impedance coupling in each frequency sample and the logarithm with base 8 had the best performance. The equation used is stated as follows:

$$OF3 = \max \sum_{k=1}^N \log_8 (|S_{11}(f_k)|) \quad (6)$$

3.4. Nonlinear Function with Mean of Cubic S_{11} Magnitude

This nonlinear function was implemented to improve the impedance coupling of an UWB antenna [12], and it uses the mean of the cubic S_{11} magnitude of the frequency components in the design range as follows:

$$OF4 = \min \mu \left\{ |S_{11}(f_k)|^3 \right\} \quad (7)$$

3.5. Nonlinear Function with Difference of the Squared Minimum Frequency

This function seeks to minimize the maximum reflection level and to control the minimum resonance frequency [13]. It is expressed as follows:

$$OF5 = \max (l_2, \dots, l_k) + \beta \left[\frac{\max(f_1 - f_{1target}, 0)}{f_{1target}} \right]^2 \quad (8)$$

where $l_k(x) = |S_{11}(f_k)| - |S_{11}|_{th}$, β is the penalty factor equal to 1000, f_1 is the minimum resonant frequency and $f_{1target}$ is the minimum frequency of the design.

4. COMPARISON OF OBJECTIVE FUNCTIONS

The comparison consisted of applying different objective functions to optimize the design of the same antenna and

to analyze the results. In this work, the design optimization problem for UWB antenna aims to obtain an adjustment of bandwidth to avoid interference with other wireless systems, and a smaller value of S_{11} magnitude as uniform as possible for improving the impedance matching in the resonant band of the antenna.

The comparison variables are the mean of S_{11} magnitude, the standard deviation of S_{11} magnitude between samples and the bandwidth adjustment, which was determined with the following equation:

$$B_{adj} = 100 \left(\frac{B}{B_d} \right) (\%) \quad (9)$$

where B is the obtained bandwidth and B_d is the design bandwidth. The bandwidth adjustment must be equal to or higher than 100% and it is considered better when its value is near 100%.

A printed rectangular monopole antenna (PRMA) with microstrip feeding method was selected because this has been the most used in applications of UWB wireless communication systems. The microstrip permits to obtain a high level of adaptability with devices that work in the microwaves frequency range. The selected substrate was the FR4-epoxy given its high level of relative permittivity that contributes to reduce the antenna's dimensions [6].

The characteristics of the substrate are: thickness $h = 1.6$ mm, relative permittivity $\epsilon_r = 4.4$ and tangent of permittivity loss $\tan(\delta) = 0.02$. The frequency range of the design was from 1.7 GHz to 3.7 GHz ($|S_{11}| \leq |S_{11}|_{th}$), which represents a design bandwidth of 2 GHz considering $|S_{11}|_{th}$ equal to -10 dB.

Before implementing the optimization method with the different objective functions, the PRMA dimensions (Figure 2) were calculated, as they determined the central values of the confinement interval of the optimization. For calculating the resonant patch dimensions, the following equation was used [19]:

$$2\pi W + L = 2\pi \left(\frac{7.2}{1.15f_L} + p \right) \text{ (cm)} \quad (10)$$

where L is the length of the patch, W is the width of the patch, p is the separation between the resonant plane and ground plane and f_L is the minimum frequency of the design in GHz.

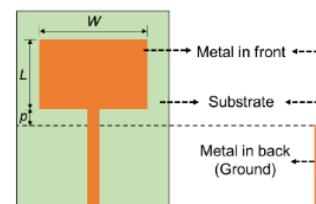


Figure 2. Geometry of the UWB PRMA

In this manner, the dimensions calculated for resonant patch were: $L = 22.06$ mm, $a = 33$ mm and $p = 2$ mm. The transmission line of the microstrip had a length of a quarter of the maximum wavelength and a width of 3 mm. With these values, the calculated antenna was simulated, obtaining a mean of S_{11} magnitude in the design range equal to -10.7 dB, a minimum frequency of 1.7 GHz and a bandwidth of 1.1 GHz, which does not comply with the design premise of the bandwidth.

Then, the setting of the PSO algorithm applied for each objective function with the following parameters was: $N = 3$ dimensions (L , W , and p), $M = 10$ particles, $N_{iter} = 120$ iterations, $N_{iter}^{cons} = 40$ iterations, w varying between 0.9 and 0.4 during the optimization process, c_1 varying from 2.8 to 2, and c_2 from 1.2 to 2. Besides, the boundary conditions were $L = 22.06 \pm 20$ mm, $W = 33 \pm 20$ mm and $p = 2 \pm 2$ mm. For the simulation, the frequency was from 1 GHz to 4.5 GHz, and the frequency step between samples was 100 MHz. In addition, 50 realizations were made for each objective function.

Table 1 shows the obtained values of the mean and standard deviation of each comparison variable for each objective function in the design range. The bandwidth adjustment was over 100% for all objective functions, achieving the design premise. With *OF1*, a low value of bandwidth adjustment was obtained but with a high standard deviation value. This function had a high value of S_{11} magnitude mean near the threshold. The *OF3* had the best performance with 127.5% followed by *OF4* with 134% but both had high deviation. In contrast, *OF5* had an adjustment of 136% with a low deviation.

The highest bandwidth adjustment, equal to 150%, was obtained with *OF2*, and it presented the highest mean of S_{11} magnitude among all functions. Therefore, this function is not recommended in the cases when it is necessary to adjust the bandwidth and to obtain a high level of coupling.

On the other hand, the nonlinear functions had a better performance regarding the mean of S_{11} magnitude, because lower values were obtained near the double of the threshold. The lowest value was achieved by *OF3*, with -22.6 dB.

Concerning the standard deviation of S_{11} magnitude, the linear functions presented low values of this variable, and the *OF2* had the lowest value. The quadratic function

was the second with lower deviation, but its mean of S_{11} magnitude was higher than other nonlinear functions, which exhibited better performance.

In general, to accomplish bandwidth adjustment it is necessary to implement linear objective functions that consider the sum or mean of S_{11} magnitude in the design range. Additionally, logarithmic functions can be used to improve this adjustment. The quadratic function had a good performance in the S_{11} magnitude and low standard deviation, which can be employed to improve the coupling performance.

5. PROPOSAL OF OBJECTIVE FUNCTION

According to the results obtained in section 4, a novel objective function was proposed. This is a nonlinear function that calculates the mean of the base 8 logarithm of the squared difference between S_{11} magnitude and its threshold when the specified criterion is accomplished, as presented in the following equation:

$$OF6 = \max \mu \left\{ C_k \cdot \log_8 \left(\left(|S_{11}(f_k)| - |S_{11}|_{th} \right)^2 \right) \right\} \quad (11)$$

$$C_k = \begin{cases} 1 & \text{if } |S_{11}(f_k)| \leq |S_{11}|_{th} \\ 0 & \text{otherwise} \end{cases}$$

The purpose of this function is to maximize the performance of the UWB antenna, considering the Equations (4), (6) and (8) for a good bandwidth adjustment, a good coupling and a low standard deviation of S_{11} magnitude, in the design bandwidth.

The UWB antenna was optimized using the PSO method with the proposed function for 50 realizations as it was made with the rest of the functions, and the results are shown in Table 2. Low values of standard deviation were obtained for each one of the comparison variables.

Figure 3 shows the box and whisker plot for the bandwidth adjustment of each objective function with its median and mean, where the proposed function has a central distribution with similar values of mean and median, and with few variations in the results obtained. This performance is also presented by the *OF2* but with a higher mean of bandwidth adjustment.

The proposed function has the second best mean value of adjustment after the *OF3*, which showed the lowest mean value but with high deviation. Regarding the mean of S_{11} magnitude for each objective function, the proposed function presented closer mean and median values in comparison with the rest of the functions, obtaining a central distribution as it is shown in Figure 4.

TABLE 1. Results of the UWB antenna optimization with the objective functions

Function	$B_{adj}(\%)$	$ S_{11} _{mean}(\text{dB})$	$ S_{11} _{std}(\text{dB})$
<i>OF1</i>	133.5±21.9	-16.8±2.6	4.9±2.4
<i>OF2</i>	150.0±0.0	-16.6±1.5	4.3±1.8
<i>OF3</i>	127.5±8.6	-22.6±0.6	8.7±1.9
<i>OF4</i>	134.0±10.8	-22.5±1.0	13.1±1.5
<i>OF5</i>	136.0±3.2	-19.8±1.4	4.9±2.6

TABLE 2. Results of the UWB antenna optimization with the proposed objective function

Function	$B_{adj}(\%)$	$ S_{11} _{mean}(\text{dB})$	$ S_{11} _{std}(\text{dB})$
<i>OF6</i>	130.0±3.3	-22.1±0.6	6.7±1.0

Furthermore, this function showed a behavior similar to the other nonlinear functions in this variable, i.e. a mean of S_{11} magnitude lower than the double of the threshold. However, it obtained the smallest standard deviation value of all functions, which implies the optimal responses are very close to each other even though they have different initial points.

Concerning the standard deviation of S_{11} magnitude in the design range, the statistical results obtained are shown in Figure 5. The $OF3$ had higher median and mean values than $OF1$ and $OF2$, $OF5$, and the proposed function ($OF6$).

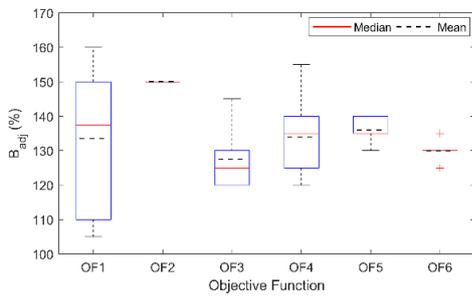


Figure 3. Box and whisker plot for the bandwidth adjustment of each objective function, indicating its median and mean

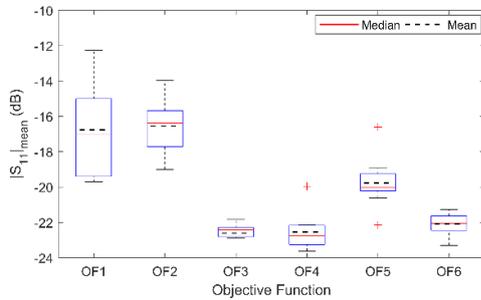


Figure 4. Box and whisker plot for the mean of S_{11} magnitude of each objective function, indicating its median and mean

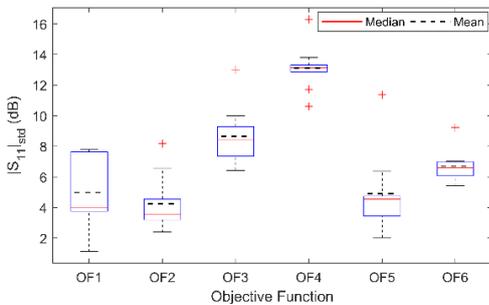


Figure 5. Box and whisker plot for the standard deviation of the S_{11} magnitude of each objective function, indicating its median and mean

This is a disadvantage of the $OF3$, which had the best response in the other comparison variables. Between the nonlinear objective functions, the proposed function had the second best performance, after the $OF5$, but it obtained a lower variation between the results of the realizations. With the function proposed in this research, a more uniform coupling of the antenna in different frequency components in the design range was obtained.

Figure 6 shows the mean of S_{11} magnitude as a function of the standard deviation of S_{11} magnitude resulting from optimization with all the objective functions and for the antenna with the calculated dimensions, which permits to affirm the performance of the proposed objective function is between the best alternatives to minimize both variables.

Moreover, Figure 7 shows the results of optimization with all the objective functions and the initially calculated antenna through the relationship of two comparison variables, the mean of S_{11} magnitude and the bandwidth adjustment. The proposed function had a good adjustment, which permits to avoid interference of other communication systems that are out of the bands.

In summary, with the proposed objective function a good performance of the UWB antenna according to the bandwidth adjustment and mean of S_{11} magnitude can be obtained. With the first one, a better adjustment than most of the objective functions was obtained. While with the second one, better uniformity of the frequency components in the design range was achieved.

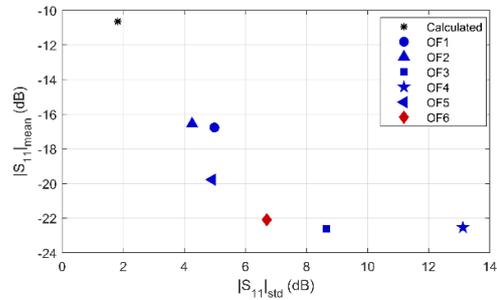


Figure 6. Mean of S_{11} magnitude as a function of the standard deviation of S_{11} magnitude for objective functions and calculated antenna

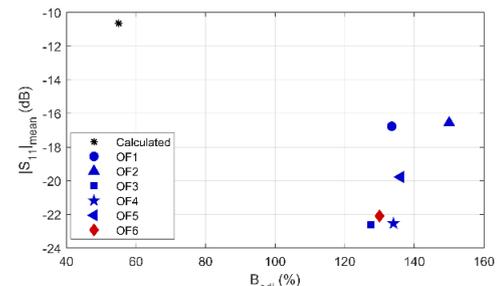


Figure 7. Mean of S_{11} magnitude as a function of bandwidth adjustment for objective functions and calculated antenna

6. CONCLUSIONS

In this work, linear and nonlinear objective functions that have been used in the process of optimization of UWB antennas were compared. The solutions from the linear objective functions had high levels of the mean of S_{11} magnitude that were near the threshold, while the nonlinear functions solutions had levels around the double of the threshold. Therefore, the last ones permit to enhance the coupling of the antenna. Concerning the standard deviation of S_{11} magnitude and the bandwidth adjustment, there was no clear trend of these variables for the different objective functions.

An objective function for the optimization of UWB antennas was proposed with basis on the relevant results of the comparison of diverse parameters. This function improves the performance of the optimized antenna according to the mean and standard deviation of S_{11} magnitude and it has a good bandwidth adjustment. It means that this is a trade-off solution of the three variables. In consequence, the proposed function can be used to avoid interference with other wireless communication systems, whose reception is not necessary, and to obtain a more uniform coupling of the antenna in the frequency components in the whole design range.

It is important to consider that these results have been obtained under a particular set of conditions, which have been described in section 4. Results could be different for PSO with a different number of particles, different dimensions, or another optimization method. Future work can be conducted for the evaluation of different configurations and methods, as well as the multi-objective formulation of the design problem, considering the bandwidth and the S_{11} magnitude simultaneously.

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Persian Abstract

چکیده

این کار یک عملکرد هدف را برای بهینه سازی آنتن فوق باند پهن برای تنظیم پهنای باند و اتصال با سایر عناصر ، بر اساس مقایسه عملکرد چندین توابع هدف از منابع ، پیشنهاد می کند. ابعاد بهینه یک آنتن تک قطبی مستطیلی چاپ شده با استفاده از روش بهینه سازی ازدحام ذرات برای مقایسه چندین توابع به دست آمد. در نتایج مقایسه ، توابع خطی دارای مقدار میانگین S11 در نزدیکی آستانه بودند ، اما انحراف استاندارد کمتری نسبت به بقیه توابع ارائه دادند. توابع لگاریتمی و مکعبی مقدار میانگین S11 بزرگتر از دو برابر آستانه را نشان دادند ، اما آنها دارای مقادیر انحراف استاندارد برتر بودند ، که با تابع درجه دوم اتفاق نیفتاد. از این رو ، تابع پیشنهادی میانگین بیان لگاریتمی با استدلال درجه دوم است. با استفاده از این عملکرد ، یک پهنای باند 130٪ ، یک میانگین اندازه S11-22.1 دسی بل و یک انحراف معادل 6.7 دسی بل بر روی باند تشدید برای آنتن طراحی شده بدست آمد. به این ترتیب می توان از عملکرد پیشنهادی برای جلوگیری از تداخل در سیستم های بی سیم دیگر و به دست آوردن یک اتصال یکنواخت آنتن استفاده کرد.
