



## The Effects of Newmark Method Parameters on Errors in Dynamic Extended Finite Element Method Using Response Surface Method

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### ABSTRACT

The Newmark method is an effective method for numerical time integration in dynamic problems. The results of Newmark method are function of its parameters ( $\beta$ ,  $\gamma$  and  $\Delta t$ ). In this paper, a stationary mode I dynamic crack problem is coded in extended finite element method (XFEM) framework in Matlab software and results are verified with analytical solution. This paper focuses on effects of main parameters in Newmark method for dynamic XFEM problems. Also use of the response surface method (RSM) a regression model is presented for estimating error of dynamic stress intensity factors (DSIF) with high validity according to results of analysis of variance (ANOVA). This work enables one to understand the effect of Newmark parameters on error of DSIFs and to find optimum  $\beta$  and  $\gamma$  for a determined number of time steps ( $N$ ). This procedure is highly effective in order to manage the computational cost and enhance the accuracy at the desired domain. The effect of the considered parameters on error, is investigated using RSM in Minitab software and optimum state for minimization of errors is illustrated.

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### NOMENCLATURE

$K_I^{dyn}$	Mode I dynamic stress intensity factor	<b>Greek Symbols</b>	
$K_{II}^{dyn}$	Mode II dynamic stress intensity factor	$\sigma_0$	Applied stress (MPa)
$t_c$	The time when applied stress wave reaches the crack tip for the first time (s)	$\rho$	Density ( $\text{kg/m}^3$ )
$\Delta t$	Time step (s)	$\nu$	Poisson's ratio
$N = 3t_c / \Delta t$	Number of time steps	$\beta$	Parameter of Newmark method
$E1$	Error of dynamic stress intensity factors in a vicinity of time $t_c$ (%)	$\gamma$	Parameter of Newmark method
$E2$	Error of dynamic stress intensity factors in a vicinity of time $3t_c$ (%)	<b>Superscripts</b>	
$C_d$	Dilatational wave speed (m/s)	$aux$	Auxiliary fields
$H(x)$	Heaviside or jump function	$dyn$	Dynamic
$F_I$	Singular or asymptotic enrichment functions	<b>Subscripts</b>	
$E$	Young modulus of elasticity (GPa)	$in$	Interaction

## 1. INTRODUCTION

Dynamic problems consist of a wide range of engineering problems in various fields [1, 2]. Finite element method is one of interesting numerical methods

for solving fracture mechanics problems [3, 4]. Dolbow et al. [5] proposed a method to model crack in framework of finite element method without remeshing and called it extended finite element method. Stolarska et al. [6] introduced coupling of level set method (LSM) and XFEM for crack growth problems. Belytschko et al. [7] used XFEM in dynamic crack propagation. Rethore et al. [8] studied stability during dynamic crack growth.

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Mohammadi [9] developed dynamic XFEM for analysis of composites. It is mandatory to select an accurate time integration algorithm in simulation of a dynamic problem. Time integration methods include the explicit and implicit approaches with different complexity, speed, efficiency and accuracy. Explicit approach is simple and fast, but it is less accurate and suffers from stability conditions. Implicit approaches are more accurate and stable, but they are computationally more expensive [9]. In wave propagation studies, small time steps must be used in order to represent the nature of problem. Therefore, very small time increments must be used, which usually enhances computation costs [10]. Implicit methods need a balance between the accuracy and the computation time [11]. Alamatian [12] introduced implicit multi time step integration on dynamic analyses with constant time step. Alamatian [13] presented an implicit time integration method with higher order accuracy for dynamic problems. Shojaee et al. [14] studied on an implicit time integration method which its advantage is eliminating conditional stability. The results of Newmark method are function of its parameters ( $\beta$ ,  $\gamma$  and  $\Delta t$ ). Stability of results depends on values of  $\beta$ ,  $\gamma$  and their accuracy depends on values of  $\beta$ ,  $\gamma$  and time increment ( $\Delta t$ ). The computation time is only depend on  $\Delta t$ . A huge part of errors in dynamic XFEM problems are errors due to time integration method. Control and management of these errors is an important challenge. Analytical solution of benchmark problems which usually used to verify numerical approaches, are based on semi-infinite or infinite plate assumption. In order to remove the effect of reflected stress wave on the crack tip, the numerical simulation of finite plates is performed until the reflected dilatational wave from another edge of the plate reaches the crack tip. The highest errors of dynamic stress intensity factors occur when:

1. Initial stress wave reaches the crack tip, which highest error in a vicinity of this time is called  $E1$  in the present work.
2. Reflected wave from the end of the plate reaches the crack tip, and most error in a vicinity of this time is called  $E2$ .

In this paper a new look at Newmark method from statistical point of view is demonstrated. The aim of this study is to determine the effects of parameters of Newmark method (i.e.  $\beta$ ,  $\gamma$  and  $N$ ) on the errors  $E1$  and  $E2$ . A dynamic fracture mechanics problem in the framework of XFEM using the response surface methodology (RSM) was studied. In addition, regression models for errors  $E1$  and  $E2$  are presented.

## 2. GOVERNING EQUATION AND FORMULATION

### 2.1. Basics of XFEM Extended finite element

method is based on enrichment of displacement field. In XFEM, finite element mesh is produced regardless of existence and location of discontinuity. Nodes around discontinuity are selected to be enriched; thus, some additional degrees of freedom are added to classical finite element at selected nodes due to enrichment of displacement field. The displacement field for a domain containing discontinuity is expressed as Equation (1) [3, 5]:

$$u^h(x) = \sum_{i \in I} \varphi_i(x) u_i + \sum_{j \in J} H(x) \varphi_j(x) a_j + \sum_{k \in K} \varphi_k(x) \left( \sum_{l=1}^4 b_l^k F_l(x) \right) \quad (1)$$

In which  $I$  is the set of total nodes,  $J$  is the set of red squared nodes and  $K$  is the set of blue circled nodes as shown in Figure 1.  $H(x)$  is Heaviside or jump function and used for nodes of completely cut elements:

$$H(x) = \begin{cases} +1 & \text{above the crack} \\ -1 & \text{below the crack} \end{cases} \quad (2)$$

The functions  $F_l(x)$  ( $l=1,2,3,4$ ) are the crack tip enrichment functions and are defined as:

$$\{F_l(r, \theta)\} = \left\{ \begin{array}{l} \sqrt{r} \sin(\theta/2), \sqrt{r} \cos(\theta/2), \\ \sqrt{r} \sin(\theta/2) \sin(\theta), \sqrt{r} \cos(\theta/2) \sin(\theta) \end{array} \right\} \quad (3)$$

where  $r$  and  $\theta$  are the polar coordinates in the local crack tip coordinate system.

In XFEM implementation two points must be considered:

1. Using Equation (1), at node  $x_i$  will have  $u(x_i) \neq u_i$  which is a contradiction. A simple idea to solve this matter is shifting enrichment functions:

$$u^h(x) = \sum_{i \in I} \varphi_i(x) u_i + \sum_{j \in J} (H(x) - H(x_j)) \varphi_j(x) a_j + \sum_{k \in K} \varphi_k(x) \left( \sum_{l=1}^4 b_l^k (F_l(x) - F_l(x_k)) \right) \quad (4)$$

2. Shared nodes between completely cut elements and crack tip elements must be enriched only by crack tip enrichment functions.

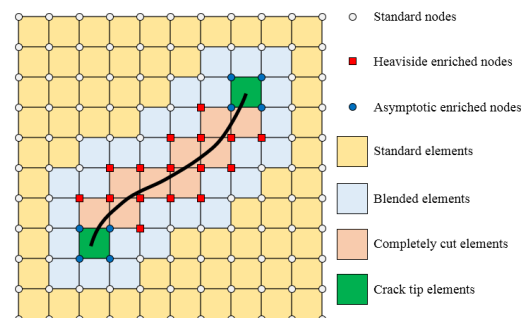


Figure 1. Selection of enriched nodes

**2.2. XFEM Dynamic Equation of Motion** The weak form of momentum equation of the initial/boundary value problem for a body with traction-free crack  $\Gamma_c$  shown in Figure 2 stated as follows:

$$\int_{\Omega} \rho \ddot{u} \delta u d\Omega + \int_{\Omega} \sigma \cdot \delta \varepsilon d\Omega = \int_{\Gamma_f} \bar{t} \delta u d\Gamma \quad (5)$$

The discretized form of Equation (5) in framework of XFEM ignoring damping effects would be given as follows:

$$[M]\{\ddot{u}^h\} + [K]\{u^h\} = \{F\} \quad (6)$$

The mass matrix  $\mathbf{M}$ , stiffness matrix  $\mathbf{K}$ , external load vector  $\mathbf{F}$  and nodal degrees of freedom  $\mathbf{u}$  for an element, respectively are defined as follows:

$$M_{ij}^e = \begin{bmatrix} M_{ij}^{uu} & M_{ij}^{ua} & M_{ij}^{ub} \\ M_{ij}^{au} & M_{ij}^{aa} & M_{ij}^{ab} \\ M_{ij}^{bu} & M_{ij}^{ba} & M_{ij}^{bb} \end{bmatrix} \quad (7)$$

$$K_{ij}^e = \begin{bmatrix} K_{ij}^{uu} & K_{ij}^{ua} & K_{ij}^{ub} \\ K_{ij}^{au} & K_{ij}^{aa} & K_{ij}^{ab} \\ K_{ij}^{bu} & K_{ij}^{ba} & K_{ij}^{bb} \end{bmatrix} \quad (8)$$

$$F_i^e = \{F_i^u, F_i^a, F_i^b\}^T \quad (9)$$

$$u^h = \{u, a, b\}^T \quad (10)$$

The components of mass matrix, stiffness matrix and force vector, respectively are given as follows:

$$M_{ij}^{\alpha\beta} = \int_{\Omega^e} h \rho (N_i^\alpha)^T (N_j^\beta) d\Omega \quad (\alpha, \beta = u, a, b) \quad (11)$$

$$K_{ij}^{\alpha\beta} = \int_{\Omega^e} h (B_i^\alpha)^T C (B_j^\beta) d\Omega \quad (\alpha, \beta = u, a, b) \quad (12)$$

$$F_i^\alpha = \int_{\Gamma} (N_i^\alpha)^T \bar{t} d\Gamma \quad (\alpha = u, a, b) \quad (13)$$

where matrices  $\mathbf{B}$  and  $\mathbf{N}$  for a four node element ( $i=1,2,3,4$ ) and using four asymptotic functions ( $l=1,2,3,4$ ) are stated as Equations (14) to (19).

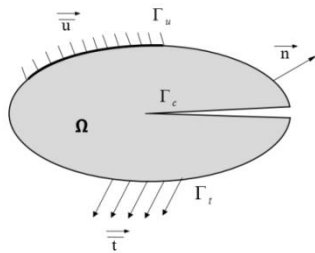


Figure 2. Initial configuration of domain

$$N_i^u = \begin{bmatrix} N_i & 0 \\ 0 & N_i \end{bmatrix} \quad (14)$$

$$N_i^a = \begin{bmatrix} HN_i & 0 \\ 0 & HN_i \end{bmatrix} \quad (15)$$

$$N_i^b = \begin{bmatrix} F_l N_i & 0 \\ 0 & F_l N_i \end{bmatrix} \quad (16)$$

$$B_i^u = \begin{bmatrix} N_{i,x} & 0 \\ 0 & N_{i,y} \\ N_{i,y} & N_{i,x} \end{bmatrix} \quad (17)$$

$$B_i^a = \begin{bmatrix} HN_{i,x} & 0 \\ 0 & HN_{i,y} \\ HN_{i,y} & HN_{i,x} \end{bmatrix} \quad (18)$$

$$B_i^b = \begin{bmatrix} F_{l,x} N_i + F_l N_{i,x} & 0 \\ 0 & F_{l,y} N_i + F_l N_{i,y} \\ F_{l,y} N_i + F_l N_{i,y} & F_{l,x} N_i + F_l N_{i,x} \end{bmatrix} \quad (19)$$

**2.3. Newmark Time Integration Method** After construction of matrices, the Newmark time integration algorithm is used to obtain nodal accelerations, velocities and displacements at each time step, respectively as below[15]:

$$\ddot{U}_n = (M + \beta \Delta t^2 K)^{-1} \left\{ F_n - K \left[ \begin{array}{c} U_{n-1} + \Delta t \dot{U}_{n-1} \\ + \frac{\Delta t^2}{2} (1-2\beta) \ddot{U}_{n-1} \end{array} \right] \right\} \quad (20)$$

$$\dot{U}_n = \dot{U}_{n-1} + (1-\gamma) \Delta t \ddot{U}_{n-1} + \gamma \Delta t \ddot{U}_n \quad (21)$$

$$U_n = U_{n-1} + \Delta t \dot{U}_{n-1} + \frac{\Delta t^2}{2} (1-2\beta) \ddot{U}_{n-1} + \beta \Delta t^2 \ddot{U}_n \quad (22)$$

where,  $\beta$  and  $\gamma$  are constant parameters which accuracy and stability of numerical scheme depends on their values. Some of common numerical schemes with different values of  $\beta$  and  $\gamma$  has been reported in reference [16] as summarized in Table 1:

TABLE 1. Some of numerical schemes

Scheme	$\beta$	$\gamma$
Central explicit	0	0.5
Average acceleration	0.25	0.5
Backward	1	1.5
Linear acceleration	0.1	0.5
Galerkin	0.8	1.5
Fox Goodwin	1/12	0.5
Linear three level difference	0.5	0.5

In XFEM problems, central explicit and average acceleration (implicit mean acceleration) schemes were used [10, 17].

**2.4. Stress Intensity Factors Computation**

Interaction integral is derived via a two-field problem consisting of state 1, the actual field and state 2, the auxiliary field:

$$J^{(1+2)} = J^{(1)} + J^{(2)} + I^{(1,2)} \tag{23}$$

A simple to implementation way for calculation of dynamic stress intensity factors is the domain-independent form of interaction integral presented by Rethore [8]:

$$I_{in}^{dyn} = - \int_A q_{k,j} [(\sigma_{mj}^{aux} u_{m,l} - \rho \dot{u}_i u_i^{aux}) \delta_{ij} - (\sigma_{ij}^{aux} u_{i,k} + \sigma_{ij} u_{i,k}^{aux})] dA + \int_A q_k [(\sigma_{ij,j}^{aux} u_{i,k} + \rho \ddot{u}_i u_{i,k}^{aux}) + (\rho \dot{u}_i^{aux} \dot{u}_{i,k} + \rho \dot{u}_i u_{i,k}^{aux})] dA \tag{24}$$

The virtual extension field  $q$  is a smooth function tangent to the crack faces, varying from  $q=0$  at the exterior boundary  $\Gamma$  at Figure 3 to  $q=1$  near the crack tip.

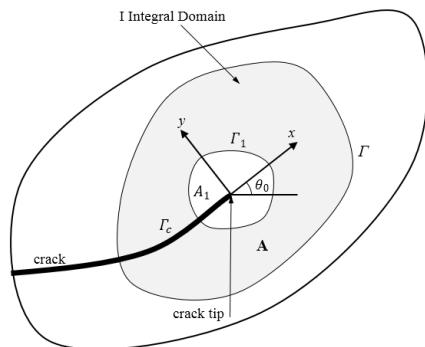
The interaction integral is related to dynamic stress intensity factors in a plane strain condition as follows:

$$I^{in} = \frac{2(1-\nu^2)}{E} (K_I^{dyn} K_I^{aux} + K_{II}^{dyn} K_{II}^{aux}) \tag{25}$$

$K_I^{dyn}$  is evaluated by defining the auxiliary state as the pure mode I ( $K_I^{aux} = 1$  and  $K_{II}^{aux} = 0$ ) and in a similar way,  $K_{II}^{dyn}$  can be obtained.

**3. DESIGN OF EXPERIMENTS (DOE)**

In order to optimize response variables in presence of various factors, design of experiments (DOE) was used. The most important objective in the DOE is to achieve the desired response with the lowest number of experiments [18].



**Figure 3.** The contour  $\Gamma$  and its interior area  $A$

In this study, for optimization of process, investigation the regression model and finding the effect of variable parameters on the output, response surface method (RSM) was used.

The RSM regression model is generally quadratic full equation or reduced form of it is stated as Equation (26) [19]:

$$y = \alpha_0 + \sum_{i=1}^n \alpha_i x_i + \sum_{i=1}^n \alpha_2 x_i^2 + \sum_i \sum_j \alpha_3 x_i x_j + \varepsilon \tag{26}$$

In Equation (26),  $y$  is the response variable,  $\alpha_0$ ,  $\alpha_1$ ,  $\alpha_2$  and  $\alpha_3$  are constant, linear, quadratic and interaction coefficients, respectively. Also  $x_i$  and  $x_j$  are the independent variables and  $\varepsilon$  is the statistical error.

After determining the regression model, efficiency of the model is checked using  $R^2$  as Equation (27) that can be obtained from analysis of variance (ANOVA) [20].

$$R^2 = 1 - \frac{SS_r}{SS_T} \tag{27}$$

where,  $SS_r$  and  $SS_T$  are the residual and total sum of squares, respectively.

In the present study,  $\beta$ ,  $\gamma$  and  $N$  are considered as variable parameters and the effects of these parameters on the  $EI$  and  $E2$  are investigated using RSM. For selecting the range of parameters, range of time steps should be determined at first in order to manage computational time, then by a few trial and error at upper and lower bound of  $N$ , range of  $\beta$  and  $\gamma$  could be determined so that the stability and error would be rational. For this purpose, by selecting the range of parameters, the design of experiments was performed according to the central composite design (CCD) with 20 experiments using Minitab software as summarized in Table 2.

**4. RESULTS AND DISCUSSION**

The example considered for verifying codes is a finite plate with a stationary crack subject to a dynamic loading as shown in Figure 4. This example is a pure mode I problem ( $K_{II}^{dyn} = 0$ ). Analytical solution of this problem is presented by Freund as stated in Equation (28) [21]:

$$K_I(t) = \begin{cases} 0 & \text{if } t < t_c \\ \frac{2\sigma_0}{1-\nu} \sqrt{\frac{C_d(1-2\nu)(t-t_c)}{\pi}} & \text{if } t \geq t_c \end{cases} \tag{28}$$

where  $t_c = h/C_d$  is the time when stress wave reaches the crack tip for the first time. This analytical solution is valid until the reflected stress wave reaches the crack tip

( $t \leq 3t_c = 3h/C_d$ ) since the assumption of the analytical solution is infinite plate.

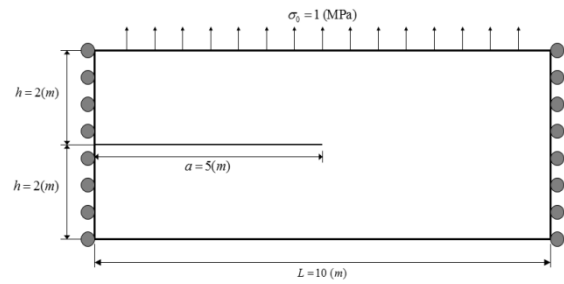
**TABLE 2.** The design of experiments according to RSM

Run	Beta	Gamma	N
1	0.8549	0.5452	181
2	0.6295	0.5102	300
3	0.4041	0.4752	181
4	0.4041	0.4752	419
5	0.6295	0.5102	300
6	0.6295	0.5700	300
7	0.6295	0.5102	300
8	1.0000	0.5102	300
9	0.8549	0.4752	181
10	0.6295	0.4500	300
11	0.6295	0.5102	300
12	0.4041	0.5452	419
13	0.6295	0.5102	500
14	0.4041	0.5452	181
15	0.6295	0.5102	100
16	0.6295	0.5102	300
17	0.8549	0.5452	419
18	0.6295	0.5102	300
19	0.2500	0.5102	300
20	0.8549	0.4752	419

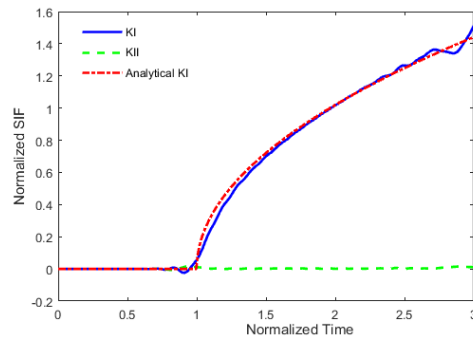
The dimensions are:  $L=10(m)$ ,  $a=5(m)$ ,  $h=2(m)$  and the material properties are the follows: Young's modulus  $E = 211(GPa)$ , poisson's ratio  $\nu = 0.3$  and density  $\rho = 7800 (kg/m^3)$ . The tensile stress applied on the top surface is  $\sigma = 1(MPa)$ . A  $40 \times 100$  regular quadrilateral meshes was used. The time integration scheme used was implicit mean acceleration scheme with 250 time steps. The dynamic stress intensity factors obtained from code has a good agreement with analytical solution as shown in Figure 5. Percentage error is also given in Figure 6. The stress intensity factors are normalized by  $\sigma_0 \sqrt{h}$  and time is normalized by  $t_c$ . According to Figure 6, the most errors are at the vicinity of  $t_{Normalized} = 1$  when stress wave reaches the crack tip for the first time, and the vicinity of  $t_{Normalized} = 3$  when the reflected stress wave reaches the crack tip [8].

The results of twenty different experiments were obtained as presented in Table 3 using verified codes.

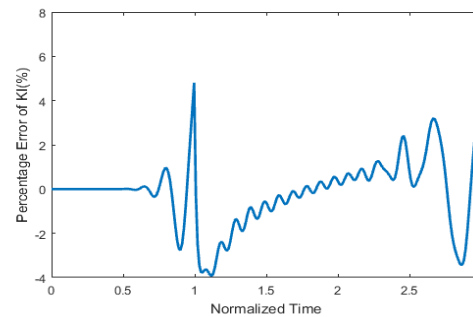
The result of normality test using SPSS software according to Kolmogorov-Smirnov method showed that P-values were equal to 0.067 and 0.293 (larger than statistical error i.e. 0.05) for  $E1$  and  $E2$ , respectively. Therefore, the obtained results were followed normal distribution [22].



**Figure 4.** Geometry and loading for the dynamic problem



**Figure 5.** Comparison between normalized DSIF obtained from code and normalized analytical solution



**Figure 6.** Percentage error of  $K_I$  using 250 time steps,  $\beta = 0.25$  and  $\gamma = 0.5$ , are  $E1 = 5\%$  and  $E2 = 6.5\%$

The regression models for  $E1$  and  $E2$  were obtained using Minitab software; as expressed in Equations (29) and (30) respectively.

$$E1 = 1.6 + 11.4\beta + 27.1\gamma - 0.0272N - 2.59\beta^2 - 27.0\gamma^2 + 0.000035N^2 - 9.1\beta\gamma - 0.00844\beta N + 0.0078\gamma N \quad (29)$$

$$E2 = 74.4 - 25.2\beta - 264\gamma + 0.0421N - 3.51\beta^2 + 243\gamma^2 - 0.000024N^2 + 45.2\beta\gamma + 0.01322\beta N - 0.0544\gamma N \quad (30)$$

The results of ANOVA are summarized in Table 4 for  $E1$  and  $E2$ .

**TABLE 3.** The results of experiments

Run	Error 1 (%)	Error 2 (%)
1	7.2685	3.1827
2	6.1386	5.0746
3	6.5216	4.5710
4	5.7148	6.3785
5	6.1386	5.0746
6	5.9644	5.3861
7	6.1386	5.0746
8	6.0876	4.5335
9	7.8360	1.4744
10	6.4355	7.4068
11	6.1386	5.0746
12	5.5628	5.7556
13	5.4536	5.8417
14	6.4558	4.6427
15	9.9051	3.3061
16	6.1386	5.0746
17	5.6856	5.5022
18	6.1386	5.0746
19	5.7559	5.5672
20	5.9072	4.9119

**TABLE 4.** The results of ANOVA

Source	Degree of freedom	P-value	
		E1	E2
Model	9	0.001	0.011
Linear	3	0.000	0.002
Square	3	0.008	0.137
Interaction	3	0.514	0.275

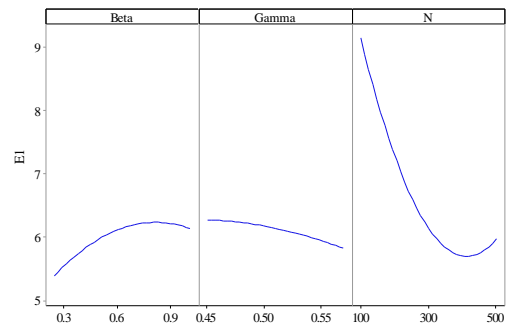
The P values less than 0.05, indicates that the desired parameters are effective [23]. It was observed that the linear and square terms were effective in the regression model of  $E1$  while the square term was not effective in model of  $E2$ . Also the results indicated that obtained regression models have high efficiency for estimating error.

The main effect of parameters on  $E1$  was obtained as illustrated in Figure 7. The results revealed that the  $E1$  changed significantly with the change in the value of  $N$ . By increasing  $N$ , the  $E1$  decreases markedly and the optimum value is in the range of 380-410 of  $N$  for minimum error. Also, the results illustrated that by increasing gamma, the  $E1$  slightly decreases. As a

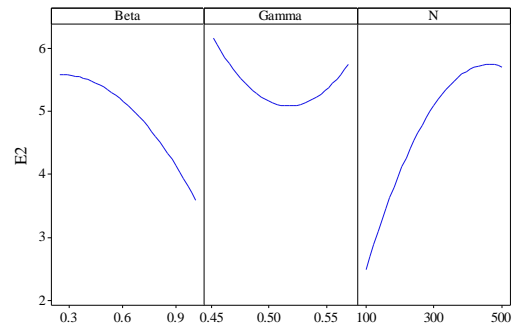
matter of fact, 0.57 is the best value for gamma for minimization of  $E1$  in the considered range. Another result that could be obtained from Figure 7 is this fact that the optimum level for beta is 0.25 at which the minimum value of  $E1$  was achieved.

Also, the main effect of parameters on  $E2$  was achieved as Figure 8. The results indicated that in contrary to  $E1$ , the  $E2$  decreases as beta increases. The optimum value for gamma is 0.52 for minimization of  $E2$ . Also, the results demonstrated that 100 is the best value for  $N$  at which  $E2$  is minimum.

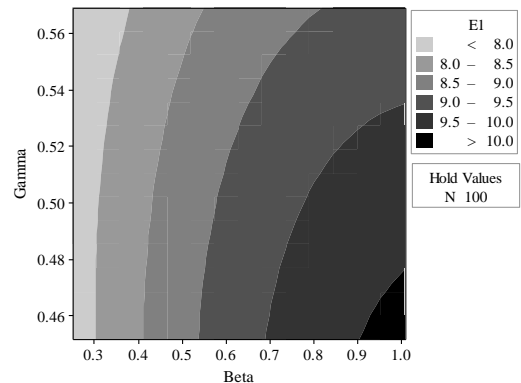
The interaction effect of beta and gamma on  $E1$  is shown in Figure 9.



**Figure 7.** The main effect of parameters on  $E1$

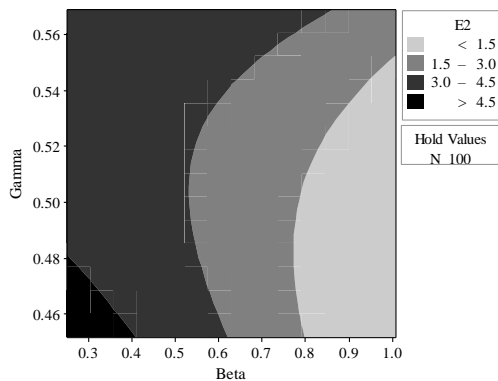


**Figure 8.** The main effect of parameters on  $E2$



**Figure 9.** Interaction effect of  $\beta * \gamma$  on  $E1$  when  $N=100$

The results demonstrate that in lowest values of beta, the  $E1$  is in optimum state and is smaller than 8% for all values of gamma. Figure 10 shows the interaction effect of beta and gamma on  $E2$ . Figure 10 reveals that in the highest values of beta ( $\beta > 0.8$ ), the  $E2$  is minimum almost for all values of gamma. Also, the results indicated that in a constant value of gamma, the  $E2$  decreases as beta increases.



**Figure 10.** Interaction effect of  $\beta \cdot \gamma$  on  $E2$  when  $N=100$

## 5. CONCLUDING REMARKS

In this study, the XFEM was implemented to model a dynamic problem with a stationary crack. Obtained dynamic stress intensity factors was verified with analytical solution. Also, the effects of parameters  $\beta$ ,  $\gamma$  and  $\Delta t$  of the Newmark time integration method on the errors at two critical times was demonstrated. These errors are called  $E1$  and  $E2$  in this paper. The regression models were introduced for errors  $E1$  and  $E2$ , which according to analysis of variance (ANOVA) results have high validity. The results showed that  $E1$  has an inverse relation with  $\gamma$  and  $N$  and direct relation with  $\beta$ , while  $E2$  has an inverse relation with  $\beta$  and the direct relation with  $N$ . Until  $\gamma = 0.52$  the relation between  $\gamma$  and  $E2$  is inverse and for greater values of  $\gamma$ , this relation is direct.

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## The Effects of Newmark Method Parameters on Errors in Dynamic Extended Finite Element Method Using Response Surface Method

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روش نیومارک روش مؤثری برای انتگرال‌گیری زمانی عددی در مسائل دینامیکی است. نتایج روش نیومارک تابعی از پارامترهای آن هستند ( $\Delta t$  و  $\gamma$ ,  $\beta$ ). در این تحقیق یک مسئله‌ی ترک دینامیکی ایستای مود I در قالب XFEM و در نرم افزار Matlab کدنویسی شده است و نتایج با حل تحلیلی صحت سنجی شده‌اند. تحقیق حاضر روی اثرات پارامترهای روش نیومارک در مسائل المان محدود توسعه یافته‌ی دینامیکی (DXFEM) تمرکز دارد. همچنین با استفاده از روش سطح پاسخ (RSM) مدل رگرسیون برای تخمین خطای ضرایب شدت تنش دینامیکی (DSIF) با دقت بالایی مطابق نتایج روش آنالیز واریانس (ANOVA) ارائه شده است. تحقیق ارائه شده محقق را قادر می‌سازد که تأثیر پارامترهای نیومارک را روی خطای ضرایب شدت تنش دینامیکی دریابد و  $\beta$  و  $\gamma$  بهینه را برای تعداد گام‌های زمانی معین بیابد. این رویه به منظور مدیریت هزینه‌ی محاسبات و افزایش دقت در ناحیه‌ی دلخواه بسیار مؤثر است. تأثیر پارامترهای در نظر گرفته شده روی خطا، با استفاده از روش RSM در نرم افزار Minitab بررسی شد و حالت بهینه برای کمینه کردن خطا نشان داده شد.

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