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TECHNICAL NOTE

Proposed Procedure for Estimating the Coefficient of Three-factor Interaction for $2^p 3^m 4^q$ Factorial Experiments

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ABSTRACT

Three-factor interaction for the two-level, three-level, and four-level factorial designs was studied. A new technique and formula based on the coefficients of orthogonal polynomial contrast were proposed to calculate the effect of the three-factor interaction The results show that the proposed technique was in agreement with the least squares method. The advantages of the new technique are 1) it is fixed, 2) it is simple and 3) it is easy to apply without the complicated matrix formula of the least squares method. This new technique will also enhance the use of the coefficients of orthogonal contrast when analyzing other experimental designs.

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1. INTRODUCTION

Factorial design is widely applied in many fields because it is economical and efficient. It allows researchers to investigate the effects of several factors simultaneously. Moreover, the joint effects of factors on selected responses can be determined. Researchers in engineering and science fields have performed various experiments using this method, e.g., ferulic acid production by co-culture by Kamaliah & Norazwina [1] and response surface methodology for optimization experiments by Singh et al. [2], Kavardi et al. [3], Yahyaei et al. [4], Moradi et al. [5] and Maluta et al. [6].

The factorial design was proposed in the 1920s by Fisher [7] and became popular among researchers. In 1937, Yates suggested a method for analyzing two-level factorial designs [8], while Davies developed a procedure to fit a second-order response model to three-level factorial designs [9]. Some researchers suggested new methods to analyze different experiments while some researchers fitted response surface models to various experiments. For instance, Margolin [10] developed a procedure to analyze and fit a response surface model to mixed-factorial designs for the two-level and three-level designs. Alkarkhi & Low [11] presented a procedure to analyze mixed experiments comprising of the two-level and three-level factorial designs by using the coefficients of the orthogonal polynomial contrast. Alqraghuli et al. [12] suggested a new method for analyzing four-level factorial designs and continued to introduce a new procedure to analyze mixed two-level and four-level factorial designs [13]. Later, Alqraghuli et al. [14] proposed a new procedure to analyze experiments of three-level and four-level designs.

The polynomial model is typically used to summarize the results gathered from experiments on mathematical models. It can similarly be used to understand the process behaviors of these models and the effects of various factors on them. Researchers applied the least squares method and matrix techniques to utilize polynomial models in factorial designs [15]. Subsequently, researchers developed a simpler but

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accurate method by using the coefficients of the orthogonal polynomial.

The new procedures presented suggest an easy and simple method to avoid the difficulties and complication of using the least squares method, especially if more than two factors are involved, which would require the use of statistical software to fit the response surface model. The coefficients of orthogonal contrasts, to date, have never been used for analyzing mixed experiments of type two-level, three-level, and four-level experiments. Therefore, the objective of this work is to propose a new procedure for analyzing and fitting response surface models to mixed experiments of two-level, three-level and four-level factorial experiments and estimating the coefficient of the three-factor interaction.

2. $2^p 3^m 4^q$ FACTORIAL DESIGN

The mixed experiment of type two-level, three-level and four-level are denoted by $2^p 3^m 4^q$, p factors each at two levels $X_1, \dots, X_p$, m factors each at three levels $Z_{1,} \dots Z_m$ and q factors each at four levels $R_1, \dots, R_s$. The simplest design for two-level, three-level and four-level, factorial design has three factors, one at two levels one at three levels and one at four levels $2^1 3^1 4^1$. The total number of runs required for $2^1 3^1 4^1$ is 24 runs for one replicate [16].

3. PROPOSED PROCEDURE

Many researchers have studied different factorial designs to develop formulas or by offering new techniques. Experiments of type $2^p 3^m 4^q$ have not been considered using the coefficients of orthogonal polynomial contrasts. Thus, this work will propose a new procedure for analyzing and fitting response surface models to this type of experiments and introduce a new formula for estimating the coefficient of the three-factor interaction.

Consider three types of factors, the first type is of two-level $(X_1, \dots, X_p)$, the second type is of three-level $(Z_{1,} \dots Z_m)$ and the last type is of four-level $(R_1 \dots, R_q)$. Thus, the design that considers all three types of factors is called experiment of type $2^p 3^m 4^q$. The proposed procedure splits the experiment into three experiments, one of type 2^p, the second is of type 3^m and the third is of type 4^q, then analyzing each experiment separately.

The coefficients of the linear effect and two-factor interaction in the response surface models are estimated using the formulas for analyzing experiments of type 2^p, experiment of type 3^m, experiment of type 4^q [14], experiments of type $2^p 3^m$ [13], experiments of type $2^p 4^q$ [10], and experiment of type $3^m 4^q$ [13], which are based on the coefficients of orthogonal polynomial contrast. We need to propose a formula for estimating the coefficient of the three-factor interaction to cover all coefficients in the model. The recommended procedure for calculating the coefficients of the three-factor interaction depends on the coefficients of orthogonal polynomial contrasts for two-level -1, 1, for three-level -1, 0, 1, and for four-level -3, -1, 1, 3.

The formulas for fitting two-level factorial designs to the response surface model are given in Equations (1) and (2) for estimating the coefficients of the linear effect and two-factor interaction respectively as:

$$b_l = \frac{contrast\ for A_l}{4n} \qquad l = 1, 2, \dots, p \tag{1}$$

$$b_{lq} = \frac{Contrast\ for\ A_l A_q}{4n} \qquad l \neq q \tag{2}$$

where n represents the number of replicates at each level or the number of replicates at the joint levels in case of interaction between different factors.

The formulas for fitting three-level factorial designs to the response surface model are given in Equations (3)-(5) for estimating the coefficients of the linear effect, two-factor interaction and the quadratic coefficients respectively as defined:

$$\gamma_r = \frac{linear\ contrast\ for\ A_r}{2n} \qquad r = 1, 2, \dots, m \tag{3}$$

$$\gamma_{rr} = \frac{Quadratic\ contrast\ for\ A_{rr}}{2n} \tag{4}$$

$$\gamma_{rL} = \frac{linear\ contrast\ for\ A_r A_L}{4n} \qquad r \neq L \tag{5}$$

The formulas for fitting four-level factorial designs to the response surface model are given in Equations (6)-(8) for estimating the coefficients of the main effect, two-factor interaction and the quadratic coefficients, respectively, as defined by Alqraghuli et al. [14].

$$\beta_s = \frac{Linear\ contrast\ for A_s}{20 \times n} \qquad s = 1, 2, \dots, q \tag{6}$$

$$\beta_{ss} = \frac{Quadratic\ contrast\ for\ A_{ss}}{16 \times n} \tag{7}$$

$$\beta_{st} = \frac{Linear\ contrast\ for\ A_s A_t}{400n} \qquad s \neq t \tag{8}$$

The formula for estimating the linear coefficient of the two-factor interaction for the mixed experiment of type two-level and three-level factorial designs is given in Equation (9) as defined by Alkarkhi & Low [11].

$$\alpha_{lr} = \frac{Linear\ contrast\ for A_l A_r}{4n} \qquad l = 1, 2, \dots, p \quad r = 1, 2, \dots, m \tag{9}$$

The formula for estimating the linear coefficient of the two-factor interaction for the mixed experiment of type two-level and four-level factorial designs is given in Equation (10) as defined by Alqraghuli et al. [13].

$$\theta_{ls} = \frac{Linear\ contrast\ for\ A_l A_s}{40\times n} \qquad l = 1,2,\dots,p \quad s = 1,2,\dots,q \tag{10}$$

The formula for estimating the linear coefficient of the two-factor interaction for the mixed experiment of type three-level and four-level factorial designs is given in Equation (11) as defined by Alqraghuli et al. [12].

$$\delta_{rs} = \frac{Linear\ contrast\ for\ A_r A_s}{40n} \tag{11}$$

$$r = 1,2,\dots,m \qquad s = 1,2,\dots,q$$

The next step is to derive a formula for calculating the linear coefficient of three-factor interaction between factor at two-level, factor at three-level and factor at four-level.

3. 1. Derive The Proposed Formula Suppose there are p factors each at two levels $X_1, X_2, \dots, X_p$, m factors each at three levels $Z_1, Z_2, \dots, Z_m$ and q factors each at four levels $R_1, R_2, \dots, R_q$, consider a response surface model in Equation (12).

$$Y_i = b_0 \sum_{l=1}^{p} b_i X_{li} + \sum\sum_{l<j} b_{lj} X_{li} X_{ji} + \sum_{r=1}^{m} \gamma_r Z_{ri} + \sum_{r=1}^{m} \gamma_{rr} Z_{ri}^2 + \sum\sum_{r<L} \gamma_{rL} Z_{ri} Z_{Li} + \sum_{s=1}^{q} \beta_s R_{si} + \sum_{s=1}^{q} \beta_{ss} R_{si}^2 + \sum\sum_{s<t} \beta_{st} R_{si} R_{ti} + \sum_{l=1}^{p}\sum_{r=1}^{m} \alpha_{lr} X_{li} Z_{ri} + \sum_{l=1}^{p}\sum_{s=1}^{q} \theta_{ls} X_{li} R_{si} + \sum_{r=1}^{m}\sum_{s=1}^{q} \delta_{rs} Z_{ri} R_{si} + \sum_{l=1}^{p}\sum_{r=1}^{m}\sum_{s=1}^{q} \tau_{lrs} X_{li} Z_{ri} R_{si} \tag{12}$$

$$i = 1,\dots,k$$

The model in Equation (12) should satisfy some constraints regarding each type of the selected factors. The constraints are based on the coefficients of orthogonal polynomial contrast.

1. The constraints for the factors at two levels are:

1. $\sum_{i=1}^{k} X_{li} = 0$ 2. $\sum_{i=1}^{k} X_{li} X_{ji} = 0$
3. $\sum_{i=1}^{k} X_{li}^2 X_{ji} = 0$ 4. $\sum_{i=1}^{k} X_{li} X_{ji}^2 = 0$
5. $\sum_{i=1}^{k} X_{li} X_{ji} X_{hi} = 0$ 6. $\sum_{i}^{k} X_i^2 = k$
7. $\sum_{i=1}^{k} (X_{li} X_{ji})^2 = k$

2. The constraints for the factors at three levels are:

1. $\sum_{i=1}^{k} Z_i = 0$ 2. $\sum_{i=1}^{k} Z_{ri} Z_{Li} = 0$
3. $\sum_{i=1}^{k} Z_{ri}^2 Z_{Li} = 0$ 4. $\sum_{i=1}^{k} Z_{ri} Z_{Li}^2 = 0$
5. $\sum_{i=1}^{k} Z_{ri} Z_{L1} Z_{hi} = 0$
6. $\sum_{i=1}^{k} Z_{ri}^2 = \sum_{i=1}^{k} Z_{ri}^4 = 2 \times 3^{m-1}$
7. $\sum_{i=1}^{k} (Z_{ri} Z_{Li})^2 = 4 \times 3^{m-2}$

3. The constraints for factors at four levels are:

1. $\sum_{i=1}^{k} R_{si} = 0$ 2. $\sum_{i=1}^{k} R_{si} R_{ti} = 0$
3. $\sum_{i=1}^{k} R_{si} R_{ti}^2 = 0$ 4. $\sum_{i=1}^{k} R_{si} R_{ti}^2 = 0$
5. $\sum_{i=1}^{k} R_{si}^2 = 20 \times 4^{q-1}$
6. $\sum_{i=1}^{k} R_{si}^4 = 164 \times 4^{q-1}$
7. $\sum_{i=1}^{k} R_{si}^2 R_{ti}^2 = 400 \times 4^{q-2}$
8. $\sum_{i=1}^{k} R_{si} R_{ti} R_{hi} = 0$
9. $\sum_{i=1}^{k} (R_{si} R_{ti})^2 = 400 \times 4^{q-2}$

4. The constraints for the joint effects between factors at two levels and factors at four levels are:

1. $\sum_{i=1}^{k} (R_{si} X_{li})^2 = 40 \times (4^{q-1} \times 2^{p-1})$
2. $\sum_{i=1}^{k} X_{li} R_{si} = \sum_{i=1}^{k} X_{li}^2 R_{si} = \sum_{i=1}^{k} X_{li} R_{si}^2 = 0$
3. $\sum_{i=1}^{k} X_{li} X_{ji} R_{si} = \sum_{i=1}^{k} R_{si} R_{ti} X_{li} = 0$

5. The constraints for the joint effects between factors at three levels and factors at four levels are:

1. $\sum_{i=1}^{k} (Z_{ri} R_{si})^2 = 40 \times (4^{q-1} \times 3^{m-1})$
2. $\sum_{i=1}^{k} Z_{ri} R_{si} = \sum_{i=1}^{k} Z_{ri}^2 R_{si} = \sum_{i=1}^{k} Z_{ri} R_{si}^2 = 0$
3. $\sum_{i=1}^{k} Z_{ri} Z_{ji} R_{si} = \sum_{i=1}^{k} R_{si} R_{ti} Z_{li} = 0$

6. The constraints for the joint effects between factors at two levels, three levels and four levels are:

1. $\sum_{i=1}^{k} X_{li} Z_{ri} R_{si} = 0$ 2. $\sum_{i=1}^{k} X_{li}^2 Z_{ri} R_{si} = 0$
3. $\sum_{h<j<l}^{k} X_{hi} Z_{ji}^2 R_{li} = 0$ 4. $\sum_{i=1}^{k} X_{li} Z_{ri} R_{si}^2 = 0$
5. $\sum_{i=1}^{k} X_{li} Z_{ri}^3 R_{si} = 0$ 6. $\sum_{i=1}^{k} X_{li} Z_{ri} R_{si}^3 = 0$
7. $\sum_{i=1}^{k} X_{li}^2 Z_{ri}^2 R_{si} = 0$ 8. $\sum_{i=1}^{k} X_{li}^2 Z_{ri} R_{si}^2$
9. $\sum_{i=1}^{k} X_{li} Z_{ri}^2 R_{si}^2$
10. $\sum_{i=1}^{k} X_{li}^2 Z_{ri}^2 R_{si}^2 = 32 \times 2^p \times 3^{m-1} \times 4^{q-1}$
11. $\sum (X_{1i} Z_{1i} R_{1i})^2 = 80 \times (2^{p-1} 3^{m-1} 4^{q-1})$

To illustrate the procedure, consider a $2^1 3^1 4^1$ experiment without losing information for the general case. Suppose there are three factors, X_1 at two levels, Z_1 at three levels and R_1 at four levels. The response surface model that describes the relationship between the selected factors and the response is given in Equation (13).

$$Y_i = b_0 + b_1 X_{1i} + \gamma_1 Z_{1i} + \gamma_{11} Z_{1i}^2 + \beta_1 R_{1i} + \beta_{11} R_{1i}^2 + \alpha_{11} X_{1i} Z_{1i} + \theta_{11} X_{1i} R_{1i} + \delta_{11} Z_{1i} R_{1i} + \tau_{111} X_{1i} Z_{1i} R_{1i} \qquad i = 1,2,\dots,k \tag{13}$$

The treatment combinations of this experiment are given in Table 1.The levels of the selected factors are represented by -1 and 1 for two-level factor, -1, 0, and 1 for three-level factor and -3, -1, 1, and 3 for four-level factor.

The proposed procedure depends on partitioning the experiment into three experiments according to the type of factors under study. Thus, two-level can be considered as 2^1 factorial design, three-level can be considered as 3^1 factorial design and four-level can be considered as 4^1 factorial design. Then, analyze each experiment separately to estimate the linear and quadratic coefficients while the coefficient of the two-factor interaction can be evaluated by studying the combination between any two types as presented earlier. Three-factor interaction between factors that have two levels, three levels and factors that have four levels can be studied by constructing $C_1^p C_1^m C_1^q$ experiments of the form $2^1 3^1 4^1$

The formula for estimating the coefficient of three-factor interaction can be derived by multiplying Equation (13) by $X_{1i} Z_{1i} R_{1i}$, summing over i and applying the constraints will result in τ_{111} in Equation (14).

TABLE 1. The results of $2^1 3^1 4^1$ factorial design

X	Z	R	Response
-1	-1	-3	Y_1
-1	0	-3	Y_2
-1	1	-3	Y_3
-1	-1	-1	Y_4
-1	0	-1	Y_5
-1	1	-1	Y_6
-1	-1	1	Y_7
-1	0	1	Y_8
-1	1	1	Y_9
-1	-1	3	Y_{10}
-1	0	3	Y_{11}
-1	1	3	Y_{12}
1	-1	-3	Y_{13}
1	0	-3	Y_{14}
1	1	-3	Y_{15}
1	-1	-1	Y_{16}
1	0	-1	Y_{17}
1	1	-1	Y_{18}
1	-1	1	Y_{19}
1	0	1	Y_{20}
1	1	1	Y_{21}
1	-1	3	Y_{22}
1	0	3	Y_{23}
1	1	3	Y_{24}

$$\sum X_{1i} Z_{1i} R_{1i} Y_i = \tau_{111} \sum (X_{1i} Z_{1i} R_{1i})^2$$

$$\tau_{111} = \frac{\sum X_{1i} Z_{1i} R_{1i} Y_i}{\sum (X_{1i} Z_{1i} R_{1i})^2} \quad (14)$$

Equation (14) can be written in the form of the coefficients of orthogonal polynomial contrast. Let us start with the denominator of Equation (14). The denominator of Equation (14) is equal to:

$\sum (X_{1i} Z_{1i} R_{1i})^2 = 80 \times (2^{p-1} 3^{m-1} 4^{q-1})$
$= 80 \times (2^{1-1} 3^{1-1} 4^{1-1}) = 80 \times 1$

where 1 represents the number of replicates.

The numerator of Equation (14) is

$\sum_{i=1}^{24} X_{1i} Z_{1i} R_{1i} Y_i =$
$(-1)(-1)(-3)Y_1 + (-1)(0)(-3)Y_2 +$
$(-1)(1)(-3)Y_3 + \cdots + (1)(1)(3)Y_{24} = -3(Y_1 + Y_{12} +$
$Y_{15} + Y_{22}) - 1(Y_4 + Y_9 + Y_{18} + Y_{19}) + 0(Y_2 + Y_5 +$
$Y_8 + Y_{11} + Y_{14} + Y_{17} + Y_{20} + Y_{23}) + 1(Y_6 + Y_7 + Y_{16} +$
$Y_{21}) + 3(Y_3 + Y_{10} + Y_{13} + Y_{24})$

The result of the numerator of Equation (14) is similar to the result of using the coefficients of linear contrasts of selected factors (linear joint contrast for X_1, Z_1 and R_1). The formula for estimating the three-factor interaction regarding the coefficients of orthogonal polynomial contrast is given in Equation (15).

$$\tau_{111} = \frac{\sum X_{1i} Z_{1i} R_{1i} Y_i}{\sum (X_{1i} Z_{1i} R_{1i})^2} = \frac{Linear\ contrast\ for\ X_1 Z_1 R_1}{80 \times 1} \quad (15)$$

In general, let n represents the number of replicates at the joint levels. Equation (15) becomes as below:

$$\tau_{lrs} = \frac{Linear\ contrast\ for A_l A_r A_s}{80 \times n} \quad (16)$$

The formula for estimating the intercept (b_0) in response surface model is given in Equation (17) which can be derived by summing Equation (13) over i and applying all the constraints presented earlier; the result will be the formula for b_0 as given in Equation (17).

$$b_0 = \bar{Y} - \gamma_{11} \bar{Z}_1 - \beta_{11} \bar{R}_1 \quad (17)$$

In general, in case of m three-level factors and q four-level factor are included in the model, Equation (17) becomes:

$$b_0 = \bar{Y} - \gamma_{11} \bar{Z}_1 - \cdots - \gamma_{mm} \bar{Z}_m - \beta_{11} \bar{R}_1 - \cdots - \beta_{qq} \bar{R}_q$$

where $\bar{Z} = \frac{\sum Z_i^2}{k}$, $\bar{R} = \frac{\sum R_i^2}{k}$, and k is the total number of observations.

4. APPLICATION

Arbitrary data was used to illustrate the new procedure for analyzing the mixed experiment of type two-level, three-level, and four-level factorial design and estimating the linear coefficient of three-factor interaction. The effect of three factors, operating time (X_1) at two levels (30 and 60 min), current applied (Z_1) at three levels (0.3, 0.4, 50 A), and inter-electrode (R_1) at four levels (0.5, 1, 1.5, 2 cm) on wastewater treatment by using electro-coagulation on power consumption (Y) was studied. Two replicates were used to run the experiment. Thus, the total number is $(2^1 3^1 4^1) \times 2 = 48$ runs. The levels of each factor in actual and coded form are given in Table 2.

Fitting a response surface model to this type of experiment requires applying the procedures presented earlier.

TABLE 2. The actual and coded form for the selected variables

Inter-electrode	0.5	1	1.5	2
Coded	-3	-1	1	3
Current applied	0.3	0.4	0.5	
Coded	-1	0	1	
Operating time	30	60		

Coded	-1	1

The response surface model that describes the relationship between power consumption (Y) and selected factors is given in Equation (18).

$$Y_i = b_0 + b_1 X_{1i} + \gamma_1 Z_{1i} + \beta_1 R_{1i} + \gamma_{11} Z_{1i}^2 + \beta_{11} R_{1i}^2 + \alpha_{11} X_{1i} Z_{1i} + \theta_{11} X_{1i} R_{1i} + \delta_{11} Z_{1i} R_{1i} + \tau_{111} X_{1i} Z_{1i} R_{1i} \quad (18)$$

Let us analyze the two-level factorial design first. The linear coefficient b_1 is:
$b_1 = \frac{Linear\ contrast\ for\ X_1}{2\times n} = \frac{35.3}{2\times 24} = 0.74$
where the linear contrast (L) for X_1 is
$L_{X1} = (-1)(1809) + (1)(1844.3) = 35.3$ and $n = 24$ represents the number of observations at each level of the two-level factorial design.
$\gamma_1 = \frac{Linear\ contrast\ for\ Z_1}{2n} = \frac{47.5}{2\times 16} = 1.484$
where $L_{Z1} = (-1)(1188.6) + (0)(1228.6) + (1)(1236.1) = 47.5$.
$\gamma_{11} = \frac{Quadratic\ contrast\ for\ Z_1}{2n} = \frac{-32.5}{2\times 16} = -1.016$
where $Q_{Z1} = (1)(1188.6) - (2)(1228.6) + (1)1236.1 = -32.5$.
and $n = 16$ represents the number of observations at each level of the three-level factorial design.
The third experiment is of type four-level factorial design. The linear (L) and quadratic contrast (Q) are:
$\beta_1 = \frac{Linear\ contrast\ for\ R_1}{20\times n} = \frac{480.9}{20\times 12} = 2.004$
where $LR_{1=} -3(729.5) + -(1064.9) + (921.2) + 3(937.7) = 480.9$
$\beta_{11} = \frac{Quadratic\ contrast\ for\ R_1}{16\times n} = \frac{-318.9}{16\times 12} = -1.66$
where $Q_{R1} = (1)(729.5) + (-1)(1064.9) + (-1)(921.2) + (1)(937.7) = -318.9$
and $n = 12$ represents the number of observations at each level of the four-level factorial design.

Next step is to analyze combined experiment to calculate the coefficient of two-factor interaction. The first two-factor interaction is between X_1 and Z.
The coefficient of two-factor interaction between X_1 and Z_1 is:
$\alpha_{11} = \frac{Linear\ contrast\ for\ X_1 Z_1}{4n} = \frac{-141.3}{4\times 8} = -4.416$
where $L_{X_1 Z_1} = (-1)(-1)(567.9) + (1)(-1)(620.7) + \cdots + (1)(573.8) = -141.3$
and $n = 8$ represents the number of observations at each joint level of the mixed two-level and three-level factorial design.
The two-factor interaction between X_1 and R_1 is:
$\theta_{11} = \frac{Linear\ contrast\ for\ X_1 R_1}{40n} = \frac{-45.1}{40\times 6} = -0.188$
where
$LX_1 R_1 = (-1)(-3)(343.5) + (-1)(-1)(555.7) + \cdots + (1)(1)(481.1) = -45.1$
and $n = 6$, represents the number of observations at each joint level of the two-level and four-level factorial design.

The coefficient of two-factor interaction between three-level and four-level factors $Z_1 R_1$ is given below:
$\delta_{11} = \frac{Linear\ contrast\ for\ Z_1 R_1}{40n} = \frac{-519.9}{40\times 4} = -3.249 =$
where $L_{Z_1 R_1 = (-1)(-3)(190.7) + \cdots + (1)(3)(259.5) = -519.9}$

The coefficient of the three-factor interaction between $X_1\ Z_1\ and\ R_1$ is calculated as defined in Equation (16).
$LX_1 Z_1 R_1 = (-1)(-1)(-3)(87.1) + \cdots + ((1)(1)(3)(111) = -4.3$
$\tau_{111} = \frac{Linear\ contrast\ for\ X_1 Z_1 R_1}{80n} = \frac{-4.3}{80\times 2} = -0.027$

The last coefficient of Equation (18) is the intercept b_0 which is calculated as defined in Equation (17):
$\bar{Y} = \frac{\sum Y}{n} = 76.1104$ $\bar{Z} = \frac{\sum Z_i^2}{k} = 0.667$
$\bar{R} = \frac{\sum R_i^2}{k} = 5$
$b_0 = \bar{Y} - \gamma_{11}\bar{Z}_1 - a_{11}\bar{R}_1$
$b_0 = 76.1104 - (-1.016)(0.667) - (-1.66)(5) = 85.0915$

The fitted response surface model that describes the relationship between the power consumption and time and current applied in Equation (18) is given below:
$Y = 85.12 + 0.74X_1 + 1.484Z_1 + 2.004R_{1-}1.016Z_1^2 - 1.66R_1^2 - 4.416X_1 Z_1 - 0.188X_1 R_{1-}3.249Z_1 R_1 - 0.027X_1 Z_1 R_1$

The experiment was also analyzed using the least squares method [18, 19]. The results of using the least squares were in agreement with the proposed procedure for three-factor interaction (τ_{111}**)** as presented in Table 3.

5. CONCLUSION

Based on the results, it can be said that the coefficients of orthogonal polynomial contrast can be used for fitting response surface models without the complication of using the least squares method.

TABLE 3. The coefficients of the model calculated by the least squares and proposed methods

Parameter	Proposed Procedure	Least squares method
b_0	85.092	85.092
b_1	0.735	0.735
γ_1	1.484	1.484
γ_{11}	-1.016	-1.016
β_1	2.004	2.004
β_{11}	-1.661	-1.661
α_{11}	-4.416	-4.416
θ_{11}	-0.188	-0.188
δ_{11}	-3.249	-3.249
$\boldsymbol{\tau_{111}}$	**-0.027**	**-0.027**

Furthermore, this result will enhance the use of coefficients of orthogonal polynomial contrast in analyzing different experimental designs and provide simple and easy formulas removing the dependence on statistical software.

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Proposed Procedure for Estimating the Coefficient of Three-factor Interaction for $2^p 3^m 4^q$ Factorial Experiments

TECHNICAL NOTE

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چکیده

تعامل سه عامل برای طرح های فاکتوریل دو سطح، سه سطح و چهار سطح مورد مطالعه قرار گرفت. یک روش و فرمول جدید مبتنی بر ضرایب کنتراست چندجمله ای متعامد برای محاسبه اثر متقابل سه عامل پیشنهاد شده است. نتایج نشان می دهد که روش پیشنهادی با روش کوچکترین مربع مطابقت دارد. مزایای روش جدید عبارتند از: ۱) آن تثبیت شده است، ۲) ساده است و ۳) استفاده از آن بدون فرمول ماتریس پیچیده روش کوچکترین مربع آسان است.همچنین، این روش جدید استفاده از ضرایب کنتراست متعادل را هنگام تحلیل سایر طرح های تجربی افزایش می دهد.

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