Modeling of A Single Turn Pulsating Heat Pipe based on Flow Boiling and Condensation Phenomena

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A B S T R A C T

Demand for high-performance cooling systems is one of the most challenging and virtual issues in the industry and Pulsating heat pipes are effective solutions for this concern. In the present study, the best predictor correlations of flow boiling and condensation are taken into account to model a single turn pulsating heat pipe mathematically. These considerations, result in derivation of more accurate results. The nucleate boiling phenomenon has been considered as the dominant mechanism of the boiling process in the evaporator. However, due to the annular flow assumption, a thin film of liquid is considered in calculation of the mass transfer out of the vapor plugs. All the fundamental relations such as momentum, mass and energy equations are solved implicitly, except the energy equation of liquid slug. The liquid slug displacement results are compared with the previous studies and the comparison indicates increase in both the frequency and the amplitude of the slug displacement. Moreover, the calculated heat flux is verified with the empirical results. The comparison shows an acceptable agreement between the findings, which is better than previous modelings without boiling and condensation. Furthermore, the effect of pipe diameter on the flow and heat transfer mechanisms has been derived. According to the results, by increasing the pipe diameter, despite a frequency decrease, the oscillation amplitude of liquid slug and total heat flux transferred into the pulsating heat pipe increases. Sensible heat contribution in the heat transfer mechanism reduces by higher pipe diameter values.


NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>A</td>
<td>Section area (m²)</td>
</tr>
<tr>
<td>Ca</td>
<td>Capillary number, νr_ν/σ</td>
</tr>
<tr>
<td>Ci</td>
<td>Friction factor for liquid, C_i=16</td>
</tr>
<tr>
<td>Cp</td>
<td>Specific heat (J/kg.K)</td>
</tr>
<tr>
<td>Cv</td>
<td>Friction factor for liquid, C_v=0.046</td>
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<tr>
<td>d</td>
<td>Diameter, (m)</td>
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<tr>
<td>E</td>
<td>Sensible heat corrector factor</td>
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<tr>
<td>f</td>
<td>Friction factor</td>
</tr>
<tr>
<td>G</td>
<td>Mass flux, (kg/m².s)</td>
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<tr>
<td>h</td>
<td>Specific enthalpy, (J/kg)</td>
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<td>hfg</td>
<td>Phase change enthalpy, (J/kg)</td>
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<tr>
<td>hlsen</td>
<td>Sensible heat coefficient, (W/m².K)</td>
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<tr>
<td>Ja</td>
<td>Jacob number, Ja= h_fg/(c_p ΔT_{sat})</td>
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<tr>
<td>x</td>
<td>Distance, (m)</td>
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<tr>
<td>Xq</td>
<td>Vapor quality</td>
</tr>
<tr>
<td>Xmt</td>
<td>Lockhart-Martineelli Parameter</td>
</tr>
<tr>
<td>We</td>
<td>Weber number</td>
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<tr>
<td>u</td>
<td>Specific internal energy, (J/kg)</td>
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<tr>
<td>V</td>
<td>Volume, (m³)</td>
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Greek symbols

<table>
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<tr>
<th>Symbol</th>
<th>Definition</th>
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</thead>
<tbody>
<tr>
<td>α</td>
<td>Thermal diffusivity (m²/s)</td>
</tr>
<tr>
<td>μ</td>
<td>Viscosity (Pa.s)</td>
</tr>
<tr>
<td>ρ</td>
<td>Density (kg/m³)</td>
</tr>
<tr>
<td>σ</td>
<td>Surface tension (N/m)</td>
</tr>
<tr>
<td>τ</td>
<td>Shear stress (Pa)</td>
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</table>

Subscripts

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1. INTRODUCTION

High-tech industries provide systems with expanded features and lead to overheating, so it must be taken away by effective cooling equipment. Pulsating heat pipes (PHPs) are one of the most applicable devices with high heat transfer rates that nowadays are used for cooling devices. The examples include a fuel cell with cooling effect reported by Clement [1] and also using PHPs to cool an electronic device by Maydanik [2].

PHP is a capillary tube that consists of one or multi bends which transfers heat from a heat source (evaporator), to a heat sink (condenser). PHPs are classified into two general structures: closed looped pulsating heat pipes (CLPHP) and open loop pulsating heat pipes (OLPHP). Although in recent years many researchers have done experimental and numerical studies on the flow and heat transfer of PHPs, the oscillation behavior of these devices has not been figured out completely.

Zhang et al. [3] studied the PHP operation with consideration of a liquid film's meniscus region in both condenser and evaporator sections. They reported reductions in amplitude of liquid slug oscillations due to a decrease in both diameter and evaporator temperatures despite the frequency of the oscillations which increases insignificantly. Shafii et al. [4] represented a mathematical model of PHPs operation based on thin liquid film model. On the basis of their results, surface tension force has no significant effect on the heat transfer contrary to the pipe diameter, any increase of which enhances the heat transfer. Besides, they showed that both of CLPHP and OLPHP have the same operational characteristics in both flow and heat transfer fields. Zhang et al. [5] investigated a single turn PHP numerically. They represented a simple model consisting of mass, momentum and energy equations and obtained their dimensionless form. Holley and Faghi [6] studied PHPs with wick structure and varying diameter. They calculated the enhancement of heat transfer by varying the pipe diameter along the length and also showed that gravity has no effect on the operation of Heat pipe. Yuan et al. [7] represented a new model of the sensible heat transfer into and out of the liquid slug based on the PHP's surrounding thermal conditions. They reported a phase difference of flow oscillation between presence and absence of the gravity effect. They also showed a high contribution (about 15%) of the sensible heat to the total heat transfer. Shao et al. [8] reported a mathematical model based on the thin film of liquid in the condenser and evaporator sections. They also highlighted the significant role of the sensible heat transfer and evaporator temperature effect on heat transfer enhancement. Ma et al. [9] modeled a single turn PHP considering the thermal energy as a driving force on the liquid slug. They reported the effect of filling ratio and pipe characteristics on the oscillation amplitude and frequency of a liquid slug. Shafii et al. [10] modeled both OLPHP and CLPHP using an explicit finite difference model. They used constant evaporative and condensation heat transfer coefficients without any discussion about the phenomena. The effects of Pipe diameter, filling ratio of PHP and evaporator temperature are studied in the paper. According to the results, liquid slug oscillation and total heat transfer mainly depend on the temperature difference of the heating and cooling walls. Based on the results, CLPHP and OLPHP have the same behavior at the specified conditions and the number of turns has no effect on the operation of heat pipe when the pressure loss is not taken into account. Arabnejad et al. [11] using Zhang et al. model [5] proposed a mathematical model, taking Chen [12] convective boiling expression into account. They considered stratified condensation in the cooling section. According to their results, latent heat has the main role in the total heat transfer into the PHP. Although they used different modeling of evaporative and condensation processes, their results of liquid slug flow are the same of Zhang et al. [5] results. Sakulchangsatjatai et al. [13] studied CLPHPs using Shafii's model [11]. They reported a corrective expression to validate numerical results with experimental ones. Furthermore, they reported the effect of pipe characteristics on the heat transfer mechanism of PHPs. Mameli et al. [14] investigated the effect of tube turns on the operation of PHPs under the condition of fixed heat flux at the pipe boundary. Moreover, different working fluids and PHP orientations were studied in the paper. Khandekar et al. [15] used empirical data and

<table>
<thead>
<tr>
<th>Ka</th>
<th>Karman number, $\text{Ka}=\text{Re}^{-\frac{1}{2}}$</th>
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<tbody>
<tr>
<td>P</td>
<td>Pressure, (Pa)</td>
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<tr>
<td>Pr</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>Q</td>
<td>Heat rate, (W)</td>
</tr>
<tr>
<td>$q''$</td>
<td>Heat flux, (W/m²)</td>
</tr>
<tr>
<td>$r_t$</td>
<td>Turn radius, (m)</td>
</tr>
<tr>
<td>S</td>
<td>Boiling heat transfer corrector</td>
</tr>
<tr>
<td>$T_w$</td>
<td>Wall temperature, (K)</td>
</tr>
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</table>

$v_{ls}$ | Liquid slug velocity, (m/s) |
$v_s$ | Single phase |
$v_l$ | Left vapor |
$v_r$ | Right vapor |

$\text{Liquid slug}$ |
$\text{Liquid}$ |
$\text{Vapor}$ |

$\text{Two phase}$ |

$\text{Prandtl number}$, $\text{Pr}$

$\text{Heat rate, (W)}$, $\text{Q}$

$\text{Heat flux, (W/m}^2\text{)}$, $q''$

$\text{Turn radius, (m)}$, $r_t$

$\text{Boiling heat transfer corrector}$, $S$

$\text{Wall temperature, (K)}$, $T_w$

$\text{Pressure, (Pa)}$, $P$

$\text{Prandtl number}$, $Pr$

$\text{Heat rate, (W)}$, $Q$

$\text{Heat flux, (W/m}^2\text{)}$, $q''$

$\text{Turn radius, (m)}$, $r_t$

$\text{Boiling heat transfer corrector}$, $S$

$\text{Wall temperature, (K)}$, $T_w$
presented a correlation to obtain heat flux in PHPs. They highlighted the importance of PHP modeling by consideration of boiling and condensation processes.

Experimental investigations of the PHPs are mainly focused on the heat transfer measurements. Flow pattern and bubble formation in closed loop pulsating heat pipes are presented in Tong et al.'s [16] study. They clearly showed the bubble formation or bubble collapsing mechanism along the pipe length and bends. Furthermore, they observed the circulation of liquid slugs in CLPHPs. Khandekar et al. [17] studied CLPHPs experimentally. Results of vertical and horizontal orientations are reported in the paper with thermodynamic considerations. Charoensawan et al. [18] used an experimental setup to study PHPs at constant evaporator and condenser wall temperature. Results are presented for different working fluids and internal diameters of pipe. It is shown that orientation of the heat pipe has a significant influence on the total heat transfer. Besides, the results emphasize that pipe diameter is an important parameter in the operation. A single turn CLPHP has been analyzed in Khandekar et al.'s [19] work. They showed that a single turn PHP represents the same thermo-physical characteristics of a multi-turn pulsating heat pipe. Mameli et al. [20] represented experimental results of CLPHP under different heat fluxes and system orientations. They reported an optimal filling ratio for a specified condition and orientation. Han et al.'s [21] experimentally investigated the effects of working fluid physical properties on PHP flow and heat transfer. The study emphasizes the role of viscosity and phase change parameters on the PHP heat transfer performance.

Most of the previous mathematical investigations, considered constant evaporative and condensation coefficients without any attention to the flow boiling or condensation. In the present study, correlations of flow boiling and condensation, which have the least deviation from the experimental data, are used for mathematical simulation of a PHP. Utilization of this model and effects of boiling and condensation result in increase of accuracy and better modeling of the problem. Furthermore, the effect of internal diameter of pipe on liquid displacement, vapor pressure, vapor mass, sensible and boiling heat transfer are investigated.

2. MATHEMATICAL MODELING

Experimental and numerical results have demonstrated that a single U-shaped pipe can show a multi-turn PHP's behavior [16, 19]. Therefore, in the present paper the operation of a single turn pulsating heat pipe is modeled mathematically. Figure 1 indicates the simple structure of a U-shaped OLPHP which was considered as a basic structure of PHP. A liquid slug, neighboring two vapor plugs, oscillates between condenser and evaporator sections. Inner diameter of pipe is \( d \) and \( x_i \) and \( x_e \) are left and right hand side positions of the liquid slug, respectively, in an arbitrary time inside the pipe. As Figure 1 shows \( L_c, L_e \) and \( L_{ls} \) are lengths of the condenser, the evaporator, and the liquid slug, respectively. The liquid slug oscillates in the pipe due to pressure difference of the adjacent vapors and heat transfers into the PHP due to the sensible and latent heat transfer. Consequently, a pulsating heat pipe is a cooling system that its theoretical modeling is due to the following main parts:

- Momentum conservation.
- Mass conservation of liquid slug and vapor plugs.
- Energy conservation of vapor plugs.
- Energy conservation of liquid slug.
- Evaporation.
- Flow condensation.

To derive the governing equations, the following assumptions are considered:

- Incompressible fluid flow.
- Smooth pipe.
- Annular flow in the condenser section (as indicated by experimental visualization).
- Uniform film thickness throughout the condenser section.
- Ideal gas assumption of vapor.

![Figure 1. Schematic of a single turn pulsating heat pipe, a) U-turn, b) Linear pipe configuration, c) Condenser section, d) Evaporator section.](image-url)
2.1. Momentum Equation of Liquid Slug  By assuming an incompressible liquid working in the PHP, the momentum conservation equation stated as follows:

\[ p_l A \frac{d(v_l v_m)}{dt} = [(P_{v_l} - P_{v_m}) - \Delta P_{\text{turn}}] A - \pi d L_{ls} \tau v_l x_l \]  

(1)

Although pressure loss at a single bend is negligible, the pressure loss will be calculated from the following experimental based relation, i.e. the 3-K model, to derive exact results [22]:

\[ \Delta P_{\text{turn}} = \frac{K}{\pi} p_l v_{ls}^2 \]  

(2)

where \( K \) is the pressure loss coefficient and can be determined from the following expression:

\[ K = \frac{K_{fs}}{Re} + K_f \left( 1 + \frac{K_d}{Re^{0.2}} \right) \]  

(3)

The constants for a single 180° turn and \( \frac{R_e}{d} = 1.25 \) are:

\( K_{fs} = 1000, \quad K_f = 0.1 \) and \( K_d = 4 \) [22].

In the momentum equation, the shear stress term can be expressed as follows:

\[ \tau = \frac{1}{\pi} f p_l v_{ls}^2 \]  

(4)

In which \( f \) is the friction factor and by consideration of smooth pipe, in both laminar and turbulent regimes, it can be determined from the following expression:

\[ f = \begin{cases} \frac{64}{Re} & \text{Re} \leq 2000 \\ \left(0.079 \ln (Re) - 1.64\right)^2 & \text{Re} > 2100 \end{cases} \]  

(5)

2.2. Mass Conservation Equations  Due to boiling or condensation processes, \( L_{ls} \) will change in each half of a cycle of oscillation. Although this change is negligible, \( L_{ls} \) is calculated from the average changes of the neighboring vapor plugs:

\[ \frac{dL_{ls}}{dt} = \frac{1}{2p_l} \left( \frac{dm_{boiling}}{dt} + \frac{dm_{condensation}}{dt} \right) \]  

(6)

The evaporation and/or condensation processes lead to the mass change of the vapor plugs. So, the continuity equation for the vapor plugs expressed as follows:

\[ \frac{dm_{boiling}}{dt} = m_{boiling} - m_{condensation} \]  

(7)

Because of small values of sensible heat coefficient into/out of the vapor plugs, mass rate transferred into and out of the vapor plugs are due to the boiling and the condensation phenomena, respectively, which will be discussed in the following sections.

2.3. Energy Conservation of Vapor Plugs  Considering ideal gas assumption and consequently, setting \( u = c_p T \) and \( h = c_p T \), energy equations of the vapor plugs will result in:

\[ m_{v_m} \frac{dT_v}{dt} = (m_{boiling} - m_{condensation})RT_v - P_v A \frac{dx_v}{dt} \]  

(8)

The last term in the above equation is worked on by vapor plug in the unit of time and \( x_v \) is the location of right or left vapor plug adjacent to the liquid slug. So, it can be concluded that \( \frac{dx_v}{dt} = \frac{dx_l}{dt} \) and \( \frac{dT_v}{dt} = - \frac{d(x_l + x_v)}{dt} \) To calculate the pressure of the vapor plug, the ideal gas relation is used:

\[ P_v V_v = m_v R T_v \]  

(9)

2.4. Sensible Heat Transfer  Considering a 1-D analysis, the energy equation for single-phase liquid slug are:

\[ \frac{1}{a_1} \frac{dT_l}{dt} = \frac{\frac{d^2T_l}{dx^2}}{k_l} \left( T_I - T_w \right) \]  

(10)

Temperature distribution of the liquid slug and sensible heat transfer into and out of the PHP can be obtained by solving the above equation, in which \( H_{\text{turn}} \) is the forced convection heat transfer coefficient of the liquid phase. It should be pointed out that regarding the flow boiling in the evaporator section, the heat transfer coefficient is corrected by an increment forced convection factor, \( E \). Hence, \( H_{\text{turn}} \) in terms of single or two phase flow is:

\[ H_{\text{turn}} = \begin{cases} h_{\text{tp}} & \text{boiling} \\ h_{\text{sp}} & \text{condensation} \end{cases} \]  

(11)

The corrector factor (\( E \)) will be defined in the flow boiling section.

By consideration of thermally developing Hagen-Poiseuille flow and taking isothermal wall condition into account, Shah and London’s equation can be used to calculate the forced convection heat transfer coefficient [23]:

\[ h_{sp} = \begin{cases} (k_l/d)(1.615x^{-0.7} - 0.7) & x' \leq 0.005 \\ (k_l/d)(1.615x^{0.2} - 0.2) & 0.005 < x' < 0.03 \\ (k_l/d)(3.657 + 0.499x) & x' > 0.03 \end{cases} \]  

(12)

The dimensionless \( x' \) is defined as:

\[ x' = \frac{h_{sp}}{k_{pr}} \]  

(13)

For the transition and turbulent flow, the Gnielinski’s relation is used since the pipe is considered smooth [23]:

\[ h_{sp} = (k_l/d) \left( \frac{1}{(Re - 1000)Pr} \right) \left( \frac{L_{ls}}{D} \right)^{0.85} \]  

(14)

where \( f \) is the friction factor that can be obtained from Equation (5). The Gnielinski’s relation governs in both transition and turbulent flow regimes.

Since, the flow boiling and condensation correlations are calibrated with Dittus-Boelter convection heat transfer coefficient (sections 2.5 and 2.6)
the following relations are used to calculate two phase sensible heat coefficient, \( h_{tp} \) [24]:

\[
h_{tp} = \begin{cases} 
3.66(k/d) & \text{Re} \leq 2000 \\
(k/d)(0.023Re^{0.8}Pr^{0.4}) & \text{Re} > 2000 
\end{cases}
\]  

(15)

Consequently, after calculating the temperature distribution, the sensible heat flux rate transferred into and out of the PHP for a liquid mesh is determined by:

\[
q''_s\text{sen, in}(x) = H_{lsen}(T_l(x,t) - T_w) \quad T_l(x,t) > T_w
\]  

(16a)

\[
q''_s\text{sen, out}(x) = H_{lsen}(T_w - T_l(x,t)) \quad T_l(x,t) < T_w
\]  

(16b)

and the total sensible heat transfer rate is obtained from the following equation:

\[
Q_{\text{sen, in}(t)} = \int H_{lsen} \pi d(T_l(x,t) - T_w) \, dx \quad T_l(x,t) > T_w
\]  

(17)

\[
Q_{\text{sen, out}(t)} = \int H_{lsen} \pi d(T_w - T_l(x,t)) \, dx \quad T_l(x,t) < T_w
\]  

(18)

### 2.5. Flow Boiling

When the liquid slug moves to the evaporator section, due to the saturation flow, boiling phenomenon and consequently mass transfer into the vapor plugs occur (Figure 1d). Flow boiling in microchannels is extreme function of the shape and size of the channel. Although the flow boiling process is very complex in microchannels, two main mechanisms are dominant in microchannels: nucleate boiling and convection boiling [25]. Due to the low Reynolds numbers of the liquid slug and low quality of vapor in the evaporator section of the PHP, nucleate boiling effects prevail and the experimental observations confirm the boiling mechanism [15, 16].

Some researchers suggested correlations to predict the flow boiling heat transfer in microchannels [12, 26–29]. Cheng [25] used experimental results to establish a database and compared the most known correlations. According to Cheng’s results, Saitoh et al. correlation [28] better estimates the flow boiling experimental results than other correlations. Therefore, Saitoh et al. relation is used in the present paper to calculate the boiling heat transfer coefficient in each liquid cell and the two phase sensible heat transfer increment factor in the evaporator (\( E \)) [28]:

\[
h_{\text{flow boiling}} = Sh_{\text{boiling}} + Eh_{tp}
\]  

(19)

In which \( E \) is defined as following:

\[
E = 1 + \frac{x_l}{d(1 - 0.5)}
\]  

(20)

the Weber number of the vapor phase is:

\[
We_v = \frac{(\sigma v_{sat}^2 x_l^2 d)}{\rho v_o}
\]  

(21)

According to definition of Lockhart-Martinelli Parameter:

\[
X_{et} = \left(1 - \frac{x_l}{d} \right) \frac{0.9}{m_o} \frac{0.5}{\mu} \frac{0.1}{\mu_o} \quad \text{Re} > 1000
\]  

(22)

\[
X_{et} = \left( \frac{c_v}{c_l} \right) \frac{0.5}{m} \frac{0.5}{\mu} \frac{0.5}{\mu_o} \frac{1}{\mu_c} \quad \text{Re} < 1000
\]  

The local vapor quality at each area section of the evaporator (\( x_q \)) can be defined as the rate of vapor mass flow into an arbitrary area section of the evaporator (mass rate of boiling) divided by the local flow rate of the liquid slug that goes in/out of the area section. So, in the boiling section of the liquid slug, \( x_q(t) \) can be found as follows:

\[
x_q(t) = \frac{\dot{m}_{\text{boiling}}(t)}{\dot{m}_{in}(t)} = \frac{\dot{m}_{\text{boiling}}(t)}{\dot{m}_{in}(t)}
\]  

(23)

According to Saitoh et al. [28] relation, \( h_{\text{boiling}} \) is boiling heat transfer coefficient that is expressed as follows:

\[
h_{\text{boiling}}(x,t) = 207 \frac{h_{in}(x,t) d_t}{\eta_t (x,t)} \frac{5.745}{\eta_t} \frac{P_t}{\eta_t} \frac{5.51}{\eta_t} \frac{1}{\eta_t}
\]  

(24)

where \( d_t(t) \) is calculated from the following relation:

\[
d_t(t) = 0.51 \left( \frac{2 \pi}{\eta_t (x,t)} \right)^{0.5}
\]  

(25)

In the above equations, \( \rho_g(t) \) is calculated by the ideal gas assumption of the vapor phase. Additionally, \( q''(x,t) \) is the sensible heat flux rate in an arbitrary location of the liquid phase in the evaporator, which can be obtained by Equation (16a). The boiling heat transfer factor, \( S \), in Equation (19) is stated as follows:

\[
S(t) = \frac{1}{1 + 0.4(10^{-4} Re E_{120})^{14}}
\]  

(26)

Consequently, the heat transfer rate into the PHP due to the boiling process is:

\[
Q_{\text{boiling}(t)} = \int S(t) h_{\text{boiling}}(x,t) \pi d(T_w - T_l(x,t)) \, dx
\]

if \( T_w > T_l(x,t) \)

(27)

Finally, the transferred mass into the vapor plugs with respect to time is calculated as follows:

\[
\dot{m}_{\text{boiling}}(t) = \frac{Q_{\text{boiling}(t)}}{h_{in}(t)}
\]  

(28)

### 2.6. Flow Condensation

While the vapor plug is transmitted to the condensation section, condensation occurs due to low wall temperature. According to the experimental results, flow pattern has a significant role on the condensation heat transfer [30]. High viscosity (due to low temperature) of liquid layer adjacent to the condenser wall causes the formation of a thin film of liquid at the perimeter of the cooling wall (Figure 1c). Therefore, one can assume annular flow at the condenser section, as has also been indicated by visualizations in experimental results [16, 17].

Several correlations are presented by researchers to evaluate the flow condensation in microchannels.
Ribatski et al. collected a database of experimental results to compare the suggested correlations [30]. Their study concludes that Cavallini et al. relation [32] provides the best predictions and the authors recommended the correlation to design heat exchangers. In the present study, Cavallini et al.’s method [32] was used to obtain the local flow condensation heat transfer coefficient:

\[
\begin{align*}
\mathbf{h}_{\text{condensation}}(x,t) &= H_{\text{lim}}(x) \\
&= 1 + 1.128x_{\delta}(x,t)^{0.817}\left(\frac{\rho}{\rho_0}\right)^{0.3657}\left(\frac{\mu}{\mu_0}\right)^{0.2160}(1 - \frac{\mu}{\mu_0})^{0.2144}Pr^{0.1}
\end{align*}
\]

where \(x_{\delta}(x,t)\) is the quality of vapor plug in an arbitrary area section of the condenser in terms of the thin liquid film thickness, \(\delta_{\text{film}}\) which can be evaluated as following:

\[
\begin{align*}
x_{\delta}(x,t) &= \frac{m_{v}(x,t)}{m_{\text{tot}}} \\
&= \frac{\nu_{v_{\delta}}}{\nu_{v_{\delta}} + \nu_{v_{\text{plug}}}} = \frac{\left(\frac{\nu_{v_{\text{plug}}}}{\nu_{v_{\delta}}} - \frac{\delta_{\text{film}}(t)}{\nu_{v_{\delta}}}\right)}{\left(\frac{\nu_{v_{\text{plug}}}}{\nu_{v_{\delta}}} - \frac{\delta_{\text{film}}(t)}{\nu_{v_{\delta}}}\right)\nu_{v_{\delta}}}
\end{align*}
\]

According to annular flow in the condenser section and creation of thin film of liquid, it is necessary to calculate the thickness of the liquid film (\(\delta_{\text{film}}\)). The empirical based correlation proposed by Han et al. [35] is used to determine the liquid film thickness. They derived the suggested relation according to experimental results which has an accuracy of 15%:

\[
\begin{align*}
\delta_{\text{film}}(t) &= \frac{0.670Ca(t)^{0.2}}{1 + 3.13Ca(t)^{0.05}0.504 Ca(t)^{0.672}Re(t)^{0.099} - 0.352We(t)^{0.629}} \\
&\times \frac{106.0(\frac{\nu_{v_{\text{plug}}}}{\nu_{v_{\delta}}})^{2}}{Re(t) \leq 2000} \\
&= \frac{1.497(\frac{\nu_{v_{\text{plug}}}}{\nu_{v_{\delta}}})^{2} + 73.92(\frac{\nu_{v_{\text{plug}}}}{\nu_{v_{\delta}}})^{0.672} - 5000(\frac{\nu_{v_{\text{plug}}}}{\nu_{v_{\delta}}})^{0.629}}{Re(t) > 2000}
\end{align*}
\]

In the present study, the thickness of the liquid film has been considered uniform through the condenser length, which seems physically acceptable because of annular flow, except the liquid-vapor interface. So, the equation represents, \(\delta_{\text{film}}\) just depends on time.

As a result, by integrating the local condensation heat transfer coefficient along the condenser length, the average condensation heat transfer coefficient will be obtained:

\[
\begin{align*}
h_{\text{condensation}}(t) &= h_{\text{pipe}} \\
&= 1 + 1.128x_{\delta}(t)^{0.817}\left(\frac{\rho}{\rho_0}\right)^{0.3657}\left(\frac{\mu}{\mu_0}\right)^{0.2160}(1 - \frac{\mu}{\mu_0})^{0.2144}Pr^{0.1}
\end{align*}
\]

To derive the above equation, it is assumed that the vapor plugs are homogenous and all quantities are uniform. \(h_{\text{pipe}}\) is the sensible heat transfer that can be calculated from Equation (15).

Consequently, the latent heat transferred out of the PHP due to the flow condensation is determined:

\[
\begin{align*}
Q_{\text{condensation}}(t) &= h_{\text{condensation}}(t)\pi d x_{\delta}(t) A_{\text{cond}}(t) - \frac{T_{\text{fg}}}{\rho_{v_0}}(t) (T_{\delta}(t) - T_{\text{fg}}) \\
&\text{if } T_{\text{fg}} < T_{\delta}(t)
\end{align*}
\]

Finally, the mass transfer out of the vapor plugs will be:

\[
\begin{align*}
m_{\text{condensation}} &= \frac{Q_{\text{condensation}}(t)}{h_{\text{fg}}}
\end{align*}
\]

3. ALGORITHM AND NUMERICAL SOLUTION

An explicit finite difference scheme in the momentum and mass conservation of the liquid slug was used to obtain the new values of the parameters at the next time step from the old values. Furthermore, the mass and energy equations of the vapor plugs are also solved by explicit scheme. Following are the implicit discretized form of the momentum and the energy equations of the liquid slug and the vapor plugs, respectively:

\[
\begin{align*}
(\rho_{v_0} v_{\delta})^{n+1} &= \frac{1}{\rho_{v_0}} \left(\frac{L_{v_{\delta}}}{A_{\text{cond}}} - \Delta P_{\text{end}} A - \Delta L_{v_0} T - \Delta x_{\delta}(t) A_{\text{cond}}(t) - \frac{T_{\text{fg}}}{\rho_{v_0}}(t) (T_{\delta}(t) - T_{\text{fg}}) \right) \\
&\text{if } T_{\text{fg}} < T_{\delta}(t)
\end{align*}
\]

\[
\begin{align*}
T_{v_0}^{n+1} &= \left(\frac{1}{m_{\text{v_0}}} \left(\frac{m_{\text{boiling-v}}}{m_{\text{condensation-v}}} + m_{\text{condensation-v}}\right)RT_{v_0}\right)^{n} - \frac{1}{m_{\text{v_0}}} \left(\frac{L_{v_{\delta}}}{A_{\text{cond}}} - \frac{T_{\text{fg}}}{\rho_{v_0}}(t) (T_{\delta}(t) - T_{\text{fg}}) \right)
\end{align*}
\]

The temperature distribution and the sensible heat transfer equation is solved using TDMA (Tri-diagonal Matrix Method) with a \(10^{-4}\) time step consideration which makes the solutions independent of time step. Mesh structure is non-uniform and consists of 1200 nodes. 400 cells belong to each end of the slug, with the length of 0.04m, and 400 cells belong to the rest of the slug which has an insignificant role in the heat transfer mechanism. All of the governing equations are solved by Matlab software.

4. RESULTS

Water thermodynamic characteristics are considered and a code is developed to solve the equations. To validate the numerical solutions, we utilized Charoenwasan et al. [18] empirical results and Khandekar et al. [15] correlation. Charoenwasan et al. [18] used an experimental setup to study the heat transferred into a multi turn PHP. Their reported results are in terms of the pipe diameter, the evaporator length and the number of turns. However, the empirical and numerical results express that the heat
transfer is not affected by the turns when the number of the turns is small [3, 16, 19]. Therefore, the mentioned results with small number of turns can be used to verify the results of the transferred heat in a single turn PHP. On the other hand, in Charoensawan et al. [18] study, the evaporator and the condenser temperatures are fixed at 80°C and 20°C (Isothermal wall condition), respectively. The evaporator length equals the condenser (results in 50% of filling ratio). Furthermore, Khandekar et al. [15] correlated Charoensawan et al. [18] empirical results and presented in the following correlation:

\[
q'' = 0.54K_a^{0.47}Pr^{0.27}T_\text{ev}^{1.43}N_{\text{t}}^{-0.27}
\]  

(37)

The authors concluded that the correlation results are in an overall drift in prediction within ±30%.

Figure 2 illustrates the comparison of the present model heat flux transferred into the PHP with those of the experimental study of Charoensawan et al. [18], Rittidech et al. [36] and correlation of Khandekar et al. [15]. In all of the case studies filling ratio is 50%, furthermore \( L_c \) has been set to 0.1m except one case, in which \( L_c = 0.15 \)m is considered. Other considered parameters are: \( \Delta T = T_c - T_e = 60^\circ C, L_e = L_o = L_i, \) and the working fluid is water. Dashed lines in this figure are the data for R123. This working fluid has only been used for validation of this model using the empirical data from Rittidech et al. [36] and in all other cases the working fluid of the PHP is water. For R123 the length of the evaporator section is set to 50mm in order to simulate the conditions used for the corresponding experimental data. The filling ratio and vapor temperature for this comparison are 50% and 50°C, respectively. Figure 2 indicates the present model which predicts heat transfer performance of the PHP with an acceptable tolerance. Additionally, all of the results indicate that the heat flux into the PHP reduces by increasing the length of the evaporator. This fact has also been concluded by other empirical results [36].

Figure 3 shows the liquid slug displacement with respect to time in comparison with those of Shafii et al. [10]. The following parameters are considered for calculation: \( P_{e}, 5628 \)Pa, filling ratio of 50%, \( T_e = 20^\circ C, \) \( h_i = 150 \frac{m}{m^2} \)W.C, \( h_c = 100 \frac{m}{m^2} \)W.C, \( \Delta T = T_c - T_e = 100^\circ C, L_e = 0.1 \)m, \( 2L_c = 0.37 \)m, \( L_i = 0.35 \)m (results in 61.4% of filling ratio) and \( d = 1.5 \)mm. As the figure illustrates, both the amplitude and the frequency of the oscillations of liquid slug calculated in the present study are higher than the results of Shafii et al. [10]. This difference can be due to the constant evaporative and condensation coefficients and neglecting flow boiling heat transfer coefficient assumptions reported by Shafii et al. [10]. In fact, high values of two-phase heat transfer coefficients cause greater mass transfer into the vapor plugs and consequently, force the slug to be transmitted rapidly between the condenser and the evaporator sections. It should be noticed that high frequency and amplitude of the liquid oscillation has an important role in the sensible heat transfer of the PHP.

Figure 4 depicts the comparison of the data in the present model with that of Shafii et al. [10] and Khandekar et al. [15]. The results show heat flux values that are derived for various temperature differences between the evaporator and condenser sections. This figure clearly shows that compared to Shafii et al.’s [10] model, this model has great advantage in prediction of the heat transfer values. The heat flux values derived using current model are close to the exact values that were presented by Khandekar et al. [15].

Figure 5 illustrates the pressure, temperature and mass of the left and the right hand side vapor plugs. The pulsating heat pipe characteristics used to derive these results are: \( \Delta T = T_e - T_c = 60^\circ C, L_c = 0.1 \)m, \( L_e = 0.2 \)m (results in 50% of filling ratio) and \( d = 2 \)mm, which will remain unchanged in the rest of the study. As the figure shows, contrary to the same frequency and amplitude of the two plugs behavior, there is a 180° phase difference in the oscillations. This is due to the nature of the PHP’s performance that whenever the liquid slug is transmitted.

![Figure 2. Comparison of the heat flux calculated in the present model with experimental results of Charoensawan et al. [18], Rittidech et al. [36] and Khandekar et al. [15] correlation data](image)

![Figure 3. Comparison of the liquid slug displacement derived with the present model with those of Shafii et al. [10]](image)
Figure 4. Comparison of the heat flux derived for various temperature differences in the present model with those of Shafii et al. [10] and Khandekar et al. [15]

Figure 5. Variation of the properties of the vapor plugs: a) Pressure b) Temperature and c) mass

to one of the vapor plugs, the physical properties of that vapor plug are affected due to the compression and boiling phenomena. This is opposite to the other plug, which experiences condensation and decompression. Figure 5c demonstrates that as the liquid slug moves to the left hand side evaporator, the boiling phenomenon induces a sharp increase in the mass of the left vapor plug. At this moment, the right vapor plug moves to the condenser section and its mass experiences a decrease due to condensation.

As stated before, one of the parameters that has a significant role in heat transfer, is the pipe diameter.

Hence, in addition to the main purpose of the present work, i.e. investigating the flow boiling and condensation, the effect of the pipe diameter has also been evaluated. In order to study the performance of the PHP with different pipe diameters, the liquid slug displacement, the pressure and the mass of the left vapor are presented in Figure 6. \( d = 1 \) mm and \( d = 2 \) mm pipe diameters are considered and other characteristics are the same as Figure 5.

As Figure 6a indicates, by increasing the diameter of the PHP, the amplitude of the liquid slug oscillation increases which is consistent with the results of other studies [3, 4, 10].

In contrast, the frequency of the slug oscillation reduces in larger pipe diameters. This phenomenon has also been reported in other studies [3, 4].

Figure 6b illustrates the left vapor pressure versus time. As the figure shows, the pressure diagram oscillates in larger values for \( d = 2 \) mm and this is in agreement with the previous works [3]. The maximum temperature of the vapor plug reaches 100°C and in this situation the maximum pressure is about 64 KPa. So, the vapor is superheated. It should be noted that, in each time step if the pressure exceeds the saturation pressure at the vapor temperature, the code will substitute it with the saturation pressure.
pressure. Figure 6c shows the mass of the left vapor for different diameters. As the figure illustrates, due to the boiling and condensation processes, the larger the pipe diameter, the larger the mean oscillation value.

Figures 7 and 8 indicate the sensible heat transfer rate for the pipe diameters of $d=2$ mm and $d=1$ mm, respectively. It is shown that increasing the pipe diameter leads to an enhance in the sensible heat transfer rate, which is consistent with the previous empirical and numerical results [10, 15, 18, 35]. This phenomenon is due to the greater heating section in larger diameters and the greater volume of the liquid slug being transmitted to the heating section as a result of the larger amplitude of the oscillation. Additionally, the frequency of the transferred heat into the PHP in $d=1$ mm is higher than $d=2$ mm that is due to the higher frequency of the slug displacement. The flow regime change, turbulent or laminar flow, causes a change in the convex or concave and the magnitude of the sensible heat transfer.

Figure 9 illustrates the comparison of the latent heat into (boiling heat) the PHP for $d=2$ mm and $d=1$ mm. The figure indicates that the latent heat into the PHP in 2 mm pipe diameter is higher than $d=1$ mm. As the figure shows, when the liquid slug end moves to the right heating section, the boiling process occurs and the boiling heat transfer jumps to the maximum value and then decreases because of increasing the temperature of the liquid cells.

The total heat flux rate transferred into the PHP with $d=2$ mm is 31.4 (KW/m$^2$) and the contribution of boiling heat transfer is 1.31 (KW/m$^2$) which is 4.1% of the total heat. On the other hand, when $d=1$ mm the total heat transferred into the PHP is 12.2 (KW/m$^2$) while the boiling heat transfer contribution is 8.3% of the total.

5. CONCLUSIONS

The performance of a pulsating heat pipe by consideration of flow boiling and condensation correlations is studied numerically. All of the fundamental equations are solved by an explicit method except the energy equation for the liquid slug. The best correlations of the flow boiling and condensation are used to evaluate the mass change and the heat transfer into and out of the PHP. According to the results, the contribution of the sensible heat is dominant in total heat input in PHP. The results showed that the frequency and the amplitude of the liquid slug displacement are larger in comparison to the previous mathematical models. The effect of the pipe diameter on the sensible and boiling heat and also on the amplitude and the frequency is investigated. The results showed that by increasing the pipe diameter, the amplitude and sensible heat and also the boiling heat increase contrary to the frequency of the PHP, which decreases. It is also shown that as the pipe diameter increases, the contribution of the sensible heat reduces. The results have been compared to the empirical and numerical results that proved good estimation of the present model.

In the present study, annular flow condensation is considered. However, this assumption is governed as long as the liquid slug velocity is high enough. Otherwise, falling condensation will be the dominant mechanism. So, for the future work, one can modify the
presented model considering falling condensation in the condenser section.

6. REFERENCES


Modeling of A Single Turn Pulsating Heat Pipe based on Flow Boiling and Condensation Phenomena

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چکیده

سیستم‌های خنک‌کننده، با کارایی بالا در اجرای نصب‌های مختلف در صنایع مختلف، در حال حاضر به عنوان یکی از چالش‌های مهم در صنعت قرار گرفته است. لوله‌های حرارتی جوشکارانه از مهم‌ترین راهکارهای موثر برای حل این چالش می‌باشند. در این مقاله، با استفاده از اصول جوشش و چگال‌ش ی جوشکارانه برای راه‌حل‌هایی که در نظر گرفته می‌شود، مدل‌سازی تحلیلی یک لوله حرارتی جوشکارانه مورد استفاده قرار می‌گیرد. این رابطه موثر در نظر گرفته شده است که نتایج صحیح‌تری بدست آورد. پدیده‌ی چگال‌ش حباب‌زا به عنوان مکانیزم غالب در فرآیند جوشش در ناحیه اواپراتور در نظر گرفته شد. در این مقاله، برای اولین بار، جریان دو فازی حلقوی در نظر گرفته شد که در نتیجه این نظریه، نتایج صحیح‌تری بدست آورده شد. انرژی محاسبه شده با نتایج تجربی برای راه‌حل‌های موجود، با پدیده‌ی انتقال حرارت از جسم به جامد در نظر گرفته شده است. این علاوه بر این، تأثیر قطر لوله بر روی جریان و انرژی محاسبه شده را نشان می‌دهد. برای مقایسه با نتایج مقایسه کرده، محلول محاسبه و نتایج مورد بررسی قرار گرفته که نتایج محاسبه شده با نتایج تجربی بسیار مطابقت داشته و تأثیر انرژی محاسبه شده با نتایج گرفته شده است. در نتیجه، می‌توان گفت که این روش برای حل این چالش‌ها کاملاً کارایی دارد.