Voltage Differencing Buffered Amplifier based Voltage Mode Four Quadrant Analog Multiplier and its Applications

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**ABSTRACT**

In this paper a voltage mode four quadrant analog multiplier (FQAM) using voltage differencing buffered amplifier (VDBA) based on quarter square algebraic identity is presented. In the proposed FQAM the passive resistor can be implemented using MOSFETs operating in saturation region thereby making it suitable for integration. The effect of non idealities of VDBA has also been analyzed in this paper. Theoretical propositions are verified through SPICE simulations at 0.18μm CMOS technology node and the simulation results are found in close agreement with theoretical values. The supply voltage is taken as ± 1V and the value of the bias current is set to 40µ A. The simulated total harmonic distortion (THD) is observed to be under 3% and the total power dissipation is found as 627µ W. The workability of the proposed FQAM is also tested through two applications, namely, an amplitude modulator and a rectifier. The simulated results corroborate the theoretical propositions.

**NOMENCLATURE**

<table>
<thead>
<tr>
<th>V</th>
<th>Volt</th>
</tr>
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<tbody>
<tr>
<td>R</td>
<td>Resistance</td>
</tr>
<tr>
<td>n</td>
<td>nano</td>
</tr>
<tr>
<td>A</td>
<td>Ampere</td>
</tr>
<tr>
<td>µ</td>
<td>Micro</td>
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<tr>
<th>Greek Symbols</th>
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<tr>
<td>ε</td>
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<tr>
<td>α&lt;sub&gt;p,n&lt;/sub&gt;</td>
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<td>β&lt;sub&gt;p,n&lt;/sub&gt;</td>
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<tr>
<td>Ω</td>
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**1. INTRODUCTION**

The FQAM performs multiplication of two bipolar signals and preserves the polarity relationship. Analog multiplier is used extensively for nonlinear applications such as a modulator, equalizer, frequency doublers, and neural computing [1]. A large number of analog multipliers are available in literature [2-14] using different analog building blocks (ABBs) having there own pros and cons.

On the other hand, researchers are continuously striving to explore different ABBs with attributes like higher bandwidth, higher slew rate, lower power consumption, and better linearity [15]. This journey initiated way back in 1966 with current conveyor [16] and still continues. As a result numerous applications using different ABBs have already being developed [17-23]. The voltage differencing buffered amplifier (VDBA) has emerged as an outcome of this consistent effort relatively recently [17]. Apart from possessing the above mentioned attributes the VDBA also provides electronic tunability through its transconductance. Various applications based on VDBA and their variants are available in the literature [15,18-21]. However, limited literature is available on VDBA based non linear applications. So the aim of this study is to present VDBA-based FQAM using the quarter square technique. All available analog multipliers [2-14] are summarized in Table 1 to identify the gap in the previous research and to justify the proposition of voltage mode VDBA based FQAM. Some of the inferences from Table 1 are:

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The structures discussed in literature [2-9, 12] are not suitable from portability view point due to higher power supply requirements. Structures discussed in literature [2-4, 10, 12 and 13] use a large number of ABBs.

The bandwidth of all the listed configurations other than reported value [10] is smaller as compared to the proposed configurations.

Multipliers reported literature [4, 5] use passive components which is not suitable for integrated circuit.

Voltage output is available at a high impedance in literature [10] making a buffer necessary to drive the voltage input circuits.

The work presented in literature [2] is a two quadrant, while the multiplier designed in another paper [5] can work as a two or four quadrant multiplier.

Thus the intention of this paper is to propose a VDBA based FQAM which operates at the low power supply consumes less power and provides higher bandwidth.

### 2. CIRCUIT DESCRIPTION

In this section, the generalized square algebraic identity based FQAM [1] is described first which has been adapted for implementation with VDBA.

#### 2.1. Generic FQAM based on Quarter Square Algebraic Identity

An FQAM can be implemented using well known quarter square algebraic identity given below:

\[(V_m1 + V_m2)^2 - (V_m1 - V_m2)^2 = 4V_m1V_m2\]  \(\text{(1)}\)

It may be observed from Equation (1) that FQAM based on quarter square algebraic identity can be designed with the help of adder, subtractor and squarer circuits arranged as depicted in Figure 1

#### 2.2. Proposed VDBA based FQAM

The circuit symbol of VDBA is shown in Figure 2, the terminals \(p\) and \(n\) are high impedance input voltage differing terminals. The \(z\) represents a high impedance output current terminal and the \(w-(w+)\) is the low impedance buffered inverted (non inverted) output terminal. Due to the high input and low output impedances, this block is well suited for voltage mode operation. The port relationship of the VDBA is described in Eq. (2).

\[
\begin{array}{cccc}
I_z & 0 & 0 & 0 \\
V_w & 0 & 0 & 0 \\
V_n & 0 & 0 & 0 \\
I_p & 0 & 0 & 0 \\
v & 0 & 0 & 0 \\
\end{array}
\begin{array}{c}
0 \\
1 \\
-1 \\
0 \\
0 \\
0 \\
\end{array}
\begin{array}{c}
g_m - g_m \\
g_m \\
g_m \\
g_m \\
g_m \\
\end{array}
\begin{array}{c}
V_m \\
I_m \\
I_m \\
I_m \\
v_m \\
\end{array}
\]  \(\text{(2)}\)

where \(g_m\) represents the transconductance of VDBA and is \(g_m\) is expressed as follows:

\[
g_m = \frac{V_m}{I_m}
\]  \(\text{(3)}\)
where \( g_m \) represents the transconductance of VDBA

It may be observed that the value of \( g_m \) can be controlled through bias current \( I_b \).

The proposed VDBA based FQAM is shown in Figure 3. The circuit comprising of VDBA I and resistance \( R_1 \) represents an adder whereas VDBA II along with resistance \( R_1 \) performs the subtraction operation.

The current and voltage outputs of VDBA I are expressed in Equations (4) and (5), respectively: Similarly, for VDBA II Equations (6) and (7) represent the current and voltage outputs.

\[
I_{c1} = g_m(V_{in1} + V_{in2})
\]  

\[
V_{w1} = -V_{w1} = g_m R_1(V_{in1} + V_{in2})
\]  

\[
I_{c2} = g_m(V_{in1} - V_{in2})
\]  

\[
V_{w2} = -V_{w2} = g_m R_1(V_{in1} - V_{in2})
\]  

**Figure 1.** Block diagram of Quarter square algebraic identity [1]

**Figure 2.** The VDBA Circuit Symbol

**Figure 3.** Proposed VDBA based FQAM

From Figure 4 current through MS3 can be written as follows:

\[
I_{MS3} = I_{MS1} + I_{MS2}
\]

As \( K_{MS1} = K_{MS2} = K = 0.5 K_{MS3} \)

\[
\frac{2K}{2}(V_{ss} - V_{n} - V_{m})^2 = K(V_{1} - V_{ss} - V_{m})^2
\]

\[
+ (-V_{1} - V_{ss} - V_{m})^2
\]

\[
V_{SO} = -\frac{V_s^2}{2(V_{ss} + 2V_{TH})} + \frac{V_{ss}}{2}
\]

where \( V_s \) is the applied input voltage. \( V_{SO} \) is the output of the squarer circuit. Using Equations (8) and (10) current through respectively can be written as follows:

\[
I_{MS3} = K(V_{SO} - V_{SS} - V_{TH})^2
\]

Now putting the value of \( V_{SO} \) from Equation (12) into Equation (13),

\[
I_{MS3} = \frac{K(V_s^4 + 2V_s^2(V_{n} + 2V_{TH})^2 + (V_{n} + 2V_{TH}))}{4(V_{n} + 2V_{TH})^2}
\]

For small signals, \( V_s^4 \approx 0 \)

\[
I_{MS3} = \frac{K}{2}V_s^2 + \frac{K}{4}(V_{ss} + 2V_{TH})^2
\]

\[
V_{SO} = \frac{I_{MS3}}{g_s} = \frac{K}{2g_s}V_s^2 + \frac{K}{4g_s}(V_{n} + 2V_{TH})^2
\]

Equation (16) suggests that the output voltage of the squarer circuit (\( V_{SO} \)) is proportional to the square of the input voltage (\( V_s \)). The outputs of VDBA I and VDBA II as expressed by Equations (5) and (7) serve as inputs to the squarer blocks of Figure 3. Using Equation (16)
the output voltages of the squarer I and squarer II, may be written as Equations (17) and (18), respectively.

\[ V_{1i} = \frac{K_v}{2g_{ms}} V_{i1}^2 + \frac{K_v}{4g_{ms}} (V_{i1} + 2V_{TH})^2 \]  
(17)

\[ V_{2i} = \frac{K_v}{2g_{ms}} V_{i2}^2 + \frac{K_v}{4g_{ms}} (V_{i2} + 2V_{TH})^2 \]  
(18)

Where \( g_{ms} \) represents the transconductance of squarer. The output voltage (\( V_{out} \)) can be determined as follows:

\[ V_{out} = V_{i1} - V_{i2} = \frac{K_v}{2g_{ms}} (V_{i1}^2 - V_{i2}^2) \]  
(19)

From Equations (5), (7) and (19)

\[ V_{out} = \frac{K_v}{2g_{ms}} R^2 ((V_{in1} + V_{in2})^2 - (V_{in1} - V_{in2})^2) = CV_{in1}V_{in2} \]  
(20)

Where \( C = \frac{2K_v}{g_{ms}} R^2 \)  
(21)

Thus, it may be concluded from Equation (21), the output voltage is proportional to the multiplication of the two input signals. The passive resistors consume a large chip area as compared to active resistors. Thus from integration view point, it is always preferred to implement a passive resistor using MOSFETs operating in the linear region.

The resistance \( R_1 \) in Figure 3 can be realized using MOSFETs as shown in Figure 5 where both the transistors \((M_{R1} \text{and } M_{R2})\) with aspect ratio as \( W_{M}/L_{M} \) are operating in saturation. The equivalent resistor \( R \) may be expressed as follows:

\[ R = \frac{V_o}{I_o} = \frac{I_{DD}}{2K_v g_{ms}(V_{DD} - V_{TH})} = \frac{1}{2K_v g_{ms}(V_{DD} - V_{TH})} \]  
(22)

Substituting the value of \( R_1 \) from Equation (22) into Equation (20) the \( V_{out} \) may be expressed as follows:

\[ V_{out} = \frac{2K_v}{g_{ms}} \left[ \frac{g_{ms}}{2} (V_{in1} - V_{in2}) \right]^2 \]  
(23)

and the C of Equation (21) modifies to following expression:

\[ C = \frac{2K_v}{g_{ms}} \left[ \frac{g_{ms}}{2} (V_{in1} - V_{in2}) \right]^2 \]  
(24)

3. NON-IDEAL ANALYSIS

In the analysis so far, the VDBA characteristics are considered to be ideal. However, in CMOS implementation of VDBA transconductance and voltage tracking errors may exist due to device mismatch which may lead to deviation from the ideal behavior. Therefore, the effect of non-idealities of VDBA needs to be considered on the performance of the proposed FQAM configuration. Taking the tracking errors into consideration Equation (2) modifies to following expression:

\[
\begin{align*}
I_e &= 0 \quad 0 \quad 0 \quad \sigma_p V_{in1} - \sigma_n V_{in2} & V_v \\
V_{en} &= \beta_p \quad 0 \quad 0 \quad 0 & I_v \\
V_{en} &= -\beta_n \quad 0 \quad 0 \quad 0 & I_v \\
I_p &= 0 \quad 0 \quad 0 \quad 0 & V_p \\
I_s &= 0 \quad 0 \quad 0 \quad 0 & V_v
\end{align*}
\]  
(25)

where \( \sigma_p(\sigma_n) \) represents transconductance transfer ratio from \( p(n) \) to \( z \) terminal and similarly \( \beta_p(\beta_n) \) represent voltage transfer ratio from \( z \) to \( w^+ (w^-) \) terminal. The non-idealities of VDBA have been summarized in Table 2.

Including the non-idealities of VDBA the outputs of VDBA I and VDBA II of Figure 3 represented by Equations (5) - (7) get modified as Equations (26)-(29):

\[ V_{n\rightarrow z} = \frac{\beta_p g_{ms}(\alpha_p V_{in1} + \alpha_n V_{in2})}{2K_v (V_{DD} - V_{TH})} \]  
(26)

<table>
<thead>
<tr>
<th>TABLE 2. VDBA Non-Idealities</th>
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<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>-----------------------------</td>
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<tr>
<td>Voltage transfer ratio</td>
</tr>
<tr>
<td>Transconductance transfer ratio</td>
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<tr>
<td>Tracking error</td>
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</tbody>
</table>

Figure 4. Squarer Circuit [13]

Figure 5. The NMOS based resistor (R) realization
\[ V_{\text{out}, \alpha} = \frac{\beta_\alpha \alpha \beta (\alpha V_{\text{in}, \alpha} + \alpha V_{\text{in}, 2})}{2K_R (V_{DD} - V_{TH})} \] (27)

\[ V_{\text{out}, \beta} = \frac{\beta_\beta \beta (\alpha V_{\text{in}, \beta} - \alpha V_{\text{in}, 2})}{2K_R (V_{DD} - V_{TH})} \] (28)

\[ V_{\text{out}, \gamma} = \frac{\beta_\gamma \gamma \beta (\alpha V_{\text{in}, \gamma} - \alpha V_{\text{in}, 2})}{2K_R (V_{DD} - V_{TH})} \] (29)

Considering \( \beta_{\text{bias}} = \beta \),

\[ V_{\text{out}, 1 + \alpha} = \frac{\beta_{\text{bias}} \alpha (\alpha V_{\text{in}, \alpha} + \alpha V_{\text{in}, 2})}{2K_R (V_{DD} - V_{TH})} = -V_{\text{out}, \alpha} \] (30)

\[ V_{\text{out}, 2 + \alpha} = \frac{\beta_{\text{bias}} \alpha (\alpha V_{\text{in}, \alpha} - \alpha V_{\text{in}, 2})}{2K_R (V_{DD} - V_{TH})} = -V_{\text{out}, \alpha} \] (31)

Using Equations (16), (30) and (31), the output voltage of the FQAM can be written as follows:

\[ V_{\text{out}, \alpha} = \left( \frac{K_\alpha}{2e} \right) \left( \frac{\beta_{\text{bias}}}{2K_R (V_{DD} - V_{TH})} \right)^2 \left( \alpha V_{\text{in}, \alpha}^2 + \alpha V_{\text{in}, 2}^2 - (\alpha V_{\text{in}, \alpha} - \alpha V_{\text{in}, 2})^2 \right) \] (32)

\[ V_{\text{out}, \beta} = \left( \frac{K_\beta}{2e} \right) \left( \frac{\beta_{\text{bias}}}{2K_R (V_{DD} - V_{TH})} \right)^2 \left( \alpha V_{\text{in}, \beta}^2 + \alpha V_{\text{in}, 2}^2 - (\alpha V_{\text{in}, \beta} - \alpha V_{\text{in}, 2})^2 \right) \] (33)

For \( \alpha_{\text{out}} = \alpha_{\text{in}} = \alpha \), the resultant output voltage may be expressed as follows:

\[ V_{\text{out}, \alpha} = (\alpha \beta)^2 CV_{\text{in}, \alpha} V_{\text{in}, 2} \] (34)

However, the deviation caused in the output due to tracking errors may be ignored as the values of \( e_{\text{gap}}, e_{\text{gm}}, e_{\text{e}}, \) and \( e_{\text{p}} \) are much smaller than unity.

4. SIMULATION RESULTS

The working of the proposed FQAM circuit is confirmed through SPICE simulations. The CMOS implementation of the VDBA [21] shown in Figure 6 is used and 0.18\( \mu \)m technology node is used for simulations.

The supply voltage used is \( \pm 1 \) V. The value of the bias current is set to 40\( \mu \)A. The aspect ratios for the transistors used in VDBA, squarer circuit, and \( R_1 \) implementation are given in Table 3.

\begin{table}[h]
\centering
\caption{Aspect Ratio of the transistors}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
Transistor & M1-M3 & M4-M6 & M5-M7 & M8-M9 & M10-M12 \\
\hline
(W/L)\( \mu \)m & 0.300/18 & 21.60/72 & 5.40/27 & 10.80/27 & 0.27/27 \\
\hline
\end{tabular}
\end{table}

The values of \( \alpha_p \) and \( \alpha_n \) are found 0.934 and 0.933, respectively. Similarly, \( \beta_p \) and \( \beta_n \) were obtained as 0.95 and 0.97, respectively.

4.1. The DC Characteristics

The DC transfer characteristics for the proposed FQAM is depicted in Figure 7(a) with the variation of \( V_{\text{in}, 2} \) from -40 mV to 40 mV. It is observed from Figure 7(a) that the proposed circuit is an FQAM. The output voltage varies linearly with \( V_{\text{in}, 1} \) as observed from Figure 7(a). The nonlinearity curve for the proposed FQAM is shown in Figure 7(b) which depicts that over the entire input range maximum nonlinearity is less than 0.03%. The simulated power consumption is found to be 627\( \mu \)W when \( V_{\text{in}, 1} \) and \( V_{\text{in}, 2} \) are kept grounded.

4.2. AC Characteristics

The simulated frequency response for output voltage is presented in Figure 8. A DC voltage source is applied to \( V_{\text{in}, 2} \) while \( V_{\text{in}, 1} \) is taken as AC source of 100 mV. The 3dB bandwidth of the proposed FQAM circuit is found to be 220 MHz.

4.3. Total Harmonic Distortion

The Total Harmonic Distortion (THD) is a measure to estimate the degree to which a system is nonlinear. Therefore the variation in THD as a function of input signal amplitude is observed. For this, a constant DC voltage of 50 mV is applied to \( V_{\text{in}, 2} \) while a sinusoidal signal of 100 kHz is applied to \( V_{\text{in}, 1} \) with amplitude varying. The measured THD is plotted in Figure 9.

Figure 6. CMOS Implementation of VDBA [21]

Figure 7. (a) DC characteristics of proposed multiplier (b) % Nonlinearity curve

Figure 8. AC characteristics
The similar measurements are obtained for two other input signals having frequencies 1 kHz and 1 MHz, respectively and found that for input signals < 50 mV the maximum THD remains under 3% for the proposed FQAM for all three cases.

4. 4 Noise Analysis The noise limits the minimum signal level that a circuit can process with acceptable quality. Therefore, the effect of noise on the proposed circuit is examined through SPICE simulations. The equivalent input noise and the equivalent output noise are plotted in Figures 10(a) and 10(b), respectively with varying input frequency. For simulations, $V_{in1}$ is taken as a 100 mV AC signal whereas $V_{in2}$ is chosen as 80 mV DC value. The observed $V_{inoise}$ and $V_{onoise}$ are 21.75 nV/√Hz and 10.38 nV/√Hz, respectively.

5. APPLICATIONS

5. 1. Amplitude Modulator An amplitude modulator can be designed provided the carrier and modulating signals are applied to the two inputs of an FQAM, respectively. To validate the functionality of proposed FQAM as an amplitude modulator, two sinusoids of 100 mV/100 kHz and 100 mV/10 kHz frequencies were applied at $V_{in1}$ and $V_{in2}$, respectively. The input transient and corresponding spectrum are shown in Figures 11(a) and 11(b), respectively. Similarly, Figure 12(a) depicts the amplitude modulated output with its spectrum in Figure 12(b). The output frequency spectrum has two frequency components of 90 kHz and 110 kHz thereby confirming the modulation operation.

5. 2. Rectifier The rectifier can be implemented using multiplier by taking one of the inputs as pulse type having frequency same as that of the signal which is to be rectified. To verify the workability of rectifier two inputs namely a sinusoidal signal and a square wave of 100 kHz/100 mV each were applied to the respective inputs of FQAM.

The input transient is shown in Figure 13(a) and the rectified output is presented in Figure 13(b). The simulated ripple factor curve is plotted in Figure 13 (c).
6. CONCLUSION

A voltage mode FQAM based on quarter square - algebraic identity employing VDBA is proposed in this paper. The circuit is suitable for integration as passive resistor may suitably be implemented using MOSFETs. The theoretical propositions have been verified through SPICE simulations using 0.18µm CMOS process parameters. The power dissipation is found low as compared to other available structures and the simulated THD is well below 3%. Applications like amplitude modulator and rectifier are predesigned using proposed structure to show its applicability and results are in total agreement with the theory.

7. REFERENCES

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چکیده
در این مقاله یک حالت ولتاژ چهار ضریب آنالوگ چهارگانه (FQAM) با استفاده از قویت کننده بافربانک (VDBA) بر اساس هویت (Quarter Square Algebraic Identity) مطرح شده است. در این مقاله مقاومت منفعل را می‌توان با استفاده از MOSFET‌های بی‌پیشنهادی، مقاومت مفعول را و نیز مقاومت از نوع فرآیندهای های معمول در محفظه اشیاء اجزا MOSFET استفاده کرد و نیز مودهای نوری فرآیندهای معمول در محفظه اشیاء اجزا MOSFET استفاده کرد. این مقاله نیز در مورد اثربخشی از طریق متغیرهای سازی SPICE در گره CMOS 0.18μm بررسی شده و نتایج آن تایید می‌شود. این مقاله نیز اثربخشی از طریق تایید SPICE و شبیه‌سازی تأکید می‌کند. شرایط این مقاله، میزان ارگانیسمی و کاهش فرآیندهای های موجود در سازه‌های مختلف و کاهش تداخلات تأثیرگذار در سیستم‌های مختلف مورد بررسی قرار گرفته است. در این مقاله نیز اثربخشی از طریق شبیه‌سازی و شبیه‌سازی شبکه‌های مختلف مورد بررسی قرار گرفته است. نتایج شبیه‌سازی این مقاله نیز اثربخشی از طریق شبیه‌سازی شبکه‌های مختلف مورد بررسی قرار گرفته است. نتایج شبیه‌سازی این مقاله نیز اثربخشی از طریق شبیه‌سازی شبکه‌های مختلف مورد بررسی قرار گرفته است.