Quaternion-based Finite-time Sliding Mode Controller Design for Attitude Tracking of a Rigid Spacecraft during High-thrust Orbital Maneuver in the Presence of Disturbance Torques

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**Abstract**

In this paper, a quaternion-based finite-time sliding mode attitude controller is designed for a spacecraft performing high-thrust orbital maneuvers, with cold gas thrusters as its actuators. The proposed controller results are compared with those of a quaternion feedback controller developed for the linearized spacecraft dynamics, in terms of settling time, steady-state error, number of thruster firings and their fuel usage. It is then proved that the sliding mode control has enough robustness against disturbances as well as a high accuracy in attitude tracking and also a low number of thruster firings. A 6 degree of freedom (DOF) total simulation, including spacecraft dynamics, guidance, navigation and control systems is also designed and the sliding mode controller performance in a sample transfer from an ecliptic orbit to a circular one is investigated. In order to solve the chattering problem caused mainly because of the discontinuity of sliding mode control algorithm and multiple switching on sliding surfaces, the sign function in the control input is replaced with a hyperbolic tangent function. Being aware of the advantages of sliding mode control method, using this algorithm in orbital transfers seems to be innovative and efficient.

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1. **Introduction**

A major part of any space mission is the orbital maneuvering process. The usage of orbital maneuvers is not only limited to the interplanetary missions, i.e., almost all space missions are accomplished by putting the spacecraft in an initial orbit and then transferring it to the final one, which results in reducing the costs of launch. With the increasing computing capabilities of space flight over the past years, extensive research has been done on spacecraft attitude control systems. Several algorithms have been designed to enhance the control system's robustness against external disturbances, uncertainties of modeling, and also to optimize the required time for the mission or the fuel consumption of the spacecraft. Typically, the main task of the attitude control system in orbital maneuvers, is to track the angular commands of the guidance system in order to make sure that the spacecraft velocity change vector \( \mathbf{V}_{\Delta} \) at the time of applying the guidance impulse, is aligned with that calculated by the guidance algorithm. Calculations of the spacecraft orbital parameters and its velocity and position in orbit, are done by the guidance system in order to determine the optimal and appropriate transfer trajectory and also the required impulse magnitudes. Considering the uncertainties in modeling of the dynamical system and also the external disturbances, it is possible that in some orbital maneuvers, the angular commands of the guidance system are such that rotations with large angles are required. Therefore, the attitude controller needs to be capable of performing large angle maneuvers. In 2011, Ming Xin and Hejia Pan [1] used a nonlinear optimal control in order to increase the performance of attitude and position maneuvers and also reduce the flexible movements caused by large angle rotations. Lu and Xia [2] considered attitude tracking of a rigid spacecraft in a finite time, under external disturbances and inertial uncertainties back in 2013.

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First, a unique fast nonsingular terminal sliding mode surface was designed without any constraints, which has not only avoided singularity, but also contained the benefits of both nonsingular terminal sliding mode and the ordinary sliding mode together. The fast nonsingular terminal sliding mode control laws were then introduced, providing finite-time convergence, robustness, faster and higher control precision. In 2007, Pini Garfìl [3] investigated the problem of orbital transfers by continuous-thrust using orbital parameters feedback from a nonlinear control method. The Gauss’s variational equations were used for modelling the spacecraft dynamics of motion in a central gravitational field. A nonlinear feedback controller was then derived, illustrating its performance through simulating orbital transfers between two geosynchronous orbits. Finally, it was proved that the proposed controller resulted in lower fuel consumption in impulsive maneuvers, compared to those performed utilizing other control methods.

The attitude dynamics equations for a rigid spacecraft are described as follows [4]:

\[ T = \dot{h} = \dot{h}_0 + \omega \times h_0 \]  

(1)

where \( T \) denotes the total applied moment vector, \( \dot{h} \) the angular momentum vector, and \( \omega \) the angular velocity vector. The subscripts \( f \) and \( B \) show the inertial frame and body frame, respectively. External moments are also considered as the sum of disturbance \( (\tau_f) \) and control \( (\tau_c) \) torques: \( T = T_c + T_f \).

In this paper, \( T_c \) refers to the control torques provided by cold gas thrusters.

2. ATTITUDE KINEMATICS AND DYNAMICS

The attitude of a three-dimensional body is defined with a set of axes fixed to the body. This set of axes is a triad of orthogonal coordinates, and is called a body coordinate frame. The attitude of a body is thought of as a coordinate transformation that transforms a defined set of reference coordinates into the body coordinates of the spacecraft [4]. The basic three-axis attitude transformation is based on the direction cosine matrix. Any attitude transformation in space is actually converted to this essential form discussed in literature [4]. Two widely used spacecraft attitude determination methods are known to be the Euler angles and quaternion methods. The Euler angle rotation is defined as successive angular rotations about the three orthogonal frame axes [4]. Supposing that the three orthogonal axes of the body frame are defined by \( i, j, \) and \( k \), and those of the reference frame by \( I, J, \) and \( K \), there is a multitude of order combinations by which the rotation can be performed. For instance, one might first perform a rotation about the \( i \) then about the \( j \), and finally about the \( k \) axis. The order of rotation could also be about \( j, i, k \), and so on. The quaternion’s basic definition on the other hand, is a consequence of the properties of the direction cosine matrix [4]. It is shown by linear algebra that a proper real orthogonal \( 3 \times 3 \) matrix has at least one eigenvector with eigenvalue of unity [4]. Quaternions inherently come along with some advantages such as no singularity - because of not including trigonometric components - and being computationally less intense compared to other attitude parameters such as Euler angles or direction cosine matrices. Although Euler angles are easy to develop and visualize, they are computationally intense. Also, a singularity problem may occur when describing spacecraft attitude kinematics in terms of Euler angles, therefore, it cannot be considered as an effective method for expressing the spacecraft attitude. The widely used quaternion representation which is based on Euler’s rotational theorem, states that the relative orientation of two coordinate systems can be described by only one rotation about a fixed axis. Therefore, considering their advantages, using quaternions seems to be the best and the most effective way for describing spacecraft attitude. The equations of motion of a spacecraft can generally be divided into kinematic and dynamic equations. The kinematic equation of the spacecraft in terms of quaternions is given by Vadali [5]:

\[ \beta = \frac{1}{2} [G(\omega)] \beta \]  

(2)

where

\[ [G(\omega)] = \begin{bmatrix} 0 & -\omega_k & -\omega_j & -\omega_i \\ \omega_k & 0 & \omega_i & -\omega_j \\ -\omega_j & -\omega_i & 0 & \omega_k \\ -\omega_i & \omega_j & -\omega_k & 0 \end{bmatrix} \]  

(3)

It is known that the quaternions are constrained by Vadali [5]:

\[ \beta' \beta = 1 \]  

(4)

The dynamic equations of motion of a rigid spacecraft has been discussed in literature [4]:

\[ I_\omega \omega = (I_2 - I_1) \omega_2 + \omega_3 + d_1 \]
\[ I_\omega \omega = (I_1 - I_2) \omega_3 + \omega_1 + d_2 \]
\[ I_\omega \omega = (I_2 - I_3) \omega_2 + \omega_3 + d_3 \]

\( \omega_1, \omega_2, \) and \( \omega_3 \) are the spacecraft angular velocities and \( I_1, I_2 \) and \( I_3 \) the spacecraft principal moments of inertia and \( \omega_1, \omega_2, \) and \( \omega_3 \) the applied control and external torques, respectively. These equations can also be written in the following form [6]:

\[ I_\omega \omega = -\omega \times (\omega \times \omega) + u + d \]

where the notation \( \omega \times \) shows a skew-symmetric matrix.
having the following structure:

\[
\omega' = \begin{bmatrix}
0 & -e_3 & e_1 \\
e_3 & 0 & -e_1 \\
-e_1 & e_3 & 0
\end{bmatrix}
\]  

(7)

It is assumed that \( f_i = -L_i' \left[ \omega^i \omega \right] \) and \( h_i = L_i' \). Therefore, Equation (6) takes the following form:

\[
\dot{\omega}_1 = f + b \beta_i + d_i ; \; i = 1,2,3
\]  

(8)

### 3. SELECTION OF SWITCHING SURFACES

During the sliding mode, the system states move on the sliding surface towards equilibrium. Such constrained motion can be described by a smaller number of variables than necessary to describe the actual system dynamics. The reduction in the number of variables is equal to the number of constraints or the number of switching surfaces. For example, if the spacecraft angular velocities are constrained to be explicit functions of the spacecraft attitude, the four variables of the vector \( q \) are sufficient to describe the motion. With this in mind, the following reduced-order optimal control problem is posed. To show that Equation (19) satisfies Equation (18), the left- and right-hand sides of Equation (18) are evaluated separately:

\[
LHS = qK^T e
\]  

(20)

\[
RHS = Q \beta + qK^T e + \sqrt{\omega} \begin{bmatrix} 0 \\ e \end{bmatrix}
\]  

(21)

Hence, it needs to be shown that:

\[
Q \beta = -\sqrt{\omega} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}
\]  

(22)

Substituting Equation (19) into Equation (12), \( \omega \) is obtained as follows:

\[
\omega = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = -\sqrt{\omega} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}
\]  

(23)

where \( \tilde{\beta} \) is the reduced Euler parameter vector. From Equation (23) and the structure of \( Q \), it can be easily
verified that Equation (22) is satisfied. Hence, the optimal switching surfaces are given by following expression:

\[ S_i = \alpha_i + k\beta_i = 0 \quad ; \quad i = 1, 2, 3 \]  

(24)

For this special case, it can also be shown that the optimal cost function is given by Vadali [5]:

\[ J^* = 2k\left[1 - \beta_i(t)\right] \]  

(25)

Although \( \beta \) and \(-\beta\) represent the same attitude, the optimal cost function given by Equation (25) for each case is different; specifically, if \( \beta_i(t) \) is negative, the cost function is more. In order to remove this ambiguity, the attitude error vector can be redefined as follows:

\[ e = [\beta_0 - \text{sgn}({\beta}_i(t))\beta_1, \beta_2, \beta_3]^T \]  

(26)

As a result, the switching surfaces will be as follows:

\[ S_i = \alpha_i + k\beta_i \text{sgn}[\beta_i(t)] = 0 \quad ; \quad i = 1, 2, 3 \]  

(27)

And the optimized cost function by the above definition is follows:

\[ J^* = 2k\left[1 - |\beta_i(t)|\right] \]  

(28)

So the sliding motion is described by the following differential equations obtained by eliminating \( \vartheta \) from Equation (2) by using Equation (27).

\[ \dot{\beta}_i = \frac{1}{2}k[1 - \beta_i^2] \text{sgn}({\beta}_i(t)) \quad , \quad i = 1, 2, 3 \]  

(29)

It can clearly be seen that these equations do not depend on any of the spacecraft parameters (namely, moments of inertia). Although a large value of \( k \) is desirable for a fast transient response, this means that the control magnitude increases with \( k \). It is also seen that the derivative \( \dot{\beta}_i \) is either positive or negative, depending on the sign of \( \beta_i(t) \). Hence, without any loss of generality, the term \( \text{sgn}[\beta_i(t)] \) in Equations (27) and (29) can be replaced by \( \text{sgn}({\beta}_i) \). Therefore, the sliding surface takes the following final form:

\[ S_i = \alpha_i + k\beta_i \text{sgn}({\beta}_i) = 0 \quad ; \quad i = 1, 2, 3 \]  

(30)

3. DESIGNING THE CONTROL LAW

The sliding mode control law contains two main parts, equivalent and switching controls. Controls that induce ideal sliding are obtained by the "equivalent control method". The equivalent control is a control that keeps the state trajectory in the vicinity of the sliding manifold. It is obtained by assuming that the switching frequency is infinite.

During ideal sliding on \( S = 0 \), \( \dot{S} \) can be set to zero, so:

\[ \dot{S} = 0 \Rightarrow \dot{\vartheta} + k\beta \text{sgn}(\beta) = 0 \]  

(31)

By using Equation (8), the equivalent control is obtained as:

\[ u_{eq} = -\frac{1}{b_i}(f_i + k\beta \text{sgn}(\beta)) \]  

(32)

The switching control, however, is considered as an extra control effort which forces the quaternion and angular velocity components to reach the sliding surface in a finite time despite of disturbances, and it is computed according to constant reaching law stated as follows:

\[ u_{sw} = -\frac{1}{b_i}c\text{sgn}(S) \]  

(33)

where:

\[ \text{sgn}(S) = \begin{cases} 1 & S > 0 \\ -1 & S < 0 \end{cases} \]  

(34)

Therefore, the control input can be obtained as follows:

\[ u_i = -\frac{1}{b_i}(f_i + k\beta \text{sgn}(\beta) + c\text{sgn}(S)) \quad ; \quad i = 1, 2, 3 \]  

(35)

And in order to reduce chattering, the sign function in the above control input can be simply replaced with a hyperbolic tangent one. Theoretically, the chattering effect is due to the presence of unmolded dynamics in the systems. Since the model used to design the controller can never capture all the system dynamics, it is not possible to obtain an absolutely "chattering free" ordinary sliding mode control. However, it can be omitted by a proper design of the control function. The initial proposed sliding mode controller was implemented with a term representing a sign function. A widely used method to avoid chattering is the use of approximations to the sign function. Saturation, sigmoid, hysteresis and hyperbolic tangent functions are often used, offering a continuous or smooth control signal. For the problem in hand in this paper, since replacing the sign function with a hyperbolic tangent function has resulted in the smoothest control signal compared to the ones obtained by using other alternatives, it is decided to use this approximation to avoid chattering as much as possible. So the final control input will have the following form:

\[ u_i = -\frac{1}{b_i}(f_i + k\beta \text{sgn}(\beta) + c\tanh(S)) \]  

(36)

The proposed control law has two design parameters \((c, k)\) which should be selected to provide stability and better performance. It should be noted that the sliding surface
slope, \( c \) is selected such that the system is stable during the sliding mode. Finally, the quaternion feedback controller for the linearized spacecraft dynamics is proposed as follows [4]:

\[
\begin{align*}
u_1 &= 2K_c \beta_{c1}\beta_{c2} + K_d \omega_1 \\
u_2 &= 2K_c \beta_{c3}\beta_{c4} + K_d \omega_2 \\
u_3 &= 2K_c \beta_{c5}\beta_{c6} + K_d \omega_3
\end{align*}
\]

(37)

where \( \beta_c \) is the quaternions error vector and \( K_c \) and \( K_d \) the controller design parameters.

4. SMC STABILITY PROOF

If any Lyapunov based control method can satisfy the Lyapunov conditions, it will be definitely stable during the control process. For this purpose, by selecting the Lyapunov function as \( V = \frac{1}{2} S^2 \); substituting the proposed control and the sliding surface in this function. Finally, simplifying it, the necessary conclusion of \( \dot{V} < 0 \) must be obtained in order to ensure the stability. By reaching this conclusion, it can be guaranteed that the control law will always be stable during the control process.

As mentioned above, a suitable Liapunov function for the closed loop system is selected [7]:

\[
V = \frac{1}{2} S^2
\]

(38)

The time derivative of \( V \) will be:

\[
\dot{V} = SS^T = S[d - c \text{sgn}(S)] = Sd - c|S| < 0
\]

(39)

which means that for the controller to be always stable, the coefficient \( c \) must be selected as \( c \geq |d_{\text{max}}| \), where \( d_{\text{max}} \) is the upper boundary of the disturbance torques.

5. COMPARING THE CONTROLLERS

An example, a three-axis attitude maneuver is presented in this section. The initial and final conditions are shown in Table 1. The nominal values of \( I_1, I_2, \) and \( I_3 \) are 500, 700, and 700 kg.m\(^2\), respectively. The coefficient \( k \) is chosen as 1.5, and \( c \) is also chosen as 0.7 based on the disturbance torque magnitudes. \( K_c \) and \( K_d \) are also selected as 8 and 40 for each three body axes.

<table>
<thead>
<tr>
<th>TABLE 1. Attitude Maneuver Boundary Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
</tr>
<tr>
<td>Roll angle</td>
</tr>
<tr>
<td>Pitch angle</td>
</tr>
<tr>
<td>Yaw angle</td>
</tr>
</tbody>
</table>

Figures 1 and 2 show the Euler angles and quaternions error respectively for each spacecraft axis. It can be seen from Figure 1 that the SMC makes the Euler angles reach the steady-state in about 20 seconds with a steady-state error of about 0.8 degree. Using this method, thrusters are fired 442 times consuming 2.1 kg of fuel, while the quaternion feedback controller takes about 50 seconds to reach the steady-state with an error of about 1 degree, with thruster activity of 750 times consuming about 2.54 kg of fuel. Figure 2 represents the quaternion errors for each controller. It can easily be seen that quaternion errors have reached the desired condition of \( \beta = [1,0,0,0]^T \) within the settling time.
Figure 3 represents the control moments during the maneuver. During the initial stages of the maneuver, due to the large deviation of the initial states from the switching surfaces, saturation occurs in the first few seconds. Once the system states reach the switching surfaces and the sliding mode is started, the control magnitudes quickly decrease.

6. ORBITAL MANEUVER SIMULATION

In order to verify the performance of the proposed sliding mode attitude controller during orbital maneuvers, a sample orbital transfer from an ecliptic parking orbit to a circular target one with parameters described in Table 2 has been performed. This maneuver is done by a simple open loop guidance algorithm in 4 phases. In phases 1 and 2, the spacecraft increases its orbital height to about 200 km via a double-impulsed Hohmann transfer orbit. In phase 3, at a true anomaly of 180 degrees, the orbit inclination is increased by 2 degrees, and finally in phase 4, there is another increase in orbital height by about 100 km with a single impulse at true anomaly of 0 which also makes the orbit close to circular. It should be noted that an ON/OFF switch is provided in the simulation process which is used to enable the control system only when the spacecraft is close to Ascending/Descending nodes, where the guidance impulse is applied.

This has been done in order to optimize the fuel usage by control thrusters. However, if continuous attitude control is demanded, this switch can be easily removed.

It can be understood from Figure 4 that the first and second impulses are applied at true anomalies of 0 and 180 degrees to increase the semi-major axis to about 6861 km. The third impulse applied at a true anomaly of 180 degrees, increases the orbit inclination to 65 degrees while other orbital elements stay unchanged. The final impulse which is again applied at a true anomaly of 180 degrees, simultaneously increases the semi-major axis to 6961 km and makes the final orbit circular. The control thrusters have fired about 4700 times with a fuel usage of almost 18 kg, while the fuel used by applying guidance impulses via the main engine was about 175 kg, as shown in Figure 5. It can also easily be understood from Figure 6 that the proposed control law could effectively eliminate the Euler angle errors whenever the spacecraft was close to Ascending/Descending nodes with an applicable control moment range shown in Figure 7.

![Figure 3. Control Torques](image)

**TABLE 2. Orbital Transfer Conditions**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Initial Conditions</th>
<th>Final Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semi-major Axis</td>
<td>6681 km</td>
<td>6981 km</td>
</tr>
<tr>
<td>Inclination</td>
<td>63°</td>
<td>65°</td>
</tr>
<tr>
<td>True Anomaly</td>
<td>-45°</td>
<td>Variable</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>0.01</td>
<td>± 0</td>
</tr>
</tbody>
</table>

![Figure 4. Changes in Angular Momentum, Eccentricity, Right Ascension of Ascending Node, Inclination, Argument of Perigee, True Anomaly and Semi-major Axis](image)
7. CONCLUSION

It was concluded that the sliding mode control method by using the above discussed algorithm in the orbital transfers seemed to be very effective, innovative and efficient as well.

8. REFERENCES

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چکیده

در این مقاله، سیستم کنترل ردیابی وضعیت زمان محدود بر پایه فیدبک کواترنیون‌های وضعیت برای فضاپیماهای صلب در طول مانور مداری تراست بالا استفاده از تراست‌های گاز سرد به عنوان وسایل نگهدارنده اکستریمیالی و سیستم کنترل لغزشی طراحی شده است. مدل دینامیکی غیرخطی قضاوت معرفی شده و سطح لغزش و نهایتاً کنترل ملد لغزشی برای این مدل دینامیکی توسعه داده شده است. همچنین نتایج کنترل طراحی شده با نتایج حاصل از یک کنترل فیدبک کواترنیون از نظر زمان نشست، خطای حالت ماندگار، تعداد روشن شدن عملکردها و همچنین مصرف سوخت آنها مقایسه شده و برتری مدل کنترلی لغزشی نتایج مدل کنترلی اثبات شده است. در پایان یک مانور مداری از مدار بیضوی به دایره با کنترل وضعیت توسط سیستم کنترل معرفی شده انجام شده و نتایج حاصل از آن ارائه و بررسی شده است.