Voltage Regulation of a Negative Output Luo Converter Using a PD-PI Type Sliding Mode Current Controller

A. Goudarzian*, A. Khosravi

Department of Electrical Engineering, Faculty of Engineering, Shahrekord Branch, Islamic Azad University, Shahrekord, Iran

1. INTRODUCTION

In recent years, power converters are commonly employed in power supplies and industrial applications [1]. Theoretically, conventional topologies such as buck-boost, Sepic, Zeta, Cuk could be employed for voltage increment. However, effects of switches, diodes, and the consequence of resistances of passive elements and losses limit the practical voltage gain of the aforementioned converters. Moreover, duty ratio of the converters will tremendously increase by increment of the output voltage. This terribly large duty ratio leads to relatively high switching losses and severe reverse recovery crisis [2]. In two last decades, the voltage lift method has been well applied for design of power converters [3]. In addition, the super lift technique geometrically enhances the voltage gain, whereas the negative output Luo converter (NOLC) does the same operation. NOLC has a high voltage gain, high power density and high efficiency. For similar conditions, the switching stress of this converter is less than the switching stress of other conventional converters. It is an extremely useful feature to reduce switching losses. Hence, NOLC is used for this study. In practice, the voltage gain of a NOLC is dependent to parametric resistances of power circuit components and operating point. Hence, a controller is needed for adjustment of the reference signal and guarantee the converter stability. From control viewpoint, the boost converters have a non-minimum phase and time-varying structure. The non-minimum phase converters have a slow dynamical response and small stability margins [4]. Hence, control of a NOLC is more complicated compared with minimum phase systems.

Small-signal analysis is presented for switching converters in literature [5, 6]. However, the duty ratio of a converter is restricted in interval of 0 and 1. The classical controllers cannot meet the dynamical performance of a non-linear converter in saturated region. Commonly, power converters are exposed to load and line variations. These oscillations may affect the

*Corresponding Author Email: alireza9071@gmail.com (A. Goudarzian)
stability of converters. It may challenge the quality, performance and stability of power systems. Therefore, a voltage regulator is required. Peak current mode method (PCM) is a type of nonlinear methods which is designed for converters [7]. However, the problem of PCM is presence of an exterior ramp signal. Consequently, the current cannot exactly reach to its desired level. An adaptive backstepping controller is proposes for a POESLLC by Abjadi et al. [8]. However, this controller needs to precise knowledge of all parameters of the converter, excepting load. Therefore, the cost increases and productivity is reduced. Also, stability analysis and construction of fuzzy controllers are reported for converters, aiming to enhance large signal characteristics [9]. However, there is no systematic method for selection of coefficients of fuzzy controllers.

In the last decade, sliding mode controller (SMC) has been widely regarded for variable structure systems [10, 11]. Two major methods exist to implement an SMC for a switching power converter, namely the indirect and direct SMC techniques. In indirect method, the control law of a converter is obtained using the equivalent control concept [12]. However, the designed sliding variable is not bounded and the system response will be deteriorated. In direct method, the switching function is directly achieved using instantaneous trajectories of the SMC. One method to generate the pulse signals for the converter is to modulate the sliding plane within a parabolic modulator (PM) with variable bandwidth [13] or a constant bandwidth hysteresis modulator (HM) [14]. However, the proposed approaches in literature [13, 14] are used for minimum phase systems. For stabilization of a non-minimum phase converter, sliding mode current control (SMCC) method should be utilized. Mamarelis et al. [15] proposes a simple SMCC for boost and Sepic converters. Although the main objective is the voltage regulation, but any output feedback was not applied to generate the reference input current. So they are weak against large load uncertainties. Furthermore, switching frequency of power electronics systems with traditional direct SMCC will be constrained, due to time delays of command circuits, analogue to analogue conversions for isolated sensors and also, parametric resistances effects of power circuits, limited bandwidth of analogue devices and etc. For power converters, low switching frequency leads to inductor core saturation, noise, large voltage ripple and etc. In addition, implementation of a simple SMCC is proposed in literature [16, 17] using digital processors. However, digital control has a serious drawbacks, which decreases sampling frequency. By observing the reported data in literature [16, 17], it is found that the converter frequency is about 10 KHz for all the tests, provided by the authors. However, the frequency must be larger than the audible frequency i.e. $f_s > 20 \text{ KHz}$. It is recommended for low power converters to work in interval of [20 200] KHz.

In view of the aforementioned discussions, this research presents a new direct SMCC controller via PD-PI type sliding surface. This solution can resolve the problems discussed in the above associated literature [11-17]. The proposed structure is presented in Figure 1. It is consisted of two loops; i) an inner loop through a proportional-integral (PI) controller. It is used to adjust the voltage and alleviate the voltage error, and ii) an outer loop via a proportional-derivative (PD) controller. It should be used for stabilizing the converter dynamics, minimizing inrush current and enhancing the switching frequency. By employing a PD compensator in the SMCC, derivative of the sliding variable (S) will be dependent to the impulse function. Therefore, the sliding surface derivative will be converged to infinite and high switching frequency operation will be achieved, despite of having a practical large hysteresis bandwidth. Other features of the controller, such as the steady state and dynamic performances are investigated in the paper. Theoretically, it is demonstrated that the utilization of a PD controller effectively enhances the switching frequency and practical tests are performed to confirm the validity of the suggested method.

The contents of the paper are as follows. In section 2, a comparison between NOLC and other converters is performed. Design, development and system stability of the proposed PD-PI SMCC for a NOLC are investigated in section 3. The practical results are represented in section 4 and the conclusions are shown in section 5.

2. COMPARISON BETWEEN NOLC AND OTHER CONVERTERS

The NOLC diagram illustrated in Figure 1, and it consists of a semiconductor switch, two diodes, three energy storage elements, resistance and input source. The converter operation investigated in two mode. At the first mode, the switch and diode $D_1$ are turned on and the diode $D_2$ is off. At the second mode, switch and diode $D_1$ are turned off and diode $D_2$ conducts. The advantages of NOLC compared with other converters is expressed in the following expression.
The switching stress is defined as follows:

\[ P_s = V_T I_T \]  

where \( P_s \) is switching losses, \( V_T \) is switch voltage drop in ON and OFF instants and \( I_T \) is switch current in ON and OFF instants. \( V_T \) and \( I_T \) are obtained for NOLC as follows:

\[ V_T = v_o \]  

\[ I_T = I_{in} = \frac{P_0}{E} \]  

where \( \Delta V_{C1} \) is the voltage ripple of the middle capacitor and \( P_0 \) is output power. By using Equations (1) and (2), the switching losses of the NOLC can be given as follows:

\[ P_s = \frac{V_o P_0}{E} = GP_0 \]  

where \( G = V_0/E \) is the voltage gain. Similarly, the switching stress of buck-boost (BB), Cuk, SEPIC and Zeta converters can be determined as follows:

\[ P_s = (G + 1)P_0 \]  

Comparing the switching stress of NOLC as expressed in Equation (3) with Equation (4). It shows that the switching stress of BB, Zeta, Cuk and SEPIC converters extremely increases in high voltage gains. But, the switching losses of the NOLC is less than others despite of increment of the voltage gain, resulting in low switching losses and high efficiency of the NOLC.

2. SMCC DESIGN

2.1. Principles The operation of a SMCC depicted in Figure 2 for a NOLC. It provides a three-dimensional phase portrait for the inductor current \( i_L \), output voltage \( v_o \) and sliding variable \( S \). For any initial operating condition that the sliding variable doesn’t start from zero, the state trajectory of the sliding plane is constituted by three regions; namely, the reaching mode, the sliding mode, and the steady mode. In the first region, the state variables are enforced to approach to the sliding variable from any initial conditions. As soon as the sliding surface touches the origin, the state space variables of the converter slide along the sliding variable and approach to their equilibrium points in the second region. Finally, the phase trajectories of the sliding variable are maintained at zero, representing the steady region with no error. The operation of a perfect SMC needs an infinite switching frequency performance of the NOLC. However, in practical applications, the sliding motion of the phase trajectories often introduces the chattering problem at vicinity of the origin. For solving this problem, hysteresis modulation is proposed. However, this method reduces the switching frequency to a limited range.

Also, time delays restrict the frequency of a SMC. One suitable method to reduce the trajectory chattering is directly use of the PD-PI type of SMC within a constant hysteresis bandwidth.

2.2. Closed Loop Analysis of the NOLC with the Proposed Pd-Pi SMC A PD-PI SMC for a non-minimum phase NOLC is depicted in Figure 1. The reduced order averaging model of a NOLC in CCM can be determined as follows:

\[
\frac{di_L}{dt} = \frac{E}{L} - \frac{(1-u)\nu_o}{L} \]  

\[
\frac{dv_o}{dt} = \frac{(1-u)\nu_o - v_o}{RC} \]  

For NOLC, the voltage value of the middle capacitor equals to input voltage for all times. A second order PD-PI type of the sliding variable is chosen as follows:

\[ S = a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4 = a_1 e_1 + a_2 e_1 + a_3 e_2 + a_4 \int e_2 dt \]  

where \( e_1 = i_L - I_d \) and \( e_2 = v_o - V_d \) are the current and voltage tracking errors, respectively. Also, \( a_1, a_2, a_3 \) and \( a_4 \) are the parameters of the controller. For the converter, the control law of the SMC is as follows:

\[ u = \begin{cases} 1 & \text{for } S < -\epsilon \\ 0 & \text{for } S > \epsilon \\ \text{unchanged } & \text{for } -\epsilon < S < \epsilon \end{cases} \]  

The sliding phase portrait of the controlled system is obtained as follows:

\[ a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4 = 0 \]  

Furthermore, \( \dot{e}_1, \dot{e}_2, \dot{e}_3, \dot{e}_4 \) can be derived as:

\[ \dot{e}_1 = \frac{E}{L} - \frac{(1-u)\nu_o}{L} \]  

\[ \dot{e}_2 = \frac{a_2}{L} - \frac{(1-u)\nu_o}{L} \]  

\[ \dot{e}_3 = v_o - V_d \]  

\[ \dot{e}_4 = v_o \]  

where \( L, C \) and \( R \) are the inductor, output capacitor and resistance of the NOLC, respectively. Also, \( i_L, v_o, E \) and
u are the inductor current, output voltage, input voltage and switching state of the converter. u is 1 for ON mode and 0 for OFF mode. By using Equations (5)-(9), the time derivative of the designed sliding variable is obtained as follows:

\[
\frac{ds}{dt} = k_1 E - k_1 (1-u)v_0 + k_2 u^2 v_0 - k_2 (1-u)v_0' + a_2 v_0' + a_4 (v_0 - V_d)
\]  

(10)

where

\[
k_1 = \frac{a_1}{4}, k_2 = \frac{a_3}{4}
\]  

(11)

For analysis of the system stability, the existence and stability conditions are checked. The first condition was investigated using a Lyapunov function. Let us define Equation (12) as a positive function for the described system:

\[
F = \frac{1}{2} s^2 > 0
\]  

(12)

Differentiating Equation (12) gives:

\[
F' = \frac{1}{2} s \left( 2a_1 E - 2a_1 (1-u)v_0 + 2a_2 u v_0 + (2a_3 - 2a_2 (1-u)) \left( \frac{1}{1+u} - \frac{v_0}{R C} \right) + 2a_4 (v_0 - V_d) \right)
\]

(13)

when \( S > \varepsilon \), switch is OFF (u=0, \( \dot{u} = -\delta(t) \)) and Equation (13) is simplified as follows:

\[
F' = \frac{1}{2} s \left( 2a_1 E - 2a_1 v_0 - 2a_2 v_0 \delta(t) + (2a_3 - 2a_2) \left( \frac{1}{1+u} - \frac{v_0}{R C} \right) + 2a_4 (v_0 - V_d) \right)
\]

(14)

when \( S < -\varepsilon \), switch is ON (u=1, \( \dot{u} = \delta(t) \)) and Equation (13) is simplified as follows:

\[
F' = \frac{1}{2} s \left( 2a_1 E + 2a_2 v_0 \delta(t) - 2a_3 \frac{v_0}{R C} + 2a_4 (v_0 - V_d) \right)
\]

(15)

By combining Equations (14) and (15), Equation (13) can be expressed as follows:

\[
F' = \frac{1}{2} s \left( 2a_1 E - 2a_3 \frac{v_0}{R C} + 2a_4 (v_0 - V_d) - a_1 v_0 + [L_3 - a_2] \frac{v_0}{R C} - |a_1 v_0 + 2a_2 v_0 \delta(t) - [L_3 - a_2] \frac{v_0}{R C} | \right)
\]

(16)

The above equation satisfies the following inequality:

\[
F' < \frac{1}{2} s \left| [2a_1 E - 2a_3 \frac{v_0}{R C} + 2a_4 (v_0 - V_d) - a_1 v_0 + [L_3 - a_2] \frac{v_0}{R C} - a_1 v_0 - 2a_2 v_0 \delta(t) + [L_3 - a_2] \frac{v_0}{R C} | \right)
\]

(17)

The following condition ensures that \( F' < 0 \):

\[
\frac{2a_1 E - 2a_3 \frac{v_0}{R C} + 2a_4 (v_0 - V_d) - a_1 v_0 + [L_3 - a_2] \frac{v_0}{R C} - a_2 v_0 \delta(t) + [L_3 - a_2] \frac{v_0}{R C} < 0
\]

(18)

leads to:

\[
0 < a_1 E + a_4 (v_0 - V_d) + a_2 v_0 \delta(t) - a_4 \frac{v_0}{R C} < a_4 v_0 - [L_3 - a_2] \frac{v_0}{R C} + a_2 v_0 \delta(t)
\]

(19)

If the initial conditions are \( v_0(0) \geq 0 \) and \( i_L(0) \geq 0 \), the existence conditions will be satisfied and the sliding surface will go to the reaching phase at a finite time. After this region, \( S' \approx 0 \) and \( S = 0 \). Also, \( i_L \approx I_d \) and \( S' = I_d' \approx 0 \). Since \( L_3 \) and \( L_4 \) are very small; thus, \( L_4 (v_0 - V_d) \approx 0 \) and \( a_2 > L_3 > 0 \). Also, \( E > 0 \), \( v_0 > E \) are satisfied for the NOLC, because of its inherent nature; then, if \( i_L > \frac{V_0}{R} \), Equation (19) is satisfied in a wide range of operating conditions.

Since the control law (7) doesn't contain control gain to be adjusted, Equation (19) is predetermined through the controller architecture. For study of the stability condition, the Filippov’s technique is used. After the sliding variable reaches to zero, the discontinuous control law (7) can be replaced by a continuous equivalent control signal \( u_{eq} \) which is obtained by solving \( S' = u' = 0 \) as:

\[
S' = k_1 E - k_1 (1-u)v_0 + k_2 u v_0 - k_2 (1-u)v_0' + a_3 v_0' + a_4 (v_0 - V_d) = 0 \Rightarrow u' = \frac{k_1 (1-u) v_0 + k_2 (1-u) v_0' - k_1 E a_3 v_0' - a_4 (v_0 - V_d)}{k_2 v_0 + k_2 v_0' + k_1 E a_3 v_0' - a_4 (v_0 - V_d)} = 0
\]

(20)

To show that \( u_{eq} \) takes value in the interval of 0 to 1 in the sliding surface, the inequality of Equation (19) must be used. This is transformed to Equation (21) after reaching time \( \delta(t) \) has

\[
L_3 \frac{v_0}{R} < a_1 E + L_4 (v_0 - V_d) + L_3 \frac{v_0}{R} - L_3 v_0 < a_1 v_0 + a_2 \frac{v_0}{R} - a_3 v_0 \Rightarrow 0 < L_3 \frac{v_0}{R} < a_1 E + L_4 (v_0 - V_d) + L_3 v_0 < a_1 v_0 + a_2 v_0 \delta(t)
\]

(21)

Dividing both sides of the inequality (21) by \( L(a_1 v_0 + a_2 v_0') > 0 \). Then, it is given that \( 0 < 1 - u_{eq} < 1 \) or \( 0 < u_{eq} < 1 \). Replacing \( u_{eq} \) into the voltage equation of (5), it yields:

\[
R i_L = \frac{(R C v_0' + V_0) (k_1 v_0 + k_2 v_0')}{k_2 v_0' + k_1 E a_3 v_0' - a_4 (v_0 - V_d)}
\]

(22)

\( i_L \) contains the time derivative of the current error and integral term of the voltage error. It is obtained from \( S \approx 0 \) as:

\[
i_L = \frac{a_1 E - a_1 e^{-a_4 E}}{a_1 + a_2 E / d t}
\]

(23)
where \( e = (v_0 - V_d) \). With \( v_0' = e' \), \( v_0'' = e'' \) and substituting (23) to (22), yields:

\[
R(a_1l_d - a_2e - a_4f) = a_3(RCe'e'' + a_5e') + a_6((RCe'' + e')k_1e' + k_2e'' + (RCe'' + e')k_1e' + k_2e'') + a_4e^2)
\]

(24)

Figure 3. Phase portrait of the closed loop system around origin

Consider three state variables be \( z_1 = \int e, z_2 = e, z_3 = e' \). Then, \( z_1' = f_1(z_1, z_2, z_3) = z_2, z_2' = f_2(z_1, z_2, z_3) = z_3 \) and \( z_3' = f_3(z_1, z_2, z_3) = e'' \). The third function can be determined from Equation (24) as follows:

\[
f_3(z_1, z_2, z_3) = [R(a_1l_d - a_2z_2 - a_4z_1)(k_1E + a_3e') + a_5e'] - (RCe'' + e')k_1e' + k_2e'' + (RCe'' + e')k_1e' + k_2e'') + a_4e^2)
\]

(25)

where:

\[
R(a_1l_d - v_d^2) = (a_1 l_d - v_d^2)
\]

(26)

Solving (26), \( p_2 = p_3 = 0 \) and also:

\[
p_1 = \frac{a_1}{a_4}(l_d - \frac{v_d^2}{R})
\]

(27)

Therefore, there is an equilibrium point \((\frac{a_1}{a_4}(l_d - \frac{v_d^2}{R}), 0, 0)\) for the closed loop error system. If the origin is a hyperbolic equilibrium point, the trajectories of a nonlinear system in a small region of neighborhood of an equilibrium point is close to the trajectories of its linearization in this region [18]. Proper selection of the PD-PI parameters ensures that the equilibrium point \((\frac{a_1}{a_4}(l_d - \frac{v_d^2}{R}), 0, 0)\) is hyperbolic. The Jacobian matrix of the system around its equilibrium point is as follows:

\[
A_1 = \begin{bmatrix}
  a_{11} & a_{12} & a_{13} \\
  a_{21} & a_{22} & a_{23} \\
  a_{31} & a_{32} & a_{33}
\end{bmatrix}, B_1 = \begin{bmatrix}
  b_{11} & b_{12} \\
  b_{21} & b_{22} \\
  b_{31} & b_{32}
\end{bmatrix}
\]

(28)

where:

\[
a_{11} = a_{13} = a_{21} = a_{22} = a_{23} = a_{31} = a_{32} = a_{33} = \frac{a_1 l_d}{a_4}
\]

(29)

and also:

\[
\begin{bmatrix}
  \dot{z}_1 \\
  \dot{z}_2 \\
  \dot{z}_3
\end{bmatrix} = A_1 \begin{bmatrix}
  z_1 \\
  z_2 \\
  z_3
\end{bmatrix} + B_1 \begin{bmatrix}
  I_d \\
  V_d
\end{bmatrix}
\]

(30)

Figure 4. (a) Waveforms of the inductor current, switching pulse, time derivative of the control law and sliding surface in steady phase during the switching operation of the SMC; (b) Theoretical switching frequency of the designed SMC

### TABLE 1. The parameters of the NOLC

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input voltage</td>
<td>( V_o )</td>
<td>12 V</td>
</tr>
<tr>
<td>Nominal output voltage</td>
<td>( V_0 )</td>
<td>36 V</td>
</tr>
<tr>
<td>Inductor</td>
<td>( L )</td>
<td>110 μH</td>
</tr>
<tr>
<td>Capacitor</td>
<td>( C_1, C_2 )</td>
<td>100 μF</td>
</tr>
<tr>
<td>Nominal load resistance</td>
<td>( R )</td>
<td>50 Ω</td>
</tr>
</tbody>
</table>
where $z_1$, $z_2$, $z_3$, $I_d$ and $\bar{P}_d$ are the perturbations of $z_1$, $z_2$, $z_3$, $I_d$ and $V_d$. Consider the desired eigenvalues of the proposed system to be $\lambda_1^*$, $\lambda_2^*$ and $\lambda_3^*$. By using the pole placement, it is given:

$$
\begin{bmatrix}
-s -a_{11} & -a_{12} & -a_{13} \\
-a_{21} & s -a_{22} & -a_{23} \\
-a_{31} & -a_{32} & s -a_{33}
\end{bmatrix} = (s - \lambda_1^*)(s - \lambda_2^*)(s - \lambda_3^*)
$$

(31)

The system parameters are shown in Table 1. By using (31), the controller parameters can be determined as follows:

$$
a_4 = 1, k_2 = 0.2, a_3 = 0.72, a_4 = 140
$$

(32)

From Equation (25), it is concluded that the system has a discontinuous subspace. It is $(z_1, -V_d, 0)$ where $z_1$ is any real point. Also, consider $I_d = \frac{V_d^2}{R}$. Figure 3 shows the phase portrait of the SMCC around $(0,0,0)$. It is obviously understood that the equilibrium point is a stable node and it has a large attraction region. For all the initial points starting at $[z_1(0), z_2(0), z_3(0)]$ with $z_2(0) > -V_d$, all the trajectories converge to origin. Therefore, it is found that the error dynamics of the closed loop system around the origin is at least semi globally stable with a large attraction region. The aforementioned phase portrait is for $S = 0$. However, in practice, the voltage error will converge to a small layer around the desired output voltage. Therefore, the error variables oscillate around 0.

Due to the property of the integral term, interior term of the integrator should be zero. Therefore, the voltage error approaches to zero. Furthermore, by integrating the voltage error from zero time to settling time, the inductor current reference can be calculated. By reaching the sliding variable and voltage error to zero, the inductor current reaches to its generated reference. It is noticed that the internal dynamic (voltage of the middle capacitor) is limited between 0 V and input voltage. Therefore, this internal dynamic is always bounded.

2.3. The Switching Frequency Analysis

According to the converter operation, it is supposed that three terms on the right side of Equation (10) are very small compared with other terms. Moreover, $u$ is a step function. Thus, $u'$ is the Dirac delta function. When $u$ steps up from 0 to 1, $u' = \delta(t)$ and when $u$ steps down from 1 to 0, $u' = -\delta(t)$. The time derivative of $S$ during the switch-OFF and switch-ON is:

$$
\begin{cases}
V_1 = \frac{dV_1}{dt}_{t=0^+} = k_1E + k_2v_0(1-D) - a_2v_0 = \frac{S_{P-O}}{\tau_{on}} > 0 \\
V_2 = \frac{dV_2}{dt}_{t=0^-} = k_1E - k_3v_0 - k_2v_0 + \frac{(a_3 - k_4)v_0}{C} = \frac{S_{P-O}}{\tau_{off}} < 0
\end{cases}
$$

(33)

where $S_{P-O}$ is the hysteresis bandwidth of the controller comparator. Also, $T_{on}$ and $T_{off}$ are the ON and OFF time intervals in a period of the switching operation, respectively. $V_1$ and $V_2$ are the speed of the designed sliding variable in the two states. The switching operation of the SMC and waveforms of $I_d$, $u$, $u'$ and $S$ are shown in Figure 4a. $S$ oscillates between $-\epsilon$ and $\epsilon$ by a hysteresis controller with speed of $V_1$ in ON state and speed of $V_2$ in OFF state around zero; i.e.,

$$
S_{P-O} = 2\epsilon
$$

(34)

From Equation (33), it is concluded that the speed of $S$ is dependent on the Dirac delta function in both ON and OFF modes. From both theoretical and practical point of view, it is understood that the sliding surface is moved between $-\epsilon$ and $\epsilon$ with infinite speed. Therefore, the high frequency operation can be achieved. By using Equation (33), the average value of $V_1$ and $V_2$ are given as follows:

$$
\begin{cases}
V_1 := k_1E + k_3u - a_2v_0 = k_1E + k_3V_d \frac{V_d}{R} \frac{V_d}{R} - a_2v_0 = k_1E - a_2v_0 \\
V_2 := k_1E - k_3V_d - k_3V_d \frac{1}{1-D} + \frac{(a_3 - k_4)v_0}{C} + (a_3 - k_4)v_0 + \frac{(a_4 - k_5)v_0}{C} \frac{(a_4 - k_5)v_0}{C}
\end{cases}
$$

(35)

Figure 5. The structure of the developed system; (a) the set-up of the system; (b) schematic diagram of the controller

Figure 6. The practical switching frequency with the proposed PD-PI SMC and conventional strategy for $V_0=36$V and different input voltages; (a) $R=100\Omega$; (b) $R=50\Omega$. 
where \( f_s \) is the switching frequency and \( D \) is the duty cycle of the converter, i.e.: 

\[
D = \frac{r_{on} + r_{off} \cdot f_s}{r_{on} + r_{off} \cdot f_s} - \frac{v_{a-E}}{v_a}
\]  

(36)

By using Equations (34)-(36), the switching frequency of the proposed SMC can be obtained as follows: 

\[
f_s = \frac{1}{r_{on} + r_{off} \cdot f_s} = \frac{1}{\frac{v_{a-E}}{r_{on}} + \frac{v_{a-E}}{r_{off} \cdot f_s} - \frac{v_{<d-E}}{k_1 V_d + k_2}}
\]  

(37)

Here, the switching frequency must be numerically obtained. We define \( \varepsilon \) as a critical hysteresis bandwidth in CCM. If \( \varepsilon < \varepsilon_C \), the switching frequency converges to infinite. If the frequency largely increases, then 

\[
\frac{k_2 v_a^2 f_s}{v_{a-E}} \gg (k_1 E - a V_{RC}^2) \quad \text{and} \quad \frac{k_2 v_a^2 f_s}{k_1 V_d - k_3 E + \left(\frac{V_{C}}{k_3} + k_2\right) (d_v - v_{<d})}).
\]

By using Equation (37) and these assumptions, \( \varepsilon \) can be determined as follows:

\[
\varepsilon = \frac{k_2 v_a}{k_1}
\]  

(38)

The switching frequency is shown in Figure 4. It is obvious that the coefficient \( k_2 \) largely impacts on the frequency. If this parameter increases, then \( \varepsilon \) will effectively increase. Therefore, high switching frequency is achieved when there exists a large time delay or hysteresis bandwidth in practice. To construct derivative of the current for the defined sliding variable as shown in Figure 1, this paper uses the following relationship:

\[
a_d \frac{d i_L}{d t} = k_2 v_L
\]  

(39)

4. PRACTICAL RESULTS

An experimental set-up of the developed system was constructed. Figure 5a shows the photograph of the NOLC with the designed PD-PI based SMC. IRFZ44N and MBR20150C are selected as power switch and diodes. Also, three parallel inductors with ferrite cores and the value of 110 \( \mu \)H/9A and two electrolytic capacitors with the value of 100 \( \mu \)F/100 V are used for power converter. The current is measured using the ACS712-20A sensor and voltage measurement is done applying the resistive dividers. The photograph of the NOLC with the controller is shown in Figure 5b. In this work, the low slew rate OP-AMP LF351 is used for the signal amplification and hysteresis modulation to show the efficacy of the proposed controller for the switching frequency enhancement. In order to show the system robustness, the desired inductor current is disabled. Furthermore, the Rigol oscilloscope is used to save the practical tests and WFM Viewer software is used to show the obtained results in computer.

Figures 6a, b show the measured switching frequency with the proposed approach and conventional SMC operated in CCM against line variations for \( R=100, 50 \Omega \), respectively. From these figures, it is observed that the practical frequency increases from the interval \( (7 \ 14) \) to \( (170 \ 200) \) kHz. Also, the ratio of the switching frequency variations to the average value of the switching frequency of the PD-PI based SMC is less than conventional SMC for the aforementioned points.

Consider \( V_o=36 \) V, \( R=100 \) \( \Omega \). Figures 7a and b show the steady waveforms of the current and generated control pulse of the NOLC in CCM for \( E=24 \) and \( E=18 \) V, respectively. It is clear that the converter under the proposed SMCC can works in CCM at low voltage gains and light loads. Furthermore, the sliding variable of the developed SMCC for \( E=18 \) V, \( V_o=36 \) V and \( R=100 \) \( \Omega \) is depicted in Figure 7c. This figure shows that the hysteresis bandwidth is about 8V. This value is so large, because of the large time delays of the circuit components. Hence, the experimental frequency of the conventional SMC effectively decreases. Moreover, the bandwidth is asymmetrical in practice.

Consider \( V_d = 36 \) V, \( E = 12 \) V and \( R = 50 \) \( \Omega \). The system starts from the initial points of \( V_0(0) = 0 \) V and \( i_L(0) = 0 \) A.
The practical result for this condition is shown in Figure 8a. In addition, Figure 8b illustrates the system response using the PD_PI type SMCC for $V_o = 36 \text{ V}$, $E = 15 \text{ V}$ and $R = 50 \text{ Ω}$.

Consider $V_o=36\text{ V}$. The load resistance steps-down from 200 to 50$\text{ Ω}$ and vice versa. Figures 9a and b depicts the system responses against load variation for $E=12$ and 18, respectively. In these figures, the voltage well follows its reference after the load change. Figure 10a shows the system response during transient region for $R=100\text{ Ω}$, $E=12\text{ V}$ and the voltage variation from about 35 to 49$\text{ V}$. This figure demonstrates that the output variable has a good behavior. Also, the overshoot is 1$\text{ V}$ with settling time of about 20ms. Figure 10b shows the practical response for $R=100\text{ Ω}$, $E=12\text{ V}$ and the voltage change from 36 to about 22$\text{ V}$. For this test, the voltage overshoot equals to about 6$\text{ V}$ and settling time of about 30ms. Figure 11c depicts the voltage behavior for $E=12\text{ V}$ and the voltage reference variation from nearly 36 to about 22$\text{ V}$. For this experiment, the settling time is about 40ms.

![Figure 8](image8.png)

Figure 8. The transient response for $R=50\text{ Ω}$; (a) $V_o=36\text{ V}$, $E=12\text{ V}$; (b) $V_o=50\text{ V}$, $E=18\text{ V}$

![Figure 9](image9.png)

Figure 9. The practical results for load variation from 200 to 50 (a) $E=12\text{ V}$, $V_o = 36\text{ V}$; (b) $E=18\text{ V}$, $V_o = 36\text{ V}$

5. CONCLUSIONS

This study proposes a PD-PI type sliding mode current controller for a NOLC in CCM operation. A systematic guideline for designing, developing and analyzing the characteristics of a cascade controller has been provided in the paper which includes proper choice of the sliding variable, investigation of the stability conditions and determination of the theoretical equation for the switching frequency. It is established that utilization of a PD compensator effectively enhances the switching frequency as compared with conventional sliding mode controller. By means of the suggested sliding variable, the minimum and maximum switching frequency increase from 7 and 14$\text{ kHz}$ to 150 and 200$\text{ kHz}$ for the aforementioned conditions, respectively. Furthermore, it is experimentally verified that the designed control scheme can well adjust the converter voltage with no steady error in presence of important disturbances in the uncertain parameters. It is said that the abrupt changes do not happen in practical applications at a same time. However, the practical experiments show high-accuracy trajectory tracking of the overall system for a strong system robustness and fast response against these abrupt variations.

6. REFERENCES

Voltage Regulation of a Negative Output Luo Converter Using a PD-PI Type Sliding Mode Current Controller

A. Goudarzian, A. Khosravi

Department of Electrical Engineering, Faculty of Engineering, Shahrekord Branch, Islamic Azad University, Shahrekord, Iran

Paper history:
Received 19 September 2018
Received in revised form 19 November 2017
Accepted 03 January 2019

Keywords:
Switching Converter
Negative Output Luo Converter
PD-PI Type Sliding Surface
Control Design

چکیده
این مقاله یک کنترل کننده جدید نوع PD-PI برای مبدل NOLC در حالت هدایت پیوسته ارائه می‌دهد. با تغییرات بار و ولتاژ وروی، دینامیک مبدل به شدت غیرخطی است. برای بهبود عملکرد مبدل و تنظیم جریان ورودی و ولتاژ خروجی مبدل، یک کنترل غیرخطی لازم است. این کنترل کننده مثلثی با داشتن پیچیده مبدل است. نتایج عملی نشان دهنده عملکرد سیار خوب کنترل‌کننده هستند که قادر است با حفظ خطا دام و ولتاژ، مبدل را در بارهای سبک و خودکنار کنی باز در نظر کاری مختلف کنترل کند.