Considering Production Planning in the Multi-period Inventory Routing Problem with Transshipment between Retailers

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ABSTRACT

Generally, the inventory routing problem occurs in a supply chain where customers consider the supplier responsible for inventory replenishment. In this situation, the supplier finds the answer to questions regarding the time and quantity of delivery to the customer as well as the sequence of customers in the routes. Considering the effect of production decisions on answering these questions, the present paper examines the integrated decision making on production, routing and inventory in a two-echelon supply chain composed of a manufacturer and multiple retailers. Transshipment, as a policy in supply chain logistic which increase integration and decrease inventory cost, is also allowed between retailers. The mathematical formulation for the problem is developed and an adaptive large neighborhood search heuristic is proposed to solve this complicated problem. The results of numerical experiments showed that the solutions yielded by the heuristic method have high efficiency.


1. INTRODUCTION

In recent years, the growing rate of competition between supply chains and attention to coordination and cooperation in supply chain management has led to further research in this field. One kind of coordination between the components of the supply chain is the coordination of inventory and routing in distribution systems. The inventory routing problem (IRP) occurs when there is a need for simultaneous decision making on inventory and vehicle routing which typically happens in vendor managed inventory (VMI) systems. The IRP which is currently used by many different industries is about distributing a product from a depot to a set of customers throughout a given planning horizon.

Since many of the parameters of the IRP including inventory level at different echelons of the supply chain as well as the amount of retailers demands which are met, depends on the amount of goods produced, production planning and production capacity play an important role in making the problem closer to the real world conditions. Also the possibility of transshipment between retailers is allowed which can reduce the total cost of the system. Transshipment means the ability to move products between locations at the same echelon of a supply chain.

Previous studies which have been done in this area will be followed in section 2. The mathematical model of the problem will be presented in the third section. In section 4, the developed heuristic method will be presented. Comparison of the performance of the heuristic algorithm with the results of the exact solution of the model will be the subject of the sixth section of this study; finally, in the seventh section, conclusions will be made and future research proposals will be presented.

2. LITERATURE

Since this problem has not been addressed in the literature so far, in this section the conducted studies on various aspects of this problem i.e. IRP, production planning in distribution systems and transshipment in inventory management systems are reviewed.
The IRP was first introduced by Bell et al. [1] in 1983, and thereafter, significant studies have been carried out in this area particularly over the last decade. Anderson et al. [2] reviewed IRP in two modes of land and sea transportation and its industrial aspects and addressed about 90 scientific articles in this field from 1983 to 2009. Coelho et al. [3] classified IRPs with regard to the structural diversity and availability of information on customer demand, aimed at completing the work discussed in literature [2]. Ghorbani and Joker [4] considered the problem of inventory routing and location in a multi-product three-echelon supply chain, and presented a heuristic method based on simulated annealing which the results were significantly better than literature studies in terms of time and error. Cordeau et al. [5] studied an IRP in multi-product condition and tried to find the answer to the problem by developing a heuristic method in three phases. Etebari and Dabiri [6] considered the IRP with different pricing for different geographic areas and solved it by developing a heuristic method. Moubed and Mehrjerdi [7] proposed a hybrid heuristic model combining dynamic programming, ant colony optimization and tabu search to solve the inventory routing problem.

Since inventory routing in production systems depends highly on production and with the aim of integrated planning, in addition to inventory management and distribution planning, production planning can be considered in this regard. So far, a few studies have been conducted in this field [8, 9]. Bard and Nananukul [9] modeled the inventory routing problem with consideration of production scheduling in single-product multi-period, and with the goal of minimizing total system costs. Adulyasak et al. [10] addressed the studies conducted on PRP and state that despite the complex structure of this problem there has been a growing tendency to study it both in the industry and in the academic level in the past decade. The advantages in the coordinated and integrated planning of production and distribution is discussed. Diaz et al. [11] in a review study examined the optimization models presented in the literature on the production and the routing problems in an integrated manner.

Transshipment is one of the main instruments in supply chain logistics which has been used in several studies to reduce the inventory costs in the two-echelon supply chains [12]. Mercer and Tao [13] examined the IRP by considering the possibility of transshipment in a case study on the distribution system of Tesco chain stores in UK, in which the products were delivered from the factory to the wholesalers and transshipment was possible among the wholesalers. Alvarez et al. [14] reviewed the inventory management system in a two-echelon spare parts supply chain with the possibility of selective transshipment to meet the demand of premium customers. Turan et al. [15] examined a two-echelon inventory management problem where there is a central warehouse at first echelon and retailers are in the second and transshipment possibility among the retailers is taken into account. Coelho et al. [16] modeled the inventory routing problem with taking transshipment into account for the first time. In this study, they used simplifying assumptions such as the one vehicle in the supplier’s transportation fleet and developed a heuristic algorithm to solve it. Patterson et al. [12] surveyed studies on inventory management systems with the possibility of transshipment. Transshipment practically occurs in the chain stores, in which, in addition to delivering products by the supplier to the stores, it is possible to transport products directly from one store to another which can reduce shortage costs when demand fluctuates [17]. Tiacci and Saetta [18] explained the reasons for the increase in the use of transshipment in various industries.

In this section, studies on inventory routing, planning production in distribution systems and inventory management with the possibility of transshipment were investigated. According to these studies, the integration of these three problems may increase the integrity and reduce the total costs in the supply chains. For this purpose, in this research, an inventory routing problem is analyzed with considering production planning and transshipment.

3. MATHEMATICAL MODEL

Here the mathematical model of the problem is presented. At first, the assumptions, parameters and variables of the mathematical model are outlined and finally the mathematical model of the problem is presented with its descriptions.

3.1. Problem Assumptions

- The two-echelon supply chain consists of a manufacturer and several retailers and is operated by vendor managed inventory system.
- Demand for retailers are certain and dynamic in different time periods.
- The number of time periods is fixed.
- A single product is considered in the problem.
- Manufacturer capacity is limited.
- The amount of production in each period can be delivered to retailers in the same period and can be used to meet the retailers’ demand in the same period. In other words, the lead time is considered as zero.
- Retailers’ warehouse capacity is limited and the shortage as the lost sales is allowed.
- Transportation is carried out by two fleets. Manufacturer fleet which is distributing products between retailers in a tour starting from the manufacturer’s site and the other one is private carrier fleet to carry out transshipment.
3.2. Signs and Parameters

Sets and indices

d, i, j: retailers indices where zero points to the manufacturer.

N: Set of retailers where \( N_0 = N \cup \{0\} \) and \(|N| = n\).

T: Set of time periods in the planning horizon in which \( |T| = \tau \).

Parameters

d_{ij}: retailer i demand in time t.

c_{ij}: The cost of transportation for a vehicle in manufacturer’s fleet from node i to node j which is calculated as a proportion of Euclidean distance between these two points. Triangular inequality exists in case of the distances and costs.

l: Shortage cost of one product in a period at the retailer store i.

b_{ij}: The transshipment cost from retailer i to retailer j as transshipment and using the private carrier fleet.

C_{p}: Production capacity.

t: Production startup cost.

\( c_e \): Production cost per unit.

h_{ij}: The cost of holding one unit of product in the manufacturer warehouse for one period.

h_r: The cost of holding one unit of product in the retailer warehouse for one period.

F_{max}: Manufacturer warehouse capacity.

F_{max;i}: Retailer i warehouse capacity.

\( \theta \): Number of vehicles in manufacturer’s fleet.

Q: The capacity of each vehicle in manufacturer’s fleet.

D_{max;i}: The upper bound on the maximum amount of product that can be loaded on a vehicle in period t, which is:

\[
D_{max;i} = \min \{Q \cdot \sum_{j \in N} d_{ij}\}
\]

It should be noted that inventory at the beginning of the planning horizon, both at the manufacturer’s warehouse and at the retailer’s warehouse, is given as the input to the model.

Decision variable

\( x_{ij} \): The binary variable which is 1 if the node i is exactly before the node j on the route of a vehicle on period t, otherwise it is zero.

\( z_t \): The binary variable which is 1 if the production is done on period t and otherwise it is zero.

q_{0i}: Delivered product to retailer i by the manufacturer’s fleet in period t.

w_{pq}: The amount of transshipped product from retailer i to retailer j in period t.

s_{0i}: The amount of shortage at the retailer i and in period t.

P_{0i}: The amount of inventory in the manufacturer’s warehouse at the end of the period t.

F_{0i}: The amount of inventory in retailer i warehouse at the end of period t.

y_{0i}: The required variable for sub-tour elimination.

3.3. Model

\[
\begin{align*}
\text{Min } Z &= \sum_{i \in N_0} \sum_{j \in N} \sum_{t \in T} c_{ij} x_{ij} + \sum_{i \in N} \sum_{t \in T} e_{it} + \sum_{i \in N} \sum_{t \in T} h_{ij} x_{ij} + \sum_{i \in N} \sum_{t \in T} h_r x_{ij} + \sum_{i \in N} \sum_{t \in T} b_{ij} w_{ij} \\
&+ \sum_{i \in N} \sum_{t \in T} b_{ij} w_{ij} + \sum_{i \in N} \sum_{t \in T} l_{ij} s_{ij} + \sum_{i \in N} \sum_{t \in T} b_{ij} w_{ij} \\
\text{Subject to: } \quad &\sum_{i \in N_0} \sum_{t \in T} c_{ij} x_{ij} + \sum_{i \in N} \sum_{t \in T} e_{it} + \sum_{i \in N} \sum_{t \in T} h_{ij} x_{ij} + \sum_{i \in N} \sum_{t \in T} h_r x_{ij} + \sum_{i \in N} \sum_{t \in T} b_{ij} w_{ij} \\
&\leq \sum_{j \in N_0} \sum_{t \in T} \sum_{i \in N} x_{ij} \quad \forall i \in N, t \in T \\
&\sum_{j \in N_0} \sum_{t \in T} x_{ij} \leq 1 \quad \forall i \in N, t \in T \\
&\sum_{j \in N_0} \sum_{t \in T} x_{ij} = \sum_{j \in N_0} \sum_{t \in T} x_{ji} \quad \forall i \in N_0, t \in T \\
&\sum_{j \in N_0} \sum_{t \in T} x_{ij} \leq \theta \quad \forall t \in T \\
&y_{ij} = \sum_{j \in N, j \neq i} x_{ij} - q_{0i} + D_{max;i} (1 - y_{ij}) \quad \forall i \in N, j \in N, i \neq j, t \in T \\
&0 \leq y_{ij} \leq Q \quad \forall i \in N, j \in N, t \in T \\
&0 \leq I_{ij}^{P} \leq I_{max}^{P}, \quad 0 \leq I_{ij}^{C} \leq I_{max}^{C} \quad \forall i \in N, t \in T \\
&x_{ij}, z_t, \in [0,1], \quad w_{ij}, p_{ij}, q_{ij}, s_{ij} \geq 0, \quad \forall i, j \in N, i \neq j, t \in T
\end{align*}
\]
Equation (1) represents the objective function of the problem which must minimize the total cost of transportation, production, holding, shortage, and transshipment. Production costs include fixed and variable costs of production and holding costs consists of holding costs at the manufacturer and the retailers’ location.

Equation (2) establishes the balance between inventory at the manufacturer’s warehouse in each period according to the inputs and outputs of the manufacturer's warehouse. Equation (3) also does this for inventory at the retailers’ warehouse. Inequality number (4) imposes a limit on the amount of production in each period which is the production capacity. Inequality (5) is a structural constraint that determines a reasonable maximum for shortage amount. Inequality (6) states that the delivery of goods to retailer i can be non-zero only if a vehicle from the manufacturer’s fleet visits this retailer. Also, in this inequality the maximum amount of goods that can be delivered to a retailer by the manufacturer’s fleet, which is the vehicle’s capacity, is also determined. Inequalities (7) to (9) deal with the routing aspect of this problem. Inequalities (7) and (8) indicate that a retailer will be visited at most once by the manufacturer's fleet in each time period. Constraint (9) refers to the maximum output edges of the manufacturer node which is at most equal to the number of vehicles in the manufacturer’s fleet. Inequalities (10) and (11) relate to the sub-tour elimination in the routing problem which are extracted from [9]. The constraint (10) tracks the load of each vehicle from manufacturer’s fleet along the route and ensures that if the retailer j is immediately after the retailer i in period t, the vehicle’s load before reaching the retailer j will be less than the vehicle's load before reaching the retailer i minus the amount delivered to retailer i. In this case, if a vehicle returns to the retailer who has already visited it, this constraint will be violated. In other words, constraint (10) excludes sub-tours that do not include the manufacturer node. Inequality (12) expresses the bounds on warehouse capacity at the manufacturer and retailers. Constraint (13) specifies the type of each of variable.

3. 4. Generating Instances According to the data reported in literature [16, 19] that each one has addressed some aspects of this problem and considering the circumstances of the problem, the required data are generated and the instances have been generated in different dimensions:

- **T**: The number of time periods from the set {3, 6}
- **n**: number of retailers:
  - Small size problems from the set {5,10}
  - Medium size problems from the set {30, 50}
  - Large size problems from the set {100,200}.

4. SOLUTION METHOD

In many studies the inventory routing problem is considered as an NP-hard problem because the vehicles routing problem can be a special case; Lenstra and Rinnooy [20] has proved the vehicle routing problem was as an NP-hard problem. Accordingly, a proper and efficient approach to the problem needs to be developed in this study. This heuristic approach has been designed within the framework of the Adaptive Large Neighborhood Search (ALNS), which was first proposed to solve the vehicle routing problem. Large Neighborhood Search was used for the first time to solve the routing problem by Shaw [21]. Ropke and Pisinger [22] modified the Large Neighborhood Search using several operators to create new solutions and called it the Adaptive Large Neighborhood Search.

The framework of the ALNS algorithm consists of five main elements [22]:

1) Large Neighborhood: In each replication, based on the various operators in the neighborhood structure of this algorithm, retailers are removed from their current routes, they could move between routes or retailers who are not on a route could be inserted. New solutions obtained for the routing problem based on this neighborhood structure are embedded in the model (as \(x_{ijt}\) variables) for solving it by CPLEX and determining the values of other decision variables (production amount, delivered product to retailers by the manufacturer’s fleet and transshipment).

2) Adaptive Search Engine: Selection of operators in each replication is done by roulette wheel mechanism. The weight of each operator in this mechanism \(w_i\) is determined by its performance in the past replications. The probability of choosing an operator i is calculated by \(\frac{w_i}{\sum_{i=1}^{h} w_i}\) if the number of operators is equal to h.

3) Adaptive Weight Adjustment: Searching in this algorithm is divided into sections that there are \(\varphi\) replications in each. The weight of each operator is determined by its performance at the end of each section. Each operator is assigned a weight and a score. At first, all weights are equal to one and the scores are zero. The scores are updated in each replication as follows; if the operator finds a better solution than the current best solution, its score increases by \(\sigma_1\); if the operator finds a better solution than the current solution, its score increases by \(\sigma_2\); and if the solution is no better than the current solution and is only an acceptable one, its score increases by \(\sigma_3\). The amount of these scores will be ascending. After \(\varphi\) replications the weight of the operators is updated on the basis of their scores in that section and their previous weights. At the end of each section the scores are again reset to zero. If \(\pi\) is the score of the operator i and \(\pi_0\) is the number of times that operator i is used in section j of the search process, the
weights are determined at the end of each section based on the following equation:

\[ w_i = \begin{cases} \frac{1}{\sum_{i} \pi_{ij}} & \text{if } o_{ij} = 1 \\ (1-\eta)w_i + \eta \frac{1}{\sum_{i} \pi_{ij}} & \text{if } o_{ij} \neq 0 \end{cases} \]

Where \( \eta \) is the reaction factor and shows the effect of the previous weight in determining the operator's new weight. As this score is lower, the previous weight of the operator will have a greater impact in the calculation of its new weight.

4) Post-Optimization: After running each operator and obtaining a new solution, routes are improved using the 2-opt algorithm. In this algorithm, all pairs of retailers on a tour are selected and then revised tours are obtained by reversing the order of the nodes between the selected pair of nodes. Finally, the order of the retailers that have the lowest transportation cost is selected.

5) Acceptance and stop criteria: The acceptance criterion is defined on the basis of the simulated annealing mechanism. In other words, if the solution is not a better one, the solution is accepted with the probability \( p = \frac{e^{-\Delta F/T_i}}{1} \) where \( T_i \) is the current temperature and \( \Delta F \) is the difference between the objective functions. The algorithm continues until the temperature reaches the minimum. In Figure 1, the introduced heuristic algorithm is shown in the form of pseudo-code.

4. NUMERICAL EXPERIMENTS

In order to evaluate the developed model, it was coded in GAMS 24.1.3 software and the CPLEX solver 12.5.1 is used to solve the model. The heuristic method is coded in MATLAB software version R2013a. The calculations are also performed on a computer with Intel Core i5 CPU 3.1GHz and 4GB RAM specifications. Regarding the parameters of the heuristic method, the values in Table 1 are assigned to the parameters which are obtained according to the studies [21, 22, 23] and the experiments on values close to the values indicated for each parameter in those studies.

The results are shown in Table 2. In the second and third columns, the number of time periods and retailers of the problem are given. In columns 4 to 7 the results of the exact solution of the model via CPLEX are presented. The fourth column represents the objective function, the fifth column indicates the runtime, and the sixth column denotes the difference between the objective function and the best lower bound.

The seventh column also presents the solution state that is either optimal or time limit. The intended time limit for this problem is 3600 seconds.

In the cases where the time limit is reached, the best solution obtained until then is recorded.

1. As it can be observed it is only possible to reach the optimal solution within 3600 seconds for the first two problems. By increasing the product \( T.n \) and consequently increasing the number of variables and the constraints of the problem, the difference between the best solution obtained with the lower limit (gap) has increased.

The results of implementing the heuristic method on problems are presented in columns 8 to 10 of the Table 2.

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Figure 1. Proposed heuristic Pseudo-code

TABLE 1. Heuristic parameters values
A Hybrid Dynamic
transshipment can be considered as an inventory Routing Problem, L.G., "Optimization of vendor-managed inventory systems in a rolling horizon

In column 10, the differences between the best solution of the heuristic method and best lower bound obtained by CPLEX are noted so that the performance of the exact solution and the heuristic method is compared using the same lower bound. As it can be observed in the case of small problems, the heuristic solutions are as good as CPLEX but in a shorter time. In the case of medium and large problems the performance of the heuristic is considerably more efficient than CPLEX both in terms of time and gap. The heuristic method has reached better solution in one-tenth of the time used by CPLEX. In the columns representing the values of time and gap of each solution method, the superior ones are highlighted. It can be concluded that CPLEX solver will be good enough for smaller problems but in larger problems which are often real world problems, the heuristic method provides remarkably better performance.

5. CONCLUSIONS

In this paper the IRP is addressed through considering production planning in a two-echelon supply chain where the manufacturer was at the first echelon and the retailers were at the second. Also the possibility of transshipment between retailers is allowed.

As a suggestion for future research, since real data on this problem in which production, routing, and inventory decisions are made simultaneously, do not exist in the literature, it is suggested to review the performance of the mathematical model under operational conditions using the real data of a supply chain. Also, the study of the interaction between the manufacturer and the retailers regarding the transshipment can be considered as a research subject especially in supply chains which decision makings are decentralized.

6. REFERENCES


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Considering Production Planning in the Multi-period Inventory Routing Problem with Transshipment between Retailers

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Abstract

This paper presents a mixed integer linear programming (MILP) model for the multi-period inventory routing problem with transshipments (IPR-T) and studies the impact of production planning on the performance of the IPR-T model. The model is designed to determine the optimal production and inventory allocation decisions over a given multi-period planning horizon. The model takes into account the transportation of inventory to retail locations, the handling of transshipments, and the production planning decisions of a manufacturer. The model is solved using the CPLEX solver in GAMS. The model is applied to a case study of a real-world food manufacturer to demonstrate its practical applicability. The results show that the proposed model can effectively support production planning decisions in the context of inventory routing and transshipment.