Transient Entropy Generation Analysis During Wustite Pellet Reduction to Sponge Iron

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**Abstract**

The present study carefully examined entropy generation during wustite pellet reduction to sponge iron. The finite volume method was used to solve the governing equations. The grain model was used to simulate the reaction rate. The reactant gases including carbon monoxide and hydrogen were converted to water and carbon dioxide after wustite reduction. Entropy is generated by heat transfer, mass transfer and chemical reactions. The rate of entropy generation is studied over a period of 150 minutes. Based on the governing equations, the share of each process in the generation of entropy was calculated. The effects of gas ratio, porosity, and tortuosity and grain diameter of wustite pellet on entropy generation were investigated. The porosity was changed from 0.2 to 0.5, tortuosity from 1 to 4, grain diameter from 7 to 20 mm and the ratio of reducing gas from 0.5 to 2. According to the results maximum value of entropy generation nearly occurs during first 20 minutes of the reduction process. It is shown that the heat transfer had the highest contribution to entropy generation. The results also indicates porosity and gas ratio are inversely proportional to the rate of entropy generation while tortuosity and grain diameter are directly proportional to entropy generation rate.


**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>C</td>
<td>Concentration</td>
</tr>
<tr>
<td>$c_p$</td>
<td>Specific heat at constant pressure</td>
</tr>
<tr>
<td>D</td>
<td>Diffusion coefficient</td>
</tr>
<tr>
<td>$d_p$</td>
<td>Pellet diameter</td>
</tr>
<tr>
<td>F</td>
<td>Local fractional reduction degree</td>
</tr>
<tr>
<td>G</td>
<td>Gibbs energy</td>
</tr>
<tr>
<td>$H_2/CO$</td>
<td>Adsorption coefficient</td>
</tr>
<tr>
<td>$\Delta H$</td>
<td>Heat of Reaction</td>
</tr>
<tr>
<td>$h$</td>
<td>Effective heat transfer coefficient</td>
</tr>
<tr>
<td>$K_0$</td>
<td>Reaction rate constant</td>
</tr>
<tr>
<td>$K_e$</td>
<td>Equilibrium constant</td>
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<tr>
<td>$k_m$</td>
<td>Mass transfer coefficient</td>
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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\epsilon$</td>
<td>Porosity of pellet</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Tortuosity of pellet</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>Mass Density</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Chemical Potential</td>
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**Greek Symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\rho_b$</td>
<td>Bulk flow pressure</td>
</tr>
<tr>
<td>$R_0$</td>
<td>Pellet diameter</td>
</tr>
<tr>
<td>$R$</td>
<td>Universal gas constant</td>
</tr>
<tr>
<td>$R_{f_c}$</td>
<td>Radii of a grain</td>
</tr>
<tr>
<td>$\delta_{b,c}$</td>
<td>Total entropy generation</td>
</tr>
<tr>
<td>$\delta_{b,c}^{T,0}$</td>
<td>Temperature, bulk &amp; initial temperature</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Volume of pellet</td>
</tr>
<tr>
<td>$X$</td>
<td>Overall fractional reduction degree</td>
</tr>
<tr>
<td>$y_{b,s}^i$</td>
<td>Initial, surface &amp; bulk mole fraction of gases</td>
</tr>
</tbody>
</table>

**Subscripts**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>b</td>
<td>Refers to bulk flow</td>
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1. INTRODUCTION

Wustite is an important intermediate product in MIDREX direct reduction technology and is converted to sponge iron in the final stage. Iron oxide (hematite) is usually reduced in three stages including reduction of hematite to magnetite, magnetite to wustite and wustite to sponge iron. Among them, wustite reduction to iron is the slowest step and controls the rate of reduction reactions [1]. Accordingly, comprehensive knowledge of wustite reduction to iron could be an important step in understanding the performance of direct reduction furnaces. Based on this knowledge, the performance of direct reduction furnaces can be improved. Consequently a number of empirical and numerical investigations have been carried out to examine the reduction process of a commercial pellet including von Bogdandy [2], McKewan [3], Turkdogan [4] and Usui et al. [5]. Some of the mentioned researches assumed rather simple formulations such as pseudosteady state, isothermal and single reactant. Some of the studies have been carried out for isothermal steady state reduction process either with pure hydrogen. Later, Valipour and Khoshandam [6] reported a significant study in which they eliminated most limitations of the previous works and studied time-dependent, non-isothermal reduction of wustite pellet with syngas. Recently, Ghadi et al. [7] investigated the capability of grain model to simulate wustite pellet reduction. They found that grain model, unlike the unreacted shrinking core model, can appropriately predict the effect of gas mixture parameters and pellet characteristics on the reduction degree. Ghadi et al. [8] also scrutinized heat and mass distribution through the wustite pellet and found their relation to reduction degree.

Despite much experimental and numerical research on wustite reduction, there is no study on entropy generation during the reduction process. Clearly, high reversible processes of heat transfer, mass transfer and chemical reactions occur simultaneously during the reduction. All these processes increase the entropy and reduce exergy (available energy or useful work generated by the system). Therefore, entropy generation should be reduced as much as possible to minimize exergy losses and to achieve the maximum usable energy. This shows the importance of entropy generation in the reduction process. To investigate this issue, the sources of entropy generation should be accurately identified to determine their contribution in entropy generation. The aim of the present study is to investigate transient entropy generation during wustite pellet reduction to sponge iron. Grain model was used to simulate the reduction rate [9, 10] and the governing equations were given with some assumptions which are considerably simplify solution of the equations. The numerical methods used to solve the equations and entropy generation calculations are given in the relevant sections. The effects of gas ratio, grain size, porosity and tortuosity on the rate of entropy generation were studied.

2. PROBLEM DESCRIPTION

The spherical wustite sample is a porous sphere which reacts with hydrogen and carbon monoxide according to the following reactions [6]:

\[ \text{FeO} + \text{CO} \rightarrow \text{Fe} + \text{CO}_2 \]  
\[ \text{FeO} + \text{H}_2 \rightarrow \text{Fe} + \text{H}_2\text{O} \]

The chemical reaction occurs on iron-wustite interface in all directions and progresses with time to convert all wustite to sponge iron. The wustite pellet is composed of many small spherical grains and the gas mixture reaches the outer surface of grains through the pores of wustite pellet. The reaction takes place within the spherical wustite and on the surfaces of small grains. The grains close to the outer surface are reduced faster and those closer to the center are reduced over a longer period to be converted into iron. The well-known grain model has been studied by many researchers and a good agreement with experimental results was observed [10]. The schematic model is shown in Figure 1.

The basic assumptions in the model used to derive the governing equations are given. It should be noted that these assumptions do not change the physics of the problem. As can be seen, the results have a good agreement with experimental results. Assumptions are stated as follows:

- The porosity of wustite pellet is assumed to be constant during the reduction process. The spherical particle is composed of a large number of spherical solid grains.
- Reduction is irreversible and its rate is controlled by reactions between gaseous species and diffusion of gases.
- The grain size does not change with the progress of reaction and no cracks (or shrinkage) are formed.
- The total pressure in the solution domain is constant during the reduction process.
- Gas-water shift reaction is neglected [1].

2.1 Governing Equations

Based on the above assumptions, heat and mass transfer and the reduction rates are as follows:
between heat and mass transfer [20-23] and chemical reaction [21-23]. These relations can be expressed by temperature, mole fraction of gas species, reaction rate and temperature- and concentration-dependent thermo-physical properties. These terms have been extended as follows:

\[ \dot{S}_b = \frac{k}{T} \left( \frac{\partial T}{\partial r} \right)^2 \]  

(9)

\[ \dot{S}_w = \dot{S}_w \sum \frac{\rho D_{ij}}{Y_i} \left( \frac{\partial Y_i}{\partial r} \right)^2 \]  

(10)

\[ \dot{S}_r = \dot{R} \frac{\rho D_{ij}}{Y_i} \left( \frac{\partial Y_i}{\partial r} \right)^2 \]  

(11)

\[ \dot{S}_t = -\frac{V}{T} \sum \mu n_{ij} \]  

(12)

The rate of entropy generation per unit volume of wustite pellet is calculated as follows [20, 21]:

\[ \dot{S}_v = 4\pi \sum_{j=0}^{K} \dot{S}_j \]  

(13)

2. 3. Additional Equations

The mass density of the gas mixture in the above equations shown can be calculated as follows:

\[ \rho = \frac{M_P}{RT} \]  

(14)

The chemical potential of the ideal gas is determined by the following equation:

\[ \mu_i(T, P) = G_i(T, P) + \int_P^{RT} \frac{R T}{P} dp \]  

(15)

After some calculation the final equation is as follows:

\[ \mu_i(T, P) = G_i(T, P) + RT \ln(Y_i) \]  

(16)

where, \( G_i \) is the molar Gibbs energy of the gas species. A comprehensive study on chemical potential and its relations to other parameters can be found in the literature [24].

2. 4. Boundary and Initial Conditions

a) Boundary Conditions

With regard to Figure 1, at the outer surface of the pellet and \( t > 0 \):

\[ -D_{ij} \frac{\partial Y_i (R_{0j}, t)}{\partial r} = k_{ij} (y_i (R_{0j}, t) - y_{0i}) \]  

(17)

\[ -\lambda_{ij} \frac{\partial T (R_{0j}, t)}{\partial r} = h(T(R_{0j}, t) - T_k) \]  

(18)

The reaction rate is calculated according to the following equation:

\[ R_{\text{eff}} = \frac{3K_{\text{eff}, i}}{r_s} \left( 1 + \frac{1}{K_{\text{eff}, i}} \right) (1 - f)^{2/3} (Y_{H_i}) \]  

(5)

\[ R_{\text{CO}} = \frac{3K_{\text{eff}, CO}}{r_s} \left( 1 + \frac{1}{K_{\text{eff}, CO}} \right) (1 - f)^{2/3} (Y_{CO}) \]  

(6)

The local reduction rate can be calculated using the following equation:

\[ \frac{\partial f}{\partial r} = \sum \left( \frac{3K_{\text{eff}, i}}{r_s} \right) \left( 1 + \frac{1}{K_{\text{eff}, i}} \right) (1 - f)^{2/3} (C_i - C_{0i}) \]  

(7)

The derivation of all equations with initial and boundary conditions and an estimate of the required quantities are fully discussed in the literature [10].
4. RESULTS AND DISCUSSION

This study estimated time-dependent entropy generation during wustite pellet reduction to sponge iron. Entropy is generated by heat and mass transfer and chemical reaction. Each process is separately evaluated to determine their contribution to entropy generation. The parameters affecting wustite reduction and therefore the rate of entropy generation include grain diameter, the ratio of reducing gases, porosity and tortuosity of wustite pellet. The rate of entropy generation was investigated over a period of 150 min. The porosity was changed from 0.2 to 0.5, tortuosity from 1 to 4, grain diameter from 7 to 20 mm and the ratio of reducing gas from 0.5 to 2.

Figure 3 shows the rate of entropy generation versus time for different porosities and various mechanisms of entropy generation. Figure 3a shows the rate of entropy generation by heat transfer versus time. As can be seen, the rate of entropy generation is sharply decreased from its maximum value and reaches value in less than 5 min. In primarily stages, there is a maximum temperature gradient due to the large difference in the temperatures of wustite pellet and the bulk flow. As a result, the rate of entropy generation is high. As the temperature of wustite pellet approaches that of the bulk flow, the rate of entropy generation is reduced. Almost in the first 5 min, the pellet temperature approaches that of the bulk flow leading to a sharp decrease in entropy generation. Then, the rate of entropy generation becomes constant for even up to 150 min. Although the reduction of wustite pellet is completed almost after 30 min, the temperature gradient (temperature difference between the surface and center of the wustite pellet) exists leading to entropy generation. On the other hand, porosity has a significant impact on the rate of thermal entropy generation.

![Figure 2](image-url)  
**Figure 2.** Comparison of model with experimental measurement from Usui et al. [5]: $Y_{H_2} = 0.55$, $Y_{H_2O} = 0.45$, $\varepsilon = 0.4$, $\tau = 4.5$, $T_{b} = 1173 \, K$, $d_{p} = 13 \, mm$
With increasing porosity, gases pass through the pores more easily because of low resistance against gas penetration. Therefore, the temperature gradient is reduced leading to a smaller temperature difference between the bulk flow and the wustite pellet. As a result, the rate of entropy generation decreases. This is clearly seen in Figure 3a. Figure 3b shows the rate of entropy generation by chemical reactions. As can be seen, entropy generation reaches its maximum value after about 10 min and then decreases. Finally, all reducing gases are oxidized and entropy generation by chemical reaction is stopped. With increasing porosity, the maximum entropy decreases and reaches zero more quickly.

Figure 3c shows the rate of entropy generation by mass transfer. As shown, entropy increases from zero to its maximum value in early stages of reduction process and then sharply decreases and reaches a constant value. The maximum entropy at this point indicates a large mass gradient. According to Valipour [26], the molar fraction of reducing gases in this region is drastically reduced due to the complex interaction of heat and mass transfer and heat and mass-dependent chemical reactions and then increases with a moderate slope leading to this behavior of entropy generation. As can be seen in Figure 3c, entropy generation approaches zero faster with increased porosity due to reduced resistance against gas permeation. Despite complete wustite reduction to iron, entropy is not zero because of a mass gradient between the surface and center of wustite pellet caused by the solid matrix leading to entropy generation. Entropy generation due to coupling between heat and mass transfer is shown in Figure 3d. As shown, there is a peak and then entropy generation reaches a constant value but does not approach zero. As can be seen, entropy generation decreases with increasing porosity. According to Figures 3a-3d, heat transfer has the highest contribution to entropy generation and after it, chemical reaction, mass transfer and coupling between heat and mass transfer have a specific contribution to entropy generation. Figure 3e shows the rate of total entropy generation by four mechanisms. Given the share of each mechanism, the total entropy is first sharply decreased and experiences a peak and finally reaches a constant value with a moderate slope. The rate of total entropy generation decreases with increasing porosity.

The following figures show the rate of entropy generation versus time similar to what was described in the previous section. In the next section, the impact of various parameters on the rate of entropy generation is discussed and additional discussion on the behavior of samples versus time is avoided.

Figure 4 shows the rate of entropy generation versus time for different tortuosities and various mechanisms of entropy generation. Figure 4a shows the
Figure 4. Entropy generation rate for different tortuosities (at $\varepsilon = 0.2, d_p = 0.01\text{mm}, GR = 1$)

rate of entropy generation by heat transfer. As can be seen, tortuosity does not have a significant impact on the rate of entropy generation. Figure 4b shows the rate of entropy generation by chemical reaction. As can be seen, the rate of entropy generation increases with an increment of tortuosity. By definition, tortuosity represents a porous medium resistance against direct flow of fluid passing through it. With increasing tortuosity, the resistance of wustite pellet against gas diffusion increases and thus the rate of entropy generation increases. Figure 4c shows the rate of entropy generation by mass transfer. As can be seen, the maximum and ultimate rates of entropy generation are increased with increasing tortuosity. Figure 4d shows entropy generation by coupling between heat and mass transfer. The rate of entropy generation increases with increasing tortuosity. Figure 4e shows the rate of total entropy versus time for different tortuosities. Clearly, the rate of total entropy generation increases with an increment of tortuosity.

Figure 5 shows the entropy generation rate versus time for various ratios of reducing gases and different mechanisms of entropy generation. Figure 5a shows the rate of thermal entropy generation for different gas ratios. As can be seen, the change in the ratio of reducing gases has a little impact on the rate of thermal entropy generation. Figure 5b shows entropy generated by chemical reaction. With increasing gas ratio (the ratio of hydrogen to carbon monoxide), the rate of entropy generation decreases due to the complex relationship between the chemical potential and the rate of reaction which is dependent on the temperature and mole fraction. Figure 5c shows the rate of entropy generation versus time for various gas ratios. Increasing the gas ratio, i.e. increasing the amount of hydrogen in the gas mixture, led to a decrease in maximum entropy generation by mass transfer.

This can be attributed to the high diffusivity of hydrogen so that by increasing the amount of hydrogen, the transport rate of the reducing gas increases. As a result, mass gradient is reduced leading to reducing entropy generation. Figure 5d shows the impact of gas ratio on the rate of entropy generation by coupling between heat and mass transfer. As can be seen, the maximum entropy generation decreases with increasing the gas ratio due to reduced thermal and mass gradients. Figure 5e shows the rate of total entropy generation versus time for different gas ratios. By increasing the amount of hydrogen in the gas mixture, entropy generation is first decreased and then experiences a peak and finally approaches a constant value with a moderate slope.

Figure 6 shows the rate of entropy generation versus time for various grain sizes of wustite pellet and different mechanisms of entropy generation. Figure 6a shows the rate of thermal entropy generation for various
Figure 5. Entropy generation rate for different Gas ratios (at $\varepsilon = 0.2, d_p = 0.01 \text{mm, } \tau = 2$)

Figure 6. Entropy generation rate for different grain sizes (at $\varepsilon = 0.2, \tau = 2, GR = 1$)
grain sizes. The rate of entropy generation is higher for large grains. This means that temperature gradient is larger for larger pellets leading to increased entropy generation. Figure 6b shows the rate of entropy generation by chemical reaction. As can be seen, increasing diameter of grains is directly related to augmentation of rate of entropy generation. Figure 6f shows the rate of mass entropy generation for different grain diameters. The rate of entropy generation increases with increasing diameter. Large grains cause a significant mass gradient in the pellet being reduced. Figure 6d shows the rate of entropy generation by coupling between heat and mass transfer. This figure indicates a direct relationship between the grain size and the rate of entropy generation. Figure 6e shows the rate of total entropy generation. As can be seen, the rate of entropy generation increases with increasing grain size. Entropy first reaches a minimum value and then increases to reach a peak and finally decreases with a moderate slope.

5. CONCLUSIONS

The rate of entropy generation per unit volume of wustite pellets versus time was examined. The grain model was selected as a conceptual model to determine the reaction rate. Entropy is generated by heat transfer, mass transfer and chemical reaction. The range of porosity, tortuosity, grain size and gas ratio is between 0.2 to 0.5, 1 to 4, 7 to 20 mm and 0.5 to 2, respectively. The rate of entropy generation is studied over a period of 150 minutes. The results obtained in this study can be summarized as follows:

- Maximum value of entropy generation nearly occurs during first 20 minutes of the reduction process.
- Heat transfer and coupling between heat and mass transfer had the highest and lowest contribution to entropy generation, respectively.
- The rate of thermal entropy generation sharply decreased and approached a constant value between 0.002 to 0.004 W/m²K after about 5 minutes due to the temperature gradient in the pellet even after complete reduction. The mass entropy generation rate experienced a peak during the first 5 minutes due to the mass of reducing gas within the pellet. Finally, entropy generation reached to a constant value due to a constant mass gradient in the reduced pellet.
- Gas diffusion within the wustite pellet was facilitated with increasing porosity, leading to a decrease in the rate of entropy generation. For example when porosity increases from 0.2 to 0.5 the peak point of entropy generation due to chemical reaction decreases from 0.0015 to 0.0006 W/m²K.
- The resistance of porous pellet against reducing gases increased with increasing tortuosity leading to an increase in the rate of entropy generation. The highest effect of tortuosity variations is on entropy generated by chemical reaction.
- With increasing the ratio of reducing gas, i.e. the mole fraction of hydrogen, the diffusivity of reducing gas increased leading to a decrease in the rate of entropy generation.
- The rate of entropy generation increased with increasing size of grains due to resistance to fluid penetration.

6. REFERENCES


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**Paper Info**

**Keywords:**
Wustite Pellet  Entropy Generation  Mathematical Model  Grain Model

**Abstract:**
In this study, entropy production during the reduction of wustite pellets to sponge iron has been investigated. The finite volume method is used to solve the governing equations. A grain model is employed to simulate the reaction rate. After completion of the wustite reduction, the gaseous species, including carbon monoxide and hydrogen, are converted to carbon dioxide. Entropy is generated through heat transfer, mass transfer, and chemical reactions. The entropy production rate is examined over 150 minutes. The effect of gas composition, torquisity, and wustite grain size on entropy production is studied. It is found that entropy production increases with time, reaching a peak of about 1.6 times the initial value at 20 minutes. Higher gas composition and lower torquisity results in lower entropy production. A maximum entropy production of 1.6 times the initial value is observed at 20 minutes.

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