Optimal Decisions in a Dual-channel Supply Chain for the Substitute Products with Special Orders under Disruption Risk and Brand Consideration

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1. INTRODUCTION

Profitability and ensuring the profitability are some of the main reasons of creating a supply chain. Each chain seeks to maximize the benefit of the entire chain instead of increasing its own profits. In fact, pricing a single item or multiple items in a system has great importance. Most of the papers published in pricing domain were about the substituted products; while, increase in the use of the product, the use of others are reduced. However, a few researchers have pointed out the substitute products with different brands. The users of these products are usually divided into two groups of indifferent and loyal customers. Since the type of products manufactured is the same in both companies, loyal customers use their own brand in each case; but the distance between the customer's position and the place of supply of the product is important to the indifferent customers. In the competitive market of the substitute products with different brands, if the product doesn't exist in the market, after a while it would be out of the market competition and since there are different risks, retailers who deliver products to their customers may not be able to meet all the orders of upper level. Therefore, the retailers must think carefully to meet the final customer demand. In general, application of the game theory in the supply chain is incorporated to the interaction between the members of the supply chain. Supply chain members may have conflicting goals so that each chain hope to maximize their profits, and this may lead to a reduction in the overall supply chain profit. For this reason, most models in the supply chain seek to interact with the supply chain members so that the total supply chain profit is maximized and the profit or loss in the supply chain is shared across all the supply chain.

Literature review is divided into two parts:

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(A. Arshadi Khamseh)
1. Reviewing researches on the pricing of substitute and complementary products and the game theory in the supply chain. 2. Reviewing researches related to the disruption risk in the supply chain and special order. Moorthy [1] showed that in the competitive market between different trading companies, the results did not depend solely on the company’s own performance and decision, but depended on how other companies used their strategy to seize the market. Taleizadeh and Nooridaryan [2] proposed a three-echelon supply chain consisting of multi-suppliers, a manufacturer, and a few retailers with reworking operation in an integrated and non-integrated structure to optimize the chain's profit in both cases by setting the optimal price and production policy which uses the Stackelberg model between the chain members. There are extensive studies on pricing of substitute products, such as: Karakul and Chan [3] examined the analytical and management effects of substitute products on pricing and procurement decisions. Their model is single-period with two products: an old product and a new one, the new product replaces an old one, if there is a shortage. Karakul and Chan [4] presented a single-period model for substitute products as a combination of pricing products and supplies for substitute products, in which each product requires logistic time and the demand for substitute products are random. Chen et al. [5] provided a pricing policy in a supply chain with substitute products, in which the manufacturer sells its products directly and through the internet. The retailer sells the substitute product which is produced by another producer. Zhaoet et al. [6] identified the problem of pricing substitute products with a producer and two retailers. The consumer demand and producer costs are uncertain with a centralized and three decentralized pricing models. Rasouli and NakhaiKamalabadi [7] presented a new mathematical model towards a joint pricing and inventory control for seasonal and substitutable products in a competitive market across a finite time planning horizon. It is assumed that the two substitute products belong to two different rival firms. Ahmadi Yazdi and Honarvar [8] presented a new model for designing integrated forward/reverse logistics based on pricing policy in direct and indirect sales channel. The proposed model includes producers, disposal center, distributors and final customers. Unlike pricing on a substitute product, few studies have been conducted on complementary products, which some of them are discussed as follows: Esmaeizadeh and Taleizadeh [9] presented the optimal price of two complementary products in a two-level supply chain in two modes. The supply chain at each level includes a retailer and two manufacturers. In the first case, they assumed that, the costs of producing complementary products at each level are the same, while in the second case it was assumed that the costs of production are different and depends on the demand. Arshadi Khamseh et al. [10] provided a pricing model for substitute products in the fuzzy supply chain with two manufacturers and a retailer with four pricing models. In most of the supply chain models, demand for products is considered constant, or demand is a random variable, and the utility demand function is used in a limited number of researches. Wong and Eyerst [11] Xia and Rajagopalan [12] used the utility function for customer demand, which is considered as the function of product price, logistic time, and customer distance from the brand. Xiao et al. [13] developed the game theory model including a manufacturer and a retailer; in which the proposed model the interaction between procurement time and price was examined. The proposed model includes a custom product in an order-based production system and the demand of the product depends upon the purpose of preparation and the selling price. The supply chain may be at risk due to various factors. One of the important risks that threatens the supply chain is the disruption risk in the supply chain. Xanthopoulos et al. [14] presented the Newsvendor model with two channels of supply, in which there is a possibility of disruption risk between the distributor and the retailer in each channel that in the event of disruption risk, only a percentage of the order quantity will be met by the distributor. MohsenzadehLedari et al. [15] presented a Newsvendor model in a multi-level supply chain with two supply channels that allows for the disruption risk between the retailer and the distributor in each of the supply channels. In that case, the event of disruption risk, the percentage of order will not be met and the retailer would deliver the amount of unsatisfied orders as special order and directly order from the manufacturer. Qi [16] presented a model in which retailers offer the possibility of supplying products from two suppliers and the first source provides the product at low prices, without the guarantee (there is a possibility of disruption risk occurrence); the second supplier provides the product at a higher price and complete reliability (there is no possibility of interruption risk occurrence).

In this paper, three-level supply chain model was developed with the possibility of disruption risk occurrence between the retailer and distributor and special ordering in the event of disruption risk, which uses the Stackelberg model for interaction between the supply chain members in both cases of the exclusive and non-exclusive markets. Thus, in exclusive market, each retailer only sells the product of the same channel manufacturer, but in non-exclusive market, retailers can offer the products of both manufacturers with different brands.

We proposed some innovations and contributions as follows, which distinguishes our investigation from previous work:
Providing a three-level supply chain that two manufacturers present the same product with different brands.

The possibility of occurring disruption risk between the retailer and the distributor, which forced us to have special order in the case of disruption.

Examination of exclusive and non-exclusive markets.

Defining utility function based upon price and distance for demands.

Reminder of this paper is organized as follows: Insection 2, problem description and assumptions are described. In section 3 notations and mathematical models are presented. A numerical example is presented to illustrate the effectiveness of the model in section 4. Finally conclusions from this research and future research are discussed in section 5.

2. PROBLEM DESCRIPTION

This paper presents a three-level supply chain that its chain members include a manufacturer, distributor and retailer. In order to supply products to the market there are two channels of supply from different manufacturers with different brands that compete with each other. Both manufacturers provide the same product, but with different brands so that products supplied in the market are replaceable and these two producers are looking for increase in their share on the market. The market is exclusive, will receive a benefit of discount percentage from distributor. Otherwise, no discount will be given. If the market is exclusive all products of a particular brand will be offered at the retail location. If the market is not exclusive, the percentage of products with a particular brand will be at the retail location and the rest will be offered at the retail location related to the competitor manufacturer. The performance of the supply chain is that the distributor buys the product from the manufacturer and sells the retailer(s). In this case due to political problems, equipment failure, natural disasters, a percentage of retail demand (s) of the retailer(s) may be not met by the distributor(s). The possibility of such case is probable that is called the disruption risk and in the case of occurrence, a percentage of demands are met. Since all demand of the market should be met by the retailer (s) and the shortage is not allowed. A percentage of the retailer(s) demands which has not satisfied by distributor (s), will be fulfilled and met in the form of a special order directly from the manufacturer of the same product at a higher price than the price of distributor. As previously mentioned, the products of both producers are interchangeable and it means that customers can use any of these products. Therefore, the tendency to buy will be related to the price of competing products, distance from the ideal price of product and customer’s distance from the product supply location. Finally, for each product demand, a utility function of the ideal price, sensitive to the distance and the brand is provided so that customers divided into loyal customers and in different customers and uniformly distributed. Also, "d" is the distance between the two retailers.

\[ U_j = r - \alpha_k \cdot r_k - t \cdot x_j \]  

(1)

2. 1. Model Assumptions

1. For supplying two substituted products, the value of the brand is taking into account.
2. The disruption risk will be accrued between the distributor, the retailer and in the case of occurrence the disruption risk, we will have special ordering.
3. Using both exclusive and non-exclusive markets in the study.
4. When have two type of customers: loyal and ordinary customers; also using the utility function for these two types of demands.
5. The shortage and lead time are not permitted.

3. MODEL DEFINITION

This section presents the mathematical model of the exclusive and non-exclusive markets; then the concave objective function related to each chain in the supply chain is described using Hessian matrix.

Parameters:

- \( r \) : The ideal price of the product
- \( \alpha_k \) : Percentage of the product j to the retailer k
- \( c_j \) : Sales price of the product j by the manufacturer to the distributor
- \( a_j \) : The production cost of the jth product
- \( U_j \) : Utility function related to the demand of the jth product
- \( t \) : The sensitivity of the customers to the brand
- \( x_i \) : Customer location (customer distance from the desired brand)
- \( p_i \) : Disruption risk in the ith distributor
- \( y_i \) : A percentage which is met by the distributor
- \( \lambda_i \) : Discount rate by the distributor i

Decision variable:

- \( r_{jk} \) : The price of the jth product at the kth retailer
- \( w_{ji} \) : The price of the jth product by the ith distributor
\( T_{jk} \): The price of the sale of the jth product by the manufacturer to the retailer k

\( d_j \): The demand for the product j

3. 1. The Problem Model in the Exclusive Market

In the exclusive market, each channel only provides the products of its manufacturer to the final customer and receives a discount. In the state of exclusive market with discount the profit of retailer 1 are as follows:

\[
\pi_{R1} = (1 - p_1)((r_{11} - (1 - \lambda_2)w_{11})),d_1 + p_1((r_{11} - (1 - \lambda_2)w_{11}),y_1, d_1, (r_{11} - T_{11}))
\]  

(2)

Profit of retailer 2 is as follow:

\[
\pi_{R2} = (1 - p_2)((r_{22} - (1 - \lambda_2)w_{22})),d_2 + p_2((r_{22} - (1 - \lambda_2)w_{22}),y_2, d_2, (r_{22} - T_{22}))
\]  

(3)

Profit of distributor 1 is as follow:

\[
\pi_{D1} = (1 - p_1)((1 - \lambda_2)w_{11} - c_1)d_1 + p_1((1 - \lambda_2)w_{11} - c_1)
\]  

(4)

Profit of distributor 2 is as follow:

\[
\pi_{D2} = (1 - p_2)((1 - \lambda_2)w_{22} - c_2)d_2 + p_2((1 - \lambda_2)w_{22} - c_2)
\]  

(5)

Profit of manufacturer 1 is as follow:

\[
\pi_{M1} = (c_1 - \alpha_1)d_1 + p_1(T_{11} - \alpha_1)(1 - y_1)d_1
\]  

(6)

Profit of manufacturer 2 is as follow:

\[
\pi_{M2} = (c_2 - \alpha_2)d_2 + p_2(T_{22} - \alpha_2)(1 - y_2)d_2
\]  

(7)

Profit of the total chain is as follow:

\[
\pi_{total} = \pi_{R1} + \pi_{R2} + \pi_{D1} + \pi_{D2} + \pi_{M1} + \pi_{M2}
\]  

(8)

Utility function of demand for the product 1 is as follows:

\[
U_1 = r - r_{11} - t.x_1
\]  

(9)

Utility function of demand for the product 2 is as follows:

\[
U_2 = r - r_{22} - t.x_2
\]  

(10)

The demand for the first and the second product is the total demand of loyal and indifferent customers to any of these products. The demand of the loyal customers is calculated by putting the utility function equals to zero for that product and the demand of the indifferent customer is equal to equivalence of the two utility function in such a way that the following expression should be considered:

Loyal customer demand functions for the first product are as follows:

\[
r - r_{11} - t.x_1 = 0
\]  

(11)

\[
x_1 = \frac{r - r_{11}}{t}
\]  

(12)

In different customer demand for the first product are as follows:

\[
r - r_{11} - t.x_1 = r - r_{22} - t.d - x_1
\]  

(13)

\[
x_1 = \frac{r_{22} - r_{11} + t.d}{2t}
\]  

(14)

Total demand of the first product is as follows:

\[
d_1 = \frac{r_{22} - r_{11} + t.d}{2t} + \frac{r - r_{11}}{t}
\]  

(15)

Loyal customer demand for the second product is as follows:

\[
x_2 = \frac{r - r_{22}}{t}
\]  

(16)

Indifferent customer demand for the second product is as follows:

\[
r - r_{21} - t.(d - x_2) = r - r_{22} - t.x_2
\]  

(17)

\[
x_2 = \frac{r_{21} - r_{22} + t.d}{2t}
\]  

(18)

The total demand of second product is as follows:

\[
d_2 = \frac{r_{21} - r_{22} + t.d}{2t} + \frac{r - r_{22}}{t}
\]  

(19)

Hessian matrix is used to illustrate the concavity of the utility function for each chain in the supply chain as follows:

\[
H_{R1} = \begin{bmatrix}
\frac{\partial^2 \pi_{R1}}{\partial r_{11}^2} & \frac{\partial^2 \pi_{R1}}{\partial r_{11} \partial r_{12}} \\
\frac{\partial^2 \pi_{R1}}{\partial r_{12} \partial r_{11}} & \frac{\partial^2 \pi_{R1}}{\partial r_{12}^2}
\end{bmatrix}
\]  

(20)

\[
H_{R2} = \begin{bmatrix}
\frac{3}{t} - p_1 & p_1(-2y_2 - \frac{3}{t} + 3(1 - y_2)) \\
0 & 0
\end{bmatrix}
\]  

(21)

As shown in the Hashin matrix of the first retailer, the profit function of the first retailer is concave.
As it is clear from the Hessian matrix of the second retailer, the profit function of the second retailer is concave. The profit functions of the distributors and producers are linear based on the sale price. Therefore, they are concave and also convex, and if we use Hessian matrix, all members of Hessian matrix will be zero. In order to obtain the product sales price at retail, following equations should be solved and the amount of the obtained sale price should be entered into the profit of the distributor chains to obtain the sales price at the distributor chain.

\[
\begin{align*}
\frac{\partial \pi_{w1}}{\partial w_{11}} &= 0 \\
\frac{\partial \pi_{w2}}{\partial w_{21}} &= 0 \\
\frac{\partial \pi_{w2}}{\partial w_{22}} &= 0
\end{align*}
\]

By replacing the optimal sales price of the product in retailer the distributor’s profit will be obtained.

\[
\begin{align*}
\frac{\partial \pi_{w1}}{\partial w_{1}} &= 0 \\
\frac{\partial \pi_{w2}}{\partial w_{2}} &= 0
\end{align*}
\]

Thus, the optimal sales price will be obtained.

\[
\begin{align*}
w_{11} &= \frac{CM - AF}{BM - AE} \tag{31}
\end{align*}
\]

By replacing the optimal sales price of the product in distributor at the producer’s profit solving the following equations, the optimal sale price at the manufacturer chain in the case of special order are obtained stated as follows:

\[
\begin{align*}
\frac{\partial \pi_{ML}}{\partial T_{11}} &= 0 \\
\frac{\partial \pi_{ML}}{\partial T_{22}} &= 0
\end{align*}
\]

\[
\begin{align*}
T_{1} &= \left( \frac{18 - \frac{8}{2} a_{1} \beta_{1} + \frac{1}{2} a_{1} \beta_{1} h_{1}}{2} \right) + \left( \frac{18 - \frac{8}{2} a_{1} \beta_{1} + \frac{1}{2} a_{1} \beta_{1} h_{1}}{2} \right) \\
T_{2} &= \left( \frac{18 - \frac{8}{2} a_{1} \beta_{1} + \frac{1}{2} a_{1} \beta_{1} h_{1}}{2} \right) + \left( \frac{18 - \frac{8}{2} a_{1} \beta_{1} + \frac{1}{2} a_{1} \beta_{1} h_{1}}{2} \right)
\end{align*}
\]

3.2. The Problem Model in the Non-exclusive Market

In the non-exclusive market, each retailer can offer both products and there is no restriction and discount for retailer. In the non-exclusive market, equations are:

Profit of retailer 1 is as follows:

\[
\begin{align*}
\pi_{R1} &= (1 - p_{1})((r_{1} - w_{11})a_{1}d_{1} + (1 - p_{1})((r_{1} - w_{11})a_{1}d_{1}) + p_{1}((r_{1} - w_{11})a_{1}d_{1} + (1 - p_{1})a_{1}d_{1}) + \\
&\quad + p_{1}((r_{1} - w_{11})a_{1}d_{1} + (1 - p_{1})a_{1}d_{1})
\end{align*}
\]

Profit of retailer 2 is as follows:

\[
\begin{align*}
\pi_{R2} &= (1 - p_{2})((r_{2} - w_{22})a_{2}d_{2} + (1 - p_{2})((r_{2} - w_{22})a_{2}d_{2}) + p_{2}((r_{2} - w_{22})a_{2}d_{2} + (1 - p_{2})a_{2}d_{2}) + \\
&\quad + p_{2}((r_{2} - w_{22})a_{2}d_{2} + (1 - p_{2})a_{2}d_{2})
\end{align*}
\]

Profit of distributor 1 is as follows:

\[
\begin{align*}
\frac{\partial \pi_{D1}}{\partial w_{11}} &= 0 \\
\frac{\partial \pi_{D2}}{\partial w_{22}} &= 0
\end{align*}
\]

Figure 2. Non-exclusive market
The total profit of supply chain is as follows:
\[
\mathcal{R}_{total} = \mathcal{R}_{r1} + \mathcal{R}_{r2} + \mathcal{R}_{p1} + \mathcal{R}_{p2} + \mathcal{R}_{M1} + \mathcal{R}_{M2}
\]  
(41)

Utility function of demand for the first product is as follows:
\[
u_1 = r - \alpha_1 \cdot r_1 - \alpha_2 \cdot r_2 - t \cdot x_1
\]  
(42)

Utility function of demand for the second product is as follows:
\[
u_2 = r - \alpha_{22} \cdot r_{22} - \alpha_{21} \cdot r_{21} - t \cdot x_2
\]  
(43)

Loyal customer demand for the first product is as follows:
\[
r - \alpha_1 \cdot r_1 - \alpha_2 \cdot t \cdot r_2 - t \cdot x_1 = 0
\]  
(44)

\[
x_i = \frac{r - \alpha_1 \cdot r_1 - \alpha_2 \cdot t \cdot r_2}{t}
\]  
(45)

Indifferent customer demand for the first product is as follows:
\[
r - \alpha_1 \cdot r_1 - \alpha_2 \cdot r_2 - t \cdot x_i = r - \alpha_{22} \cdot r_{22} - \alpha_{21} \cdot r_{21} - t \cdot (d - x_i)
\]  
(46)

\[
x_i = \frac{r - \alpha_{22} \cdot r_{22} + \alpha_{21} \cdot r_{21} + t \cdot d - \alpha_1 \cdot r_1 - \alpha_2 \cdot r_2}{2t}
\]  
(47)

The total demand of first product is as follows:
\[
d_i = \frac{\alpha_{22} \cdot r_{22} + \alpha_{21} \cdot r_{21} + t \cdot d - \alpha_1 \cdot r_1 - \alpha_2 \cdot r_2}{t}
\]  
(48)

Loyal customer demand for the second product is as follows:
\[
r - \alpha_{22} \cdot r_{22} - \alpha_{21} \cdot r_{21} - t \cdot x_2 = 0
\]  
(49)

\[
x_2 = \frac{r - \alpha_{22} \cdot r_{22} - \alpha_{21} \cdot r_{21}}{t}
\]  
(50)

Indifferent customer demand for the second product is as follows:
\[
r - \alpha_1 \cdot r_1 - \alpha_2 \cdot r_2 - t \cdot (d - x_2) = r - \alpha_{22} \cdot r_{22} - \alpha_{21} \cdot r_{21} - t \cdot x_2
\]  
(51)

As Hessian matrix of the first retailer profit shows, its first minor determinant is negative and the second minor determinant which is the determinant of Hessian matrix is equal to \(\frac{\partial^2 \pi_{r1}}{\partial \alpha_{21}^2} \); that is a positive value. Therefore, Hessian matrix of the retailer profit is concave. The
profit function of the distributors and producers is linear based on the sale price, so it is concave and convex as well and if we use Hessian matrix, all members of the Hessian matrix will be zero. In order to obtain the product sale price at retail, following equations should be solved and the amount of the obtained sale price should be entered into the profit of the distributor chains to obtain the sales price at the distributor chain. The equations are as follows:

\[
\begin{align*}
\frac{\partial \pi_{M1}}{\partial T_{11}} &= 0 \\
\frac{\partial \pi_{M1}}{\partial T_{12}} &= 0 \\
\frac{\partial \pi_{M1}}{\partial T_{21}} &= 0 \\
\frac{\partial \pi_{M1}}{\partial T_{22}} &= 0
\end{align*}
\]  

(58)

Optimal values of retailer prices are as follows:

\[
\begin{align*}
t_1 &= (2r + dt + 4z_1, w_{11} - 2a_1, w_{12} + 4T_{11} + a_1, p_i - 2T_{12} + a_1, p_i - 4T_{11} + a_1, p_i + 2T_{12} + a_1, p_i + 4T_{11} + a_1, p_i ) / 6a_1 \\
t_2 &= (2r + dt + 4z_1, w_{11} - 2a_1, w_{12} + 4T_{11} + a_1, p_i - 2T_{12} + a_1, p_i - 4T_{11} + a_1, p_i + 2T_{12} + a_1, p_i + 4T_{11} + a_1, p_i ) / 6a_1
\end{align*}
\]

(60)

By replacing the optimal sales price of the product in the retailer the distributor's profit and solving the below equations, the optimal sale price at the distributor chain will be obtained. Optimal values of distributor prices are as follows:

\[
\begin{align*}
\frac{\partial \pi_{M1}}{\partial a_1} &= 0 \\
\frac{\partial \pi_{M1}}{\partial a_2} &= 0 \\
\frac{\partial \pi_{M1}}{\partial p_1} &= 0 \\
\frac{\partial \pi_{M1}}{\partial p_2} &= 0
\end{align*}
\]

(63)

The above equations are solved to obtain the sales price at the distributor chain. The optimal values of manufacturer prices are as follows:

\[
\begin{align*}
\frac{\partial \pi_{M1}}{\partial T_{11}} &= 0 \\
\frac{\partial \pi_{M1}}{\partial T_{12}} &= 0 \\
\frac{\partial \pi_{M1}}{\partial T_{21}} &= 0 \\
\frac{\partial \pi_{M1}}{\partial T_{22}} &= 0
\end{align*}
\]

(66)

By replacing the optimal sales price of the product in the distributor at the producer's profit and solving the following equations, the optimal sale price at the manufacturer chain in the case of special order is obtained. Optimal values of manufacturer prices are as follows:

\[
\begin{align*}
\frac{\partial \pi_{M1}}{\partial a_1} &= 0 \\
\frac{\partial \pi_{M1}}{\partial a_2} &= 0 \\
\frac{\partial \pi_{M1}}{\partial p_1} &= 0 \\
\frac{\partial \pi_{M1}}{\partial p_2} &= 0
\end{align*}
\]

(67)

4. NUMERICAL EXAMPLE

Five numerical examples are solved to demonstrate the functionality and performance of the proposed models. In all examples data were randomly generated. In each instance, it was shown that by changing the important parameters of the problem, the sales price of the product per chain of the supply chain, the demand for each product and the supply chain profit will be changed.

4.1. Numerical Example in the Exclusive Market

For the different values of the parameters, the values of the decision variables are listed in Table 1. It has shown that how the changes in the important parameters affect the decision variables as well as the total profit of the chain.

In examples 1, 2 and 3 it has shown that by a corresponding increase of \(c_1, c_2, a_1, a_2, p_1, p_2, y_1, y_2\); the amount of selling prices also increased in all chains of the supply chain and rising prices lead to reduce the demands for both products. Thus, the profits of the entire chain are reduced. In examples 4 and 5 only the cost of producing and the manufacturer's selling price \((c_1, c_2, a_1, a_2)\) are increased that cause increasing the average of selling price in all chains of the supply chain whereas the demand for both products
and profitability are reduced. As shown in the above table, with a simultaneous increase in \( c_1, c_2, \alpha_1, \alpha_2, p_1, p_2, \gamma_1, \gamma_2; \) the selling prices are increased in all the chains of the supply chain and rising prices lead to reduce the demands for both products. Thus, the profits of the entire chain are reduced and also in cases where only \( c_1, c_2, \alpha_1, \alpha_2 \) increase, the selling prices increase in all the chains, the demands for both products and the profits decrease. In Table 3, the up arrow is an increase sign and the down arrow is a decrease sign that show the summary results in Figure 3.

### TABLE 1. Parameters for the model in the exclusive and non-exclusive market

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
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<td>40</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
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<td>16</td>
<td>16</td>
<td>16</td>
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<tr>
<td>( d )</td>
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<td>1</td>
<td>1</td>
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<td>( \alpha_{11} )</td>
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<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>( \alpha_{22} )</td>
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<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>( c_1 )</td>
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</tr>
<tr>
<td>( c_2 )</td>
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<td>10</td>
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<td>6</td>
<td>8</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>4</td>
<td>5</td>
<td>7</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>( p_1 )</td>
<td>0.2</td>
<td>0.4</td>
<td>0.7</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>( p_2 )</td>
<td>0.3</td>
<td>0.6</td>
<td>0.65</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>( \gamma_1 )</td>
<td>0.7</td>
<td>0.8</td>
<td>0.9</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>0.75</td>
<td>0.85</td>
<td>0.9</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>( \lambda_1 )</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>( \lambda_2 )</td>
<td>0.14</td>
<td>0.14</td>
<td>0.14</td>
<td>0.14</td>
<td>0.14</td>
</tr>
</tbody>
</table>

### TABLE 2. Optimum values of the decision-variables in the exclusive market

<table>
<thead>
<tr>
<th>Decision variables</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_1 )</td>
<td>34.5744</td>
<td>35.3454</td>
<td>36.1777</td>
<td>36.9423</td>
<td>39.3103</td>
</tr>
<tr>
<td>( r_{22} )</td>
<td>34.3715</td>
<td>35.0929</td>
<td>35.8236</td>
<td>36.5367</td>
<td>38.7018</td>
</tr>
<tr>
<td>( w_{11} )</td>
<td>31.1240</td>
<td>32.5775</td>
<td>34.3373</td>
<td>35.1173</td>
<td>39.1106</td>
</tr>
<tr>
<td>( w_{22} )</td>
<td>30.6660</td>
<td>32.1605</td>
<td>32.7144</td>
<td>34.0820</td>
<td>37.4980</td>
</tr>
<tr>
<td>( T_{11} )</td>
<td>13.7254</td>
<td>18.9705</td>
<td>18.1512</td>
<td>27.4509</td>
<td>41.1764</td>
</tr>
<tr>
<td>( T_{22} )</td>
<td>10.9803</td>
<td>13.7254</td>
<td>19.7285</td>
<td>21.9607</td>
<td>32.9411</td>
</tr>
<tr>
<td>( d_1 )</td>
<td>0.8327</td>
<td>0.78300</td>
<td>0.7278</td>
<td>0.6784</td>
<td>0.5209</td>
</tr>
<tr>
<td>( d_2 )</td>
<td>0.8581</td>
<td>0.8145</td>
<td>0.7720</td>
<td>0.6001</td>
<td>0.6001</td>
</tr>
<tr>
<td>Total profit</td>
<td>50.9193</td>
<td>47.8989</td>
<td>43.1172</td>
<td>39.4669</td>
<td>29.22463</td>
</tr>
</tbody>
</table>

### Figure 3. The impacts of changing parameters on the decision variables and benefit of supply chain

### TABLE 3. The impacts of changing parameters on the decision variables and benefit of supply chain

<table>
<thead>
<tr>
<th>Change of parameters</th>
<th>Impact on the decision variables</th>
<th>Total profit of supply chain</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_1 ), ( c_2 ), ( a_1 ), ( a_2 ), ( p_1 ), ( p_2 ), ( \gamma_1 ), ( \gamma_2 ), ( \lambda_1 ), ( \lambda_2 )</td>
<td>( \uparrow ) ( \uparrow ) ( \uparrow ) ( \uparrow ) ( \uparrow ) ( \uparrow ) ( \uparrow ) ( \uparrow ) ( \uparrow ) ( \uparrow ) ( \uparrow ) ( \uparrow ) ( \downarrow ) ( \downarrow )</td>
<td>( \downarrow ) ( \downarrow )</td>
</tr>
</tbody>
</table>
4. 2. Numerical Examples in the Non-exclusive Market

For the different values of the parameters, the values of the decision variables were obtained and results are shown in Table 4. In fact, it has shown that how the changes in the important parameters affected on the decision variables as well as the total profit of the chain. In examples 1, 2 and 3 it has shown that by a corresponding increase of \( c_1, c_2, \alpha_1, \alpha_2, p_1, p_2, y_1, y_2, \) the amount of selling prices were also increased in all chains of the supply chain and rising prices lead to reduce the demand for both products and thus the profits of the entire chain are reduced. In examples 4 and 5 only the cost of producing and the manufacturer's selling price \( (c_1, c_2, \alpha_1, \alpha_2) \) are increased that caused increase in the manufacturer's selling price in all chains of the supply chain whereas the demand for both products and profitability were reduced.

As shown in the above table, with a simultaneous increase in \( c_1, c_2, \alpha_1, \alpha_2, p_1, p_2, y_1, y_2, \) the selling prices are increased in all the chains of the supply chain and rising prices lead to reduce the demand for both products. Thus, the profits of the entire chain are reduced and also in cases where only \( c_1, c_2, \alpha_1, \alpha_2 \) increased, the selling prices increased in all chains, the demand for both products and the profitability also decreased. In Table 5, the up arrow is an increase sign and the down arrow is a decrease sign; these arrows show the summary results in Figure 4.

| TABLE 4. Optimum values of the decision variables in the non-exclusive market |
|-----------------|-----|-----|-----|-----|-----|
| Decision variables | Case 1 | Case 2 | Case 3 | Case 4 | Case 5 |
| \( r_{11} \) | 43.2251 | 43.3140 | 43.5239 | 43.7148 | 44.2045 |
| \( r_{12} \) | 43.2251 | 43.3140 | 43.5239 | 43.7148 | 44.2045 |
| \( r_{21} \) | 43.1623 | 43.2575 | 43.4670 | 43.5896 | 44.0160 |

| TABLE 5. The effect of changing parameters on the decision variables and the profit of the supply chain |
|-----------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Change of parameters |
| \( c_1 \) | \( c_2 \) | \( \alpha_1 \) | \( \alpha_2 \) | \( p_1 \) | \( p_2 \) | \( y_1 \) | \( y_2 \) | \( k \) | \( k \) | \( T_1 \) | \( T_2 \) | \( d_1 \) | \( d_2 \) |
| \( \uparrow \) | \( \uparrow \) | \( \uparrow \) | \( \uparrow \) | \( \uparrow \) | \( \uparrow \) | \( \uparrow \) | \( \uparrow \) | \( \uparrow \) | \( \uparrow \) | \( \uparrow \) | \( \uparrow \) | \( \downarrow \) | \( \downarrow \) |
| \( \uparrow \) | \( \uparrow \) | \( \uparrow \) | \( \uparrow \) | \( \downarrow \) | \( \downarrow \) | \( \downarrow \) | \( \downarrow \) | \( \downarrow \) | \( \downarrow \) | \( \downarrow \) | \( \downarrow \) | \( \downarrow \) | \( \downarrow \) |

5. CONCLUSION

In this paper, a three-echelon supply chain problem for the pricing of substitute products taking into account the brand value and the disconnection risk between the distributor and the retailer was developed in two states of exclusive and non-exclusive markets; in which the event of disruption risk, the retailer provides its required products by special order and directly from the manufacturer. We showed that in both cases, when the...
cost of production in the manufacturer chain increases, it causes an increase in the price of the products in the chains of the distributor and retailer which reduces the demand and profitability. In addition, production costs, when the possibility of the disruption risk rises, the product price also increases which causes reduction in the demand and also supply chain profitability in both cases of the exclusive and non-exclusive markets. In this work, the utility function of the demand was used to determine the demand for products. In future research we can use random variable for demand function and the lack of lag and lost sales when the shortage occurs. Based on the obtained results, it can be concluded that in the competitive market, where the competition is on the quality and price, manufacturer selection has the great importance and the retailers can offer several products from these manufacturers to optimize their benefits.

6. REFERENCES

Optimal Decisions in a Dual-channel Supply Chain for the Substitute Products with the Special Orders under Disruption Risk and Brand Consideration

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