A Multi-objective Hierarchical Location-allocation Model for the Healthcare Network Design Considering a Referral System

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1. INTRODUCTION

In the network design, facility location is very crucial in both industries and healthcare systems. In healthcare, small numbers of facilities available for people or unfit locations of facilities have been reported to increase costs, as well as mortality and disease rate [1]. Moreover, the main purpose of the healthcare network design model is to determine the optimal location of healthcare facilities and to allocate patients to these opened facilities to receive services. The basic location-allocation models are divided into three groups including set covering model, maximal covering model, and $p$-median model.

Most of the papers related to the healthcare network design presented in the last years have considered the network as a single level and have paid less attention to a hierarchical structure. Also, real-world referral systems have been implemented in healthcare facility networks in order to reduce patient expenses in recent years.

In a referral system, healthcare service providing centers are divided into three levels. The first level is primary health centers (family physician centers) that are the entry point of patients to the healthcare system and are usually sited in the closest locations to patients living zones. The second level is specialized centers that are responsible for providing specialized services to the referred patients of the first level. The third level is super specialty centers that provide super specialty services to the referred patients of the second level.

In the healthcare network design, capacity planning for different facilities in the network is another important issue that must be determined in order to reduce the variable cost of the healthcare system. However, most authors of the published papers in recent years have not considered this issue in their proposed models. When designing a healthcare network, the
decision-makers need to consider different objective functions including accessibility improvement, equity maximization and cost minimization [2]. In designing a healthcare network, considering maximum workload in each health center is an important issue that directly affects both physicians’ and patients’ satisfaction. This objective function had not been observed in most published papers in recent years.

The main contributions of our paper differentiating it from other relevant papers can be summarized as follows:

- Proposing a novel mixed-integer nonlinear programming model with capacity planning to design a healthcare network by considering a referral system.
- Considering two objective functions that deal with minimizing the total cost and total traveled distance as well as minimizing the maximum workload.
- Considering three levels for the referral system in the network including family physician, specialized, and super specialty centers.

The remainder of the paper is organized as follows. Section 2 reviews the relevant literature. Section 3 illustrates the problem definition in details. Section 4 describes the mathematical formula of the model. Section 5 presents solution procedures and describes computational results and sensitivity analysis. Finally, Section 6 presents some concluding remarks of the paper and some recommendations for future research.

2. LITERATURE REVIEW

Due to the extensive literature on location-allocation models or referral systems for the healthcare facilities network design. Kim and Kim [3] presented a mixed-integer linear programming model for determining the location of long-term care facilities with the objective function of minimizing the maximum allocated patients to each facility. They presented a branch-and-bound algorithm for solving their proposed model. Syam and Cote [4] proposed a location-allocation model for specialized health care services, such as traumatic brain injury (TBI) treatment. The objective function in their study was to minimize the total cost. They used a simulated annealing algorithm to solve their proposed model. Then, they improved their early model in a recent paper [5] by considering the severity of injury for patients. Also, the penalty cost related to the reception of additional patients was added to the previous objective function. Sharif et al. [6] presented a capacitated maximal covering location model for the location-allocation problem of healthcare facilities at one district in Malaysia. In order to solve their proposed model, they proposed a new solution approach based on genetic algorithm. Gunes et al. [7] proposed a novel integer programming model for planning a primary care facility network based on patient preferences including distance to facilities and equity, as well as based on physician preferences including income and workload, equity, professional support and a collegial environment. Kim and Kim [8] presented an integer programming model for the healthcare facilities location problem with the limited budget. In order to solve the model, they used a heuristic algorithm based on Lagrangian relaxation and subgradient optimization methods for a number of problem instances. Ghaderi and Jabalameli [9] presented a multi-period and uncapacitated model to design a network to locate facilities with budget-constraint. They used two efficient heuristics to solve the model.

Mohammadi et al. [10] studied a reliable healthcare network design problem that proposed a bi-objective and multi-service model with uncertainty related to the covering threshold and a number of patients. In order to cope with the uncertainty, they used two meta-heuristic algorithms including simulated annealing and imperialist competitive algorithm. Zarrinpoor et al. [11] developed a reliable bi-level facility location model for health service networks considering risks of unexpected disruptive events, congestion, service quality, service capacity, uncertainty related to the reliability, demand, service and geographical accessibility. In their proposed model, the objective function was to minimize the total costs. Shishebori [12] presented a novel single-objective mixed-integer non-linear programming model for a single-level facility location network design problem considering reliability associated with facilities. In the objective function, they considered different costs. Schweikhart and Smith-Daniels [13] considered a two-level hierarchical referral delivery network and proposed a non-linear integer model for determining the number of locations and service offering of facilities with a limited budget and capacity in order to maximize the profit.

Galvao et al. [14] proposed a tri-level hierarchical model for locating maternal and perinatal healthcare facilities in Rio de Janeiro. The objective function was to minimize total traveled distances by mothers. In order to solve the model, they developed some relaxations and heuristics. Galvao et al. [15] developed their early model by taking into account a capacity constraint. They solved this novel model with a Lagrangian method using Rio de Janeiro data. Mestre et al. [2] presented two multi-service, multi-period, multi-objective and hierarchical location-allocation models for hospital network planning under uncertainty associated with demands. The first objective of the model was to minimize the total cost and the second one was to improve geographical access. Zarrinpoor et al. [16] presented a novel reliable hierarchical location-
allocation model considering heterogeneous probabilistic disruptions of facilities. In order to solve the proposed model, a Benders decomposition algorithm was developed. Mousazadeh et al. [17] proposed a novel model for the health service network design under epistemic uncertainty and considered two levels of referral systems. The objective function was to minimize two criteria including the total cost of establishing new facilities and the total traveled distance by patients. Although two levels of the referral system are considered in this paper, the assumption of capacity planning and the objective function of minimizing the maximum workload has not been raised.

3. PROBLEM DESCRIPTION

The respective problem is a multi-objective, multi-service and multi-level health facilities network design with capacity planning for facilities of the network including patients zones, family physician centers, specialized clinics, specialized hospitals, super specialty clinics and super specialized hospitals. At first, each patient zone is assigned to one family physician center in the network that is responsible for delivering elemental services to the assigned people. After visiting the family physician center, three probable conditions could occur: 1) patients leave the network because no more treatments were needed, 2) patients were referred to specialized clinics for receiving the specialized non-surgery cure, and 30 patients were referred to specialized hospitals for receiving specialized surgery cure. After referring patients to specialized clinics or hospitals, another three probable conditions could occur: 1) patients leave the network because no more treatments were needed, 2) patients were referred to super-specialty clinics for receiving super specialized non-surgery cure, and 3) patients were referred to super specialized hospitals for receiving super specialized surgery cure. The proposed model should determine optimal selection of the location for different facilities, optimal flow of patients zones to family physician centers, optimal flow between family physician centers and specialized facilities, optimal flow between specialized facilities and super specialty facilities and optimal capacity for different specialized and super specialty facilities as the main strategic and tactical decisions.

The assumptions of our problem are described as follows:

- At the beginning of planning horizon, there is no opened facility in the network.
- The potential locations for establishing different facilities are certainly specified.
- The number and types of various services are known (non-urgency services are considered in this article).
- The demands of each patient` s zone for each service is specific and certain.
- All demands of patient`s zone is satisfied by various facilities in the network.
- The maximum allowed capacity for each service in different facilities is considered limited in the network.
- Setting up some services in some specialized centers or super specialty centers were not possible.
- Each patient`s zone is served by one family physician center in the network without paying attention to the type of service required by patients of this zone.
- Each family physician center refers patients to one specialized hospital and one specialized clinic for each service. Also, each specialized hospital or each specialized clinic is allowed to refer patients to one super specialty hospital and one super specialty clinic.

4. MATHEMATICAL MODEL

4. 1. Notations The sets, parameters and decision variables are defined below:

**sets**
- \( I \) Set of patients zones
- \( J \) set out of candidate locations for family physician centers
- \( L \) Set of candidate locations for specialized hospitals
- \( K \) Set of candidate locations for specialized clinics
- \( F \) Set of potential locations for the specialized facility \((f = l \cup k)\)
- \( M \) Set of candidate locations for superspecialty hospitals
- \( N \) Set of candidate locations for superspecialty clinics
- \( E \) Set of potential locations for the super specialty facility \((e = m \cup n)\)
- \( S \) Set of types of services

**Parameters**
- \( f_j \) Fixed cost of opening a family physician center at candidate location \( j \).
- \( g_h \) Fixed cost of opening a specialized facility at the candidate location \( h \).
- \( p_e \) Fixed cost of opening a super specialty facility at the candidate location \( e \).
- \( d_{is} \) Demand of patient`s zone for service type \( s \).
- \( yd_{ij} \) Travel distance between patient`s zone \( i \) and family physician center \( j \).
- \( wd_{jh} \) Travel distance between family physician center \( j \) and specialized facility \( h \).
- \( zd_{ih} \) Travel distance between the specialized facility \( h \).
and super specialty facility $e$.

$cay_j$ Maximum capacity of opened family physician center at candidate location $j$.

$caw_h$ Maximum capacity of an opened specialized facility at candidate location $h$ for service type $s$.

$ca_{e}^{i}$ Maximum capacity of a super specialty facility at candidate location $e$ for service type $s$.

Fraction of people in patient’s zone $i$ that are referred to a specialized hospital by family physician center for service type $s$.

Fraction of people in patient’s zone $i$ that are referred to a specialized clinic by family physician center for service type $s$.

Fraction of people in family physician center $j$ that are referred to a super specialty hospital by specialized hospital for service type $s$.

Fraction of people in family physician center $j$ that are referred to a super specialty hospital by a specialized clinic for service type $s$.

Fraction of people in family physician center $j$ that are referred to a super specialty hospital by a specialized clinic for service type $s$.

Cost of setting up for each capacity unit service type $s$ in a specialized facility $h$.

Cost of setting up for each capacity unit service type $s$ in a super specialty facility $e$.

1, if setting up the service type $s$ in specialized facility $h$ is possible; 0, otherwise.

1, if setting up the service type $s$ in super specialty facility $e$ is possible; 0, Otherwise.

Maximum amount of the total cost related to establishing various facilities in the network.

Maximum amount of the total cost related to setting up various services in the specialized facilities and super specialty facilities.

Maximum amount of the traveled distance by patients to reach facilities.

Variables

$y_j$ 1, if a family physician center is opened at candidate location $j$; 0, otherwise.

$w_h$ 1, if a specialized facility is opened at candidate location $h$; 0, otherwise.

$z_e$ 1, if a super specialty facility is opened at candidate location $e$; 0, otherwise.

$wy_h$ 1, if service types in a specialized facility $h$ to be provided; 0, otherwise.

$zy_e$ 1, if service types in super specialty facility $e$ to be provided; 0, otherwise.

$ay_{ij}$ the flow of patients from patients zone $i$ to family physician center $j$ for service type $s$.

$by_{jh}$ the flow of patients from family physician center $j$ to specialized facility $h$ for service type $s$.

$cy_{he}$ the flow of patients from specialized facility $h$ to super specialty facility $e$ for service type $s$.

$au_{ij}$ 1, if patients zone $i$ is assigned to family physician center $j$; 0, otherwise.

$bu_{jh}$ 1, if family physician center $j$ will refer patients to a specialized facility $h$ for service type $s$; 0, otherwise.

$cu_{he}$ 1, if the specialized facility $h$ will refer patients to a super specialty facility $e$ for service type $s$; 0, otherwise.

$ew_{h}^{se}$ capacity expansion the service type $s$ in a specialized facility $h$.

$ez_{e}^{s}$ capacity expansion the service type $s$ in a super specialty facility $e$.

4. 2. Model The mixed-integer nonlinear programming model for the three-level health facilities network design is as follows:

Subject to:

$$\sum_{i \in I} \text{ay}_{ij} \leq \text{cay}_j \cdot y_j \quad \forall j \in J$$

$$\sum_{j \in J} \text{by}_{jh} \leq \text{ew}_h^{s} \quad \forall h \in H, s \in S$$

$$\sum_{h \in H} \text{cy}_{he} \leq \text{ez}_e^{s} \quad \forall e \in E, s \in S$$

$$\text{ew}_h^{s} \leq \text{caw}_h^{s} \cdot \text{wy}_h^{s} \quad \forall h \in H, s \in S$$

$$\text{ez}_e^{s} \leq \text{ca}_{e}^{i} \cdot \text{zy}_e^{s} \quad \forall e \in E, s \in S$$

$$\text{wy}_h^{s} \leq \text{wp}_h^{s} \quad \forall h \in H, s \in S$$

$$\text{zy}_e^{s} \leq \text{zp}_e^{s} \quad \forall e \in E, s \in S$$
The objective function (1) is composed of three terms. The first term relates to minimization of the total opening cost for family physician centers, specialized hospitals, specialized clinics, super specialty hospitals and super-specialty clinics. The second terms minimizes the setup cost of different types of services in each facility. In order to improve patients accessibility to different facilities in the network, the third term minimizes the total distance traveled by patients to reach each facility. Since the objectives have different units, they are normalized. The objective function (2) is aimed at minimizing the maximum workload for each opened facility in each level of the network. Since it is assumed that each facility has a different capacity, therefore, total coming flow to each opened facility is normalized. Constraint (3) guarantees that the total input flow to each family physician center does not exceed the relevant capacity. Constraints (4) and (5) are related to capacity planning for different services in the specialized and super specialty facilities. Constraints (6) and (7) ensure that the planned capacity for each specialized and super specialty facility does not exceed the maximum capacity. Constraints (8) and (9) state that a type of service can be set up in one of the specialized or super specialty facilities if it is possible. Constraints (10) and (11) guarantee that each service can be set up in specialized or super specialty facilities only if there is any demand in those centers.

Constraints (12) and (13) ensure that each service can be assigned to specialized or super specialty facilities only if suchfacilities are built in one of the candidate locations. Equations (14) and (15) state that a new specialized or super specialty facility is only built when at least one type of service is set up in the related center. Constraints (16)-(22) guarantee that all patients demands should be satisfied by family physician centers, specialized hospitals, specialized clinics, super specialty hospitals and super specialty clinics, respectively. Constraints (23)-(25) state that only a patient flow is possible from patients zones to each family physician center, from each family physician center to each specialized facility or from each specialized and super specialty facility.
if the corresponding facilities have already been opened. Constraint (26) guarantees that each patients zone must be assigned only to one family physician center. Constraints (27) and (28) guarantee that each family physician center is allowed to refer patients to the most specialized hospital and/or specialized clinic. Constraints (29) and (30) guarantee that each specialized hospital and each specialized clinic for each service is allowed to refer patients to the most super specialty hospital and one super specialty clinic, respectively. Constraint (31) is related to the definition of binary and integer decision variables in the proposed model.

4. 2. 1. Model Linearization The presented model is a mixed-integer nonlinear programming (MINLP) model regarding second objective function. However, it can be easily linearized by introducing new variables and additional constraints as follows:

\[
\text{Max} \sum_{i,d} \left( \sum_{j} \alpha_{ij} y_{ij} \right) = ma \quad (32)
\]

\[
\text{Max} \sum_{h \in H} \left( \sum_{j \in J} \beta_{jh} x_{jh} \right) = me \quad (33)
\]

\[
\text{Max} \sum_{e \in E} \left( \sum_{s \in S} \gamma_{es} z_{es} \right) = mb \quad (34)
\]

As well as equivalent minimization maximum workload for the mixed-integer linear programming model (MILP) is formulated by:

\[
\text{Min}(ma + me + mb) \quad (35)
\]

The additional constraint for linearization is as follows:

\[
\sum_{j \in J} \alpha_{ij} y_{ij} \leq ma \quad \forall j \in J \quad (36)
\]

\[
\sum_{j \in J} \beta_{jh} x_{jh} \leq me \quad \forall h \in H \quad (37)
\]

\[
\sum_{s \in S} \gamma_{es} z_{es} \leq mb \quad \forall e \in E \quad (38)
\]

5. COMPUTATIONAL RESULTS

This section presents the computational results and analysis of the proposed model using numerical examples. The parameters values in instances were generated from Ghaderi et al. [9]. Table 1 gives details on simulation of the input data. In these tables, \([a, b]\) states that the data follows discrete uniform distribution in the interval \([a, b]\). Table 2 illustrates different problem sizes that are used for the sensitivity analysis. The presented model is implemented and solved with the augmented \(\varepsilon\)-constraint method on numerical examples.

<table>
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<tr>
<th>TABLE 1. Parameters values in instances</th>
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<td>(f_j)</td>
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<th>TABLE 2. Conditions of instances used for the results</th>
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<td>Numerical example</td>
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The model was coded in GAMS software version 24.1.2 and solved with CPLEX solver on a computer (Intel Quad Core i5-2.50 GHz, 2450M CPU, and 4GB RAM). For solving this model in the augmented \( \varepsilon \)-constraint method, the first objective function related to minimizing the total opening cost, total setup cost and total travelled distance is considered as the first priority, and the workload minimization objective function is considered as the second priority. In order to implement the augmented \( \varepsilon \)-constraint method, the interval between the worst and best value for each objective function is divided into five sections \((q_i = 5)\). Figure 1 shows the conflict between the two intended objective functions in numerical example 3.

According to Figure 1, since both the objective functions are intended to minimize some criteria, the negative slope between the values of the objective functions shows the conflict and Pareto space between the objective functions.

5.1. Sensitivity Analysis

In the proposed model, one important parameter in management decisions is the maximum allowed capacity for each facility, which limits the capacity expansion in each opened facility. Therefore, Figures 2 and 3 demonstrate the effects of increasing the maximum allowed capacity of each objective function, as compared to when the maximum allowed capacity is fixed for the numerical example 2. The maximum allowed capacity increases in three steps, including 0%, 50%, and 100%. According to Figures 2 and 3, the decision makers could realize how the maximum capacity can be varied in each Pareto solution in order to reach better objective function values. For example, if the decision maker selects the Pareto solution 6, the decision would not be appropriate to increase the maximum capacity as much as 100% or 50%. The reason is that, although the second objective function related to the maximum workload has improved with this increase in this Pareto solution but the first objective function related to the total cost and total traveled distance has increased.

The obtained result could also be attributed to the fact that if a decision maker selects the Pareto solution 1, the decision to increase as much as 50% or 100% is an appropriate decision because any increase in this Pareto solution leads to improvement in both the objective functions.

In this section, the sensitivity analysis has been conducted on the increasing demand. Figures 4 and 5 show these results for the numerical example 2. The demand increases in three steps including 0%, 50%, and 100%. As can be seen in all the Pareto solutions, with increasing in the demand, the first objective function related to the total cost and total traveled distance increases because of establishing more facilities and setting up more services. Similarly, by increasing the demand, the second objective function related to the maximum workload increases because of more congestion patients in each facility.

In terms of managerial insights of this work managers of healthcare organization can use the proposed model to make a trade-off between total costs and maximum workload.
They can benefit from the obtained Pareto solutions and choose an option which is in line with their goals and policies. Following the proposed model has a great effect on reducing costs and maximum workload.

6. CONCLUSIONS

This paper presented an optimization MINLP model for location-allocation healthcare facilities and capacity planning for these facilities by considering a referral system. In this network, three levels of the referral system were considered with different healthcare facilities including family physician centers, specialized hospitals, specialized clinics, super specialty hospitals and super specialty clinics. The proposed model has two objective functions that has been described in Section 4. The first objective function in the model consists of three criteria. The first criterion relates to the minimization of the total opening cost for different facilities in the network. The second criterion aims to minimize the total setup cost of different types of services in each facility. The third criterion relates to the minimization of the total traveled distance by patients to reach each facility in order to improve patients accessibility to different facilities in the network. The second objective function intends to minimize the maximum workload in each level. For solving the presented model, it was first linearized and then, is solved with the augmented $\varepsilon$-constraint method on numerical examples in different sizes in Sections 5 and 6 respectively. Finally, sensitivity analyses were carried out on some of the model parameters including maximum capacity and demand and based on our results, we conclude that:

- Because the network structure considered in this study is based on the referral system in Iran, there is an effective coordination between different levels of the network for providing a service to a patient. Consequently, compared to a network single-level reduces dramatically the cost and maximum workload.
- One of the important parameters of the proposed model that influences net density is the maximum allowable capacity for each facility. If the maximum allowable capacity is low, the need for activation of more and more of the facilities is needed and the network is dispersed, and vice versa.
- If demand for demographic regions increases, network costs will be worsened by increasing the population density in each of the centers due to the construction of more facilities and maximum workload in various facilities.

Areas of further research are human resources planning (health service providers) in each facility, considering uncertainty related to patients demands, fixed opening cost and setting up cost and developing heuristic and meta-heuristic algorithms to solve large-scale instances for the proposed model in order to reach the near-optimal solution when an exact solver does not reach an optimal solution in a reasonable time.

7. REFERENCES

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A Multi-objective Hierarchical Location-allocation Model for the Healthcare Network Design Considering a Referral System

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Abstract:

In this study, a multi-objective hierarchical location-allocation model for healthcare network design considering a referral system is developed. The model is designed to optimize the allocation of healthcare facilities while considering patient preferences and service quality. The methodology is illustrated through a case study in a real-world scenario.

1. Introduction

1.1. Background

The healthcare network design problem has been a subject of extensive research in recent years. Various approaches have been proposed to address this problem, such as linear programming, integer programming, and heuristic methods. However, the complexity of healthcare networks requires a more sophisticated approach to consider multiple objectives and constraints.

1.2. Objectives

The main objectives of this study are to develop a multi-objective hierarchical location-allocation model for healthcare network design considering a referral system.

1.3. Methodology

The methodology involves the development of a mathematical model that incorporates multiple objectives and constraints. The model is solved using a multi-objective optimization algorithm.

1.4. Case Study

The developed model is illustrated through a case study in a real-world scenario.

2. Results

The results of the case study demonstrate the effectiveness of the developed model.

3. Conclusion

The developed multi-objective hierarchical location-allocation model for healthcare network design considering a referral system is a valuable tool for healthcare planners and decision-makers.

4. Future Work

Future work includes the development of a decision support system based on the developed model and the assessment of its impact on healthcare service quality and patient satisfaction.

References


